In the paper under review the condition that every point-finite family of open subsets of a topological space is countable (called point-finite countability condition, or property pcc) is studied. Similarly, a Banach space $E$ is said to be half-pcc if every point-finite family consisting of half-spaces of $E$ is countable. The authors show that a Banach space $E$ is half-pcc if, and only if, for any set $\Gamma$, every bounded linear map $E \to c_0(\Gamma)$ has separable range. Not every half-pcc space is separable. The authors give various additional conditions for a half-pcc Banach space to be separable. It turns out that hereditarily half-pcc Banach spaces are related to Banach spaces with the Kunen-Shelah property. Similar properties for $C(K)$ spaces with the topology of the pointwise convergence are considered. In particular, the authors construct interesting $C(K)$ spaces which answer some questions of A. V. Arkhangel’kii [in Recent progress in general topology (Prague, 1991), 1–56, North-Holland, Amsterdam, 1992; MR1229122], S. A. Argyros et al. [Proc. London Math. Soc. (3) 85 (2002), no. 3, 742–768; MR1936819 (2003k:46022)] and P. Holický, M. Šmídek and L. Zajíček [Comment. Math. Univ. Carolin. 39 (1998), no. 3, 469–482; MR1666778 (99k:46073)] concerning pcc.

{Reviewer’s remark: Some results of the paper were obtained independently in [V. V. Mikhaïlyuk, Acta Math. Hungar. 117 (2007), no. 4, 315–323; MR2357415 (2008i:54028)]. In particular, Propositions 2.8 and 2.11 (if $K$ is the continuous image of a Valdivia compact space (or is linearly ordered) and $C(K)$ is pointwise pcc then $K$ is metrizable) are similar to the corresponding results of Mikhaïlyuk’s paper.}

Reviewed by Anatolij M. Plichko (Kraków)

References


8. A.V. Arkhangel'skii, V.V. Tkachuk, Calibers and point-finite cellularity of the space $C_p(X)$ and some questions of S. Gul’ko and M. Husek, Topology Appl. 23 (1986) 65–72. MR0849094 (87h:54036)


Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.