Consumer Choice Model For Forecasting Demand And Designing Incentives For Solar Technology

Ruben Lobel
Operations Research Center, MIT, Cambridge, MA 02139, rlobel@mit.edu

Georgia Perakis
Sloan School of Management, MIT, Cambridge, MA 02139, georgiap@mit.edu

In this paper, we develop a model for the adoption of solar photovoltaic technology by residential consumers. In particular, we assume consumers purchase these solar panels according to a discrete choice model. The technology adoption process is reinforced by network externalities such as imitating customer behavior and cost improvements through learning-by-doing. Using this model, we develop a framework for policy makers to find optimal subsidies in order to achieve a desired adoption target with minimum cost for the system. We discuss the structure of the optimal subsidy policy and how the overall system cost changes with the adoption target. Furthermore, we validate the model through an empirical study of the German solar market, where we estimate the model parameters, generate adoption forecasts and demonstrate how to solve the policy design problem. We use this framework to show that the current policies in Germany are not efficient. In particular, our study suggests that their subsidies should be higher in the near future and the gradual phase-out of the subsidies should occur faster.

History: First draft - released December 2010, Updated January 2011 *

1. Introduction

Solar Photovoltaic (PV) technology has greatly improved over the last two decades. With today's technology, cheap and efficient rooftop solar panels are available to residential consumers with the help of public subsidy programs. As the market for these solar panels develops worldwide, many questions remain unanswered about how this technology is adopted by customers and how to design incentives for its adoption. In this paper we will develop a framework to model and control the adoption process of solar technology. Furthermore, we will test our framework by developing an empirical study based on the history of the German solar market.

Forecasting demand can be particularly challenging for new technologies that are not fully mature yet. More specifically, the cost of a new solar panel installation decreases as more solar panels are sold, mainly due to improvements in the installation network and manufacturing of the PV

* Preliminary version - Please do not circulate without consent of the authors.

modules. These cost improvements are mainly induced by demand-driven economies of scale and competition effects that stimulate cost cuts and research. Additionally, consumer awareness about the technology will improve with the number of installed systems, which creates a second positive feedback in the adoption process. In other words, the cost improvements and the information spread through the consumer market will reinforce the demand process over time. These are commonly referred to as network externalities. These effects are particularly dominant at the early stages of technological development. Also because of these network effects, governments interested in accelerating this adoption process may often want to subsidize early adopters.

In particular, one of the reasons why governments subsidize the installation of solar panels is to promote the development of the solar technology so that it will become cost competitive with traditional sources of generation, therefore economically self-sustainable. The point where electricity generated from a solar installation reaches the electricity grid price is usually called grid-parity, either at the wholesale market price or at the higher end-consumer retail price. Estimates for reaching this grid-parity point can be as early as 2013-2014 at the retail level or 2023-2024 at the wholesale level (see Bhandari and Stadler (2009)). Therefore, the main question that needs to be addressed by policy makers today is not whether solar technology will eventually takeoff, but rather when it will takeoff and whether something should be done to accelerate this process.

Most of the incentives devised today for solar technology come from governmental subsidies in the form of installation rebates, feed-in-tariffs or subsidized loans. Recent reports by IEA (2004), EEG (2007), BMU (2008) and EPIA (2009) provide great insights about the current status solar PV technology as well as the history of the subsidy programs behind it.

In this paper, we study the problem from the perspective of the policy maker, where the goal is to find the optimal subsidy value to offer customers willing to adopt these rooftop solar panels. More specifically, we assume the government has a particular adoption target, within a given time frame, and is able to offer a rebate to the customers adopting the technology. These targets are very common among policy makers, as in the "1,000 rooftops" and "100,000 rooftops" programs in Germany. Jager-Waldau (2007) summarizes some target levels for renewable energy production and photovoltaic adoption through Europe, as decided by the European Commission. One particular example of such policies comes from EUC (1997), a white paper from the European Commission, which proposed a target adoption level of 500,000 PV solar installations across the European Union by 2010. This number was surpassed by Germany alone, as the proposed target was very conservative.

The main reasons for these adoption targets include stimulating technological progress, as explained before, but also diversifying the generation portfolio with renewable and carbon-free energy and reducing the peak load of the electricity grid. Much has been discussed about the reasons for subsidizing altogether, see for instance Sanden (2005). It is also debatable whether demand-side subsidies are the most effective way to stimulate the technological progress, as opposed to using only direct research investments (see Duke and Kammen 1999, Nemet and Baker 2008). Also, governments are typically in favor of stimulating domestic manufacturing over importing solar panels when it comes to defining the subsidy policy. Jager-Waldau (2007) displays a snapshot of the PV solar manufacturing industry worldwide. Also, Taylor and Plambeck (2007) and Islegen and Plambeck (2010) have recently explored how supply-chain contracts influence manufacturing capacity investments, including applications to the solar industry. The growth of cheap foreign manufacturing of solar modules is usually mentioned as one of the causes for the collapse of the Spanish subsidy program. How manufacturers react to policy makers is certainly an important effect and it should be considered when a government decides the adoption targets.

In our modeling framework we do not focus on the origin of the adoption targets, but rather assume they are given. We then discuss the effects of changing these targets on the subsidizing cost for the government. We also develop a relationship between optimizing the subsidies for reaching an adoption target versus optimizing subsidies to maximize social welfare, which is a more common objective in policy design. Given these adoption targets, understanding the purchasing behavior of potential solar panel customers and how this will affect the overall spread of the technology is crucial for minimizing the subsidy costs paid by the tax-payers. In particular, we use a discrete choice approach to model customer behavior as a function of the price incentives offered by the government (installation rebates).

In this paper, we provide a modeling framework to tackle the policy design problem and also develop an empirical study of the German solar market to validate our assumptions and demonstrate how to apply this framework in a realistic practical setting. More specifically, we calibrate our model using data from the solar market in Germany, which has a history of strong subsidy policies. We further demonstrate how to forecast the future adoption levels in Germany and how they can use this model to find optimal subsidy levels as a function of their future adoption targets, as well as quantify the trade-off between adoption levels and subsidy costs. Finally, we investigate the efficiency of the current subsidy policy in Germany.

The outline of this paper can be described as: In the remainder of Section 1, we summarize the main contributions of this paper and discuss some of the relevant literature for this research area. In Section 2, we define the demand model and the policy optimization problem. In Section 3, we conduct the empirical study on the German market. In Section 4, we analytically explore the structure of the policy design problem to develop insights about the optimal rebate policy. In Section 5, we summarize the results of this paper and outline some possible directions of research that extend this work.

1.1. Contributions

In summary, the goal of this paper is to propose a framework for designing subsidy policies for the adoption of new technologies, while considering the impact of consumer behavior and cost evolution dynamics in the overall adoption process. The first contribution of this paper is the development of a new practical policy optimization tool that policy-makers can apply to create comprehensive policy recommendations. Furthermore, we test the applicability of this model with an empirical study of the German solar market, where we demonstrate how to estimate the model parameters using real data, solve the policy design problem and discuss insights about the state of the current policy in Germany.

In the empirical study of the German solar market, we estimate our demand model using market data from 1991-2007. We then use the fitted model to produce adoption level forecasts until the year 2030 and solve hypothetical policy design questions in order to obtain insights about the structure of the system cost, the optimal solution and the efficiency of the current policy. We show that the system cost of subsidizing is a convex function of the adoption target level. In other words, the cost of subsidizing becomes increasingly more expensive as the adoption target increases. This is partially due to the fact that the network externality benefits of early subsidies become saturated. We observe this effect empirically in the German market study. We also prove it for the general setting during our analytical study of the optimal solution.

Finally, we demonstrate in our empirical analysis that the current subsidy policies in Germany are not economically efficient. We show this by solving the policy optimization model for any possible adoption target in the baseline adoption forecast of the current system. In all these cases, there exists a way to reach the given target and still achieve a lower cost for the system, in particular by raising earlier subsidies and lowering future subsidies. This means that if the German government is trying to achieve a certain target adoption level at some point in the future, the current subsidies are suboptimal. We further prove that because of the decreasing nature of the optimal rebate policy, if the German government is actually trying to optimize a social welfare objective, the current policies are still suboptimal. We would like to bring special attention to the novelty of this adoption target optimization approach for studying welfare efficiency without any knowledge about the actual welfare function or solving the optimal welfare problem. In summary, these results definitely raise a warning about the efficiency of the current policies in Germany. The current Feed-in-Tariff program in place today in Germany is already in a phasing-out stage (see Figure 1 or the report EEG (2007) for further details). Our assessment indicates that these subsidies should be higher now and lower in the future, while the magnitude of this change depends on the actual objective of the German government. In other words, they should increase current subsidies and also the rate of decay for future subsidies (faster phase-out).

1.2. Literature Review

Historically, the economics literature has focused on models for policy design with a social welfare objective, not adoption target levels (see Ireland and Stoneman (1986), Joskow and Rose (1989), Stoneman and Diederen (1994), Acemoglu et al. (2010) for further references). On the other hand, the marketing literature has focused primarily on diffusion models for new technologies without a policy design focus. For further reference in diffusion models, see the seminal work of Rogers (1962), Bass (1969) or more recent review papers by Mahajan, Muller, and Bass (1990), Geroski (2000), Rao and Kishore (2010). Finally, the operations management literature has recently devoted attention to models that deal with customer choice behavior (for example Su and Zhang (2008), Cachon and Swinney (2009), Cachon and Feldman (2010), Allon et al. (2010b,a), Musalem et al. (2010)). Some of these are particularly focused on empirical work. In our paper, we combine ideas from all these fields to develop a policy optimization framework for solar technology using a choice model of demand that incorporates network externalities. To the best of our knowledge, this is the first paper to approach the policy-making problem from the target adoption level perspective and apply it to real market data in an empirical setting.

The models for innovation with network externalities are quite familiar to economists. Farrell and Saloner (1985, 1986), Katz and Shapiro (1986, 1992) began exploring the issue of technology standardization and compatibility, which provide positive network externalities at first, but may later inhibit innovation. This effect can be particularly important in the adoption of computer software and telecommunications, but not so much in photovoltaic technology. Chou and Shy (1990) argue that returns to scale on the production level can also produce the same behavior as observed in cases of consumers with preferences that are affected by direct network externalities. The cost reductions that follow the adoption process is commonly known as the learning-by-doing effect. This effect has been widely studied since the seminal paper by Arrow (1962), and also more specifically for the case of photovoltaics, see Harmon (2000), IEA (2000), McDonald and Schrattenholze (2001), Nemet (2006), Sderholm and Sundqvist (2007), Bhandari and Stadler (2009), Yu et al. (2009).

The way that information spreads through the network of customers is another important effect that has been given a lot of attention. How consumers become aware of a new product or gather information about it's quality may determine the successful take-off of a new technology. Ellison and Fudenberg (1995) propose a model of word-of-mouth information spread across consumers, where the structure of the social network may lead to an inefficient herding behavior. Vives (1997) develops a model for social learning of public information where he shows that the rate of information gathering is slower than socially optimal, with examples both in a learning-by-doing case and consumers learning about a new product. In particular, Vives (1997) develops a theoretical model for social learning. In this model, the precision of public knowledge increases at a rate $t^{1/3}$, where t is the number of time periods. He admits that this particular functional form is a direct result of his modeling choices, but the general idea that information gathering is concave should remain valid regardless of the model. Ulu and Smith (2009) have recently developed a model for how information spread affects the adoption behavior of consumers, where they concluded that better information sources increases the consumers' value function for adopting the technology but perhaps induces them to wait longer for information gathering. Aral et al. (2009) argue that traditional methods for estimating social contagion may be overestimating this network effect, while homophily between consumers can explain a large portion of the technology spread.

The theoretical models mentioned above can provide intuition for the information spread effect, but are generally not applicable in practice. When studying this effect in a practical empirical setting, it is common to assume a functional form for how consumer utility is affected by some proxy measure of the information spread. For example, Shurmer and Swann (1995) advocate for the use of either a linear or log relation between the network size and consumer utility in a simulation based study of the spreadsheet software market, whereas they note that a basic model without this effect makes very bad market forecasts. Berndt et al. (2003) also considers both a linear and a log effect of depreciated adoption levels when estimating the diffusion of a pharmaceutical product. Doganoglu and Grzybowski (2007) used a linear function to model the network externality effect on consumers' utility for an empirical study of the German mobile telephony market. Swann (2002) also studies the functional form of network externalities in consumers' utility of adoption, proposing conditions for linear or S-shaped functions, focusing on cases where there is a concrete benefit of network size after the purchase, like in telecommunication networks. Using a more detailed model of the network, Tucker (2008) uses individual measures of the agents' position on the social network to analyze their impact on the overall adoption of video-conference technology within a bank. Her empirical study demonstrates how agents that more "central" and/or "boundary-spanners" (between disconnected groups) are more influential in the technological adoption process by creating a larger network externality. Goolsbee and Klenow (2002) develop an empirical study of the diffusion of home computers in the US, with emphasis on local network externality effects using geographic information of adoption. In particular, they show that people were more likely to buy a computer if more people in their local area had already adopted the technology. Jager (2006) reaches a similar conclusion through a behavioral study among adopters of solar PV technology, using a survey of residents of a city in the Netherlands. In our paper, we will use the log effect, as suggested in Shurmer and Swann (1995), Berndt et al. (2003), because it satisfies the concave behavior that we want to model. We additionally tried other similar functional forms during our empirical study, but the log effect presented the best fit.

Within the broader marketing literature, Hauser et al. (2006) enumerates the multiple directions of future research that should to be explored by the marketing community. One such direction is to improve our understanding of consumer response to innovation. Our paper tries to address this issue by using a diffusion model based on the logit demand model. Our particular diffusion model can be placed within the broader class of proportional hazard rate models. Developed by Cox (1972) for modeling the life time of an agent in a larger population, the hazard rate model has been widely used in biostatistics and its application in marketing has been well documented in Helsen and Schmittlein (1993). In our case, the agent's life-time duration is the moment he/she makes the purchase decision and adopts the technology. We diverge from the original Cox model in the particular functional form of the adoption probability, where we use the logit demand derivation for the probability of purchase. All these models will result in the familiar S-shaped diffusion pattern. The particular functional form chosen for this paper is derived from the characteristics of consumer purchasing behavior. Additionally, it was chosen because it provides good estimation results with the German market data and analytical tractability.

Lately, the operations management community has been developing ground-breaking empirical work, in particular on the area of customer choice models. For example, Allon et al. (2010a) estimate how customers value waiting times in fast-food restaurants. Musalem et al. (2010) estimate how out-of-stocks in the retail market affect customer purchasing decisions, further using this model to suggest promotion policies to the retailers. The use of structural estimation techniques has enabled the development of comprehensive decision-making tools and provided useful insights, as we have done in this paper for the solar subsidy design problem.

Recent work by Benthem et al. (2008) estimated a demand model with a similar learning-by-doing effect on a study of the California Solar Initiative and Wand and Leuthold (2010) used the same model on a study of the German market. These two papers assume there is a known environmental externality cost that is avoided by PV installations and assume the government tries to maximize the net social welfare of the system when deciding the subsidy policies. On the other hand, these environmental costs are mostly deduced from the global impact of climate change. Until there is a world-wide efficient emissions trading market (cap-and-trade or carbon taxing), no particular government has an incentive to pay these costs of avoiding climate change. Furthermore, according to their model, the net social welfare cost of solar incentives is zero as all the money paid by the government goes back to consumers through their investment subsidies. In our research, we do not attempt to quantify the social benefits of solar technology, but instead directly use adoption targets as our policy optimization objective, which we believe is a more realistic setting. To the best of our knowledge, our paper is the first work to apply a consumer choice modeling approach to understand the adoption of solar technology with fixed adoption targets. We are also able to use this framework of adoption targets to evaluate the welfare efficiency of the current German policy without making assumptions on the environmental benefits of solar installations.

2. Model

We consider a modeling framework that can be divided in two parts: the demand model and the policy-maker's problem. In Section 2.1 we develop a solar panel demand model based on the customers' purchasing behavior and the network externalities that increase the incentives for future purchases the more consumers adopt the technology. In Section 2.2, we propose an optimization model for solving the subsidy policy design question. We have included a notation summary in Appendix A, which may be a useful reference while reading this modeling section of the paper.

2.1. Demand Model

The first step to understand the adoption process of a certain technology is to understand the customer behavior. At each time step (for example each year), we consider every household as a potential customer who is given a choice between purchasing a solar panel or not. Let M_t be the market size (number of households) and x_t be the number of customers at a given time t that have already adopted the technology, in this case rooftop photovoltaic solar panels. Define r_t as the rebate level offered by the government, which is the policy maker's decision variable in order to control the adoption rate of the technology. Let $q_t(x_t, r_t)$ be the demand for solar panels at time t. The technology adoption in the population is given by the discrete time diffusion process: $x_{t+1} = x_t + q_t(x_t, r_t)$.

The demand model we propose in this paper is centered around the average consumer's utility profile, namely $V_t(x_t, r_t)$. In order to maintain tractability and also due to the lack of additional data for the empirical study, we will make the following assumptions:

Assumption 1.

a) At each time period t, a customer will either buy an average sized solar installation (denoted AvgSize) or not buy anything;

b) After purchase, this customer is out of the market: no depreciation, resell or additional purchase options;

c) The solar yield (electricity generation) and installation costs are homogenous across the entire country;

d) Demand q_t for solar panels at time t follows a logit demand model, which is a function of the utility that consumers have for purchasing a solar panel at time t.

In particular, Assumption 1.d defines the demand q_t as a logit demand function, which is equal to the number of remaining potential customers times the probability that each of these customers will make the purchase decision at time t. This customer purchase probability is what we also call adoption or diffusion rate. For a review on diffusion models, see Mahajan, Muller, and Bass (1990).

The motivation behind the logit demand model comes from customers being rational utility maximizing agents. With this in mind, define $V_t(x_t, r_t)$ as the nominal utility that the average consumer has for purchasing a solar panel at time t. It is a function of the current state of the system x_t and the rebate levels r_t. Additionally, define $\epsilon_{t,i}$ as a random utility factor for a given customer i at t that captures the heterogeneity of consumers' utility. It represents the utility impact of all characteristics that different consumers have, for example geographic location, household sizes, discount rate differences, or environmental conscience. Let $U_{t,i}$ be customer i's perceived utility for purchasing a solar panel at time t. This is given by:

$$U_{t,i} = V_t(x_t, r_t) + \epsilon_{t,i} \tag{1}$$

The logit demand model is one of the most common demand models used in the discrete choice literature (see for example Ben-Akiva and Lerman (1993), Train (2003) for further references on discrete choice models). This is mainly due to its analytical tractability. This is also the reason we use this model in our framework. The logit model assumes consumers are utility maximizing agents and the heterogenous component $\epsilon_{t,i}$ comes from an extreme value distribution. Therefore, at each point in time, customers are given a random draw $\epsilon_{t,i}$ and will decide to purchase the solar panel if the utility of purchase $U_{t,i}$ is greater than zero (where zero is the utility of no purchase). Therefore, the probability of adoption for any given consumer can be obtained by integrating the distribution of $\epsilon_{t,i}$ over the region $U_{t,i} > 0$. This gives us the well-known logit demand model:

$$q_t(x_t, r_t) = (M_t - x_t)\frac{e^{V_t(x_t,r_t)}}{1 + e^{V_t(x_t,r_t)}} \tag{2}$$

The first term $(M_t - x_t)$ represents the number of left-over consumers who have not purchased a solar panel yet at time t and the remaining term is the probability of adoption for any of these customers. Additionally, we need to assume the following:

Assumption 2.

a) Consumers do not make a strategic timing decision when purchasing the panel. If their private signal $\epsilon_{t,i}$ is strong enough so that $U_{t,i} > 0$, then they will make the purchase at that time t.

b) The heterogeneity random components $\epsilon_{t,i}$ are uncorrelated across time periods.

One may argue that Assumption 2.a is a strong assumption, as consumers might be tempted to wait for panels to become cheaper. Note that Kapur (1995), Goldenberg et al. (2010) show that the presence of network externalities might encourage agents to wait and delay overall adoption process. On the other hand, Choi and Thum (1998) suggest that in the presence of network externalities consumers do not wait enough to adopt a new technology, settling for what is available instead of waiting for an improvement. Nevertheless, as we observed in the German market data of Section 3 (see Figure 2), the Feed-in-Tariffs offered by the government are decreasing faster than the installation costs after 2005. This gives the incentive for consumers to not be strategic about their timing decisions. Before 2005, this was not the case, which suggests that strategic timing behavior of consumers might have influenced the demand for panels and a more complex model of consumer behavior might be necessary. On the other hand, using more complex models may lead to estimation and tractability problems. Assumption 2.a can be interpreted as consumers being short-sighted agents that can only maximize their utility within a given time period.

Assumption 2.b considers that each customer is given a draw of it's private utility shock $\epsilon_{t,i}$ for that year, which is independent from the private shocks in previous years ($\epsilon_{\tau,i}$ for $\tau < t$). This can be a stringent assumption, as people who have not adopted solar panels because they have lower income or live in a region that has low solar irradiation will tend to not adopt in future periods as well. These problems can be reduced by introducing demographic data into the demand model, which would make $\epsilon_{t,i}$ capture less of these fixed components of heterogeneity. For example, one way to use demographic data into the demand model would be to introduce random coefficients as in the BLP model (introduced in Berry (1994) and Berry, Levinsohn, and Pakes (1995)). This approach has been quite popular in the industrial organization literature recently. The main idea behind the random coefficients model is that consumer's sensitivity to the monetary value of the solar panel, the NPV of the project, should be heterogenous due to those demographic differences between consumers. By using the distribution of each of these demographic components within the entire population, we can use computational methods to find better estimates of the probability of adoption. Although this would possibly lead to a more precise demand model, we choose to avoid the BLP approach for the following reasons: we do not have sufficient data points to introduce more parameters to estimate, given that we are working with yearly data over a span of 17 years; we wish to maintain the analytical closed-form tractability of the demand function, as it will allow us to explore insights about the structure of the optimal solution of the policy design problem (see Section 4).

To fully specify the demand function described in (2), we need to define the nominal utility of the average consumer, denoted by $V_t(x_t, r_t)$. Consumers' perceived utility for adopting a solar panel should be a function of many parameters, including the monetary value of the investment and the awareness that customers have of the given technology. The first component of $V_t(x_t, r_t)$ is the monetary value of an average solar installation purchased at time t, namely NPV_t. This component is equal to the government installation rebate to consumers r_t plus the future discounted cash flows d_t minus the installation cost k_t, all this is multiplied by the size of an average household installation, denoted by $AvgSize$. That is:

$$NPV_t(x_t, r_t) = (-k_t(x_t) + r_t + d_t) AvgSize \tag{3}$$

In particular, we model the installation cost $k(x_t)$ as a decreasing function of the number of solar panels sold x_t. This is consistent with the learning-by-doing effect, that is, the more people adopt a given technology, the cheaper this technology becomes in the future. In other words, the installation cost can be expressed as a decreasing function of the installed capacity. For further references of learning-by-doing in photovoltaics, see Harmon (2000), IEA (2000), McDonald and Schrattenholze (2001), Nemet (2006), Sderholm and Sundqvist (2007), Bhandari and Stadler (2009), Yu et al. (2009).

In the model we introduce in this paper, we represent the solar installation costs with a single term $k_t(x_t)$, expressed in €/Wp of installed capacity (nominal solar installation sizes are measured in Watt-peak, i.e. Wp, which represents the electricity peak generation capacity in Watts under standard laboratory conditions). In practice, there are many different parts in an installation of a solar panel. These include the solar module, additional electronic components (also known as Balance-Of-System) and labor costs. Ideally, we would want to model the evolution of each cost separately, since the module costs evolve according to the global demand for solar panels, while the other costs decrease with the number of local installations. Nevertheless, given that we only have information on the total installation costs for our empirical study, we simplify the cost evolution dynamics by defining a single cost function for the solar installation. This cost function decreases with the number of installations in the country. In particular, the log-log learning curve is the standard model in the learning-by-doing literature. Let a_I and b_I be the installation cost parameters and ν_t represent a random technological improvement factor for time t. Then the cost dynamics can be described as:

$$\log(k_t(x_t)) = a_I + b_I \log(x_t) + \nu_t \tag{4}$$

Finally, the discounted cash flow d_t denotes the present value of the cash payments the customer will receive after purchasing the panel at time t. Note that in countries like Germany, where a Feed-in-Tariff system is implemented, the customer will lock the price of the tariff on the year he purchases the panel and will keep selling electricity at that price for the duration of the contract. For example, in Germany this contract lasts 20 years (see the report EEG (2007) for further reference). Most estimates for the lifetime of solar panels suggest that they would last 30 years or more, but given that this value is discounted to the present and the Feed-in-Tariff expires in 20 years, possibly bringing the selling price to retail levels, the residual cash flow after 20 years will be very small compared to earlier ones. For simplicity, we choose to consider the discounted cash flow of the panel only for the duration of the Feed-in-Tariff contract. Let FIT_t (in €/kWp) be the revenue earned at each year for a panel bought at time t. This is the value of the Feed-in-Tariff contract times the average annual electricity output of a 1 kWp nominal capacity solar panel. Besides the electricity selling revenue, the consumer needs to pay yearly operation and maintenance (OM_t) costs (this is about 2% of k_t every year). We further assume a discount rate of δ_c (which is about 3 to 5%). Then the discounted cash flow is given by:

$$d_t = \sum_{\tau=1}^{T_{mod}} \frac{1}{(1+\delta_c)^\tau} [FIT_t - OM_t] \tag{5}$$

With the discounted cash flow described in (5), the installation costs given in (4) and the government installation rebate level r_t, we obtain the net present value of the installation NPV_t (see definition in (3)). For the remainder of this paper, we consider d_t as a given constant, as defined in (5). In other words, we take the Feed-in-Tariff subsidies as data and the government can further subsidize only by introducing upfront rebates r_t. Negative rebates can also be used in our model. This would imply a tax increase on the sale of the panels. Either way, for a fixed discount rate δ_c, both forms of subsidies are equivalent from an average consumer's perspective. Behaviorally, different forms of subsidies evoke diverse responses among customers and the net present value might not be the best way to capture how consumers perceive this investment. Another possible measure of investment quality would be the payback period (time until investment cost is recovered). For this paper, we will not focus on the discussions about the cognitive limitations of consumers or possible trade-offs between different forms of subsidy. We use the more common economic definition of utility that consumers are directly affected by the net present value of their investment.

As mentioned before, the second component that affects the consumer's perceived utility toward the solar panel purchase is the awareness level of the customer about the technology. In particular, there are two network externalities that we want to emphasize in our model: learning-by-doing and information spread. Because of these network effects, it might be cheaper for the government to subsidize the early adopters in order to promote a faster overall adoption of the technology. The first network externality, learning-by-doing on the installation costs, is modeled in (4).

The second externality is what we call information spread effect, or imitating customer behavior, and it can be usually observed for most new technology adoption processes. It has been well documented in the marketing literature (see for example Mahajan, Muller, and Bass (1990)) and in the behavioral economics literature (see for example Jager (2006)). We provide a more in depth discussion of the literature on information spread in Section 1.2. In summary, this effect happens because consumers become increasingly more aware about a new technology as more people buy the product. In our case, the more rooftop panels are adopted in a neighborhood, other consumers in the same neighborhood will be more likely to adopt the technology as well (see Goolsbee and Klenow (2002), Jager (2006)). On the other hand, the marginal impact of this information spread on the remaining customers should naturally decrease with the number of adoptions (see Vives (1997)). Therefore, the effect of this externality on consumer purchases should be a concave function of the number of customers that have already adopted this technology.

As mentioned in Section 1.2, theoretical agent-based models for information spread through a network provide useful insights about the overall effect, but are generally not practical for empirical applications. In order to conduct empirical work on this subject, it is often common to assume a particular functional form for how such network externalities affects consumer utility and aggregate purchase behavior, as in Shurmer and Swann (1995), Swann (2002), Berndt et al. (2003), Doganoglu and Grzybowski (2007). In this part of the model, the effect we want to capture is the development of consumer awareness and how it affects consumers' perceived utility of purchase. In particular, we model this effect as a penalty function on the proportion of adopted customers x_t/M_t, which lies between 0 and 1. We propose the following limiting conditions for this penalty function: If nobody has adopted the new technology, consumers are generally unaware of the product and their perceived utility of purchase should go to $-\infty$; If everyone has adopted the technology, $x_t/M_t = 1$, then this penalty should go to zero. Together with the concavity condition mentioned before, we propose the use of a logarithmic relation between average consumer's perceived utility and the adopted share of the population: $V_t \sim \log(x_t/M_t)$. This functional form is consistent with previous empirical work on technology adoption, in particular Shurmer and Swann (1995), Berndt et al. (2003), where they also test a logarithmic relation between network externalities and consumer utility of purchase.

Assumption 3. *The average customer's perceived utility for purchasing a solar panel is proportional to the log of the ratio of adopted customers in the population, due to information spread and consumer awareness of the technology:* $V_t \sim \log(x_t/M_t)$

This functional relation in Assumption 3 is by no means the only choice for modeling the information spread effect while satisfying the concavity and limiting conditions. It was chosen mainly due to the good fit demonstrated in our empirical study of the German solar market, compared to other functional forms we tested (for example $V_t \sim 1 - (x_t/M_t)^{-1}$). Its simplicity and tractability are additional advantages of this modeling choice, which are important both for estimation purposes in Section 3.1 and for the analytical results of Section 4.

Gathering all the utility components described so far, define the average customer's perceived utility for purchasing a solar panel at time t given by:

$$V_t(x_t, r_t) = a_D NPV_t(x_t, r_t) + b_D \log(x_t/M_t) + c_D + \xi_t \tag{6}$$

The first part $a_D NPV_t(x_t, r_t)$ denotes the monetary component of the utility, $b_D \log(x_t/M_t)$ denotes the impact of the information spread in the consumer's utility, c_D denotes the baseline utility for making a solar panel purchase, and finally ξ_t is a random demand shock for year t. Note that a_D, b_D and c_D are demand parameters that need to be estimated, while ξ_t is a random utility component. In particular, ξ_t represents all unobserved demand shocks for a given year that affect all consumers and cannot be captured in our data. This could for example represent a demand shock due to a strong advertising campaign in that year. The definition of consumer i's utility function is then given by adding the nominal average consumer utility with the heterogeneity component:

$$U_{t,i} = a_D NPV_t(x_t, r_t) + b_D \log(x_t/M_t) + c_D + \xi_t + \epsilon_{t,i} \tag{7}$$

As defined before in (2), we can now explicitly write the demand model as:

$$q_t(x_t, r_t) = (M_t - x_t) \frac{e^{a_D NPV_t(x_t,r_t) + b_D \log(x_t/M_t) + c_D + \xi_t}}{1 + e^{a_D NPV_t(x_t,r_t) + b_D \log(x_t/M_t) + c_D + \xi_t}} \tag{8}$$

In order to use the demand model defined in (8) and obtain statistically significant estimation results, we need to make some assumptions.

Assumption 4.

a) ξ_t is not correlated with $NPV_t(x_t, r_t)$ or $\log(x_t/M_t)$;

b) ξ_t is not autocorrelated with ξ_τ, for all $\tau < t$;

The correlation described in Assumption 4.a can be a problem for the estimation procedure, but is usually treated with the use of instrumental variables. Autocorrelation, as described in Assumption 4.b, is a very common problem when estimating time series data. This problem can usually be solved by fitting an auto-regressive model for these demand shocks together with the demand model (for example, $\xi_t = \alpha \xi_{t-1} + \eta_t$). In order to maintain simplicity of the model and minimize the number of parameters to be estimated, we have assumed correlations are zero. Furthermore, we have tested this assumption in the empirical study of the German market data and the estimation output demonstrated no significant correlation.

In summary, the full demand model can be described by the following set of equations, for all $t = 1, ..., T-1$:

$$\begin{array}{ll} \text{Diffusion Process:} & x_{t+1} = x_t + q_t(x_t, r_t) \\ \text{Logit Demand:} & q(x_t, r_t) = (M_t - x_t) \dfrac{e^{a_D NPV_t(x_t,r_t) + b_D \log(x_t/M_t) + c_D + \xi_t}}{1 + e^{a_D NPV_t(x_t,r_t) + b_D \log(x_t/M_t) + c_D + \xi_t}} \\ \text{Net Present Value:} & NPV_t(x_t, r_t) = (-k_t(x_t) + r_t + d_t) AvgSize \\ \text{Learning-by-Doing:} & k(x_t) = e^{a_I + b_I \log(x_t) + \nu_t} \end{array} \tag{9}$$

2.2. Policy-Maker's Problem

The discrete choice model framework described in the previous section can be used to solve a variety of management and policy problems. In particular, policy makers are traditionally faced with the problem of setting subsidy levels to stimulate the adoption of a technology up to a target level within a certain time frame. Jager-Waldau (2007) provides some examples of renewable energy/photovoltaic target levels for the European Union, although without clear implementation guidelines. As an example of these target policies within a country, there was Germany's pioneer "1000 Solar Rooftops" program in the early 90's. The next program, known as "100,000 Solar Rooftops", started in 1999 with subsidized loans and expected to install around 300MWp of solar panels within 6 years. The program ended before schedule in 2003 when the target was reached, suggesting the subsidy might have been higher than necessary. This seems to suggest that without further understanding of customer behavior and the dynamics of the adoption process, the policies can become short-sighted, possibly under/over-subsidizing.

Using our demand model defined in (9), the policy maker should find the optimal rebate levels r_t for $t = 1, ..., T-1$ in order to minimize the total present value of the rebate costs, while still achieving the target adoption level x_T at the end of the planning horizon. In this paper, we only consider a deterministic model and therefore the random components ν_t and ξ_t (from equations (4) and (8)) will be set to zero. This work, to the best of our knowledge, is the first one to deal with target policy optimization with a choice model approach and network externalities. Introducing uncertainty into the policy optimization framework adds an extra level of complexity to the model that would overshadow some of the insights that we are trying to obtain in this paper. Nevertheless, we believe this is actually a very promising direction to extend this work. For this paper we will focus only on the deterministic counterpart of the policy problem defined in the following optimization model:

$$\begin{aligned} Cost_1(x_1, x_T) = \min_{r_1,\dots,r_{T-1}} \ & \sum_{t=1}^{T-1} \delta_g^{t-1} r_t q_t(x_t, r_t) \\ s.t. \quad & x_{t+1} = x_t + q_t(x_t, r_t), && \forall t = 1, ..., T-1 \\ & q(x_t, r_t) = (M_t - x_t)\frac{e^{a_D(r_t - k(x_t) + d_t) + b_D \log(x_t/M_t) + c_D}}{1 + e^{a_D(r_t - k(x_t) + d_t) + b_D \log(x_t/M_t) + c_D}}, && \forall t = 1, ..., T-1 \\ & k(x_t) = e^{a_I + b_I \log(x_t)}, && \forall t = 1, ..., T-1 \end{aligned} \tag{10}$$

Note that parameters M_t, d_t, δ_g and x_1 are given data, denoting respectively the market size at time t, the discounted future cash flow of solar installations purchased at time t, the government's discount rate, and the initial number of household solar installations sold before time t. The set of parameters (a_I, b_I) and (a_D, b_D, c_D) allow us to define the cost evolution dynamics and the demand function, respectively. These parameters need to be estimated using a historical data set, as we demonstrate in Section 3.1. Note that we replace the $NPV_t(x_t, r_t)$ by $(r_t - k(x_t) + d_t)$, in order to make the notation more concise. The average installation size originally included in the NPV definition of (3) can be suppressed because we are estimating a_D and this only causes a proportional shift in the estimate.

The problem described in (10) can be solved numerically using a dynamic programming reformulation, where the state of the system is x_t (the number of solar panels sold up to time t). The adoption target condition can be enforced with a terminal system cost of zero if the target adoption level has been achieved and set to infinity otherwise.

$$Cost_T(y, x_T) = \left\{ \begin{array}{l} 0, \;\; \textit{if } y \geq x_T \\ \infty, \;\; o.w. \end{array} \right\} \tag{11}$$

At each step, the policy maker decides the rebate level r_t. The immediate rebate cost observed at each period is the rebate times the amount of people who adopted at that given time step: $r_t q_t(x_t, r_t)$. The objective of the government at each time step is to minimize the immediate rebate cost plus discounted future rebate costs. Define for $t = 1, ..., T-1$ the following cost-to-go functions:

$$Cost_t(x_t, x_T) = \min_{r_t} r_t q_t(x_t, r_t) + \delta_g Cost_{t+1}(x_t + q_t(x_t, r_t), x_T) \tag{12}$$

It is easy to see that the solution of the dynamic program in (12) leads to a solution of the original problem in (10). This is due to the fact that the state variable x_t decouples the problem across multiple time periods. The second term in the cost-to-go function, x_T, is used here as a fixed parameter, i.e., some policy target that is known beforehand. We use this notation because we will later explore the implications of changing the target adoption levels in the overall system cost (see Section 4).

Note that the DP formulation in (12) can be numerically solved by discretizing the state-space. Given that we only carry one state variable, we can perform a line search over r_t at each period t and efficiently compute the cost-to-go functions by backwards induction.

We can easily add further complexity levels to the model in (12), such as constraints on the rebate levels and quantity caps on the number of subsidized panels (these are actually commonplace in many countries). For example, in order to avoid strategic timing behavior of the customers, we have argued that subsidy levels decrease at a faster rate than the costs improve. In order to maintain that argument, we might need to introduce a decreasing rebate constraint $r_t \leq r_{t-1}$. For that, we need to add another dimension to the state-space of the dynamic program to keep track of previous rebate levels. Nevertheless, this is not much harder to solve, as 2-state DP is still numerically tractable. In the remainder of this paper, we focus only on the base model defined in (10), without these extensions of the problem. We have implemented and tested some of these constraints. Nevertheless, they do not add additional insight into the policy design problem that we are dealing with.

3. Empirical Analysis of the German Solar Market

In this section we perform an empirical study of the German solar market by estimating the demand model described in Section 2.1 and using this model to produce forecasts for future adoption levels of the solar technology and validate the model. Furthermore, we use these forecasts and the DP formulation of the policy-maker's problem described in Section 2.2 to produce policy recommendations.

We have gathered the following information on the German PV solar market data:

a) Number of households in Germany from 1991 to 2007

b) Forecasted number of households in Germany from 2008 to 2030

c) Feed-in-Tariff rates (€/kWh) from 1991 to 2007

d) Feed-in-Tariff forecasted rates (€/kWh) from 2008 to 2030

e) Average solar installation cost (€/kWp)from 1991 to 2007

f) Nameplate peak capacity (MWp) of solar panels installed in Germany from 1991 to 2007

g) Distribution of PV solar installation sizes made in 2009

h) Discount rate used by customers and government

i) Average annual PV solar electricity yield (annual kWh/kWp)

Sources for the data collected for this study include IEA (2004), Schaeffer et al. (2004), Wissing (2006), EEG (2007), PVPS (2008), Frondel et al. (2008), EPIA (2009), Bhandari and Stadler (2009), as well as the databases of the Eurostat (European Commission Statistical Office) and the Federal Statistics Office of Germany.

The data for the Feed-in-Tariff rates both past and forecasted can be seen in Figure 1. The average solar installation cost k_t is displayed in Figure 2, together with the discounted cash flow d_t for a solar installation. This discounted cash flow data is displayed in Figure 2 and can be obtained using the Feed-in-Tariff rates, discount rates and the annual solar yield data. The discount rate used by customers and government is assumed to be $\delta_c = \delta_g = 95\%$ (approximately equivalent to the 5% interbank interest rate). Finally, we assume that average annual solar yield is 750 kWh per kWp of installed peak capacity, which is derived by the average total amount of PV electricity generated divided by the installed capacity in each year.

The nameplate peak capacity (MWp) of solar panels installed from 1991-2007, together with the distribution of installations in 2009, will be used to estimate the number of residential installations done between 1991-2007. The resulting estimated number of solar household installations together with the number of households in Germany is displayed in Figure 3. The details of these calculations will be discussed next.

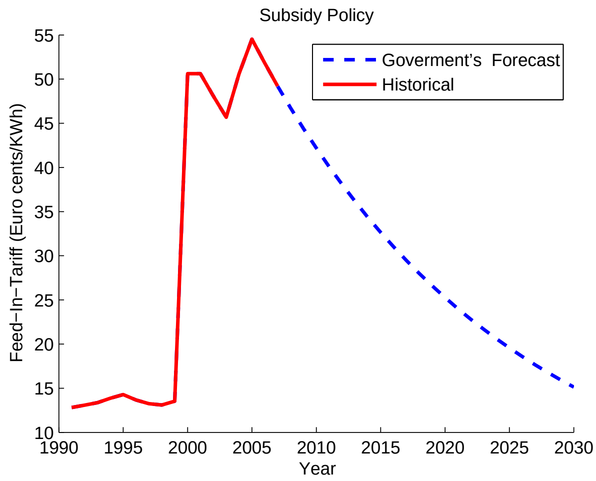

Figure 1 **Current subsidy policy in Germany**

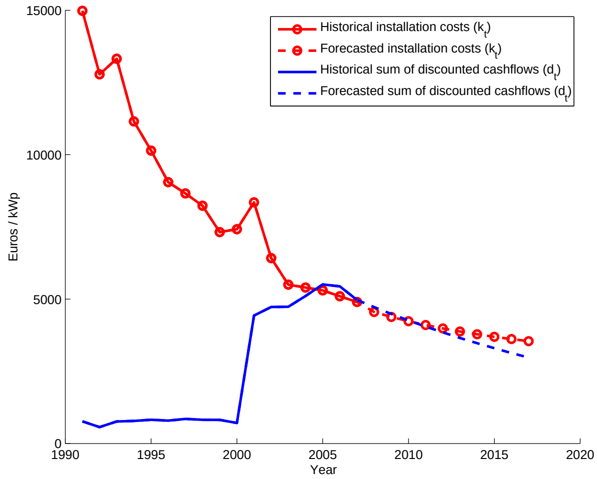

Figure 2 **Installation costs (k_t) vs. Discounted cash flows (d_t)**

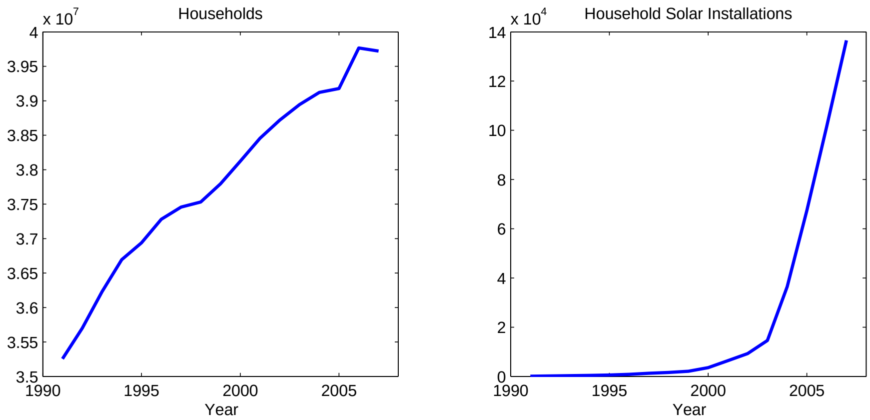

Figure 3 **Number of households (M_t) and number household of solar installations (x_t) in Germany**

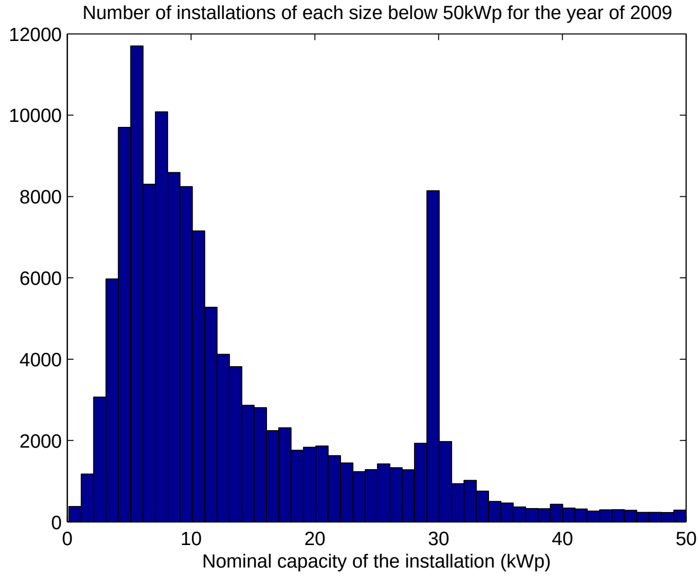

Figure 4 Histogram of PV installations in 2009

The data obtained for the amount of historical solar installations was in the total cumulative installed capacity of PV solar panels in Germany, including both rooftop and open-space installations. In general, these two types of installations are very different both in terms of size and incentive tariffs. Modeling the adoption of both types of installations with the same demand model can be inaccurate, but we could not obtain differentiated data about the size of installations from 1991-2007. According to Reichmuth et al. (2010), such information was not even collected for rooftop systems for this data range. Starting in 2009, Germany's Federal Network Agency (Bundesnetzagentur) requires all new PV installations to register their installed capacity. Using this new database, we obtain the size of all new solar installations performed in 2009. In Figure 4, we display a histogram with the installations in 2009 for each size ranging from $1kW_p$ to $50kW_p$. In fact the full data for 2009 includes very large installations, including some solar farms of approximately $50MW_p$ and $20MW_p$. Residential rooftop installations are usually considered to be under $30kW_p$. This is also the criterion used in the Renewable Energy Sources Act (EEG) to define the feed-in-tariffs for small scale installations. In fact, we observe in Figure 4 a sharp increase in installation numbers exactly at $30kW_p$, as customers have a strong incentive not to go over this limit in order to obtain the higher feed-in rates.

In order to differentiate residential installations from open-space installations, we will use the proportion of installations sizes in 2009 to infer the number of residential installations from the total aggregate installed capacity from 1991 to 2007. We understand this is a strong assumption, but it the best we can do with the information that is available. In fact, the sum of all PV systems installed in 2009 is $3,429kW_p$, while 42.84% of these were from installations under $30kW_p$. The total number of installations under $30kW_p$ was $122,863$ (out of a total of $141,278$ new PV systems) and the average size of these residential systems was $11.95kW_p$. To put things in perspective, these new rooftop installations in 2009 broke yet another record for the number of installations in the country and yet covered approximately only 0.32% of the households in Germany.

Using the historical series of total installed nameplate capacity of solar panels in Germany (both residential and not) together with the ratio of residential installations of 42.84% and the average system size of $11.95kW_p$, we extrapolate the historical x_t adoption level, i.e. the number of residential customers that had purchased a solar panel before each year between 1991-2007. The result is displayed in Figure 3.

3.1. Fitting the Installation Cost and Demand Model

There are basically two estimations to be made from the data that was gathered: the cost function and the consumer utility model. In particular, we need to estimate those five coefficients (a_I, b_I) and (a_D, b_D, c_D). Note that the cost function appears inside the consumer utility model through the NPV of the solar installation. If we try to estimate both relations together, the estimation will have problems with the endogeneity of the system cost evolution in the adoption process. Therefore, we can first estimate the dynamics for the cost of solar installations and then estimate the utility model afterwards. The cost improvement function was estimated with a simple regression on the log-log relationship between k_t and x_t, as defined in (4). Table 1 displays the estimation results.

	Estimate	Std. Error
a_I	3.05	0.0635
b_I	-0.127	0.0065
R^2	0.907	

Table 1 Estimation Results for the Installation Cost (Learning-by-Doing effect)

The results of the cost dynamics fitted above in Table 1 can be translated into a perhaps more common terminology of Learning Rate (LR) and Progress Ratio (PR). In particular, $PR = 2^{b_I} = 92\%$ and $LR = 1 - PR = 8\%$.

The demand model defined in (8) can be expressed as a linear function of the utility parameters that we want to estimate. Let $\lambda_t = \frac{x_{t+1}-x_t}{M_t-x_t} = \frac{e^{a_D NPV_t + b_D \log(x_t/M_t) + c_D + \xi_t}}{1+e^{a_D NPV_t + b_D \log(x_t/M_t) + c_D + \xi_t}}$. Then $1 - \lambda_t = \frac{M_t - x_{t+1}}{M_t - x_t} = \frac{1}{1+e^{a_D NPV_t + b_D \log(x_t/M_t) + c_D + \xi_t}}$. Therefore:

$$\log\left(\frac{x_{t+1}-x_t}{M_t - x_{t+1}}\right) = \log\left(\frac{\lambda_t}{1-\lambda_t}\right) = a_D NPV_t + b_D \log(x_t/M_t) + c_D + \xi_t \quad (13)$$

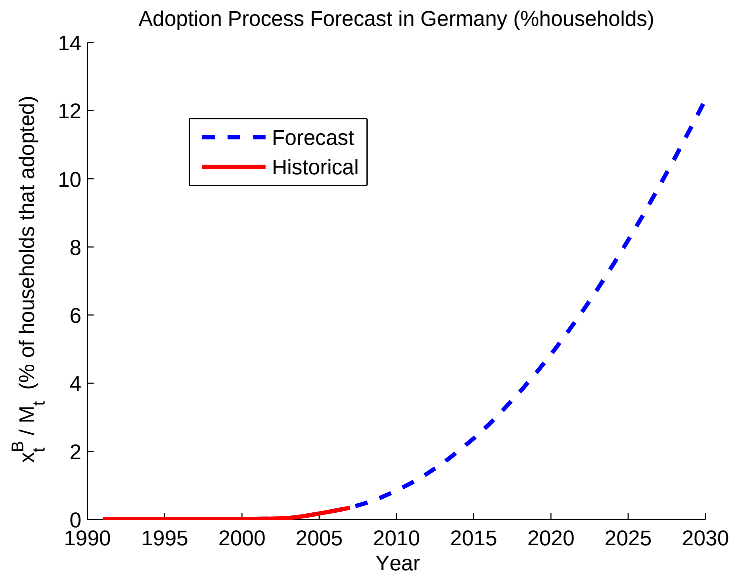

Figure 5 **Baseline forecast of adoption ($r_t = 0$) in Germany as a proportion of the households**

We consider the unobserved demand shock ξ_t as the error measure and use a generalized method of moments approach to estimate the relation in (13). Note that a necessary condition for (13) to hold is $M_t > x_{t+1} > x_t$. This condition is generally true for the adoption of any new technology, since demand is always positive and the market size is still far from the number of adopted customers (see Figure 3). Table 2 displays the estimation results.

	Estimate	Std. Error
a_D	1.636×10^{-4}	1.120×10^{-4}
b_D	0.657	0.240
c_D	-2.891	1.592
R^2	0.957	

Table 2 **Estimation Results for the Demand Model**

The estimation results from Tables 1 and 2 seem to present a good fit to the historical data for the cost and demand curves. We can now use our calibrated model to forecast future adoption levels and solve the policy-making problem to obtain insights about the situation of the German market.

3.2. Forecasting and Policy Optimization

Using the model estimated in Section 3.1, in Figure 5 we forecast the future baseline adoption levels, x_t^B, using forecasts of the number of households and future feed-in rates. Define this baseline adoption path as the natural adoption process if we do not intervene on the subsidy levels ($r_t = 0$).

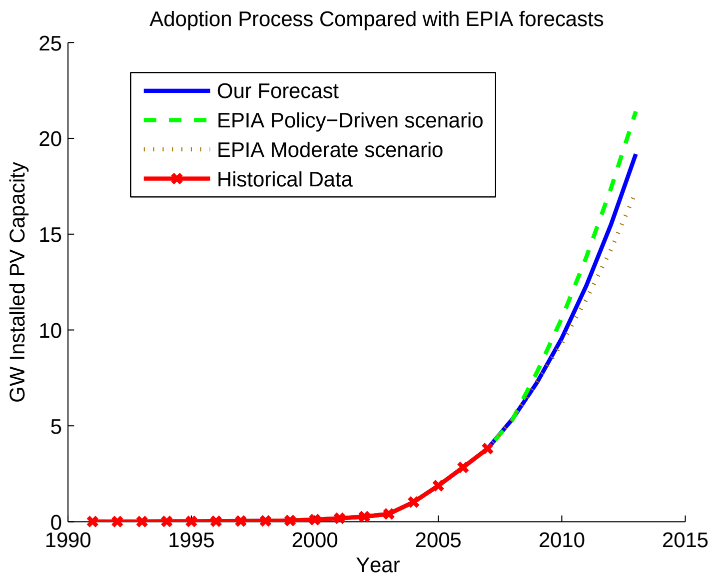

Figure 6 **Comparing our baseline forecast with EPIA benchmark**

Using the installation distribution in 2009, we can infer the total (both residential and non-residential) PV installed capacity for the following few years, 2008-2013. In Figure 6, we compare our results with a well recognized forecast benchmark from the European Photovoltaic Industry Association (EPIA). Our baseline predictions for the total installed solar generation capacity in Germany by 2013 are 11.5% above the EPIA conservative (status-quo) forecast and 10.4% below the EPIA aggressive (stronger policy) forecast. This comparison serves as a sanity check for us to trust the forecasting ability of our model.

Using the estimated model, we also demonstrate how to use the policy design tool developed in Section 2.2 with a hypothetical adoption target. Starting from 2008, consider the target adoption level for 2030 to be at our baseline adoption forecast at $x_T^B = 12.3\%$. In this case, we observe that by readjusting the current subsidy policy, we can obtain net present value savings of 32.5 billion Euros, over the next 22 years. In Figure 7, we display the optimal rebate strategy. This strategy is computed by numerically solving the dynamic program in (12) and it displays positive rebates in the early stages and negative rebates after 2015. In other words, this rebate structure could translate into an increase in subsidies in the first few years and the removal of some of the current subsidies later on (possibly smaller Feed-in-Tariffs or higher sales taxes). The jerkiness of this plot is due to the rough discretization used to solve the dynamic program. Note also that the rebate structure is decreasing over time. This is consistent with the assumption that consumers should have no incentives to be strategic about their purchase timing decision.

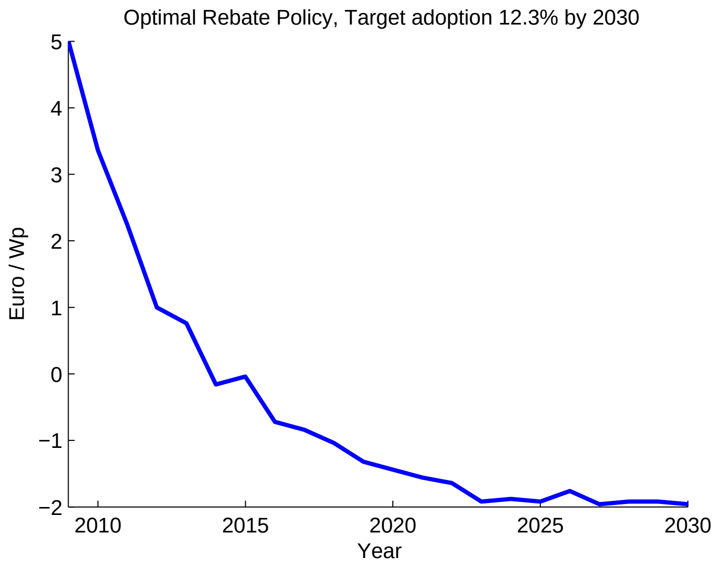

Figure 7 Optimal Rebate Policy: Target 12.3% by 2030

By looking at the structure of the optimal rebate path in Figure 7, we can see that there are three forces defining this optimal policy: The first one increases the subsidies at the beginning of the planning horizon, in order to kick-start the effects of the positive network externalities. The second contradicting force comes from the discounted nature of the problem, favoring later subsidies. The third force is a free-riding effect, where subsidizing is cheaper at later periods because network externalities have already taken effect. The combination of these three effects will make the optimal rebate path distribute rebates in a non-trivial manner. In other words, it is not optimal to waste all our subsidizing efforts at the first stage, but instead there is an efficient way to distribute the rebates along the time horizon with minimal cost to the system.

Figure 8 displays the adoption forecast using this optimal rebate policy and compare it to the baseline adoption path. Additionally, Figure 9 displays the forecasted evolution of installation costs under the baseline path and with optimized policies.

In order to understand the trade-off between the adoption target level established for the year 2030 and the cost it will incur for the government, we ran the policy optimization for multiple target levels, ranging from 1% to 25%. In Figure 10, we observe that below a 16.3% adoption level, the government can actually save money by optimally managing the subsidy policy. This is consistent with Figures 7-8, where we display the optimal rebate and adoption path for a particular adoption target of 12.3%.

In Section 4, we explore further the analytical structure of the optimal policy and target cost function. More specifically, in Theorem 1 we prove that the government's cost function is convex as

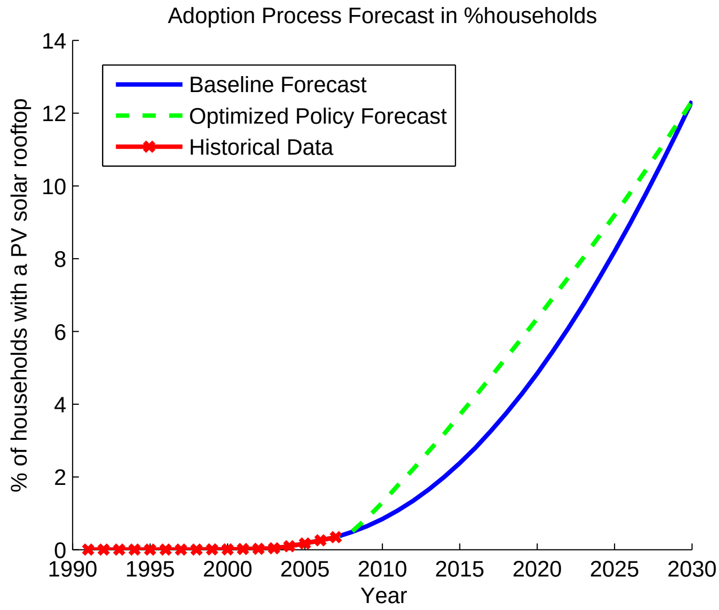

Figure 8 **Adoption Forecast: Target 12.3% by 2030**

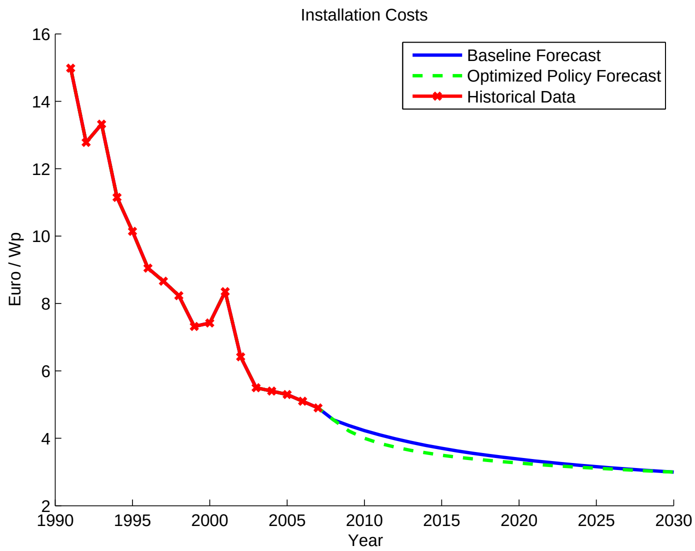

Figure 9 **Installation Costs: Target 12.3% by 2030**

a function of the adoption target, as we observed in Figure 10. This convexity result requires some mild assumptions on the installation cost and demand model parameters which are clearly satisfied for our empirical study. We also further analyze in Section 4.1 the behavior of the optimal solution as we change the target adoption level for a two-stage problem, where we conclude that one of the reasons why increasing the adoption targets becomes increasingly more expensive is because of the saturation of the network externality benefits.

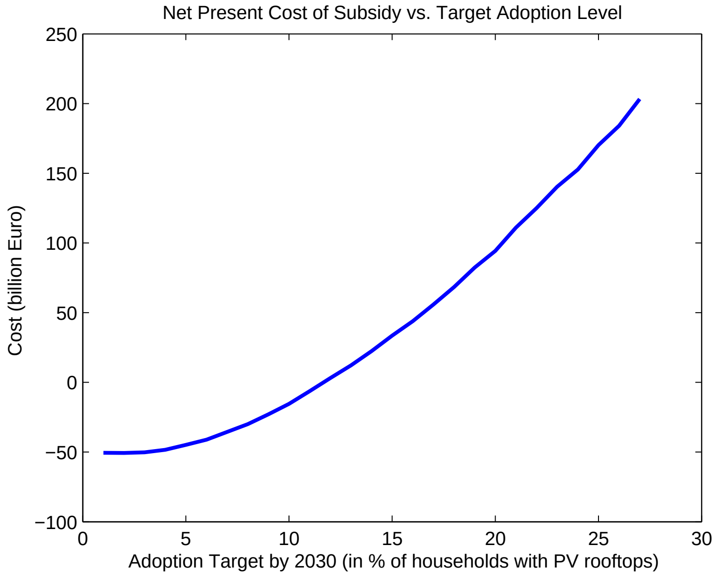

Figure 10 **Trade-off between adoption target by 2030 and net system cost (or savings)**

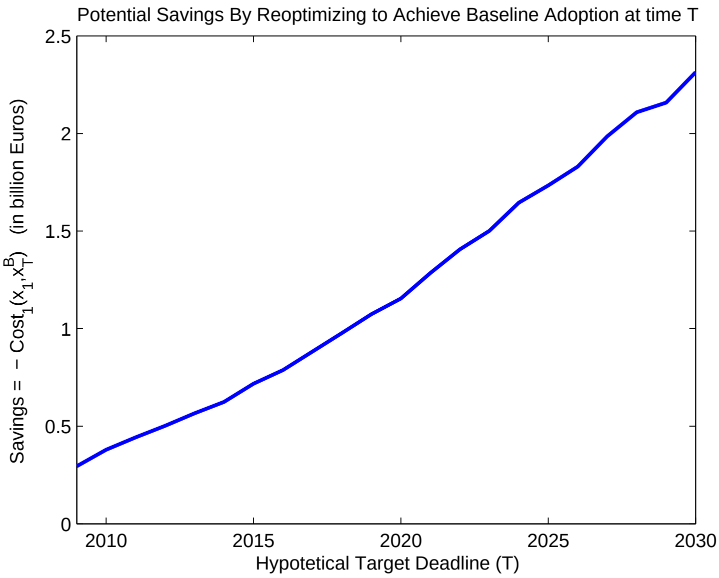

Figure 11 **Cost saving by optimizing subsidies when assuming target adoption x_T to be given by baseline x_T^B**

For our last experiment, we varied the target adoption deadline T from 2009 to 2030 and assumed that the target adoption rate was our baseline estimate for the adoption level at that given year, x_T^B, as seen in Figure 5. We then optimized the subsidy policy for that given target x_T^B and observed the government cost for achieving that same target level by time T. The motivation of this experiment is to reverse engineer what could potentially be the government's current subsidy policy motivation or determine if the current policy is suboptimal. If the government was in fact optimally designing the current subsidies to reach an adoption target at any of these years, in theory, our baseline should forecast the optimal adoption path to that target. In other words, the optimal rebate from the optimization model should be $r_t = 0$, for all $t \leq T$, and the potential cost improvement of changing the subsidy policy should also be zero.

In fact, we observe that for any target deadline T between 2009 and 2030, there is a cheaper way to achieve the same adoption level as the baseline forecast predicts, x_T^B. Figure 11 displays the result of this experiment. We note that the potential cost savings is always positive for any target adoption level in the baseline forecasted adoption path. This indicates that the current design of the subsidies are not optimally designed for any potential adoption target.

Another hypothesis is that the government is actually maximizing some measure of social welfare, as opposed to trying to achieve some given target, and could potentially be optimal under that objective. We discuss the welfare problem in more detail in Section 4.2. From Theorem 2, we show that because the optimal rebate structure found by our dynamic program is decreasing in time, i.e., $r_t \geq r_{t+1}$ (see for example, Figure 7), then our optimized adoption path is always above the baseline adoption path $x_t^* \geq x_t^B$ (see Figure 8). Therefore the cumulative welfare benefits from solar panel adoption will always be higher in our optimized solution. Our new solution will not be necessarily the optimal welfare solution, but it will certainly provide higher social welfare than the current baseline path. This shows that the government is still acting suboptimally, even from a social welfare perspective.

The result that the current subsidy policy is suboptimal, both from a target adoption and from a welfare perspective, needs to be evaluated carefully. Throughout our modeling process we have made many assumptions about consumer behavior and the demand structure that need further exploration. Also, a few simplifying assumptions were made simply because of the lack of detailed data on the solar market. That being said, we believe this empirical study developed a first step in analyzing this issue and raises a clear warning sign about the economic efficiency of the current policy. We believe these experiments can be further improved by updating the data set and possibly using a more detailed demand model.

4. Analysis of the Policy-Maker's Problem

In this section, we explore some of the theoretical insights that can be obtained by analyzing the structure of the optimization model we developed in Section 2.2 for the policy design problem. Consider the problem faced by the policy maker in (10), where x_1 is the initial number of solar panels sold and x_T is the given adoption target. As before, we control the adoption levels by adjusting the rebate rates r_t. For a full notation summary, see Appendix A. We will make a few technical assumptions about the parameters of the model that are necessary for the analysis:

ASSUMPTION 5.

a) $M_t > x_{t+1} > x_t$*, for all* $t = 1, ..., T-1$*.*

b) $b_I < 0$*.*

c) $a_D > 0$*,* $b_D > 0$*, and* $a_D + b_D \leq 1$*.*

Assumption 5.a means that we are only concerned with the problems where the market is still under development and the potential market size is greater than the number of panels sold at any point of our decision horizon. Assumption 5.b is true by the nature of the learning-by-doing effect, which decreases installation cost with the number of installed panels. Assumption 5.c is a technical assumption that we use to obtain convexity of the government's cost function. Assumptions $a_D > 0$ and $b_D > 0$ hold due to the nature of the demand model, but the last part $a_D + b_D \leq 1$ is not as obvious. The top-level intuition behind it is that the benefits of network externalities such as learning-by-doing and information spread may have a concave impact on the system cost. Without these network effects, the nature of the logit model alone could guarantee convexity for the cost function. This assumption, $a_D + b_D \leq 1$, guarantees that the concave network effects do not overshadow the convexity of the demand model. In fact, this condition is easily satisfied in the empirical study of Section 3.1.

We will now show that the total present system cost $Cost_1(x_1, x_T)$ is convex in the future target adoption level. As the demand function $q(x_t, r_t)$ is monotonically increasing in r_t, we can easily invert the relation and express the rebate as a function of the desired demand, q_t.

$$r_t(x_t, q_t) = k(x_t) - d_t - \frac{b_D \log(x_t/M_t) + c_D + \xi_t - \log(-q_t/(q_t - M_t + x_t))}{a_D}$$

Note also, that demand is determined for a given a adoption path $q_t = x_{t+1} - x_t$

$$r_t(x_t, x_{t+1}) = k(x_t) - d_t - \frac{b_D \log(x_t/M_t) + c_D + \xi_t - \log\left(\dfrac{x_{t+1} - x_t}{M_t - x_{t+1}}\right)}{a_D} \tag{14}$$

For simplicity, consider the following 3-period model (T=3), where x_1 is the initial state and x_3 is the final target state. The only decision to be made is where the middle state x_2 should be placed. Once x_2 is decided, the rebates for both periods will be determined by $r_1(x_1, x_2)$ and $r_2(x_2, x_3)$ according to equation (14). By controlling directly the adoption path and not the rebates, we can deal with a single variable unconstrained problem, instead of a two-variable problem with balance constraints. Define the inside cost function:

$$J_1(x_2,x_3) = r_1(x_1,x_2)(x_2 - x_1) + \delta r_2(x_2,x_3)(x_3 - x_2)$$

Then the policy maker's problem can be reformulated as:

$$Cost_1(x_1,x_3) = \min_{x_2} J_1(x_2,x_3) \tag{15}$$

The following lemma will be used to show the convexity of the total system cost function.

LEMMA 1. *$J_1(x_2,x_3)$ is jointly convex in x_2 and x_3.*

The proof of Lemma 1 is very heavy in algebraic manipulations. To improve the reading of the paper, we placed this proof in Appendix B. Given the convexity of $J_1(x_2,x_3)$ in x_2, we know that the optimal solution $x_2^*(x_3)$ comes from the solution of the first order optimality condition, $\frac{dJ}{dx_2}(x_2^*(x_3),x_3) = 0$. From the joint convexity of J_1, we can also obtain the following results.

COROLLARY 1. *Let $x_2^*(x_3)$ be the optimal adoption path for a given target x_3. Then $Cost_1(x_1,x_3) = J_1(x_2^*(x_3),x_3)$ is a convex function of x_3.*

The proof of this Corollary 1 is a well known result from convex analysis and it comes directly from the joint convexity of the inner function $J_1(x_2,x_3)$ (for further reference see Boyd and Vandenbergh (2004)). We will use Corollary 1 in the proof of convexity for the T-period case in Theorem 1.

Another interesting outcome of Lemma 1 is stated below in Corollary 2. This corollary is derived from the Implicit Function Theorem (see Bertsekas (1995))

COROLLARY 2. *Let $x_2^*(x_3)$ be the optimal adoption path for given target x_3. Then the first order optimality condition on x_3 implies: $\frac{dx_2^*}{dx_3}(x_3) = -\frac{d^2J}{dx_2dx_3}(x_2^*(x_3),x_3)\left(\frac{d^2J}{dx_2^2}(x_2^*(x_3),x_3)\right)^{-1}$.*

The above result, Corollary 2, will be later used to develop insights about the structure of the optimal solution. For now, we will focus on the convexity result. In the original T-period problem, we obtain the following result.

THEOREM 1. *$Cost_1(x_1,x_T)$ is convex in x_T.*

The intuition behind the proof of Theorem 1 is to show convexity for an additional period $T = 4$ and $Cost_1(x_1,x_4) = \min_{x_3} Cost_1(x_1,x_3) + \delta^2 r_3(x_3,x_4)(x_4 - x_3)$. By induction, we can show that the cost for any time horizon T is a convex function of the target. Once again, the derivation of this proof is relegated to Appendix C.

This result may be useful in order to extend this model into many future research directions, including solving the problem with uncertainty (with randomness in demand and/or technological progress) or introducing multiple products (for example, different installation sizes). In these cases, one possible approach would be to use approximate dynamic programming, which may require some convexity structure of the value function.

4.1. Insights on the optimal solution

By examining at the 3-period problem defined in (15), we can obtain some insights on the structure of the optimal solution and optimal system cost. Consider the first order condition: $\frac{dJ}{dx_2}(x_2^*(x_3), x_3) = 0$. By rearranging the terms of this equation, we obtain:

$$\frac{M_1 - x_1}{a_D(M_1 - x_2^*(x_3))} + r_1(x_1, x_2^*(x_3)) + \delta\left[k'(x_2^*(x_3)) - \frac{b_D}{a_D x_2^*(x_3)} - \frac{1}{a_D} - r_2(x_2^*(x_3), x_3)\right] = 0$$

This can be used to find the optimal mid-point adoption level $x_2^*(x_3)$, located where the marginal cost of increasing the level in the first period is the same as the marginal benefit from the second period. In the first period, for each marginal unit of x_2 that we increase over the optimal, we incur a marginal cost of the rebate price $r_1(x_1, x_2^*(x_3))$, plus a rebate adjustment of $\frac{M_1 - x_1}{a_D(M_1 - x_2^*(x_3))}$ needed to meet the higher demand in this first period. In the second period, the marginal unit increase in x_2 will lower the overall system cost (with discounting δ) by the rebate $r_2(x_2^*(x_3), x_3)$ adjusted for the network externalities gain $k'(x_2^*(x_3)) - \frac{b_D}{a_D x_2^*(x_3)}$ and also for the fact that we need to serve a lower demand, which also affects the rebate level by $\frac{1}{a_D}$. Strict convexity of $J(x_2, x_3)$ in x_2 guarantees that this equation has a monotonically increasing left hand side, which means that there is a unique optimal solution and it can be easily computed numerically.

It is only natural to ask how the optimal mid-point adoption level changes with the adoption target level. From Corollary 2, we have that $\frac{dx_2^*}{dx_3}(x_3) = -\frac{d^2 J}{dx_2 dx_3}(x_2^*(x_3), x_3)\left(\frac{d^2 J}{dx_2^2}(x_2^*(x_3), x_3)\right)^{-1}$. We can then show the following properties of the optimal solution:

Proposition 1. *Let $x_2^*(x_3)$ be the optimal mid-point level. Then $\frac{dx_2^*}{dx_3}(x_3) > 0$, which implies that the optimal x_2 is strictly increasing in the target x_3. If we also have that $x_2 - x_1 \leq M_2 - x_3$, then we can also show $\frac{dx_2^*}{dx_3}(x_3) < 1$.*

See Appendix D for a proof of Proposition 1. We use this result to get intuition about how the system cost changes as a function of the adoption target.

Consider the variation in the optimal rebate levels. We can express them as:

$$\frac{dr_1}{dx_3}(x_1, x_2^*(x_3)) = \left[\frac{1}{a_D(x_2^*(x_3) - x_1)} + \frac{1}{a_D(M_1 - x_2^*(x_3))}\right]\frac{dx_2^*}{dx_3}(x_3)$$

$$\frac{dr_2}{dx_3}(x_2^*(x_3), x_3) = \frac{1}{a_D(x_3 - x_2^*(x_3))} + \frac{1}{a_D(M_2 - x_3)} + \left[k'(x_2^*(x_3)) - \frac{b_D}{a_D x_2^*(x_3)} - \frac{1}{a_D(x_3 - x_2^*(x_3))}\right]\frac{dx_2^*}{dx_3}(x_3)$$

The derivative of the system cost can also be expressed as:

$$\frac{dCost_1}{dx_3}(x_1,x_3) = \frac{dr_1}{dx_3}(x_1,x_2^*(x_3))(x_2^*(x_3)-x_1) + r_1(x_1,x_2^*(x_3))\frac{dx_2^*}{dx_3}(x_3) \\ +\delta\frac{dr_2}{dx_3}(x_2^*(x_3),x_3)(x_3-x_2^*(x_3)) + \delta r_2(x_2^*(x_3),x_3)\left(1-\frac{dx_2^*}{dx_3}(x_3)\right)$$

In order to develop intuition about the optimal cost variation, assume we are working under the regime where $M_t - x_{t+1} >> x_{t+1} - x_t$, for both $t = 1,2$. This is the case in any solar market today and for the foreseeable future. For example, in the German case studied in the previous section the amount of solar capacity installed is not even 1% of the potential market size. For this reason we reformulate the optimal rebate derivatives with an approximation, where the terms $\frac{1}{a_D(M_1 - x_2^*(x_3))}$ and $\frac{1}{a_D(M_2-x_3)}$ go to zero. The cost derivative can be approximated by:

$$\frac{dCost_1}{dx_3}(x_1,x_3) \cong \left[\frac{1}{a_D} + r_1(x_1,x_2^*(x_3))\right]\frac{dx_2^*}{dx_3}(x_3) \\ +\delta\left[\frac{1}{a_D} + r_2(x_2^*(x_3),x_3)\right]\left(1-\frac{dx_2^*}{dx_3}(x_3)\right) + \delta(x_3 - x_2^*(x_3))\left[k'(x_2^*(x_3)) - \frac{b_D}{a_D x_2^*(x_3)}\right]\frac{dx_2^*}{dx_3}(x_3) \tag{16}$$

Each new marginal unit of target adoption level x_3 will need to be distributed into the first and second period of sales determined by $\frac{dx_2^*}{dx_3}(x_3)$ and $(1 - \frac{dx_2^*}{dx_3}(x_3))$ respectively. Note from the first term in the above equation that a marginal increase in the target level will increase the mid-point level by $\frac{dx_2^*}{dx_3}(x_3)$ and each additional unit of mid-point adoption level will cost an additional $\frac{1}{a_D} + r_1(x_1,x_2^*(x_3))$ to the system, where $\frac{1}{a_D}$ is the rebate adjustment due to increased demand in the first period. The cost of the additional target level units allocated to the second period will be $\left[\frac{1}{a_D} + r_2(x_2^*(x_3),x_3)\right]\left(1-\frac{dx_2^*}{dx_3}(x_3)\right)$, where $\frac{1}{a_D}$ is the adjustment in the second rebate price due to higher demand. On the other hand, each unit of mid-point level increase will save the system some money on the second period because of network externalities, which is represented by the last term on the equation. The externality benefits affect all sales made in the second period $(x_3 - x_2^*(x_3))$, not just the new additional units required for the marginal target increase.

With the relation in (16), the policy maker can obtain the trade-offs of raising the target adoption level, without having to resolve the entire system cost. If x_3 is already very high, it is likely that the cost benefits due to the network externalities are saturated, as $k'(x_2^*(x_3)) - \frac{b_D}{a_D x_2^*(x_3)}$ will increase and approach zero as we increase x_3 (note that $k'(x) < 0$ and $k''(x) > 0$). Then raising the target levels become increasingly more expensive, which is one reason why the cost function $Cost_1(x_1,x_3)$ is convex, as we concluded in Theorem 1.

4.2. Welfare Maximization

Perhaps more common in the economics literature, the objective of a policy optimization problem can be expressed as a social welfare maximization problem. In this particular case, it is debatable how one should quantify the benefits of developing the solar industry for a particular government.

There is obviously a global benefit for clean electricity generation, but the local benefits from avoiding carbon emissions cannot be rewarded to a single state or country unless we develop an efficient global carbon market. Other pollutants have more local impact, but in general pollution avoidance cannot solely justify solar technology, as there are other technologies that are much more cost efficient (from wind generation to building retro-fitting). On the other hand, there are less tangible benefits of stimulating the solar technology by a particular government. These include generation portfolio diversification, peak-load reduction, development of a local solar manufacturing and installation industry.

For the reasons above, we have so far waived the welfare discussion and assumed that policy-makers have a given strategic adoption target. We further demonstrated how the cost behaves for different target levels, which could potentially aid policy-makers when setting such targets. If we could quantify all the benefits of solar adoption, then we could use welfare maximization to find these targets with a slight modification of our optimization model.

Suppose we are given a benefit function $Benefit_t(x_t)$ which depends on the realized adoption at each stage. This would be the case if there was a given price for CO_{2_t} at time t and every solar panel installed saves the country that amount of money for avoiding carbon emissions. See for example Benthem et al. (2008) for an example of policy optimization with carbon externality costs. Naturally, this function $Benefit_t(x_t)$ should be increasing on the adoption level at x_t. Define the social welfare problem as:

$$Welfare(x_1,...,x_T) = \sum_{t=1}^{T-1} \left[Benefit_t(x_t) - \delta^{t-1} r_t(x_t, x_{t+1})(x_{t+1} - x_t) \right]$$

The optimal welfare problem can be solved by maximizing the expression above, which can be done numerically by solving a dynamic program. Next, we define a useful result connecting the optimization for adoption targets with social welfare efficiency.

THEOREM 2. *Let x_t^B be the baseline adoption path, where $r_t = 0$, for all $t = 1,...,T-1$. Consider x_T^B as the adoption target in the optimization model (10) and let x_t^* be the optimal adoption path for this model. If the overall system cost is negative, $Cost_1(x_1, x_T^B) < 0$, and the optimal rebate path is non-increasing, $r_t(x_t^*, x_{t+1}^*) \geq r_{t+1}(x_{t+1}^*, x_{t+2}^*)$, then the welfare of the new optimized path is greater than the welfare under the baseline path, i.e.,*

$$Welfare(x_1, x_2^*, ..., x_{T-1}^*, x_T^B) > Welfare(x_1, x_2^B ..., x_{T-1}^B, x_T^B)$$

Note that Theorem 2 connects with our findings in the empirical study at the end of Section 3.2. In that part of the study, we argue that the current subsidy policy for the German solar market is suboptimal, even from a welfare maximization perspective. To avoid cluttering the paper, we display the proof of Theorem 2 in Appendix E.

5. Conclusions

In summary, we model the adoption of solar photovoltaic technology as a diffusion process where customers are assumed to be rational agents following a discrete choice model. We show how this framework can be used by a policy maker to design optimal incentives in order to achieve a desired adoption target with minimum cost for the system. In particular, this policy design model takes into consideration network externalities such as information spread and cost improvements through learning-by-doing. To demonstrate the applicability of this framework, we develop an empirical study of the German photovoltaic market and show how this model can be fitted to actual market data and how it can be used for forecasting and subsidy policy design. Finally, we analyze the structure of the optimal solution of the subsidy design problem to obtain insights about the government's subsidizing cost and to understand how this adoption target optimization problem can be related to the welfare maximization problem.

We show in our numerical experiments that in the early stages of the adoption process, it is optimal for the government to provide strong subsidies, which take advantage of network externalities to reach the target adoption level at a lower cost. As the adoption level increases, these network externalities become saturated and the price paid for raising the adoption target becomes increasingly more expensive. In particular, we are able to prove analytically that the system cost is a convex function of the adoption target. This convex trade-off between adoption targets and subsidy cost was also evident in our empirical study. We believe that this framework for quantifying the cost of adoption targets could be a very useful tool for the policy-makers that design these targets.

We also observe in this empirical study that the current subsidy policy in Germany is not being efficiently managed. We can argue the suboptimality of the current policy because for every possible adoption target chosen along the baseline adoption path, there is a better way to reach the same target at a lower cost for the system. More specifically, we can achieve that by raising early subsidies and lowering future subsidies. Finally, we proved that even if the government is maximizing social welfare instead of minimizing cost of achieving a target, the current subsidy policy is still suboptimal. We believe that this model should be further developed with more levels of detail in the demand model and data collection given the potential real-world impact of these policy recommendations.

5.1. Future Research

We are currently exploring multiple directions to improve this model. For instance, solar module manufacturers need to decide investment levels for their factories, which in turn determines the production output of solar modules. This affects prices and the overall adoption of the technology. Modeling the interaction between the policy-makers, consumers and solar panel manufacturers would provide a very interesting extension of our model. The field of industrial organization is also rich in models of industry dynamics (see for examples Ericson and Pakes (1995), Bajari et al. (2007), Weintraub et al. (2008)). Nevertheless, these models do not have a policy-making perspective using adoption targets.

On another note, it would be interesting to improve the way we optimize for policies by taking into account uncertainty in demand and in technological development. For instance, one of the key assumptions in our model, the learning curve of the installation costs, is known to have a large margin of error, as noted in van Sark et al. (2008). Policies that take into account the uncertainty in technological development and learning-by-doing effects have not been widely explored or implemented in the real-world.

Furthermore, it would be good to explore alternative ways to model consumer behavior and the information spread effect. We made an assumption about customers being short-sighted. This assumption can be waived given the state of policies in the system today, but is probably not accurate for the early stages of the technological development. Finally, investigating the impact of information spread on the consumers' purchasing behavior is another very promising direction to extend this work.

Acknowledgments

We would like to thank the following people for the private conversations we had, where we obtained great insight about the problem, references, data sources and very useful feedback on our model: Itai Ashlagi, Eduardo Azevedo, Kostas Bimpikis, Michael Caramanis, Stephen Graves, Ilan Lobel, Marcelo Olivares, Ariel Pakes, Ignacio Perez-Arriaga, Alexander Rikun, Nikolaos Trichakis, and the attendants of the 2010 MIT Operations Management Seminar Series, 2010 MSOM Conference, 2009/2010 INFORMS Annual Meetings and the MIT Electricity Students Research Group.

Appendix A: Notation

The following notation summary will be useful for reference throughout the paper.

M_t : Market size at year t, equal to number of households
x_t : Number of household solar panels installed up to year t
x_T : Target adoption level
T : Length of the policy time horizon, also known as target adoption deadline
q_t : Demand for household solar panels installed at year t (kWp)
r_t : Government installation rebate (€/kWp)
k_t : Solar installation cost at year t, including labor, module and hardware (€/Wp)
δ_c, δ_g : Discount rate of the customers and government, respectively
d_t : Discounted future cash flows of a solar installation (€/kWp)
FIT_t : Feed-in-Tariff value at year t times the average annual electricity output (€/kWp)
OM_t : Operation and Maintenance cost (€/kWp)
$AvgSize$: Average household installation size (kWp)
T_{mod} : Lifetime of the module
NPV_t : Net Present Value of an average sized solar installation purchased at t (€)
a_I, b_I : Installation cost parameters, from learning-by-doing effect
a_D, b_D, c_D : Demand model parameters or consumer utility function parameters
ξ_t : Unobserved demand shocks at time t
$\epsilon_{t,i}$: Random utility component for customer i at time t
$U_{t,i}$: Utility of purchasing a solar panel for customer i at time t
V_t : Nominal utility that the average consumer has for purchasing a solar panel
$Cost_t(x_t, x_T)$: Subsidy cost at time t, starting from adoption level x_t and finishing at adoption target x_T

Appendix B: Proof of Lemma 1

The definition of $J_1(x_2, x_3)$ is given by:

$$J_1(x_2,x_3) = r_1(x_1,x_2)(x_2 - x_1) + \delta r_2(x_2,x_3)(x_3 - x_2) = f_1(x_2) + \delta f_2(x_2,x_3)$$

We can prove the joint convexity by parts. The first part: $f_1(x_2) = r_1(x_1,x_2)(x_2 - x_1)$ needs only to be proved convex in x_2. By taking the second derivative of this term in x_2 we obtain:

$$\frac{d^2 f_1}{dx_2^2} = (M_1 - x_1)^2/(a_D(M_1 - x_2)^2(x_2 - x_1)) > 0$$

To prove joint convexity of $J_1(x_2, x_3)$ it remains to show that the principal components of the Hessian of $f_2(x_2, x_3)$ are also positive. In particular we will split up the function f_2 in two parts. Choose $\alpha_1, \alpha_2 > 0$ such that $\alpha_1 + \alpha_2 = 1$.

$$g_1(x_2,x_3) = \left(k(x_2) + \frac{\alpha_1}{a_D}\log\left(\frac{x_3 - x_2}{M_2 - x_3}\right)\right)(x_3 - x_2)$$
$$g_2(x_2,x_3) = \left(-\frac{b_D \log(x_2)}{a_D} + \frac{\alpha_2}{a_D}\log\left(\frac{x_3 - x_2}{M_2 - x_3}\right)\right)(x_3 - x_2)$$

We can rewrite $f_2(x_2, x_3)$ as:

$$f_2(x_2,x_3) = g_1(x_2,x_3) + g_2(x_2,x_3) - \left(d_2 + \frac{c_D}{a_D} - \frac{b_D \log(M_2)}{a_D}\right)(x_3 - x_2)$$

At this point, we just need to prove joint convexity of g_1 and g_2. In particular, we will show that the principal components of the Hessian of g_1 and g_2 are positive. For the first term, the first second derivative is given by:

$$\frac{d^2 g_1}{dx_2^2} = k''(x_2) - 2k'(x_t) + \frac{\alpha_1}{a_D(x_3 - x_2)} > 0$$

This is positive simply by verifying each term. Note here that $k''(x_2) > 0$ and $k'(x_2) < 0$ because of the decreasing and convex nature of the learning function given by $b_I < 0$.

It remains to show that the determinant of the Hessian of g_1 is positive:

$$\frac{d^2 g_1}{dx_2^2}\frac{d^2 g_1}{dx_3^2} - \left(\frac{d^2 g_1}{dx_2 dx_3}\right)^2 \geq 0$$

With some algebraic manipulations, we can show that the expression above reduces to:

$$k''(x_2)\frac{\alpha_1}{a_D}\left(\frac{M_2 - x_2}{M_2 - x_3}\right)^2 - 2k'(x_2)\frac{\alpha_1}{a_D}\frac{M_2 - x_2}{(M_2 - x_3)^2} - (k'(x_2))^2 \geq 0$$

In particular, $k''(x_2) = \frac{b_I(b_I-1)}{x_2^2}k(x_2)$ and $k'(x_2) = \frac{b_I}{x_2}k(x_2) = \frac{x_2}{b_I-1}k''(x_2)$. Also $k'(x_2)^2 = \frac{b_I}{b_I-1}k''(x_2)$ Then the expression above becomes:

$$k''(x_2)\frac{\alpha_1}{a_D}\left(\frac{M_2 - x_2}{M_2 - x_3}\right)^2 - 2\frac{x_2}{b_I - 1}k''(x_2)\frac{\alpha_1}{a_D}\frac{M_2 - x_2}{(M_2 - x_3)^2} - \frac{b_I}{b_I - 1}k''(x_2) \geq 0$$

This can be reduced to:

$$\frac{\alpha_1}{a_D}\frac{M_2 - x_2}{(M_2 - x_3)^2}\left(M_2 - x_2 - \frac{2x_2}{b_I - 1}\right) \geq \frac{b_I}{b_I - 1}$$

In particular $-\frac{2x_2}{b_I-1} > 0$, then $\frac{M_2-x_2}{(M_2-x_3)^2}\left(M_2 - x_2 - \frac{2x_2}{b_I-1}\right) > \left(\frac{M_2-x_2}{M_2-x_3}\right)^2 > 1$ where the last inequality comes from $M_2 > x_3 > x_2$ of Assumption 5.a. Also from Assumption 5.b, we have that $1 > \frac{b_I}{b_I-1}$. Therefore the equation we want to prove can be implied if $\frac{\alpha_1}{a_D} \geq 1$, since:

$$\frac{\alpha_1}{a_D}\frac{M_2 - x_2}{(M_2 - x_3)^2}\left(M_2 - x_2 - \frac{2x_2}{b_I - 1}\right) > \frac{\alpha_1}{a_D} \geq 1 > \frac{b_I}{b_I - 1}$$

Let $\alpha_1 = a_D$ and we have proven the joint convexity of g_1. For the joint convexity of g_2 we need to do a similar algebraic manipulation. Note that the first term in g_2 is $-b_D \log(x_2)/a_D$ is the equivalent of $k(x_2)$ in the proof of g_1. The first component of the Hessian matrix will be:

$$\frac{d^2 g_2}{dx_2^2} = b_D/(a_D x_2^2) + 2b_D/(a_D x_2) + \frac{\alpha_2}{a_D(x_3 - x_2)} > 0$$

This is also positive. The determinant of the Hessian of $g_2(x_2, x_3)$ can be expressed as:

$$(a_D/b_D)(-b_D/(a_D x_2))^2\frac{\alpha_2}{a_D}\left(\frac{M_2 - x_2}{M_2 - x_3}\right)^2 - 2(-b_D/(a_D x_2))\frac{\alpha_2}{a_D}\frac{M_2 - x_2}{(M_2 - x_3)^2} - (-b_D/(a_D x_2))^2 \geq 0$$

This is equivalent to:

$$(a_D/b_D)(-b_D/(a_D x_2))\frac{\alpha_2}{a_D}\left(\frac{M_2 - x_2}{M_2 - x_3}\right)^2 - 2\frac{\alpha_2}{a_D}\frac{M_2 - x_2}{(M_2 - x_3)^2} - (-b_D/(a_D x_2)) \leq 0$$

$$(-b_D/(a_D x_2))\left(\frac{(a_D/b_D)\alpha_2}{a_D}\left(\frac{M_2 - x_2}{M_2 - x_3}\right)^2 - 1\right) - 2\frac{\alpha_2}{a_D}\frac{M_2 - x_2}{(M_2 - x_3)^2} \leq 0$$

The expressions above will be true if $\left(\frac{(a_D/b_D)\alpha_2}{a_D}\left(\frac{M_2-x_2}{M_2-x_3}\right)^2 - 1\right) \geq 0$, which can be reformulated into:

$$\frac{\alpha_2}{b_D} \geq \left(\frac{M_2 - x_3}{M_2 - x_2}\right)^2$$

Since $M_2 > x_3 > x_2$, we need to show only that $\alpha_2 \geq b_D$, then:

$$\frac{\alpha_2}{b_D} \geq 1 > \left(\frac{M_2 - x_3}{M_2 - x_2}\right)^2$$

In particular we had chosen $\alpha_2 = 1 - \alpha_1 = 1 - a_D$. From Assumption 5.c, we have that $a_D + b_D \leq 1$, which implies that $\alpha_2 \geq b_D$, which concludes our proof that $g_2(x_2, x_3)$ is jointly convex in (x_2, x_3). Together with the earlier proofs that g_1 and f_1 are jointly convex as well, we have that $J_1(x_2, x_3)$ is jointly convex.

Appendix C: Proof of Theorem 1

Consider the 4-period problem ($T = 4$):

$$\begin{aligned} Cost_1(x_1, x_4) = & \min_{x_2, x_3} r_1(x_1, x_2)(x_2 - x_1) + \delta r_2(x_2, x_3)(x_3 - x_2) + \delta^2 r_3(x_3, x_4)(x_4 - x_3) \\ & \min_{x_3} Cost_1(x_1, x_3) + \delta^2 r_3(x_3, x_4)(x_4 - x_3) \end{aligned}$$

We need to show that $Cost_1(x_1, x_3) + \delta^2 r_3(x_3, x_4)(x_4 - x_3)$ is jointly convex in (x_3, x_4). We already know that $Cost_1(x_1, x_3)$ is convex in x_3 from Corollary 1. Joint convexity of $r_3(x_3, x_4)(x_4 - x_3)$ can be proven in the say way as we did for $f_2(x_2, x_3)$ (see Appendix B). The combination of these results implies that $Cost_1(x_1, x_4)$ is convex in x_4. Define $Cost_1(x_1, x_T)$:

$$Cost_1(x_1, x_T) = \min_{x_{T-1}} Cost_1(x_1, x_{T-1}) + \delta^{T-2} r_{T-1}(x_{T-1}, x_T)(x_T - x_{T-1})$$

By induction we can easily show that $Cost_1(x_1, x_T)$ is convex in x_T.

Appendix D: Proof of Proposition 1

From Corollary 2, we have that $\frac{dx_2^*}{dx_3}(x_3) = -\frac{d^2 J}{dx_2 dx_3}(x_2^*(x_3), x_3)\left(\frac{d^2 J}{dx_2^2}(x_2^*(x_3), x_3)\right)^{-1}$. In particular:

$$\frac{d^2 J}{dx_2 dx_3}(x_2^*(x_3), x_3) = \delta\left(k'(x_2) - \frac{b_D}{a_D x_2} - \frac{1}{a_D(M_2 - x_3)} - \frac{1}{a_D(x_3 - x_2)}\right) < 0$$

Furthermore,

$$\frac{d^2 J}{dx_2^2}(x_2^*(x_3), x_3) = \delta\left(k''(x_2) - 2k'(x_2) + \frac{b_D}{a_D x_2^2} + 2\frac{b_D}{a_D x_2} - \frac{1}{a_D(x_3 - x_2)}\right) + \frac{(M_1 - x_1)^2}{a_D(x_2 - x_1)(M_1 - x_2)^2} > 0$$

These equations will directly imply $\frac{dx_2^*}{dx_3}(x_3) > 0$. In order to obtain $\frac{dx_2^*}{dx_3}(x_3) < 1$, we need to show $\frac{d^2 J}{dx_2^2}(x_2^*(x_3), x_3) > -\frac{d^2 J}{dx_2 dx_3}(x_2^*(x_3), x_3)$, which can be seen by analyzing each term separately:

$$k''(x_2) - 2k'(x_2) > -k'(x_2)$$

$$\frac{b_D}{a_D x_2^2} + 2\frac{b_D}{a_D x_2} > \frac{b_D}{a_D x_2}$$

$$\frac{(M_1 - x_1)^2}{a_D(x_2 - x_1)(M_1 - x_2)^2} \geq \frac{\delta}{a_D(M_2 - x_3)}$$

The first two expressions are easy to verify. The third one is generally true from the dimensions of the problem we are dealing with, but we introduce one more assumption to be rigorous: $x_2 - x_1 \leq M_2 - x_3$. We know that $\frac{(M_1 - x_1)^2}{(M_1 - x_2)^2} > 1 > \delta$, therefore with the additional assumption $x_2 - x_1 \leq M_2 - x_3$, we conclude that $\frac{dx_2^*}{dx_3}(x_3) < 1$.

Appendix E: Proof of Theorem 2

Let x_t^B be the baseline adoption path, where $r_t = 0$ for all $t \geq 1$. Pick some arbitrary time $T \geq 3$ and set the adoption target for the optimization model (10) at x_T^B and let x_t^* be the new optimal adoption path, with negative system cost $Cost_1(x_1, x_T^B) < 0$, and the optimal rebate path is non-increasing $r_t(x_t^*, x_{t+1}^*) \geq r_{t+1}(x_{t+1}^*, x_{t+2}^*)$.

If the system cost is negative, there must be a negative rebate along the rebate path. If all rebates are negative, then the new adoption path x_t^* is strictly below the baseline path x_t^B, which doesn't reach the target adoption at x_T^B, therefore is a contradiction. Then there must be a rebate that is positive as well.

From monotonicity of rebates, we can infer that the first rebate must be positive and the last rebate negative and that there is a cross point in time $1 \leq \bar{t} \leq T$ such that all rebates $r_t > 0$ for all $t \leq \bar{t}$ and all rebates are non-positive $r_t \leq 0$ for $t > \bar{t}$. We can further infer that if the adoption level at the cross point is lower than the baseline level, $x_{\bar{t}}^* < x_{\bar{t}}^B$, then from the non-positive rebates after the cross point we cannot reach the target adoption x_T^B, which is a contradiction. For the same reason, we can infer that $x_t^* \geq x_t^B$ for $t \geq \bar{t}$. Since all the rebates are positive before the cross point, we can further infer that $x_t^* \geq x_t^B$ for $t \leq \bar{t}$ as well. This implies that the new the adoption path dominates the baseline path at every time step: $x_t^* \geq x_t^B$.

From the definition of the welfare function, we have that:

$$Welfare(x_1, ..., x_T) = \sum_{t=1}^{T-1} \left[Benefit_t(x_t) - \delta^{t-1} r_t(x_t, x_{t+1})(x_{t+1} - x_t) \right]$$

We know that:

$$Cost_1(x_1, x_T^B) = \sum_{t=1}^{T-1} \left[\delta^{t-1} r_t(x_t^*, x_{t+1}^*)(x_{t+1}^* - x_t^*) \right] < 0 = \sum_{t=1}^{T-1} \left[\delta^{t-1} r_t(x_t^B, x_{t+1}^B)(x_{t+1}^B - x_t^B) \right]$$

Furthermore, we know that $x_t^* \geq x_t^B$ and the welfare benefit functions are increasing, therefore:

$$Benefit_t(x_t^*) \geq Benefit_t(x_t^B)$$

By adding all the welfare benefit terms and the cost, we obtain the original expression and conclude the proof:

$$Welfare(x_1, x_2^*, ..., x_{T-1}^*, x_T^B) > Welfare(x_1, x_2^B ..., x_{T-1}^B, x_T^B)$$

References

Acemoglu, Daron, Philippe Aghion, Leonardo Bursztyn, David Hemous. 2010. The environment and directed technical change Working paper, Harvard University.

Allon, G., A. Federgruen, M. Pierson. 2010a. How much is a reduction of your customers wait worth? an empirical study of the fast-food drive-thru industry based on structural estimation methods. *to appear on M&SOM* .

Allon, G., A. Federgruen, M. Pierson. 2010b. Price competition under multinomial logit demand function with random coefficients. *to appear on Econometrica* .

Aral, Sinan, Lev Muchnik, Arun Sundararajan. 2009. Distinguishing influence-based contagion from homophily-driven diffusion in dynamic networks. *Proceedings of the National Academy of Sciences* **106**(51) 21544–21549.

Arrow, Kenneth J. 1962. The economic implications of learning by doing. *The Review of Economic Studies* **29**(3) pp. 155–173.

Bajari, Patrick, C. Lanier Benkard, Jonathan Levin. 2007. Estimating dynamic models of imperfect competition. *Econometrica* **75**(5) 1331–1370.

Bass, F. M. 1969. A new product growth for model consumer durables. *Management Science* **15**(5) 215–227.

Ben-Akiva, M., S.R. Lerman. 1993. *Discrete choice analysis: theory and application to travel demand*. MIT, Cambridge, MA.

Benthem, A., K. Gillingham, J. Sweeney. 2008. Learning-by-doing and the optimal solar policy in california. *The Energy Journal* **29**(3) 131–151.

Berndt, Ernst R., Robert S. Pindyck, Pierre Azoulay. 2003. Consumption externalities and diffusion in pharmaceutical markets: Antiulcer drugs. *The Journal of Industrial Economics* **51**(2) 243–270.

Berry, Steven, James Levinsohn, Ariel Pakes. 1995. Automobile prices in market equilibrium. *Econometrica* **63**(4) pp. 841–890.

Berry, Steven T. 1994. Estimating discrete-choice models of product differentiation. *The RAND Journal of Economics* **25**(2) 242–262.

Bertsekas, D. 1995. *Nonlinear Optimization*. Athena Scientific.

Bhandari, Ramchandra, Ingo Stadler. 2009. Grid parity analysis of solar photovoltaic systems in germany using experience curves. *Solar Energy* **83**(9) 1634 – 1644.

BMU. 2008. Lead study 2008 - further development of the strategy to increase the use of renewable energies within the context of the current climate protection goals of germany and europe. Study commissioned by the German Federal Ministry for the Environment, Nature Conservation and Nuclear Safety (BMU).

Boyd, S., L. Vandenbergh. 2004. *Convex Optimization*. Cambridge University Press.

Cachon, Gérard P., Pnina Feldman. 2010. Dynamic versus static pricing in the presence of strategic consumers. *working paper* .

Cachon, Gérard P., Robert Swinney. 2009. Purchasing, pricing, and quick response in the presence of strategic consumers. *Management Science* **55**(3) 497511.

Choi, Jay Pil, Marcel Thum. 1998. Market structure and the timing of technology adoption with network externalities. *European Economic Review* **42**(2) 225 – 244.

Chou, C., O. Shy. 1990. Network effects without network externalities. *International Journal of Industrial Organization* **8**(2) 259 – 270.

Cox, D. R. 1972. Regression models and life-tables. *Journal of the Royal Statistical Society. Series B (Methodological)* **34**(2) 187–220.

Doganoglu, Toker, Lukasz Grzybowski. 2007. Estimating network effects in mobile telephony in germany. *Information Economics and Policy* **19**(1) 65 – 79.

Duke, R., D. Kammen. 1999. The economics of energy market transformation programs. *Energy Journal* .

EEG. 2007. Renewable energy sources act (EEG) - progress report 2007 .

Ellison, Glenn, Drew Fudenberg. 1995. Word-of-mouth communication and social learning. *The Quarterly Journal of Economics* **110**(1) 93–125.

EPIA. 2009. Global market outlook for photovoltaics until 2013 .

Ericson, Richard, Ariel Pakes. 1995. Markov-perfect industry dynamics: A framework for empirical work. *The Review of Economic Studies* **62**(1) 53–82.

EUC. 1997. Energy for the future: renewable sources of energy white paper for a community strategy and action plan .

Farrell, Joseph, Garth Saloner. 1985. Standardization, compatibility, and innovation. *The RAND Journal of Economics* **16**(1) 70–83.

Farrell, Joseph, Garth Saloner. 1986. Installed base and compatibility: Innovation, product preannouncements, and predation. *The American Economic Review* **76**(5) 940–955.

Frondel, M., N. Ritter, C. Schmidt. 2008. Germany's solar cell promotion: Dark clouds on the horizon. *Ruhr Economic Papers* .

Geroski, P. A. 2000. Models of technology diffusion. *Research Policy* **29**(4-5) 603 – 625.

Goldenberg, Jacob, Barak Libai, Eitan Muller. 2010. The chilling effects of network externalities. *International Journal of Research in Marketing* **27**(1) 4 – 15.

Goolsbee, Austan, Peter J. Klenow. 2002. Evidence on learning and network externalities in the diffusion of home computers. *Journal of Law and Economics* **45**(2) 317–343.

Harmon, C. 2000. Experience curves of photovoltaic technology. .

Hauser, John, Gerard J. Tellis, Abbie Griffin. 2006. Research on innovation: A review and agenda for marketing science. *Marketing Science* **25**(6) 687 – 717.

Helsen, Kristiaan, David C. Schmittlein. 1993. Analyzing duration times in marketing: Evidence for the effectiveness of hazard rate models. *Marketing Science* **12**(4) 395–414.

IEA. 2000. Experience curves for energy technology policy .

IEA. 2004. Renewable energy - market & policy trends in IEA countries .

Ireland, N., P. Stoneman. 1986. Technological diffusion, expectations and welfare. *Oxford Economic Papers* **38**(2) pp. 283–304.

Islegen, O., E.L. Plambeck. 2010. Capacity leadership in supply chains with asymmetric demand information and noncontractible capacity .

Jager, Wander. 2006. Stimulating the diffusion of photovoltaic systems: A behavioural perspective. *Energy Policy* **34**(14) 1935 – 1943.

Jager-Waldau, Arnulf. 2007. Photovoltaics and renewable energies in europe. *Renewable and Sustainable Energy Reviews* **11**(7) 1414 – 1437.

Joskow, Paul L., Nancy L. Rose. 1989. The effects of economic regulation. *Handbook of Industrial Organization* **2** 1449 – 1506.

Kapur, Sandeep. 1995. Technological diffusion with social learning. *The Journal of Industrial Economics* **43**(2) 173–195.

Katz, Michael L., Carl Shapiro. 1986. Technology adoption in the presence of network externalities. *The Journal of Political Economy* **94**(4) 822–841.

Katz, Michael L., Carl Shapiro. 1992. Product introduction with network externalities. *The Journal of Industrial Economics* **40**(1) 55–83.

Mahajan, Vijay, Eitan Muller, Frank M. Bass. 1990. New product diffusion models in marketing: A review and directions for research. *The Journal of Marketing* **54**(1) 1–26.

McDonald, A., L. Schrattenholze. 2001. Learning rates for energy technologies. *Energy Policy* .

Musalem, Andres, Marcelo Olivares, Eric T. Bradlow, Christian Terwiesch, Daniel Corsten. 2010. Structural estimation of the effect of out-of-stocks. *Management Science* **56**(7) 1180–1197.

Nemet, G., E. Baker. 2008. Demand subsidies versus r&d: Comparing the uncertain impacts of policy on a pre-commercial low-carbon energy technology. Working Paper Series, La Follette School of Public Affairs, University of Wisconsin-Madison.

Nemet, Gregory F. 2006. Beyond the learning curve: factors influencing cost reductions in photovoltaics. *Energy Policy* **34**(17) 3218 – 3232.

PVPS. 2008. IEA PVPS - annual report .

Rao, K. Usha, V.V.N. Kishore. 2010. A review of technology diffusion models with special reference to renewable energy technologies. *Renewable and Sustainable Energy Reviews* **14**(3) 1070 – 1078.

Reichmuth, M., I. Erfurt, A. Schiffler, T. Kelm, M. Schmidt. 2010. Analyse zur mglichen anpassung der eeg-vergtung fr photovoltaik-anlagen .

Rogers, E. M. 1962. *Diffusion of Innovations*. Free Press, New York.

Sanden, B. 2005. The economic and institutional rationale of pv subsidies. *Solar Energy* **78**(2) 137 – 146. ISES Solar World Congress 2003.

Schaeffer, G., E. Alsema, A. Seebregts, L. Beurskens, H. de Moor, W. van Sark, M. Durstewitz, M. Perrin, P. Boulanger, H. Laukamp, C. Zuccaro. 2004. Learning from the sun - analysis of the use of experience curves for energy policy purposes: The case of photovoltaic power. final report of the photex project. .

Sderholm, Patrik, Thomas Sundqvist. 2007. Empirical challenges in the use of learning curves for assessing the economic prospects of renewable energy technologies. *Renewable Energy* **32**(15) 2559 – 2578.

Shurmer, M., P. Swann. 1995. An analysis of the process generating de facto standards in the pc spreadsheet software market. *Journal of Evolutionary Economics* **5**(2) 119.

Stoneman, Paul, Paul Diederen. 1994. Technology diffusion and public policy. *The Economic Journal* **104**(425) pp. 918–930.

Su, Xuanming, Fuqiang Zhang. 2008. Strategic Customer Behavior, Commitment, and Supply Chain Performance. *Management Science* **54**(10) 1759–1773.

Swann, G. M. Peter. 2002. The functional form of network effects. *Information Economics and Policy* **14**(3) 417 – 429.

Taylor, Terry A., Erica L. Plambeck. 2007. Supply Chain Relationships and Contracts: The Impact of Repeated Interaction on Capacity Investment and Procurement. *Management Science* **53**(10) 1577–1593.

Train, K. 2003. *Discrete Choice Methods with Simulation*. Cambridge University Press.

Tucker, Catherine. 2008. Identifying formal and informal influence in technology adoption with network externalities. *Management Science* **54**(12) 2024 – 2038.

Ulu, Canan, James E. Smith. 2009. Uncertainty, Information Acquisition, and Technology Adoption. *Operations Research* **57**(3) 740–752.

van Sark, W., E. Alsema, H. Junginger, H. de Moor, G. Schaeffer. 2008. Accuracy of progress ratios determined from experience curves: the case of crystalline silicon photovoltaic module technology development. *Progress in Photovoltaics: Research and Applications* **16**(5).

Vives, Xavier. 1997. Learning from others: A welfare analysis. *Games and Economic Behavior* **20**(2) 177 – 200.

Wand, R., F. Leuthold. 2010. Feed-in tariffs for photovoltaics: Learning by doing in germany? Working Paper.

Weintraub, G., C. Benkard, B. Van Roy. 2008. Markov perfect industry dynamics with many firms. *Econometrica* **76**(6) 1375–1411.

Wissing, L. 2006. Exchange and dissemination of information on pv power systems. *National Survey Report of PV Power Applications in Germany* Prepared on behalf of BMU German Federal Ministry for the Environment, Nature Conservation and Nuclear Safety.

Yu, C., W. van Sark, E. Alsema. 2009. Incorporation of input prices and scale effects in a multi-factor learning curve for photovoltaic technology. *Proceedings of the 10th IAEE European Conference, Vienna.* .