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The Hyperinflation Model of Money Demand (or Cagan Revisited): Some New Empirical Evidence from the 1990s*

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Abstract

This paper employs cointegration techniques to examine three recent hyperinflationary episodes in transition economies, which, with the exception of Russia (1992-1994), have been largely overlooked in the literature. More specifically, these episodes include Bulgaria during 1995-1997 and Ukraine during 1993-1995. We use the well-known maximum likelihood estimator due to Johansen (1988, 1991) and Stock and Watson's (1993) dynamic ordinary least squares (DOLS) estimator to complement each other and obtain consistent estimates of the semi-elasticity of real money demand with respect to inflation. The empirical results obtained in this study support the Cagan model of money demand in the East European hyperinflation experiences of the 1990s. However, our results do not indicate that the rational expectations hypothesis holds during these episodes. In addition, we also test the hypothesis that monetary policy in these three hyperinflations was conducted with the sole intent of maximizing the inflation tax revenue for the government.

Keywords: Cagan, cointegration, inflation tax, transition economies, stabilizations

JEL Classification: C45, C62, E31, E63, E65

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1. Introduction

Cagan's (1956) model is the seminal work in the empirical analysis of hyperinflations. Recently, cointegration methods have been applied to the model in which the demand for real balances is predominantly determined by the expected rate of future inflation (Taylor, 1991; Phylakis and Taylor, 1993; Engsted, 1993, 1994, 1998; Michael, Nobay, and Peel, 1994; Choudhry, 1998; and Petrovic, Bogetic and Vujosevic, 1999). The main insight of this literature is that through the careful analysis of the stochastic properties of the data using cointegration theory, one can validate (or reject) the model and obtain efficient estimates of key parameters. In addition, cointegration properties inherent in the hyperinflation model allow researchers to suitably represent and test hypotheses with only very weak assumptions about the expectations formation mechanism. The main findings of this work provide some evidence to support a Cagan-type money demand function for most hyperinflations. However, most studies fail to find robust support for the rational expectations hypothesis in this context across countries and sample periods.

In what follows, we examine the evidence from the most recent experiences of three transition economies: Bulgaria, Russia and Ukraine. The Cagan model is empirically tested using cointegration methodology. Since the specified model is without any *a priori* assumption about expectations formation, we proceed with a test of the rational expectations hypothesis (spelled out in a precise way below). The paper concludes, following Cagan (1956) and Taylor (1991), with a test of the hypothesis that the percentage rate of increase in money supply and prices which maximizes the revenue from seigniorage creation is in fact equal to the average rate of inflation in the sample.

This paper employs cointegration techniques to examine three recent hyperinflation episodes, which, with the exception of Russia (1992-1994), have been largely overlooked in the literature. More specifically, these episodes include Bulgaria during 1995-1997 and Ukraine during 1993-1995. We use the well-known maximum likelihood estimator due to Johansen (1988, 1991) and Stock and Watson's (1993) dynamic ordinary least squares (DOLS) estimator to complement each other and obtain consistent estimates of the semi-elasticity of real money demand with respect to inflation. As Michael, Nobay, and Peel (1994, p. 9) note, since small samples are biased in favor of rejecting cointegration (the null hypothesis being noncointegration), finding evidence of it lends strong support to its existence in well-defined money demand functions during hyperinflation. Hakkio and Rush (1991) also argue that while the frequency of observations is not crucial, the "long run" may in some cases "be a matter of months." For hyperinflation episodes, a few months are all that matters.

In addition, we present some test results using Cagan's data for both the Austrian and the first Russian hyperinflations of the 1920s. The results are somewhat different from those in the earlier literature (Taylor, 1991) and the episodes remain a useful reminder of the circumstances in which hyperinflations emerge and subside. We also provide consistent DOLS estimates of the parameters of interest and compare those to the Johansen estimates found in previous work.

Next, given the cointegrating estimates, if they exist, we test the rational expectations hypothesis using two different definitions of money: M2 (currency and deposits) and C^P (currency in circulation). The restrictions are stated as orthogonality assumptions of the prediction error onto the information available at time t in a single equation framework. Taylor (1991, p. 346) establishes the equivalence between this set of restrictions and cross-equation restrictions in a VAR model of the cointegrating relationship and the rate of growth of inflation. Finally, the paper tests the hypothesis that monetary policy in these three hyperinflations was conducted with the sole intent of maximizing the inflation tax revenue for the government.

2. Model and Data

Cagan's model posits that under conditions of severe inflation, movements in real money balances will be dominated principally by fluctuations in inflationary expectations. Following Taylor (1991) and Phylakis and Taylor (1993), the model, omitting a constant term, can be written as:

$$(m-p)_t = -\gamma \Delta p_{t+1}{}^e + \xi_t \ , \tag{1}$$

where the natural logarithm of nominal money balances and price level are denoted by *m* and *p*, respectively. The superscript indicates expectations formed at time t, and ξ_t captures all relevant factors not specified in the money demand function (1). These, it is assumed, have only small influence during hyperinflation and, if stationary, imply that the error term is also stationary though possibly serially correlated. As is conventional, Δ is the difference operator. Finally, γ is the semi-elasticity of real money demand with respect to expected inflation. Hence, the full expected inflation rate elasticity of real money balances is $\left|\gamma \Delta p_{t+1}{}^e\right|$.

By substituting actual rates of inflation for expected inflation, and adding $\gamma\Delta p_t$ to both sides of (1), we obtain the following equation:

$$(m-p)_t + \gamma \Delta p_t = -\gamma \Delta^2 p_{t+1} + \psi_{t+1} \ , \tag{2}$$

The error term in (2) is $\psi_{t+1} = \left[\xi_t + \gamma\,(\Delta p_{t+1} - \Delta p_{t+1}{}^e)\right]$. Suppose that under hyperinflation both the rate of change in real money balances and inflation are stationary processes. This implies Δp_t and $(m\text{-}p)_t$ are each integrated of order one (I(1)). Taylor (1991, p. 329) asserts that if the expectational errors are stationary regardless of the method of forming expectations, since ψ_{t+1} and $\gamma\Delta^2 p_{t+1}$ are stationary, (2) suggests that the linear combination $\left[(m-p)_t + \gamma\Delta p_t\right]$ should also be stationary. This means that real money balances and inflation are cointegrated in the sense of Engle and Granger (1987). The normalized cointegrating parameter is exactly the parameter of interest in the Cagan study (γ here). A failure, however, to find cointegration would indicate a rejection of Cagan's model. If cointegration exists, then the variables have a common trend and the estimate of the parameter that relates them is "superconsistent."[1] The presence of cointegration has also the intuitive meaning that although prices and money may both accelerate dramatically, they tend to move together. We use two estimators to study the stochastic properties of money and prices in the recent hyperinflation episodes: those of Johansen (1988, 1991) and Stock and Watson (1993), as discussed below.

In the empirical tests we apply monthly data available for Bulgaria from July 1995 to September 1997; for Russia from January 1992 to September 1994; and for Ukraine from January 1993 to August 1995. The actual sample size used in estimation within these periods (see Tables in appendix) is determined by observing months in which the monthly rates of inflation increased above 15 to 20 percent (in some cases, Ukraine, much higher) and remained at these rates for several months.[2]

[1] "Superconsistency" refers to the speed of convergence of the estimated parameters to their true values, at the rate T, instead of the usual $T^{1/2}$. This is because if a variable is I(1), its sum of squares is $O_p(T^{1/2})$. Cointegrated vectors have the same superconsistency property (Stock, 1987 and Maddala and Kim, 1998).

[2] See Dornbusch, et. al. (1990). Throughout, we use these authors' definition of hyperinflation ("extreme inflation"). Cagan's (1956, p. 25) original definition referred to much higher rates, price increase rising above 12000 % per year: "beginning in the month the rise in prices exceeds 50 percent

The end is the month in which those price increases declined below 15-20 percent and were maintained at a lower rate for at least twelve months without any reversal of this trend. The Bulgarian data come from the *Monthly Bulletin* of the Bulgarian National Bank. The Russian data are obtained from various monthly issues of *Russian Economic Trends*. The Ukrainian data are due to the Institute of Economic Research and Policy Consulting in Kiev and the central bank.[3] M2 and currency in circulation (C^P) are the two monetary variables used in the estimation process. To obtain real money balances, we deflated M2 and C^P by the Consumer Price Index (CPI) for each country. The monthly inflation rates are computed as the difference between successive monthly values of the natural logarithm of the CPI.

3. Recent Hyperinflations: A Comparison

In the 1990s, most countries in Eastern Europe and the former Soviet Union experienced high and variable rates of inflation (Cukierman, et. al., 2002). While reasons differ across countries, the main issue relates to the inflationary financing of the deficit. Large transition shocks, liberalization of previously administered or subsidized prices, delayed structural reforms, and crucially the inability of central governments to collect taxes and to finance expenditures externally through capital markets brought about worsening of conditions. The huge strain on public finances imposed upon transition governments by enterprises still remaining under state control and non-functioning banking system further exacerbated the need for more seigniorage, which in turn led to even higher rates of money growth.

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and as ending in the month before the monthly rise in prices drops below that amount and stays below for at least a year."

[3] I am particularly indebted to R. Guicci at the Deutsche Institute fuer Wirtschaftsforschung for gaining access to the Ukrainian data and generously sharing it. In addition, those data are compared to series

Figures 1-3 provide a visual summary of the behavior of inflation, money growth and real balances. The relative movement of the logarithm of CPI and the logarithm of the monetary variables indicates a close relationship, common in high inflations, i.e., higher money growth for a number of reasons including endogeneity may be associated with higher inflation. For the periods under study, Table 1 shows that higher average inflation was accompanied by higher average growth rates of M2 as well. In every case, the ratio of those two series is higher than unity and suggests that indeed as inflation increased agents tended to reduce (economize on) their money holdings. In other words, the velocity of money rose rapidly with inflation.

Table 2 gives an overview of some of the underlying inflationary trends for Bulgaria, Russia and the Ukraine from 1990-1998. What is apparent from this table is that high and increasing inflation (in excess of 20%-25% per month) in all three countries was accompanied by higher rates of money creation as well. Fiscal deficits were higher in periods preceding the hyperinflation or directly associated with it. For example, in Russia, during 1992-1994, the budget deficit was primarily financed through central bank credits.[4] Real output declined sharply and government spending remained relatively high for most of the decade. As is evident, most of these expenditures were financed through seigniorage.

In mid-1996, after two years of real output growth, Bulgaria experienced renewed economic turmoil, several bank runs occurred, confidence and support for the government declined. The country entered a period of hyperinflation. Output declined by more than 10 percent and a banking crisis ensued. The inconsistent

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independently compiled through UEPLAC. The results are very similar, thus we rely on the latter source. T. Choudhry also willingly provided his Russian dataset to compare with mine.

[4] Further details of the Russian hyperinflation can be found in Choudry (1998) and Lavigne (1999).

policies of the central bank caused a huge depletion of official reserves and failed to shore up a rapid decline in money demand.[5] Inflation soared to almost 500 percent (on an annual basis) in January 1997 and exceeded 2000 percent in March 1997. From December 1996 to February 1997, real balances fell by more that 60 percent. The main cause for the rapidly accelerating inflation was the continued central bank financing of the budget deficit. The government resigned in December 1996, and a supplementary budget law was passed that required the central bank to finance the gap in the budget. The gap amounted to more than 9 percent of GDP (IMF Country Report, 2001). The "fiscal news," i.e., that the budget deficit would be monetized, increased inflationary expectations and the public lost confidence in any immediate restoration to normality.

After leaving the ruble zone in 1992, Ukraine experienced a severe trade shock and disruption and "disorganization" of output.[6] Real output collapsed precipitously and has not recovered to its pre-transition levels since (Table 2). These acute supply shocks and lack of coordinated policy within the ruble zone initially had provided incentives for the government to monetize large deficits. Havrylyshyn et. al. (1994) explain that the continued inflationary bias in government policies was reinforced by aspects of domestic political economy and lack of consensus towards reform and stabilization.[7] Rent-seeking behavior by managers of state-owned enterprises and the government's unwillingness to stop providing credits to old industry through the budget further delayed the prospects of recovery. Inflation soared to more than 4000 percent (on an annual basis) in 1993 and real balances fell

[5] For further details of the ensuing hyperinflation and the circumstances around it, see Gulde (1999), Miller (1999) and Carlson and Valev (2001).

[6] See Blanchard and Kramer (1996) for a lucid discussion of the circumstances in which most transition economies underwent a huge initial drop in real output.

substantially.[8] The amount of seigniorage ran to an estimated 29 percent of GDP (Havrylyshyn et. al., 1994, p.361). Ukraine finally introduced a currency reform in August 1996.

In comparison, Table 3 lists selected hyperinflations over the last century. It is evident that the hyperinflation phenomenon is relatively short in duration and characterized by highly unstable and explosive paths of inflation and money growth (Bruno, 1991). In most instances, there was a complete collapse of the currency and the monetary system. Most of those hyperinflations are well documented. The important common feature that emerges is the fiscal difficulties that accompanied each episode. In some sense, hyperinflations have been largely fiscal in origin. The way these economies regained stability was only after a credible and abrupt change in fiscal and monetary regimes. This attests to the importance of inflationary expectations in successful stabilizations. It is only after a drastic revision of the public's expectations of future government policies that inflation is promptly curtailed (Sargent, 1993). In Germany, inflation was finally brought down suddenly after the implementation of a new fiscal regime in November 1923.[9] Bolivia, for example, managed to end its hyperinflation by unifying and stabilizing the exchange rate while at the same committing to adjusting the fundamental fiscal imbalances in the economy (Sachs, 1987).

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[7] Aslund, Boone, and Johnston (1996) present a political economy model, which accounts for why many post-communist governments pursued less than socially optimal stabilization policies. They provide the empirical evidence as well.

[8] When the public realizes that the path of deficit financing is unsustainable, Roberts (1993) claims "the prediction is that there is a universal abandonment of money as soon as the market immediately perceives the deficit is too high." This is what followed in Ukraine.

[9] Vegh (1992) describes the experiences of effectively stopping hyperinflations in 1920s Europe and 1980s Latin America. His evidence suggests that the exchange rate served as the nominal anchor, and hyperinflations were stopped "almost overnight" without significant loss in output. Webb (1986) also asserts that inflationary expectations were driven in an important way during the German hyperinflation by fiscal news, i.e., the steady increase and anticipation of budget deficits.

4. Unit Root Tests and Stochastic Properties of the Data

Estimating cointegrating vectors and tests conventioanlly require that we examine the stochastic properties of the data first. Thus, we try to identify whether the variables of interest are integrated of order one, i.e., whether they possess a unit root. To do this, we attempt to find a parsimonious representation of the data rather than precisely determine its order of integration. We employ three tests to achieve this objective. To overview our findings, the conjecture that real money balances and inflation are both I(1) during hyperinflation is largely borne out by the data, with the exception of Bulgaria, where the inflation series appears stationary according to some but not all tests, and Ukraine, where the real money balances may possess a second unit root. However, for the purposes of our study, we interpret the results to suggest that these series are I(1). In addition, revisiting the Cagan data set of the classic hyperinflations of the interwar period, we document a new finding, i.e., in the first Russian hyperinflation the real balances series appears stationary in first differences. This is somewhat at variance with earlier work (Taylor, 1991, Table 1-2 and Engsted, 1994). For comparison, the results are presented in Appendix 2.

First, real money balances (RM2 and RC^P) and inflation are tested for the existence of a unit root using a modified Dickey-Fuller test (ADF-GLS) due to Elliot, Rothenberg, and Stock (ERS) (1996).[10] Second, we compute the Schmidt-Phillips $\tilde{\tau}$, which is a Lagrange multiplier (LM) principle test and has a different parameterization than the Dickey-Fuller type tests. The difference arises in the residual upon which the first difference of the series is regressed as shown below (see Schmidt and Phillips, 1992, p. 260). Third, we use, as confirmatory analysis, the

[10] Phillips and Xiao (1998) and Maddala and Kim (1998) provide an exhaustive overview of a variety of unit root tests. The latter conclude that DF, ADF, and PP lack power against meaningful alternatives and recommend that they be replaced with other tests and used less in practice (p. 92).

Kwiatkowski, Phillips, Schmidt, and Shin (KPSS) test (η_μ, η_τ) in which level or trend stationarity is formulated as the null hypothesis.[11] The results from these tests are reported in Tables 4 and 5.

To start with the last test first, the KPSS is defined in the following way. Assume a model (or data generating process) for a variable y_t:

$$y_t = \xi t + r_t + \varepsilon_t, \quad r_t = r_{t-1} + u_t,$$

where r_t is a random walk and the u_t are iid$(0,\sigma_u^2)$. The null hypothesis is then formulated as: $H_0 : \sigma_u^2 = 0 \quad or \quad r_t$ is a constant, or a null hypothesis of stationarity. It is important to note that the KPSS test statistic is robust to general specification of the error process. The critical values are provided in Kwiatkowski, et. al. (1992, Table 1, p. 166). Next, the Schmidt-Phillips $\tilde{\tau}$ test is constructed using a similar Bhargava-type formulation where the data generating process is: $y_t = \varphi + \xi t + X_t$, $X_t = \rho X_{t-1} + \varepsilon_t$. The null hypothesis of a unit root corresponds to ρ=1. The model yields the following equation:

$$\Delta y_t = b_0 + b_1 t + \phi y_{t-1} + e_t.$$

The LM statistic tabulated in Schmidt and Phillips (1992, p. 264) is a t-statistic for ϕ=0 in:

$$\Delta y_t = \text{int}\,ercept + \phi \tilde{S}_{t-1} + error,$$

where $\tilde{S}_{t-1}$ is the residual in a regression of y_{t-1} on an intercept and a time trend. Monte Carlo results show that their test statistic is more powerful than DF tests.[12]

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[11] For an accessible description and intuition behind these tests, see Maddala and Kim (1998). They also provide a comparison to other existing unit root tests.
[12] For more on this point see Schmidt and Phillips (1992) and Maddala and Kim (1998, p.85).

ERS define the model to be as above, but they performed their modified ADF-GLS t-test on a regression:

$$\Delta y_t^{\,d} = a_0 y_{t-1}^{\,d} + \sum_{i=1}^{p} a_i \Delta y_{t-1}^{\,d} + error \ ,$$

where testing $a_0 = 0$ is the unit root null hypothesis, and y_t^d is the detrended series y_t. The procedure depends on whether we consider a model with a drift or a linear trend: $y_t^{\,d} = y_t - \hat{b}' z_t$, where $\hat{b}$'s are obtained in a regression of $y_{\bar{a}}$ on $Z_{\bar{a}}$; alphas are a predetermined constant specified in ERS; $Z_t = (1, t)'$. ERS show in their experiments that ADF-GLS tests conducted on the detrended series have "best overall performance in terms of small-sample size and power" (Elliott, et. al., p. 827-830).[13] Thus, in our examination of the hyperinflation series, we place some emphasis on these tests.

In all the tests, we first check for the presence of two unit roots and include a time trend in all single unit root tests. We do not include a time trend when testing for two unit roots and compare to the appropriate critical values. Given Kwiatkowski, et. al. (1992) study, we use a maximum of six lags in the KPSS test. However, we report only up to four in Table 4 and two in Table 9; results do not change if all lags are applied.

5. Cointegration Tests and A Rational Expectations Test

A system of nonstationary [I(1)] variables can share a common stochastic trend and be cointegrated. The main idea in the context of hyperinflation is that

[13] Elliott (1999) proposes some additional improvements on the ADF-GLS tests while relaxing the assumptions and initial conditions on the error process in the model.

although real money balances and inflation (or expected rate of inflation) may vary substantially, there exists a long-run relationship between them, such as the one described by the Cagan model of money demand. If cointegration exists, then we may find the common stochastic trend relating those two variables and study its short-run dynamics. We apply the Johansen (1991, 1992) vector error correction model (VECM) and the Stock and Watson (1993) dynamic OLS approach to study the movement of real money balances and inflation in the most recent episodes of hyperinflation.

The Johansen VECM approach models the data in a multivariate representation, which takes into account the dynamics of the relationship among the variables being studied. This procedure allows for more robust results than other existing methods, and hypotheses tests can be conducted using standard asymptotic chi-square tests (Gonzalo, 1994). It computes a maximum likelihood to determine the presence of cointegrating vectors in nonstationary series in a VAR representation. Let Y_t be a data vector such that

$$Y_t' = \left[(m_t - p_t) \ \ \Delta p_t \right].$$

A vector autoregressive representation of Y_t can be rearranged to get:

$$\Delta Y_t = \sum_{j=1}^{k-1} \Gamma_j \Delta Y_{t-j} + \Pi Y_{t-1} + \mu + \varepsilon_t \ , \tag{3}$$

where $\Pi = -(I - \sum_{i=1}^{k} \Pi_i)$, and ε_t is identically and normally distributed. Since we argue that real money balances and inflation may have a common stochastic growth pattern, there is a separate drift term μ in (3). This is the case of stochastic cointegration.

Johansen and Juselius (1990) show that the coefficient matrix Π contains information about the long-run properties of the variables in Y_t. The hypothesis of cointegration is formulated as a reduced rank of the coefficient matrix. In Equation (3), the rank (the number of non-zero characteristic roots, or eigenvalues) of the matrix Π is equal to the number of cointegration relationships and indicates the long-term effects of ε_t on Y_t. As shown by Johansen (1991) and Johansen and Juselius (1990), one important property of the impact matrix is that it can be written as a product of two n by k matrices of rank k<n, i.e., $\Pi = \alpha\gamma'$:

$$\Pi Y'_{t-1} = \alpha\gamma Y'_{t-1} = \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} \gamma' \begin{pmatrix} (m-p)_{t-1} \\ \Delta p_{t-1} \end{pmatrix}. \tag{4}$$

The α's are the speed-of-adjustment coefficients to long-run equilibia and describe the short-run dynamics of the system. We report the whole impact matrix for each country in Table 4 and interpret the results.

Standard decomposition of the drift term in Equation (3) and using Equation (4) yields the following error correction model (ECM) of the real money balances equation, which we report in Table 6:

$$\Delta(m-p)_t = \sum_{i=1}^{k-1} \Gamma_{i(m-p)} \Delta(m-p)_{t-i} + \sum_{i=1}^{k-1} \Gamma_{i\Delta p} \Delta p_{t-i} + \alpha_1((m-p)_{t-1} - (-\gamma \Delta p_{t-1})) + \varepsilon_{1t}. \tag{5}$$

The Johansen tests are Likelihood Ratio (LR) tests that determine the rank of Π: *Trace* and *Maximal*. The trace test statistics (studies have shown to be preferred) tests the null hypothesis of no cointegration (r=0) vs. the general alternative. Hence, it is used to test for the existence of cointegration. The maximal test statistics provide evidence for the null hypothesis of the number of cointegrating vectors being equal to r vs. r+1 (one vs. two, etc.). The estimated statistics are then compared against the critical values tabulated in Osterwald-Lenum (1992) to make inferences regarding the existence of cointegration in our model. In the process, and where applicable, we adjust these critical values by a scaling factor T/(T-nk), which expresses the ratio of the approximate finite-sample critical value and the asymptotic critical value provided by Johansen's method at the corresponding significance level. We correct the critical values in Osterwald-Lemun (1992) to fit our specific sample. The ratio is viewed as a measure of finite sample bias (Cheung and Lai, 1993, p. 318-319).

The Stock-Watson dynamic OLS (DOLS) are based on the triangular representation of cointegrated systems introduced by Phillips (1991). The assumption here is a pretest. This is examined in some depth in (Hayashi, 2000). Consider the following system of I(1) variables:

$$\begin{aligned} y_{1t} &= \mu + \gamma' y_{2t} + \upsilon_{1t} \\ y_{2t} &= y_{2t-1} + \upsilon_{2t} \quad , \end{aligned} \tag{6}$$

and let y_{1t} and y_{2t} be I(1), and ν_{1t} and ν_{2t} be I(0). Then, n-dimensional vector $\begin{pmatrix} y_{1t} & y_{2t}' \end{pmatrix}'$ is cointegrated with a cointegrating vector captured by γ. To correct for endogeneity and correlation between the errors, we add leads and lags of Δy_{2t}. This yields the following single equation representation, which we use to estimate the Cagan model (compare with ECM in (5)):

$$(m-p)_t = \mu + \gamma \Delta p_t + \sum_{i=-k}^{k} \Phi_{i\Delta p} \Delta p_{t-i} + \omega_t , \tag{7}$$

where from Equation (6) $y_{1t}=(m-p)_t$ and $y_{2t}=\Delta p_t$. γ is the semi-elasticity of real money balances with respect to inflation, the parameter of interest in the Cagan model. Here it is defined to be negative (see the results reported in Tables 6 and 7). We compare and contrast the two estimates in the next section.

If cointegration exists in the specified models as discussed above, and we are able to obtain a "superconsistent" estimate of γ, then one may retrieve the prediction errors and treat the coefficient of interest as known. If expectations were rationally formed, and assuming Sargent's (1977) restriction imposed on Cagan's model $E(x_t \mid I_t) = 0$, then the errors in (8):

$$e_{t+1} = \gamma \Delta p_{t+1} + (m - p)_t \tag{8}$$

would not depend on any information available at t, i.e., $E(e_t \mid I_t) = 0$. One way to test the rational expectations hypothesis in this context is to regress the predicted error on lagged values from the I_t set, and test for the joint insignificance of the coefficients. [14] Results are exhibited in Table 8.

6. Empirical Results and Discussion

Tables 4 and 5 list the results from the three unit root tests described in Section 3. These tests are applied to the real money balances and inflation in different periods and episodes of hyperinflation. The picture that emerges from these tests is consistent with the assumption that real balances and inflation are integrated of order one for each of the countries. The hypothesis that there are two unit roots in the series is rejected for most of the cases in each country at five percent and this finding is confirmed by the three tests. The exceptions are the real balances in Bulgaria (RCP series) and Ukraine, where the hypothesis of only one unit root is rejected at five percent, and this occurs only when the Schmidt and Phillips test is applied for the particular time period.

We find, in the case of the first Russian hyperinflation, similar results for real balances, however this also occurs for the ADF-GLS results. The hypothesis of only one unit root present cannot be rejected at five percent when the confirmatory KPSS test is used. Given previous results that disagree on this issue (Taylor, 1991) and Engsted (1994), it is with some amount of confidence that it can be asserted that the real balances are I(1) in this case at ten percent significance level. Thus, when first differenced our series appear to be stationary, as confirmed by the three types of tests reported.

The results reported in Tables 6 and 7 reveal some evidence of cointegration of current inflation and real money balances in the case of the three East European hyperinflations of the 1990s. However, in the Russian case, when the Chaing-Lai correction is employed, the null hypothesis of no cointegration is rejected at only ten percent and only in the case of RCP specification. The hypothesis of at least one cointegrating vector is in no case rejected. Although this provides evidence in support of the Cagan model as applied to these countries, the estimated coefficients and signs do not allow for easy interpretation. In the case of Russia, we fail to identify uniquely a stable money demand function. None of the coefficients has the theoretically expected sign. This is also the case when the RM2 specification is estimated in Ukraine.

These results are somewhat at variance with the ones provided in Choudry (1998).[15] However, his reported estimates do not necessarily correspond to the

[14] Taylor (1991) provides an equivalent procedure to test the same hypothesis. Similar results are shown in Chow (1989) under a number of alternative forms of expectations formation for present value models, discussed in Shiller and Campbell (1987).

[15] In addition, when currency depreciation is included in the system as proxy for the expected rate (Abel, et. al., 1979) or expected rate of return on foreign assets (Taylor, 1991), the results do not change. While the existence of cointegration cannot be rejected at standard significance levels, the coefficient of currency depreciation appears insignificant or of smaller magnitude, implying that real balances are mainly determined by the influence of expected domestic inflation in the sample periods. This finding is interesting since it is different from the findings in Choudry (1998) for Russia or the intuition provided in Aarle and Budina (1995).

highest eigenvalue in the VECM.[16] We decided not to report the vector (although of the correct sign) associated with the lower eigenvalue in the case of Russia since that may not be appropriate as revealed by our DOLS estimates. The results from the Russian hyperinflation in the 1920s, however, reveal point estimates of γ which are of the correct sign and of plausible size. This also applies to the Austrian estimates; in both cases it is worth noting that the two different estimates (Johansen and DOLS) are quite similar, as expected. While the Russian results are indeed within the range of the estimates in Cagan (1956) and Engsted (1994), the Austrian inflation elasticity of real money balances is smaller in absolute value. As discussed below, even then the authorities seemed to have increased money creation faster than warranted. This is evident from the likelihood ratio tests presented in Table 10.

The Bulgarian results provide strong support for the Cagan model; the point estimates are of the correct sign and plausible magnitude (see Tables 6 and 7, second column).[17] These are also confirmed by the DOLS estimates. In the sample period, there is some evidence that the rate of change in prices had a larger impact on the demand for real M2 than it did on real currency. This is somewhat surprising given the huge depreciation of the foreign exchange rate in Bulgaria and massive bank runs at the time. Table 7 shows, however, that the semi-elasticity of real currency with respect to inflation is slightly higher than that for real M2.

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[16] The number of lags used in Choudry (1998) is also different than what we use. Given results provided by AIC and likelihood ratio tests, the VECM is better specified with two lags than the suggested four or six. The value of the likelihood ratio test of four vs. two lags is 9.04 in rm2 specification and six vs. two lags 26.47 in rcp specification. However, when more lags are added, we still fail to identify a plausible money demand function in this case.

[17] A coefficient of 13.840 in the Bulgarian hyperinflation episode implies an average inflation rate elasticity of 2.537. This high inflation semi-elasticity is also found in Phylakis and Taylor (1993), who study similar Latin American experiences; for Argentina they obtain an estimate of 12.69, and for Chile 16.87.

Given the model in (1), Cagan (1956, p. 77-80) shows that the maximum revenue the government may elicit from the inflation tax is equal to $-\frac{1}{g}$. Given the average inflation rate in Table 1, the last row in Table 6 gives the likelihood ratio test of the null hypothesis that the governments in hyperinflationary East European countries expanded money creation with the intent of maximizing revenue. Clearly, this hypothesis cannot be rejected in the case of Bulgaria; in the case of Ukraine, the estimates do not warrant the same conclusion.

Finally, we test the Cagan model under rational expectations, where applicable. The results are reported in Table 8. We obtain the prediction error, as explained earlier, using two estimates of the elasticity. One is the cointegration estimate of g , $\hat{g}$, and the other is computed assuming authorities maximized revenue from printing (issuing) money, $\hat{g} = -\frac{1}{p}$, where π is the average inflation rate in the specific sample period. We use two and four lags in estimating the regression and test jointly for zero coefficients. In all of the cases the data reject the hypothesis that expectations were rational in the turbulent episodes of high and variable inflation. This finding is consistent with those reported in earlier studies for the Latin American experience [Phylaktis and Taylor (1991)], and the classic hyperinflations Engsted (1994) and Taylor (1991). Choudry (1998) also rejects the null hypothesis in this context in Russia, as do Petrovic and Vujosevic (1996) for the Yugoslav hyperinflation (1991-1993).

7. Conclusion

This paper finds empirical support for the Cagan model of money demand in the East European experiences of the 1990s. The vindication comes from the presence of cointegration between real money balances and inflation in these countries. However, we identify a stable money demand function in the case of Bulgaria only, with coefficients that are of plausible size and correct sign, and to a lesser extent, Ukraine. We fail to find a stable money demand function in the case of Russia. In addition, our results do not provide evidence supporting the rational expectations hypothesis for these countries and time periods. The conduct of monetary policy in the case of Bulgaria seems to have been motivated by maximizing inflation tax revenue for the government.

These countries experienced major supply and demand shocks in the transition process, accompanied by a variety of regime shifts that might have affected the money creation and inflation processes. This analysis is only an attempt to describe an aspect of their monetary experiences, and in the case of Russia the estimates do not yield to an easy interpretation. In addition, Elliott (1998) cautions against the use of inference on long-run relationships between economic variables and questions the robustness of cointegration tests when unit roots in variables of interest are near but necessarily one. Thus, hypothesis tests could be affected adversely, and he suggests other methods of using instruments for the endogenous regressors in the system or using bootstrap techniques as in Pesaran and Shin (1995).

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Appendix 1

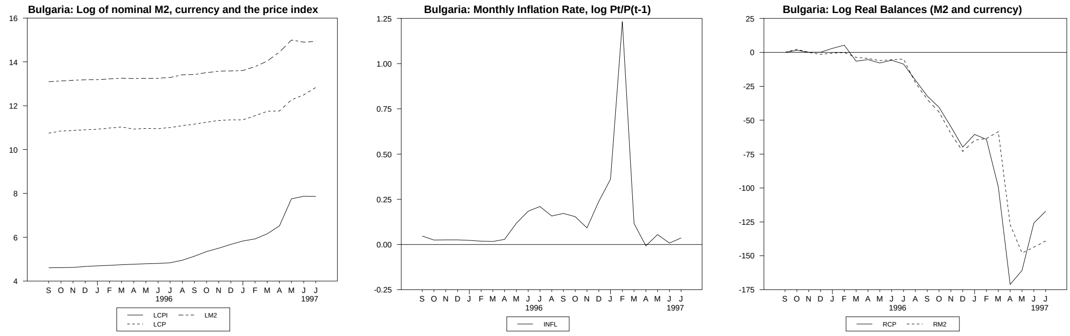

Figure 1: Bulgarian Hyperinflation (1996-1997): Money Creation, Prices, Inflation and Real Balances

Figure 2: Russian Hyperinflation (1992-1994): Money Creation, Prices, Inflation and Real Balances

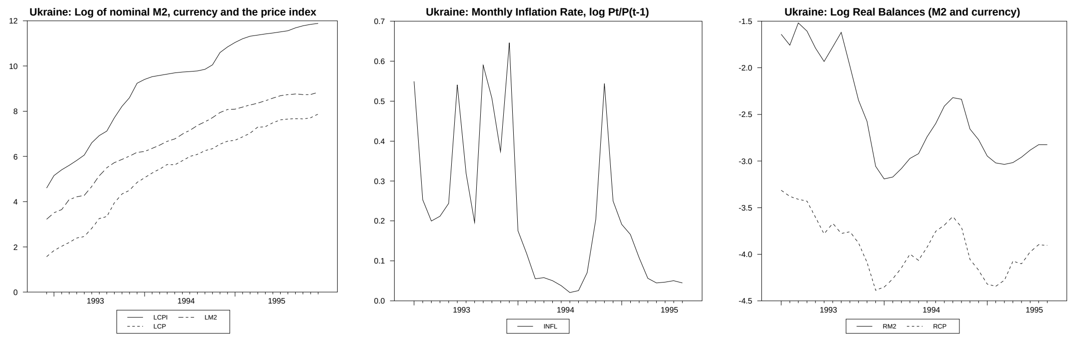

Figure 3: Hyperinflation in Ukraine (1993-1995): Money Creation, Prices, Inflation and Real Balances

Note: RCP is the real currency in circulation

TABLE 1. AVERAGE MONTHLY INFLATION AND MONEY GROWTH RATES (%)

	Period	AVG INFL	AVG M2	Ratio
BULGARIA	1995:9-1997:3	18.33	10.59	1.73
RUSSIA	1992:1-1994:9	16.29	14.25	1.14
UKRAINE	1992:12-1995:3	27.09	19.68	1.38

Note: The average rates are continuously compounded in the following way: $X_T=X_1(1+g/100)^T$, where X_i is nominal money stock and price level in period i and T is the sample size (number of months); g is the growth rate.

TABLE 2. CHARACTERISTICS OF HIGH INFLATION ECONOMIES IN THE 1990S

	YEAR	Real GDP Growth Rate (percentage change)	Inflation Rate (percentage change)	Fiscal Balance (percent of GDP)	Government Expenditure (percentage of GDP	Broad Money (percentage change)
Bulgaria	1990	-9.1	26.3	-4	65.9	17
	1991	-11.7	333.5	-3.8	45.6	110
	1992	-7.3	82	-5.2	43.6	53.6
	1993	-2.4	73	-10.9	48.1	47.6
	1994	1.8	96.3	-5.8	45.7	78.6
	1995	2.1	62	-6.4	43	39.6
	1996	-10.9	123	-13.4	47	124.5
	1997	-7	1082	-6.3	33.5	359.3
	1998	3.5	22.2	0.9	35.8	9.6
Russia	1990	n.a	5.6	n.a.	n.a.	17.6
	1991	-5	93	n.a.	n.a.	126
	1992	-14.5	1526	-18.9	65.8	568
	1993	-8.7	875	-7.3	43.6	409
	1994	-12.7	311.4	-10.4	45.1	200
	1995	-4.1	197.7	-6.0	38.5	125.8
	1996	-3.5	47.8	-8.9	41.9	30.6
	1997	0.8	14.7	-7.6	42.9	29.8
	1998	-4.6	27.6	-8.0	39.7	19.8
Ukraine	1990	-3.4	4.2	n.a.	n.a.	n.a.
	1991	-11.6	91	n.a.	n.a.	n.a.
	1992	-13.7	1210	-25.4	58.4	n.a.
	1993	-14.2	4735	-16.2	54.5	785
	1994	-23	891	-7.8	51.4	568
	1995	-12.2	377	-4.9	33.0	173
	1996	-10.1	80	-3.2	42.8	32
	1997	-3.0	15.9	-5.4	43.0	34
	1998	-1.9	10.6	-2.8	39.0	25

Sources: National Statistical Institute, Bulgarian National Bank, UEPLAC, Derzhkomstat, IMF Country Surveys, Russian Economic Trends various issues, EBRD Transition Report 2000.

TABLE 3. SELECTED HYPERINFLATIONS: KEY FACTS

Country	Average Monthly Inflation Rate	Peak Monthly Inflation Rate (Date)	Number of Months with Inflation >50% (>25%)
Austria	47.1	134	4
		(8/1922)	(10)
Germany	322	32400	11
		(10/1923)	(20)
Poland	81.4	275	9
		(10/1923)	(16)
Russia	57.0	213	10
		(1/1924)	(18)
Bolivia	48.1	182	9
		(2/1985)	(16)
Argentina	66	200	3
		(7/1989)	(16)
Brazil	19.7	73	3
		(1/1990)	(16)
Ukraine	30.6	90.8	6
		(12/1993)	(5)
Georgia	44.1	211.1	9
		(9/1994)	(4)
Bulgaria	24.5	242.8	1
		(2/1997)	(4)

Sources: Own calculations, Bruno (1990), Cagan (1956)

TABLE 4

Unit Root Tests					
Country		Two roots		One root	
		$\tilde{\tau}$	ADF-GLS	$\tilde{\tau}$	ADF-GLS
Bulgaria	RM2	-4.24	-3.47	-1.88	-1.60
(1995:09 - 1997:03)	RC^P	-3.08	-3.28	-1.74	-1.28
	INFL	-5.60	-4.38	-3.16	-1.60
Russia	RM2	-3.27	-3.53	-1.29	-2.41
(1992:02 - 1994:09)	RC^P	-3.48	-4.11	-1.41	-2.52
	INFL	-3.67	-4.63	-1.75	-2.31
Ukraine	RM2	-2.89*	-2.60	-1.13	-2.16
(1993:01 – 1994:11)	RC^P	-2.92*	-2.25	-1.20	-1.97
	INFL	-3.23	-3.90	-2.62	-1.99

Note: * indicates significance at 10%. Given sample size, critical values are as follows: Phillips-Schmidt tau test: -2.85 (10%), -3.18 (5%), -3.90 (1%) Schmidt and Phillips (1992, p. 264). ADF-GLS test has critical values of –2.89 (10%), -3.19 (5%), -3.77 (1%) when there is a linear trend included and –1.62 (10%), -1.95 (5%), -2.58 (1%) when only constant is involved, Elliott, Rothenberg and Stock (1996, p. 825)

TABLE 5

KPSS Tests							
Country		Two roots			One root		
		Lag 0	Lag 2	Lag 4	Lag 0	Lag 2	Lag 4
Bulgaria	RM2	0.362	0.330	0.394	0.257	0.162	0.159
(1996:01 - 1997:09)	RC^P	0.352	0.348	0.383	0.328	0.174	0.157
	INFL	0.038	0.085	0.151	0.151	0.127	0.145
Russia	RM2	0.159	0.099	0.094	0.327	0.130	0.099
(1992:02 - 1994:09)	RC^P	0.136	0.093	0.093	0.326	0.132	0.105
	INFL	0.127	0.109	0.108	0.299	0.136	0.110
Ukraine	RM2	0.237	0.145	0.131	0.407	0.159	0.121
(1993:01 – 1994:11)	RC^P	0.215	0.141	0.146	0.447	0.180	0.138
	INFL	0.150	0.225	0.228	0.140*	0.092	0.084

Note: * indicates significance at 10%. Critical values are in Kwiatkowski, Phillips, Schmidt and Shin (1992). For a linear trend in the model, the critical values are 0.119 (10%), 0.146 (5%), 0.216 (1%); for the model with intercept only, 0.347 (10%), 0.463 (5%), 0.739 (1%).

TABLE 6. COINTEGRATING VECTORS: JOHANSEN VECTOR ERROR CORRECTION MODEL

Country	BULGARIA		RUSSIA		UKRAINE	
	(1)	(2)	(1)	(2)	(1)	(2)
No. of Lags in VECM	4	4	2	2	4	6
Q-statistic (p-value)	3.46 (0.48)	6.871 (0.14)	1.642 (0.80)	2.799 (0.59)	3.159 (0.53)	0.83 (0.93)
$\lambda^{largest}$	0.7679	0.8421	0.4420	0.5066	0.5914	0.8999
Trace	39.83	46.76	20.17^{+}	26.43^{*}	22.04	42.25
Maximal	21.91	27.68	17.50^{*}	21.19^{+}	17.01	39.13
$\hat{\gamma}$	13.840	5.555	-7.290	-0.754	-6.494	0.370
α_1	-0.245	-0.843	0.141	-0.322	-0.008	-0.620
α_2	0.325	0.252	0.059	0.205	0.203	1.989
$\alpha_1 - \alpha_2$	-0.570	-1.095	0.082	-0.527	-0.211	-2.609
LR (H_0: $\gamma = -100/\pi$) (p-value)	1.06 (0.30)	0.01 (0.92)	n.a.	n.a.	n.a.	35.63 (0.00)

Notes: (1) indicates the real M2 equation specification and (2) is the real currency equation. The appropriate lag length for the VECM is determined according to Akaike Information criterion and a Likelihood Ratio test. We test each system for residual autocorrelation using three LM type tests; reported is the Q-statistic for the forth-order residual autocorrelation, which is χ^2 distributed. The marginal significance levels are found in parentheses. $\lambda^{largest}$ indicates the largest calculated eigenvalue. Trace is the *trace* test statistic under the null of no cointegration and Maximal is the *maximum eigenvalue* test statistic. γ is the normalized coefficient in the cointegrating vector. α_1 is the adjustment coefficient for the cointegrating vector in the $\Delta(m-p)_t$ -equation; α_2 is the adjustment coefficient for the cointegrating vector in the Δp_t –equation.. The Likelihood Ratio (LR) test statistic for the null hypothesis: $\gamma=-100/\pi$ is constructed as in Johansen and Juselius (1990), marginal significance level is in parentheses. The test is distributed as chi-square with one degree of freedom under the null hypothesis. n.a. means not applicable. * indicates results not significant when adjusted critical values used (Cheung-Lai correction). + 10% significance when Cheung-Lai correction used. Critical values are those from Osterwald-Lenum (1992).

TABLE 7. ESTIMATED COINTEGRATING RELATIONS: THE STOCK-WATSON DYNAMIC OLS

Estimates \ Country	BULGARIA (1)	BULGARIA (2)	RUSSIA (1)	RUSSIA (2)	UKRAINE (1)	UKRAINE (2)
Number of Lags and Leads	4	4	3	3	4	4
Test for Normality* (p-value)	2.69 (0.26)	1.97 (0.37)	2.21 (0.33)	1.63 (0.44)	1.28 (0.52)	0.79 (0.68)
Point Estimates (Standard Error) $\hat{\gamma}$	-3.924 (0.031)	-4.816 (0.104)	3.977 (1.273)	0.657 (0.219)	2.881 (0.245)	0.494 (0.057)

Notes: *Jarque-Bera Test for normality is distributed as $\chi^2_{(2)}$. (1) is the real M2 equation specification and (2) is the real currency equation. AR(2) was used for the error to estimate the long-run covariance matrix and the Wald test statistic estimates of coefficient restrictions (not reported in the table)

TABLE 8. TESTS OF THE HYPERINFLATION MODEL UNDER RATIONAL EXPECTATIONS

F-test in Eq. (8) \ Country		BULGARIA	RUSSIA	UKRAINE
With Cointegration estimate $\hat{\gamma}$	F_1 (p-value)	30.12 (0.00)	n.a.	17.12 (0.00)
	F_2 (p-value)	21.37 (0.00)	n.a.	5.72 (0.01)
With $\gamma=-100/\pi$	F_1 (p-value)	16.39 (0.00)	54.37 (0.00)	13.59 (0.00)
	F_2 (p-value)	9.71 (0.00)	28.81 (0.00)	6.36 (0.00)

Notes: F_1 tests the significance of two lags and F_2 of four lags in the information set of the error term.

Appendix 2

The Cagan Data Revisited

Figure 4: Hyperinflation in the 1920s

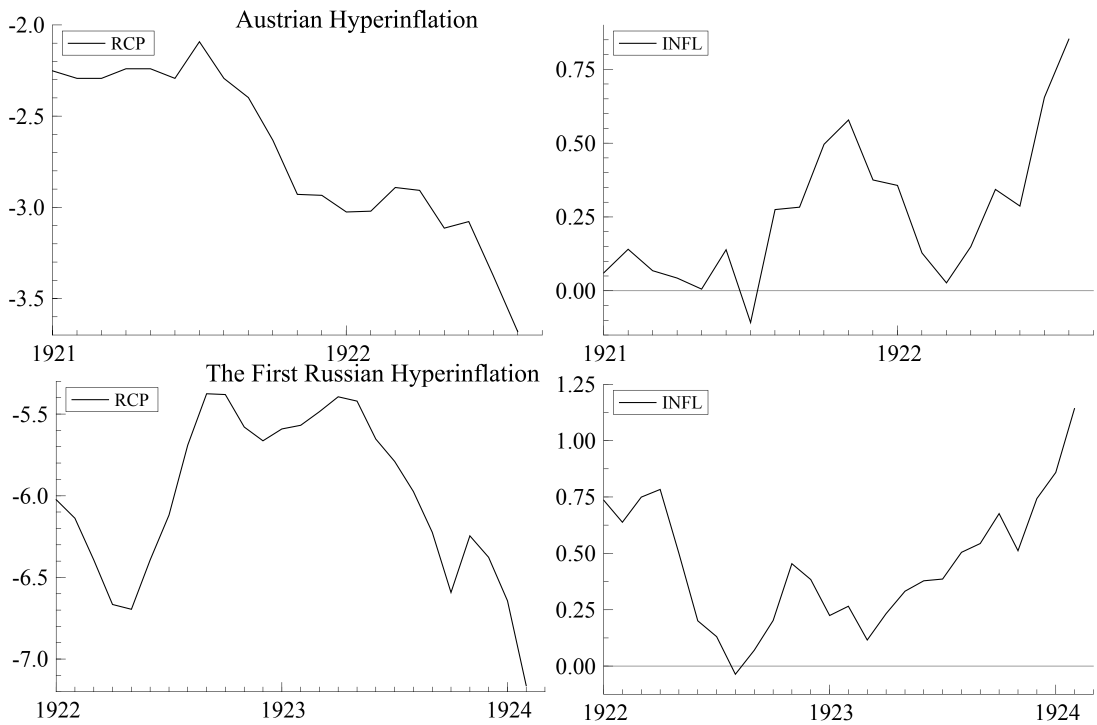

TABLE 9

Unit Root Tests									
Country		Two roots				One root			
		$\tilde{\tau}$	ADF-GLS	KPSS 0	KPSS 2	$\tilde{\tau}$	ADF-GLS	KPSS 0	KPSS 2
Austria	RC^P	-3.56	-3.45	0.279	0.225	-1.93	-1.03	0.168	0.092
(1921:01 - 1922:08)	INFL	-4.74	-4.50	0.147	0.148	-1.92	-1.86	0.116	0.067
Russia	RC^P	-2.90*	-2.24	0.482	0.288	-1.05	-1.35	0.519	0.213
(1922:01 - 1924:02)	INFL	-4.01	-3.59	0.540	0.399	-1.51	-1.29	0.458	0.204

Note: * indicates significance at 10%. See Table 2 and 3.

TABLE 10 CAGAN DATA REVISITED

	RUSSIA (1922-1924)	AUSTRIA (1921-1922)
Johansen estimate	3.369	3.768
DOLS (Standard Error)	-3.665 (0.573)	-3.362 (0.260)
LR (H_0: γ=-100/π) (p-value)	6.72 (0.01)	0.07 (0.80)
F_1 (p-value)	21.36 (0.00)	5.87 (0.01)
F_2 (p-value)	17.85 (0.00)	3.66 (0.09)

Note: See Tables 4 and 5.