Krister Segerberg and Rysiek Sliwinski (eds.), *Logic, law, morality: thirteen essays in practical philosophy in honour of Lennart Åqvist*, pp. 167–180. *Uppsala philosophical studies 51*. Uppsala: Department of Philosophy, Uppsala University, 2003.

THE VERY REPUGNANT CONCLUSION

Gustaf Arrhenius

1. Introduction

Population axiology concerns how to evaluate populations in regard to their goodness, that is, how to order populations by the relations "is better than" and "is as good as". This field has been riddled with "paradoxes" which seem to show that our considered beliefs are inconsistent in cases where the number of people and their welfare varies. Already in Derek Parfit's seminal contribution to the topic, an informal paradox — the Mere Addition Paradox — was presented and later contributions have proved similar results.[1] All of these contributions, however, have one thing in common: They all involve an adequacy condition that rules out Parfit's Repugnant Conclusion:

> *The Repugnant Conclusion*: For any perfectly equal population with very high positive welfare, there is a population with very low positive welfare which is better, other things being equal.[2]

A number of theorists, however, have argued that we should accept the Repugnant Conclusion and hence that avoidance of this conclusion is not a convincing adequacy condition for a population axiology. Torbjörn Tännsjö, for example, argues that the Repugnant Conclusion is not at all repugnant but rather "an unsought, but acceptable, consequence of hedonistic utilitarianism":[3]

> How we judge the Repugnant Conclusion must in the end and perhaps primarily depend upon where we think the line between a life worth living and a life not worth living more precisely should be drawn. Where are we situated in relation to this level? The Repugnant Conclusion will not be especially repugnant if we think

1 See Parfit (1984), pp. 419ff. For an informal proof of a similar result with stronger assumptions, see Ng (1989), p. 240. A formal proof with slightly stronger assumptions than Ng's can be found in Blackorby and Donaldson (1991). For theorems with much weaker assumptions, see my (1999), (2000b), and especially (2000a), (2001).

2 See Parfit (1984), p. 388. My formulation is more general than Parfit's and he doesn't demand that the people with very high welfare are equally well off.

3 Tännsjö (1998), p. 162.

> that normally most people are quite close to this level and that they perhaps often fall below this level.[4]

In the theorem we shall present here, we shall replace avoidance of the Repugnant Conclusion with a weaker condition, namely avoidance of the following conclusion:

> *The Very Repugnant Conclusion*: For any perfectly equal population A with very high positive welfare, and for any number of lives with very negative welfare, there is a population B consisting of the lives with negative welfare and lives with very low positive welfare which is better than population A, other things being equal.

This conclusion seems much harder to accept than the Repugnant Conclusions. Here we are comparing one population where everybody enjoys very high quality of lives with another population where people either have very low positive welfare or very negative welfare. Even if we were to accept (I don't) Tännsjö's argument that the Repugnant Conclusion is acceptable because most people today have very low positive welfare, we are not forced to accept the Very Repugnant Conclusion. Since the latter conclusion also involves people with very negative welfare, Tännsjö' argument, which only concerns the axiological evaluation of lives with very low positive welfare, is not applicable. We might, for example, accept the Repugnant Conclusion but not the Very Repugnant Conclusion because we give greater moral weight to suffering than to positive welfare.

I shall make use of one controversial principle in the theorem below, namely a version of the Mere Addition Principle. This reason for this is just brevity, however. As I showed in Arrhenius (2000a, 2001), one can replace this condition with other conditions that are logically weaker and intuitively much more compelling. The aim here is to show how one can replace avoidance of the Repugnant Conclusion with avoidance of the Very Repugnant Conclusion in a version of the Mere Addition Paradox.

2. The Basic Structure

For the purpose of proving the theorem, it will be useful to state some definitions and assumptions, and introduce some notational conventions. A *life* is individuated by the person whose life it is and the kind of life it is. A *population* is

---

[4] Tännsjö (1991), pp. 42-3 (my translation). Cf. Hare (1993), p. 74 and Ryberg (1996), p. 154.

a finite set of lives in a possible world.[5] We shall assume that for any natural number n and any welfare level $\mathbb{X}$, there is a possible population of n people with welfare $\mathbb{X}$. Two populations are identical if and only if they consist of the same lives. Since the same person can exist (be instantiated) and lead the same kind of life in many different possible worlds, the same life can exist in many possible worlds. Moreover, since two populations are identical exactly if they consist of the same lives, the same population can exist in many possible worlds. A *population axiology* is an "at least as good as" quasi-ordering of all possible populations, that is, a reflexive, transitive, but not necessarily complete ordering of populations in regard to their goodness.

A, B, C,… A_1, A_2,…, A_n, $A \cup B$, and so on, denote populations of finite size. The number of lives in a population X (X's population size) is given by the function N(X). We shall adopt the convention that populations represented by different letters, or the same letter but different indexes, are pairwise disjoint.

The relation "*has at least as high welfare as*" quasi-orders (reflexive, transitive, but not necessarily complete) the set $\mathcal{L}$ of all possible lives. A life p_1 has higher welfare than another life p_2 if and only if p_1 has at least as high welfare as p_2 and it is not the case that p_2 has at least as high welfare as p_1. p_1 has the same welfare as p_2 if and only if p_1 has at least as high welfare as p_2 and p_2 has at least as high welfare as p_1. We shall say that a life has *neutral welfare* if and only if it has the same welfare as a life without any good or bad welfare-components, and that a life has *positive* (*negative*) welfare if and only if it has higher (lower) welfare than a life with neutral welfare.

By a *welfare level* $\mathbb{A}$ we shall mean a set such that if a life a is in $\mathbb{A}$, then a life b is in $\mathbb{A}$ if and only if b has the same welfare as a. In other words, a welfare level is an equivalence class on $\mathcal{L}$. Let a^* be a life which is representative of the welfare level $\mathbb{A}$. We shall say that a welfare level $\mathbb{A}$ is higher (lower, the same) than (as) a level $\mathbb{B}$ if and only if a^* has higher (lower, the same) welfare than (as) b^*; that a welfare level $\mathbb{A}$ is positive (negative, neutral) if and only if a^* has positive (negative, neutral) welfare; and that a life b has welfare below (above, at) $\mathbb{A}$ if and only if b has higher (lower, the same) welfare than (as) a^*.

We shall assume that *Discreteness* is true of the set of all possible lives $\mathcal{L}$ or some subset of $\mathcal{L}$:

[5] For some possible constraints on possible populations, see Arrhenius (2000a), ch. 2.

> *Discreteness*: For any pair of welfare levels $\mathbb{X}$ and $\mathbb{Y}$, $\mathbb{X}$ higher than $\mathbb{Y}$, the set consisting of all welfare levels $\mathbb{Z}$ such that $\mathbb{X}$ is higher than $\mathbb{Z}$, and $\mathbb{Z}$ is higher than $\mathbb{Y}$, has a finite number of members.

The statement of the informal version of some of the adequacy conditions below, for example the Non-Elitism Condition, involve the not so exact relation "slightly higher welfare than". In the exact statements of those adequacy conditions, we shall instead make use of two consecutive welfare levels, that is, two welfare levels such that there is no welfare level in between them. Discreteness ensures that there are such welfare levels. Intuitively speaking, if $\mathbb{A}$ and $\mathbb{B}$ are two consecutive welfare levels, $\mathbb{A}$ higher than $\mathbb{B}$, then $\mathbb{A}$ is just slightly higher than $\mathbb{B}$. More importantly, the intuitive plausibility of the adequacy conditions is preserved. Of course, this presupposes that the order of welfare levels is fine-grained, which is exactly what is suggested by expressions such as "Jan is slightly better off than Gert" and the like.[6] Notice that Discreteness doesn't exclude the view that for any welfare level, there is a higher and a lower welfare level (compare with the natural numbers).

Given Discreteness, we can index welfare levels with integers in a natural manner. Discreteness in conjunction with the existence of a neutral welfare level and a quasi-ordering of lives implies that there is at least one positive welfare level in $\mathcal{L}$ such that there is no lower positive welfare level.[7] Let $\mathbb{W}_1$, $\mathbb{W}_2$, $\mathbb{W}_3$,… and so forth represent positive welfare levels, starting with one of the positive welfare level for which there is no lower positive one, such that for any pair of welfare levels $\mathbb{W}_n$ and $\mathbb{W}_{n+1}$, $\mathbb{W}_{n+1}$ is higher than $\mathbb{W}_n$, and there is no welfare level $\mathbb{X}$ such that $\mathbb{W}_{n+1}$ is higher than $\mathbb{X}$, and $\mathbb{X}$ is higher than $\mathbb{W}_n$. Analogously, let $\mathbb{W}_{-1}$, $\mathbb{W}_{-2}$, $\mathbb{W}_{-3}$,… and so on represent negative welfare levels. The neutral welfare level is represented by $\mathbb{W}_0$.

A *welfare range* $\mathbb{R}$(x, y) is a union of at least *three* welfare levels defined by two welfare levels $\mathbb{W}_x$ and $\mathbb{W}_y$, $x < y$, such that for any welfare level $\mathbb{W}_z$, $\mathbb{W}_z$ is a subset of $\mathbb{R}$(x, y) if and only if $x \leq z \leq y$.[8] We shall say that a welfare range $\mathbb{R}$(x, y) is higher (lower) than another range $\mathbb{R}$(z, w) if and only if $x > w$ ($y < z$); that a welfare range $\mathbb{R}$(x, y) is positive (negative) if and only if $x > 0$ ($y < 0$); and

---

[6] For a defense of Discreteness, see Arrhenius (2000a), ch 10, section 9.

[7] There might be more than one since we only have an quasi-ordering of lives, that is, there might be lives and thus welfare levels which are incommensurable in regard to welfare.

[8] The reason for restricting welfare ranges to unions of at least three welfare levels, as opposed to at least two welfare levels, is that this restriction allows us to simplify the exact statements of the adequacy conditions.

that a life *p* has welfare above (below, in) $\mathbb{R}$(x, y) if and only if *p* is in some $\mathbb{W}_z$ such that $z > y$ ($z < x$, $y \geq z \geq x$). A welfare range $\mathbb{R}$(x, Ω) is a union of all the welfare levels $\mathbb{W}_i$, $i \geq x$, and a welfare range $\mathbb{R}$(Ω, y) is a union of all the welfare levels $\mathbb{W}_i$, $i \leq y$.

3. Adequacy Conditions

We shall make use of the following five adequacy conditions:

(1) *The Egalitarian Dominance Condition*: If population A is a perfectly equal population of the same size as population B, and every person in A has higher welfare than every person in B, then A is better than B, other things being equal.

(1*) *The Egalitarian Dominance Condition (exact formulation)*: For any populations A and B, *N*(A)=*N*(B), and any welfare level $\mathbb{W}_x$, **if** B⊂$\mathbb{R}$(Ω, x-1) and A⊂$\mathbb{W}_x$, **then** A is better than B, other things being equal.

(2) *The Dominance Addition Condition*: If population A and B are of the same size and everyone in A has lower welfare than everyone in B, then a population consisting of the B-lives and any number of lives with positive welfare is at least as good as A, other things being equal.

(2*) *The Dominance Addition Condition (exact formulation)*: For any populations A, B, and C, and any welfare level $\mathbb{W}_x$, **if** A⊂$\mathbb{R}$(Ω, x-1), B⊂$\mathbb{R}$(x, Ω), *N*(A)=*N*(B), and C⊂$\mathbb{R}$(1, Ω), **then** B∪C is at least as good as A, other things being equal.

(3) *The General Non-Extreme Priority Condition*: There is a number *n* of lives such that for any population X, and any welfare level $\mathbb{A}$, a population consisting of the X-lives, *n* lives with very high welfare, and one life with welfare $\mathbb{A}$, is at least as good as a population consisting of the X-lives, *n* lives with very low positive welfare, and one life with welfare slightly above $\mathbb{A}$, other things being equal.

(3*) *The General Non-Extreme Priority Condition (exact formulation)*: For any $\mathbb{W}_z$, there is a positive welfare level $\mathbb{W}_u$, and a positive welfare range $\mathbb{R}$(1, y), $u > y$, and a number of lives $n > 0$ such that **if** A⊂$\mathbb{W}_x$, $x \geq u$, B⊂$\mathbb{R}$(1, y), *N*(A)=*N*(B)=*n*, C⊂$\mathbb{W}_z$, D$\subset\mathbb{W}_{z+1}$, N(C)= N(D)=1, **then,** for any E, A$\cup$C$\cup$E is at least as good as B$\cup$D$\cup$E, other things being equal.

(4) *The Non-Elitism Condition*: For any triplet of welfare levels $\mathbb{A}$, $\mathbb{B}$, and $\mathbb{C}$, $\mathbb{A}$ slightly higher than $\mathbb{B}$, and $\mathbb{B}$ higher than $\mathbb{C}$, and for any one-life population A with welfare $\mathbb{A}$, there is a population C with welfare $\mathbb{C}$, and a population B of the same size as A$\cup$C and with welfare $\mathbb{B}$, such that for any population X, B$\cup$X is at least as good as A$\cup$C$\cup$X, other things being equal.

(4*) *The Non-Elitism Condition (exact formulation)*: For any welfare levels $\mathbb{W}_x$, $\mathbb{W}_y$, x-1 > y, there is a number of lives $n > 0$ such that **if** A$\subset\mathbb{W}_x$, N(A)=1, B$\subset\mathbb{W}_y$, N(B)=n, and C$\subset\mathbb{W}_{x-1}$, N(C)=$n+1$, **then**, for any D, C$\cup$D is at least as good as A$\cup$B$\cup$D, other things being equal.

(5) *Avoidance of the Very Repugnant Conclusion*: There is a perfectly equal population with very high positive welfare, and a very negative welfare level, and a number of lives at this level, such that the high welfare population is at least as good as any population consisting of the lives with negative welfare and any number of lives with very low positive welfare, other things being equal.

(5*) *Avoidance of the Very Repugnant Conclusion*: *(exact formulation)*: There is a negative welfare level $\mathbb{W}_x$, $x < 0$, two positive welfare ranges $\mathbb{R}$(u, v) and $\mathbb{R}$(1, y), $u > y$, and two population sizes $n > 0$, $m > 0$, such that **if** A$\subset\mathbb{W}_z$, $z \geq u$ N(A)=n, B$\subset\mathbb{R}$(1, y), C$\subset\mathbb{W}_x$, N(C)=m **then** A is at least as good as B$\cup$C, other things being equal.

Notice that in the exact formulation of the adequacy conditions, we have eliminated concepts such as "very high positive welfare", "very low positive welfare", "very negative welfare", and the like. Hence, such concepts are not essential for our discussion and results. For example, in the exact formulation of Avoidance of the Very Repugnant Conclusion, we have eliminated the concepts "very low positive welfare" and "very high positive welfare" and replaced them with two non-fixed positive welfare ranges, one starting at the lowest positive welfare level, and the other one starting anywhere above the first range.

4. The Impossibility Theorem

> *The Impossibility Theorem*: There is no population axiology which satisfies the Egalitarian Dominance, the Non-Elitism, the General Non-Extreme Priority, Avoidance of the Very Repugnant Conclusion, and the Dominance Addition Condition.

Proof: We shall show that the contrary assumption leads to a contradiction. We shall first make use of two lemmas to the effect that the Non-Elitism and the General Non-Extreme Priority Condition each imply another condition. Then we shall show that there is no population axiology which satisfies these two new conditions in conjunction with the Egalitarian Dominance, Avoidance of the Very Repugnant Conclusion, and the Dominance Addition Condition.

4.1 Lemma 1

> LEMMA 1*:* The Non-Elitism Condition implies Condition β:
>
> *Condition β*: For any triplet $\mathbb{W}_x$, $\mathbb{W}_y$, $\mathbb{W}_z$ of welfare levels, $x > y > z$, and any number of lives $n > 0$, there is a number of lives $m > n$ such that **if** $A \subset \mathbb{W}_x$, $N(A)=n$, $B \subset \mathbb{W}_z$, $N(B)=m$, and $C \subset \mathbb{W}_y$, $N(C)=m+n$, **then,** for any D, $C \cup D$ is at least as good as $A \cup B \cup D$, other things being equal.

We shall prove lemma 1 by first proving

> LEMMA 1.1: The Non-Elitism Condition entails Condition α.
>
> *Condition α*: For any welfare levels $\mathbb{W}_x$, $\mathbb{W}_y$, $x\text{-}1 > y$, and for any number of lives $n > 0$, there is a number of lives $m \geq n$ such that **if** $A \subset \mathbb{W}_x$, $N(A)=n$, $B \subset \mathbb{W}_y$, $N(B)=m$, $C \subset \mathbb{W}_{x-1}$, $N(C)=m+n$, **then**, for any D, $C \cup D$ is at least as good as $A \cup B \cup D$, other things being equal.

Proof: Let

(1) $\mathbb{W}_x$ and $\mathbb{W}_y$ be any welfare levels such that $x\text{-}1 > y$;
(2) n be any number of lives such that $n > 0$;
(3) $p > 0$ be a number which satisfies the Non-Elitism Condition for $\mathbb{W}_x$ and $\mathbb{W}_y$.

Let $A_1,\ldots, A_{n+1}$, $B_1,\ldots, B_{n+1}$, and $C_0,\ldots, C_n$, be any three sequences of populations satisfying

(4) $A_i \subset \mathbb{W}_x$; $N(A_i)=1$ for all i, $1 \leq i \leq n$; $A_{n+1}=\varnothing$;
(5) $B_i \subset \mathbb{W}_y$; $N(B_i)=p$, for all i, $1 \leq i \leq n$; $B_{n+1}=\varnothing$;

(6) $C_i \subset \mathbb{W}_{x-1}$; $N(C_i)=p+1$, for all i, $1 \leq i \leq n$; $C_0=\varnothing$.

Finally, let

(7) D be any population.

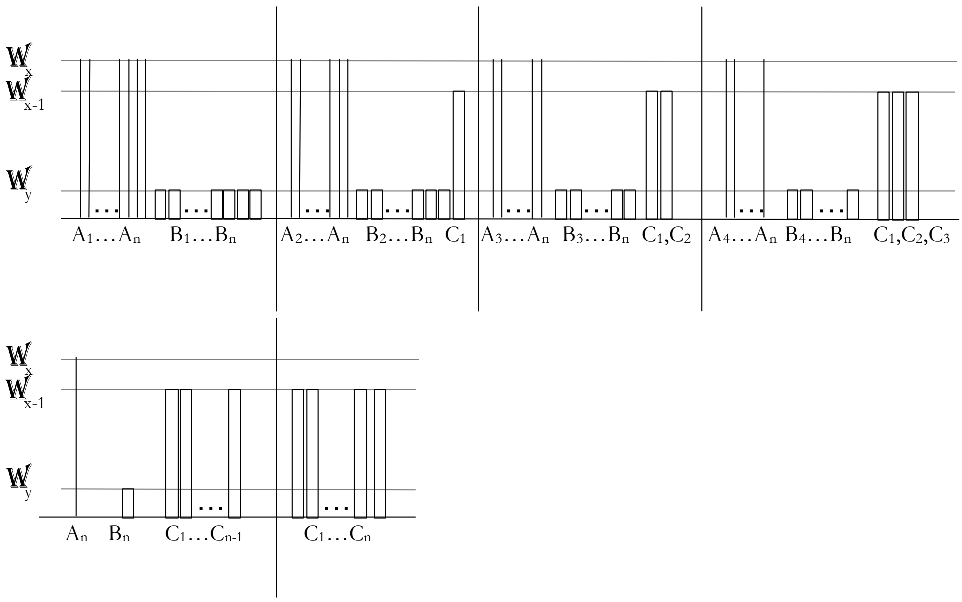

Diagram 1

The above diagram shows a selection of the involved populations in a case where $n \geq 6$. The width of each block represents the number of people, the height represents their lifetime welfare. Dots in between two blocks indicate that there is a number of same sized blocks which have been omitted from the diagram. Population D is omitted throughout.

Since $\mathbb{W}_x$ and $\mathbb{W}_y$ can be any pair of welfare levels separated by at least one welfare level, and D can be any population, and $N(A_1 \cup \ldots \cup A_n)=n$ (by (4)) can be any number of lives greater than zero, and $N(B_1 \cup \ldots \cup B_n)=np \geq n$ (by (5)), we can show that lemma 1.1 is true by showing that $C_1 \cup \ldots \cup C_n \cup D$ is at least as good as $A_1 \cup \ldots \cup A_n \cup B_1 \cup \ldots \cup B_n \cup D$. This suffices since $A_1, \ldots, A_{n+1}$, $B_1, \ldots, B_{n+1}$, $C_1, \ldots, C_n$, and D are arbitrary populations satisfying (4)-(7).

It follows from (3)-(6) and the Non-Elitism Condition that

(8) $C_i \cup E$ is at least as good as $A_i \cup B_i \cup E$
for all i, $1 \leq i \leq n$ and any $E \subset \mathbb{R}(y, x)$

and from (4)-(7) that

(9) $A_{i+1} \cup \ldots \cup A_{n+1} \cup B_{i+1} \cup \ldots \cup B_{n+1} \cup C_0 \cup \ldots \cup C_{i-1} \cup D \subset \mathbb{R}(y, x)$
for all i, $1 \leq i \leq n$.

Letting $E = A_{i+1} \cup \ldots \cup A_{n+1} \cup B_{i+1} \cup \ldots \cup B_{n+1} \cup C_0 \cup \ldots \cup C_{i-1} \cup D$, (8) and (9) imply that

(10) $C_i \cup [A_{i+1} \cup \ldots \cup A_{n+1} \cup B_{i+1} \cup \ldots \cup B_{n+1} \cup C_0 \cup \ldots \cup C_{i-1} \cup D]$
is at least as good as
$A_i \cup B_i \cup [A_{i+1} \cup \ldots \cup A_{n+1} \cup B_{i+1} \cup \ldots \cup B_{n+1} \cup C_0 \cup \ldots \cup C_{i-1} \cup D]$
for all i, $1 \leq i \leq n$ (see Diagram 1).

Transitivity and (10) yield that

(11) $C_n \cup A_{n+1} \cup B_{n+1} \cup C_0 \cup \ldots \cup C_{n-1} \cup D$ is at least as good as
$A_1 \cup B_1 \cup A_2 \cup \ldots \cup A_{n+1} \cup B_2 \cup \ldots \cup B_{n+1} \cup C_0 \cup D$

and since $A_{n+1} = B_{n+1} = C_0 = \varnothing$ (4-6), line (11) is equivalent to (see Diagram 1)

(12) $C_1 \cup \ldots \cup C_n \cup D$ is at least as good as $A_1 \cup \ldots \cup A_n \cup B_1 \cup \ldots \cup B_n \cup D$.

Q.E.D.

To show that Lemma 1 is true, we now need to prove

LEMMA 1.2: Condition α entails Condition β.

Proof. Let

(1) $\mathbb{W}_x$, $\mathbb{W}_y$, $\mathbb{W}_z$ be any three welfare levels such that $x > y > z$;
(2) $r = x\text{-}y$.

Let $A_1, \ldots, A_{r+1}$ and $B_1, \ldots, B_{r+1}$ be any two sequences of populations, $m_0, \ldots, m_r$ any sequence of integers, and f a function satisfying

(3) $m_0 > 0$;
(4) $f(m_i) = m_0 + m_1 + \ldots + m_i$, for all i, $0 \leq i \leq r$,
(5) $m_i \geq f(m_{i-1})$ satisfies Condition α for $\mathbb{W}_{x-(i-1)}$, $\mathbb{W}_z$, and $f(m_{i-1})$ for all i, $1 \leq i \leq r$;
(6) $A_i \subset \mathbb{W}_{x-(i-1)}$, $N(A_i) = f(m_{i-1})$ for all i, $1 \leq i \leq r+1$;
(7) $B_i \subset \mathbb{W}_z$, $N(B_i) = m_i$, for all i, $1 \leq i \leq r$; $B_{r+1} = \varnothing$.

Finally, let

(8) D be any population.

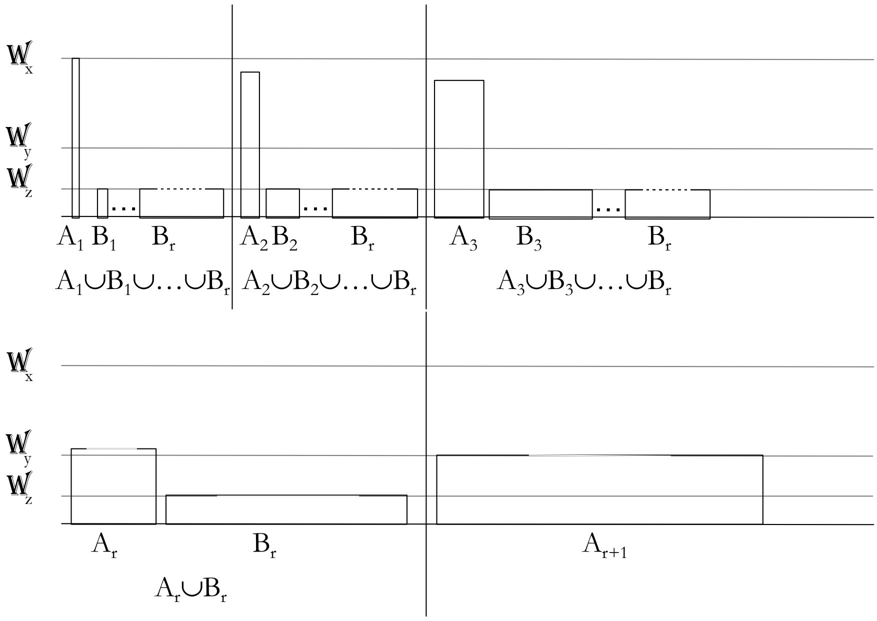

Diagram 2

The above diagram shows a selection of the involved populations in a case where $r \geq 4$. A broken line indicates that the corresponding block is larger than it appears in the diagram. Population D is omitted throughout.

We can conclude from (3)-(7) that $N(B_1\cup\ldots\cup B_r) > m_0=N(A_1)$. Consequently, since $\mathbb{W}_x$, $\mathbb{W}_y$, and $\mathbb{W}_z$ can be any welfare levels such that $x > y > z$, and D can be any population, we can show that Condition α implies Condition β by showing that $A_{r+1}\cup D$ is at least as good as $A_1\cup B_1\cup\ldots\cup B_r\cup D$. This suffices since $A_1,\ldots, A_{r+1}$, $B_1,\ldots, B_r$, and D are arbitrary populations satisfying (6)-(8).

From (3)-(7) and Condition α, it follows that

(9) $A_{i+1}\cup E$ is at least as good as $A_i\cup B_i\cup E$ for all i, $1 \leq i \leq r$ and any $E\subset\mathbb{R}(z, y+1)$.

and from (7) and (8) that

(10) $B_{i+1}\cup\ldots\cup B_{r+1}\cup D\subset\mathbb{R}(z, y+1)$ for all i, $1 \leq i \leq r$.

Consequently, letting $E=B_{i+1}\cup\ldots\cup B_{r+1}\cup D$, (9) and (10) imply that

(11) $A_{i+1}\cup[B_{i+1}\cup\ldots\cup B_{r+1}\cup D]$ is at least as good as $A_i\cup B_i\cup[B_{i+1}\cup\ldots\cup B_{r+1}\cup D]$ for all i, $1 \leq i \leq r$ (see Diagram 2).

Transitivity and (11) yield that

(12) $A_{r+1}\cup B_{r+1}\cup D$ is at least as good as $A_1\cup B_1\cup\ldots\cup B_{r+1}\cup D$

and since $B_{r+1}=\varnothing$ (7), line (12) is equivalent to (see Diagram 2)

(13) $A_{r+1} \cup D$ is at least as good as $A_1 \cup B_1 \cup \ldots \cup B_r \cup D$.

Q.E.D.

It follows trivially from lemma 1.1 and 1.2 that lemma 1 is true.
Q.E.D.

4.2 Lemma 2

LEMMA 2*:* The General Non-Extreme Priority Condition implies Condition δ.

Condition δ: For any $\mathbb{W}_z$, $z < 0$, and any number of lives $m > 0$, there is a positive welfare level $\mathbb{W}_u$, and a positive welfare range $\mathbb{R}(1, y)$, $u > y$, and a number of lives $n > 0$ such that **if** $A \subset \mathbb{W}_x$, $x \geq u$, $B \subset \mathbb{R}(1, y)$, $N(A)=N(B)=n$, $C \subset \mathbb{W}_z$, $D \subset \mathbb{W}_3$, $N(C)=N(D)=m$, **then,** for any E, $A \cup C \cup E$ is at least as good as $B \cup D \cup E$, other things being equal.

Proof: See lemma 5.2 in Arrhenius (2000a) or lemma 2 in Arrhenius (2001).

4.3 Lemma 3

Finally, we shall show that the impossibility theorem is true by proving

LEMMA 3: There is no population axiology which satisfies the Egalitarian Dominance Condition, Avoidance of the Repugnant Conclusion, the Dominance Addition Condition, Condition β, and Condition δ.

Proof. We show that the contrary assumption leads to a contradiction. Let

(1) $\mathbb{W}_z$ be a negative welfare level, m a population size, $\mathbb{R}(w, t)$ and $\mathbb{R}(1, v)$, $w > v$, two welfare ranges, and p a population size, which satisfy Avoidance of the Very Repugnant Conclusion;

(2) $\mathbb{W}_u$ be a welfare level, $\mathbb{R}(1, y)$ a welfare range, and n a number of lives, which satisfy Condition δ for $\mathbb{W}_z$ and m;

(3) $\mathbb{W}_x$ be a welfare level such that $x \geq w$ and $x \geq u$;

(4) $r > n+p$ be a number of lives which satisfies Condition β for the three welfare levels $\mathbb{W}_{x+2}$, $\mathbb{W}_3$, and $\mathbb{W}_1$ and for $n+p$ lives at $\mathbb{W}_{x+2}$;

(5) $A_1 \subset \mathbb{W}_x$, $A_2 \subset \mathbb{W}_{x+1}$, $A_3 \subset \mathbb{W}_{x+2}$, $N(A_1)=N(A_2)=N(A_3)=p$;

(6) $B_1 \subset \mathbb{W}_1$, $B_2 \subset \mathbb{W}_{x+2}$, $N(B_1)=N(B_2)=n$,

(7) $C_1 \subset \mathbb{W}_1$, $N(C_1)=r$;

(8) $D_1 \subset \mathbb{W}_3$, $D_2 \subset \mathbb{W}_z$, $N(D_1)=N(D_2)=m$.

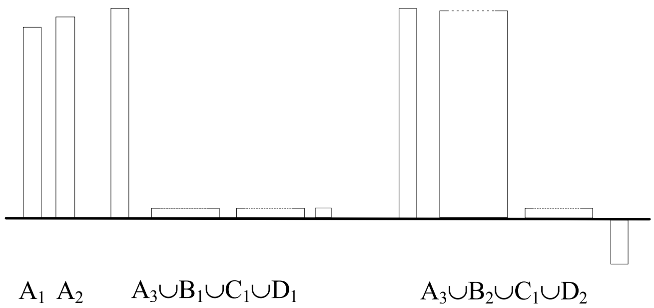

Diagram 3

Again, the width of the blocks in the diagram represents the number of lives in the population, the height represents their lifetime welfare. Dashes indicates that the block in question should intuitively be much wider than shown.

Since $A_1 \subset \mathbb{W}_x$, $A_2 \subset \mathbb{W}_{x+1}$ and $N(A_1)=N(A_2)$ (by (5)), the Egalitarian Dominance Condition implies that

(9) A_2 is better than A_1 (see Diagram 3).

Since $A_2 \subset \mathbb{W}_{x+1}$, $A_3 \subset \mathbb{W}_{x+2}$, $N(A_2)=N(A_3)$ (by (5)), and $B_1 \cup C_1 \cup D_1$ only consists of lives with positive welfare (by (6)-(8)), it follows from the Dominance Addition Condition that

(10) $A_3 \cup B_1 \cup C_1 \cup D_1$ is at least as good as A_2 (see Diagram 3).

From (6) we get that $N(B_1)=N(B_2)=n$, $B_1 \subset \mathbb{R}(1, y)$, and $B_2 \subset \mathbb{W}_{x+2}$. From (3) we get that $x+2 > u$. Accordingly, from (2), (8) and Condition δ we get that

(11) $A_3 \cup B_2 \cup C_1 \cup D_2$ is at least as good as $A_3 \cup B_1 \cup C_1 \cup D_1$ (see Diagram 3).

Let

(12) $A_4 \subset \mathbb{W}_3$, $N(A_4)=p$;
(13) $B_3 \subset \mathbb{W}_3$, $N(B_3)=n$;
(14) $C_2 \subset \mathbb{W}_3$, $N(C_2)=r$;

Since $A_3 \cup B_2 \subset \mathbb{W}_{x+2}$, and $N(A_3 \cup B_2)=n+p$ (by (5) and (6)), and since $C_1 \subset \mathbb{W}_1$, $N(C_1)=r$ (by (7)), it follows from (12)-(14) and Condition β that

(15) $A_4 \cup B_3 \cup C_2 \cup D_2$ is at least as good as $A_3 \cup B_2 \cup C_1 \cup D_2$ (see Diagram 4).

Diagram 4

It follows from (12)-(13) and the definition of a welfare range that $A_4 \cup B_3 \cup C_2 \subset \mathbb{R}(1, v)$. From (5) we get that $A_1 \subset \mathbb{W}_x$, $N(A_1)=p$, and from (3) that $x \geq w$. Since $D_2 \subset \mathbb{W}_z$, $N(D_2)=m$ (by (8)), it follows from (1) and Avoidance of the Very Repugnant Conclusion that

(16) A_1 is at least as good as $A_4 \cup B_3 \cup C_2 \cup D_2$ (see Diagram 4).

By transitivity, it follows from (9)-(11) and (15) that

(17) $A_4 \cup B_3 \cup C_2 \cup D_2$ is better than A_1

which contradicts (16).
Q.E.D.

It follows trivially from lemma 1, 2 and 3 that the impossibility theorem is true.
Q.E.D.[9]

[9] I would like to thank Kaj Børge Hansen for his very helpful and detailed comments on my earlier theorems in this area which facilitated the writing of this paper. Financial support from the Swedish Foundation for International Cooperation in Research and Higher Education (STINT) is gratefully acknowledged.

References

Gustaf Arrhenius, "An Impossibility Theorem in Population Axiology with Weak Ordering Assumptions", in Rysiek Sliwinski (ed.), *Philosophical Crumbs*, Uppsala Philosophical Studies 49, Uppsala: Department of Philosophy, Uppsala University, 1999.

Gustaf Arrhenius, *Future Generations: A Challenge for Moral Theory*, F.D.-Diss., Uppsala: University Printers, 2000a.

Gustaf Arrhenius, "An Impossibility Theorem for Welfarist Axiologies", *Economics and Philosophy*, October 2000b.

Gustaf Arrhenius, "What Österberg's Population Theory has in Common with Plato's", in Erik Carlson & Rysiek Sliwinski (eds.) *Omnium-gatherum*, Uppsala Philosophical Studies 50, Uppsala: Department of Philosophy, Uppsala University, 2001.

Charles Blackorby and David Donaldson, "Normative Population Theory: A Comment", *Social Choice and Welfare*, 8, 261-7, 1991.

Richard M. Hare, "Possible People" in *Essays on Bioethics*, Oxford: Clarendon Press, 1993.

Yew-Kwang Ng, "What Should We Do About Future Generations? Impossibility of Parfit's Theory X", *Economics and Philosophy* 5(2), 235-253, 1989.

Derek Parfit, *Reasons and Persons*, Oxford: Oxford University Press, 1984.

Jesper Ryberg, *Topics on Population Ethics*, Ph.D. Diss., University of Copenhagen, 1996.

Torbjörn Tännsjö, *Göra Barn*, Borås: Sesam förlag, 1991.

Torbjörn Tännsjö, *Hedonistic Utilitarianism*, Edinburgh: Edinburgh University Press, 1998.