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Abstract

While many analyses of monetary policy consider only a target for a short-term nominal interest rate, other dimensions of policy have recently been of greater importance: changes in the supply of bank reserves, changes in the assets acquired by central banks, and changes in the interest rate paid on reserves. We first extend a standard New Keynesian model to allow a role for the central bank's balance sheet in equilibrium determination and then consider the connections between these alternative policy dimensions and traditional interest rate policy. We distinguish between "quantitative easing" in the strict sense and targeted asset purchases by a central bank, arguing that, according to our model, while the former is likely to be ineffective at all times, the latter can be effective when financial markets are sufficiently disrupted. Neither is a perfect substitute for conventional interest rate policy, but purchases of illiquid assets are particularly likely to improve welfare when the zero lower bound on the policy rate is reached. We also consider optimal policy with regard to the payment of interest on reserves; in our model, this requires that the interest rate on reserves be kept near the target for the policy rate at all times.

Key words: credit policy, quantitative easing, zero lower bound

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The recent global financial crisis has confronted central banks with a number of questions beyond the scope of standard accounts of the theory of monetary policy. Monetary policy is ordinarily considered solely in terms of the choice of an operating target for a short-term nominal interest rate, such as the federal funds rate in the case of the Federal Reserve. Yet during the recent crisis, other dimensions of policy have occupied much of the attention of central bankers. One is the question of the appropriate size of the central bank's balance sheet. In fact, the Fed's balance sheet has grown dramatically in size since the fall of 2008 (Figures 1 and 2).

As shown in Figure 1, the component of the Fed's liabilities constituted by reserves held by depository institutions has changed in an especially remarkable way: by the fall of 2008 reserves were more than 100 times larger than they had been only a few months earlier. This explosive growth has led some commentators to suggest that the main instrument of US monetary policy has changed, from an interest-rate policy to one often described as "quantitative easing." Does it make sense to regard the supply of bank reserves (or perhaps the monetary base) as an alternative or superior operating target for monetary policy? Does this (as some would argue) become the only important monetary policy decision once the overnight rate (the federal funds rate) has reached the zero lower bound, as it effectively has in the US since December 2008? And now that the Federal Reserve has legal authorization to pay interest on reserves (under the Emergency Economic Stabilization Act of 2008), how should this additional potential dimension of policy be used?

The past two years have also seen dramatic developments with regard to the composition of the asset side of the Fed's balance sheet (Figure 2). Whereas the Fed had largely held Treasury securities on its balance sheet prior to the fall of 2007, other kinds of assets — a variety of new "liquidity facilities", new programs under which the Fed essentially became a direct lender to certain sectors of the economy, and finally targeted purchases of certain kinds of assets, including more than a trillion dollars' worth of mortgage-backed securities — have rapidly grown in importance, and decisions about the management of these programs have occupied much of the attention of policymakers during the recent period. How should one think about the aims of these programs, and the relation of this new component of Fed policy to traditional interest-rate policy? Is Federal Reserve credit policy a substitute

for interest-rate policy, or should it be directed to different goals than those toward which interest-rate policy is directed?

These are clearly questions that a theory of monetary policy adequate to our present circumstances must address. Yet not only have they been the focus of relatively little attention until recently, but the very models commonly used to evaluate the effects of alternative prescriptions for monetary policy have little to say about them. Many models used for monetary policy analysis — both theoretical models used in normative discussions of ideal monetary policy commitments, and quantitative models used for numerical simulation of alternative policies — abstract altogether from the central bank’s balance sheet, simply treating a short-term nominal interest rate as if it were under the direct control of the monetary authorities, and analyzing how that interest rate should be adjusted.¹ But such a framework rules out the kinds of questions that have recently preoccupied central bankers from the start.

In this paper, we extend a basic New Keynesian model of the monetary transmission mechanism to explicitly include the central bank’s balance sheet as part of the model. In addition to making more explicit the ways in which a central bank is able to (indirectly) exert control over the policy rate, the extended model allows us to address questions about other dimensions of policy of the sort just posed. In order to make these questions non-trivial, we also introduce non-trivial heterogeneity in spending opportunities, rather than adopting the familiar device of the “representative household,” so that financial intermediation matters for the allocation of resources; we introduce imperfections in private financial intermediation, and the possibility of disruptions to the efficiency of intermediation, for reasons taken here as exogenous, so that we can examine how such disturbances affect the desirability of central-bank credit policy; and we allow central-bank liabilities to supply transactions services, so that they are not assumed to be perfect substitutes for privately-issued financial instruments of similar maturity and with similar state-contingent payoffs. Finally, we consider the conduct of policy both when the zero lower bound on the policy rate is not a binding constraint, and also when it is.

In section 1, we begin with a general discussion of whether (and when) one should expect aspects of the central bank’s balance sheet to matter for equilibrium determination. This

¹This approach is developed in detail in Woodford (2003).

is intended both to motivate our modeling exercise, by explaining what features a model must have in order for policies affecting the balance sheet to be of possible significance, and to introduce some important distinctions among alternative dimensions of policy. Section 2 outlines the structure of our model, with primary attention to the way that we model financial intermediation and the policy choices available to the central bank. Section 3 then uses the model to discuss changes in the supply of bank reserves as a dimension of policy, and the related question of the rate of interest that should be paid on reserves. Section 4 turns to the question of the optimal composition of the central bank's asset portfolio, considering the conditions under which the traditional "Treasuries only" policy would be optimal in the context of our model. Section 5 then considers the optimal size and duration of central-bank credit policy in those cases where "Treasuries only" is not the optimal policy, and section 6 concludes.

1 When Does the Central-Bank Balance Sheet Matter?

It might be thought that monetary policy analysis would *have* to be involve explicit consideration of the central bank's balance sheet, at least to the extent that one believes in the importance of general-equilibrium analysis. Yet monetary DSGE models often abstract from any discussion of the central bank's balance sheet. In fact, this is not only possible (in the sense that the models have at least a logically consistent structure), but is quite innocuous, under a certain idealized view of the functioning of financial markets. Neither the size nor the composition of the central bank's balance sheet matter for equilibrium prices or quantities *except* because of financial imperfections. This is important, both to understand why additional dimensions of policy may suddenly become relevant when the smooth functioning of financial markets can no longer be taken for granted, and to understand why we emphasize certain types of frictions in our analysis below. The introduction of credit frictions requires a significant complication of our analysis, and before undertaking this modeling effort, it may be useful to clarify why our assumptions about credit frictions are essential to the conclusions

that we obtain.

1.1 An Irrelevance Result

It is often supposed that open-market purchases of securities by the central bank must inevitably affect the market prices of those securities (and hence other prices and quantities as well), through what is called a “portfolio-balance effect”: if the central bank holds less of certain assets and more of others, then the private sector is forced (as a requirement for equilibrium) to hold more of the former and less of the latter, and a change in the relative prices of the assets will almost always be required to induce the private parties to change the portfolios that they prefer. In order for such an effect to exist, it is thought to suffice that private parties not be perfectly indifferent between the two types of assets; and there are all sorts of reasons why differences in the risky payoffs associated with different assets should make them not perfect substitutes, even in a world with frictionless financial markets.²

But this doctrine is inconsistent with the general-equilibrium theory of asset prices, at least to the extent that financial markets are modeled as frictionless. It is clearly inconsistent with a representative-household asset pricing theory (even though the argument sketched above makes no obvious reference to any heterogeneity on the part of private investors). In the representative-household theory, the market price of any asset should be determined by the present value of the random returns to which it is a claim, where the present value is calculated using an asset pricing kernel (stochastic discount factor) derived from the representative household’s marginal utility of income in different future states of the world. Insofar as a mere re-shuffling of assets between the central bank and the private sector should not change the real quantity of resources available for consumption in each state of the world, the representative household’s marginal utility of income in different states of the world should not change. Hence the pricing kernel should not change, and the market price of one unit of a given asset should not change, either, assuming that the risky returns to which the asset represents a claim have not changed.

The flaw in the “portfolio-balance” theory is the following. It assumes that if the private

²Explanations by central banks of what they believe is accomplished by targeted asset purchases frequently invoke this mechanism.

sector is forced to hold a portfolio that includes more exposure to a particular risk — say, a low return in the event of a real-estate crash — then private investors’ willingness to hold that particular risk will be reduced: investors will anticipate a higher marginal utility of income in the state in which the real-estate crash occurs, and so will pay less than before for securities that have especially low returns in that state. But the fact that the central bank takes the real-estate risk onto its own balance sheet, and allows the representative household to hold only securities that pay as much in the event of a crash as in other states, does not make the risk disappear from the economy. The central bank’s earnings on its portfolio will be lower in the crash state as a result of the asset exchange, and this will mean lower earnings distributed to the Treasury, which will in turn mean that higher taxes will have to be collected by the government from the private sector in that state; so the representative household’s after-tax income will be just as dependent on the real-estate risk as before. This is why the asset pricing kernel does not change, and why asset prices are unaffected by the open-market operation.³

The irrelevance result is easiest to derive in the context of a representative-household model, but in fact it does not depend on the existence of a representative household, nor upon the existence of a complete set of financial markets. All that one needs for the argument are the assumptions that (i) the assets in question *are valued only for their pecuniary returns* — they may not be perfect substitutes from the standpoint of investors, owing to different risk characteristics, but not for any other reason — and that (ii) *all investors can purchase arbitrary quantities of the same assets at the same (market) prices*. Under these assumptions, the irrelevance of central-bank open-market operations is essentially a Modigliani-Miller result, as noted by Wallace (1981). If the central bank buys more of asset x by selling shares of asset y , private investors should wish purchase more of asset y and divest themselves of asset x , by exactly the amounts that undo the effects of the central bank’s trades. The reason that they optimally choose to do this is in order to hedge the additional tax/transfer income risk that they take on as a result of the change in the central bank’s portfolio. If share θ_h of the returns on the central bank’s portfolio are distributed to household h , where

³Eggertsson and Woodford (2003) show in the context of a representative-household model that it does not matter which assets a central bank purchases in its open-market operations, on precisely this ground.

the $\{\theta_h\}$ are a set of weights that sum to 1, then household h should choose a trade that cancels exactly fraction θ_h of the central bank's trade, in order to afford exactly the same state-contingent consumption stream as before. Summing over all households, the private sector chooses trades that in aggregate precisely cancel the central bank's trade. The result obtains even if different households have very different attitudes toward risk, different time profiles of income, different types of non-tradeable income risk that they need to hedge, and so on, and regardless of how large or small the set of marketed securities may be. One can easily introduce heterogeneity of the kind that is often invoked as an explanation of time-varying risk premia without this implying that any "portfolio-balance" effects of central-bank transactions should exist.

As Wallace (1981) notes, this implies that *both* the size and the composition of the central-bank balance sheet should be irrelevant for market equilibrium in a world with frictionless financial markets (more precisely, a world in which the two postulates hold). This does *not*, however, mean, as is sometimes thought, that *monetary policy* is irrelevant in such a world; it simply means that monetary policy cannot be implemented through open-market operations. Control of a short-term nominal interest rate by the central bank remains possible in the frictionless environment. The central bank is still free to determine the nominal interest rate on overnight balances at the central bank as an additional dimension of policy (alongside its decisions about the quantity of liabilities to issue and the particular types of assets that it buys with them).⁴ This interest rate must then be linked in equilibrium to other short-term interest rates, through arbitrage relations; and hence the central bank can determine the level of short-term nominal interest rates in general. Moreover, the central bank's adjustment of nominal interest rates matters for the economy. Even in an endowment economy with flexible prices for all goods, the central bank's interest-rate policy can determine the evolution of the general level of prices in the economy; in a production economy with sticky prices and/or wages, it can have important real effects as well.⁵

⁴The existence of these three independent dimensions of central-bank policy is discussed further below in section 2.3, in the context of an explicit model.

⁵Both the way in which the central bank can determine the level of short-term interest rates in a frictionless economy, and the macroeconomic implications of interest-rate policy in such models, are treated in detail in Woodford (2003, chaps. 2, 4).

In analyzing monetary policy options for a world of this kind, there would be no need to include the central bank's balance sheet in one's model at all: it would suffice that the short-term nominal interest rate be one of the asset prices in the model, and that it be treated as under the control of a monetary authority. This provides a potential justification for the use of "cashless" models in monetary policy analysis, in which no balance-sheet quantities at all appear.⁶ At the same time, the assumptions required for the irrelevance result are still fairly strong (even if not so special as discussions of "portfolio-balance" effects often seem to assume), and it worth considering the consequences of relaxing them.

1.2 Allowing a Transactions Role for Central-Bank Liabilities

Many readers of Wallace (1981) are likely to have found the result paradoxical, and doubted the practical relevance of the entire line of reasoning, for one reason in particular. Wallace's result implied, not only that exchanges of Treasuries for mortgage-backed securities by the Federal Reserve, holding fixed the overall size of the Fed's balance sheet, should have no effect, but also that increases in the supply of bank reserves as a result of open-market purchases of Treasuries should have no effect. Yet the latter kind of operation had long been routinely used by the Fed to bring about desired changes in the federal funds rate, as every undergraduate learns. The theory seemed patently inapplicable to the operations of actual central banks in actual market economies.

Moreover, it is clear that overnight balances at the Fed have often been held despite being dominated in rate of return; until October 2008, these balances earned a zero nominal return, while other overnight interest rates (such as the federal funds rate) were invariably higher, for reasons that cannot be attributed purely to default risk. A natural (and thoroughly

⁶Of course, no one has proposed that any actual economies literally satisfy the two postulates; in particular, some central-bank liabilities are clearly valued in ways that are inconsistent with the two postulates. The use of "cashless" models for practical monetary policy analysis accordingly requires a further argument, which is that the transactions frictions that account for the observed demand for base money are not likely to make a large quantitative difference for the structural relations that matter for the analysis of alternative interest-rate policies, even if they matter a great deal for the precise way in which a central bank is able to implement interest-rate policy. See, *e.g.*, McCallum (2000, 2001), Woodford (2003, chap. 2, sec. 3, and chap. 4, sec. 3), and Ireland (2004).

conventional) inference is that this particular asset is (or at least, has often been) held for reasons beyond its pecuniary return alone; we may suppose that reserves at the Fed (and base money more generally) supply transactions services, by relaxing constraints that would otherwise restrict the transactions in which the holders of the asset can engage. The existence of these non-pecuniary returns — which may be modeled using any of a variety of familiar devices — will invalidate the Wallace (1981) neutrality result, at least insofar as open-market purchases of securities that increase the supply of reserves are concerned.

We can introduce a transactions role for reserves, or for liabilities of the central bank more generally, however, while still entertaining the hypothesis that with regard to *all assets other than monetary liabilities of the central bank*, the two postulates still hold: assets other than “money” are valued only for their pecuniary returns, and all investors can purchase arbitrary quantities of any of these assets at the same (market) prices. In this case, a weaker irrelevance result for central-bank trades still applies. No open-market operation that changes the composition of the central bank’s asset portfolio, while keeping unchanged the outstanding volume of the monetary liabilities of the central bank, should have any effects on asset prices, goods prices, or the allocation of resources.⁷ Again, the argument is essentially a Modigliani-Miller theorem, and holds despite an arbitrary degree of heterogeneity in the situations of different households, and regardless of the size of the set of traded securities.

The result in this case validates the classic monetarist position: the supply of monetary liabilities by the central bank matters for macroeconomic equilibrium, but it does not matter at all what kinds of assets might “back” those liabilities on the other side of the central bank’s balance sheet, or how the base money gets to be in circulation. Hence a generation or two of texts in monetary economics have found it convenient to analyze monetary policy using models in which there is no central-bank balance sheet — merely a government printing press which creates additional “money” at a greater or lesser rate, which is then put in the hands of private parties, perhaps by dropping it from helicopters. Again, the omission is completely justifiable, if financial markets function efficiently enough for the two postulates to hold, except for the qualification regarding the special properties of “money.”

⁷This is the result obtained by Eggertsson and Woodford (2003), in the context of a representative-household model with transactions services represented by money in the utility function.

Under this view, there would be still be no ground for viewing targeted asset purchases as a relevant dimension of central-bank policy, though variations in the supply of monetary central-bank liabilities would matter. It might seem, then, Bernanke's (2009) assertion that the Federal Reserve's expansion of its balance sheet in the fall of 2008 represented "credit easing" rather than "quantitative easing" had matters backward: that the only real effect that should have been expected would have resulted from the expansion in the supply of bank reserves, regardless of the nature of the lending financed by that expansion in reserves.

Yet this is not the conclusion that we draw at all. Expansion of the supply of bank reserves stimulates aggregate demand under normal circumstances, because it has ordinarily been the means by which the Fed has lowered the federal funds rate; yet once the supply of reserves is sufficient to drive the funds rate essentially to zero (as has been the case since late in 2008), there is no reason to expect further increases in the supply of reserves to increase aggregate demand any further, as we explain in the context of an explicit model in section 3. Once banks are no longer foregoing any otherwise available pecuniary return in order to hold reserves, there is no reason to believe that reserves continue to supply any liquidity services at the margin; and if they do not, the Modigliani-Miller reasoning applies once again to open market operations that increase the supply of reserves, just as in the model of Wallace.

While this reasoning cannot correctly be applied to conclude that open-market securities purchases that increase the supply of reserves can *never* have any effect, it can quite plausibly be used to conclude that such purchases should have no effect once the opportunity cost of reserves has fallen to zero. (It is fairly obvious that an open-market purchase of riskless short-term Treasury securities by issuing similarly riskless nominal short-term liabilities of the Fed should have no effect, once there is no longer any shortage of cash. But if pure changes in the central bank's asset portfolio have no effect, then an increase in reserves to purchase short-term Treasuries should have the same effect as an increase in reserves to purchase some other asset, which effect must then be zero.) Hence it is not plausible that "quantitative easing" can be an effective strategy for providing further monetary stimulus once the zero lower bound is reached.

1.3 Do Targeted Asset Purchases Ever Matter?

The previous analysis suggests that Bernanke (2009) was right to be skeptical about the effectiveness of “quantitative easing.” But is there any more reason to expect “credit easing” to be of avail? Under the two postulates mentioned above, the answer is no. Yet there is some evidence suggesting that at least some of the Fed’s special credit facilities, and similar programs of other central banks, have affected asset prices⁸

As a simple example, Figure 3 shows the behavior of the spreads between yields on various categories of commercial paper and the one-month overnight interest-rate swap rate (essentially, a market forecast of the average federal funds rate over that horizon), over the period just before and after the introduction of the Fed’s Commercial Paper Funding Facility at the beginning of October 2008. (The darkest solid line shows the quantity of purchases of commercial paper by the Fed, which spikes up sharply at the introduction of the new facility.) The reason for the introduction of the new facility had been a significant disruption of the commercial paper market, indicated by the explosion of spreads in September 2008 for all four of the types of commercial paper shown in the figure. The figure also shows that spreads came back down again immediately with the introduction of the new facility, for three of the classes of paper (all except the A2/P2 paper) — these three series being precisely the ones for commercial paper of types that qualified for purchases under the CPFF. The spread for the A2/P2 paper instead remained high for several more months, though this spread as well has returned to more normal levels eventually, with the general improvement of financial conditions.⁹

Not only the sudden reduction in spreads for the other three types of paper, but the fact that spreads did *not* decline in the case of paper not eligible for purchase by the new facility, suggests that targeted asset purchases by the Fed did change the market prices of the assets in question. Hence some further modification of the two postulates is required in

⁸See, e.g., Ashcraft *et al.* (2010), Baba *et al.* (2006), Gagnon *et al.* (2010), Sarkar (2009) and Sarkar and Shrader (2010). For skeptical readings of the evidence, see instead Taylor (2009) and Stroebel and Taylor (2009).

⁹For further discussion of the crisis in the commercial paper market, this Fed program, and its effects, see Adrian *et al.* (2010) and Kacperczyk and Schnabl (2010).

order to allow a realistic analysis of the effects of programs of this kind. We propose that it is the assumption that all investors have equal opportunities to invest in all assets on the same terms that must be modified. If only certain specialists have the expertise required to invest in commercial paper, then developments that adversely affect the capital of those specialists (or their ability to fund themselves) can result in an increase in commercial paper yields relative to those on other instruments of similar maturity, as occurred in the fall of 2008. And central-bank purchases of commercial paper will not be offset by a corresponding reduction in private-sector purchases of that specific asset — even if the implications of the policy for state-contingent tax liabilities are correctly understood by everyone — if the parties whose state-contingent tax liabilities change are largely investors who *cannot* invest in commercial paper in any event, and so cannot hold less of it even if the central bank’s action causes them to bear more income risk that is correlated with commercial-paper returns.

Hence in developing a model to assess the conditions under which “unconventional” dimensions of monetary policy may be relevant, it is important that we not assume that all financial-market participants can costlessly trade the same set of financial instruments. (A fortiori, it is important that we not adopt the simplification of assuming a representative agent.) In the next section, we sketch a relatively simple model that possesses the minimal elements required for a non-trivial discussion of the issues raised in the introduction.¹⁰

We assume heterogeneity in the spending opportunities available to different households at any point in time, so that some will have a motive to borrow while others are willing to save; and we assume that the households without current urgent needs for funds lack the expertise required to directly extend credit themselves to the borrowing households, so that they must instead deposit funds with competitive intermediaries who are in turn able to offer loan contracts to the borrowing households. We also allow the central bank a choice between holding “liquid” assets (Treasury debt) that can also be held by saving households on the same terms, and “illiquid” assets (the debt of private borrowers) that can otherwise only be held by the specialist intermediaries. Finally, we allow the central bank to create liabilities (reserves) that supply liquidity services, and so may be held in equilibrium even

¹⁰Other recent examples of DSGE models that can be used to address some of these same issues include Gertler and Karadi (2009), Gertler and Kiyotaki (2010), and Del Negro *et al.* (2010).

when they earn a lower interest rate than that on Treasury debt.

We model these various frictions in relatively reduced-form ways; our interest here is not in illuminating the sources of the frictions, but in exploring their general-equilibrium consequences. In particular, we wish to understand the extent to which they give rise to a multiplicity of independent dimensions for central-bank policy, and the interrelations that exist between variations of policy along these separate dimensions.

2 A Model with Multiple Dimensions of Monetary Policy

Here we sketch the key elements of our model, which extends the model introduced in Cúrdia and Woodford (2009a) to introduce the additional dimensions of policy associated with the central bank's balance sheet. (The reader is referred to our earlier paper, and especially its technical appendix, for more details.)

2.1 Heterogeneity and the Allocative Consequences of Credit Spreads

Our model is a relatively simple generalization of the basic New Keynesian model used for the analysis of optimal monetary policy in sources such as Goodfriend and King (1997) and Woodford (2003). The model is still highly stylized in many respects; for example, we abstract from the distinction between the household and firm sectors of the economy, and instead treat all private expenditure as the expenditure of infinite-lived household-firms, and we similarly abstract from the consequences of investment spending for the evolution of the economy's productive capacity, instead treating all private expenditure as if it were non-durable consumer expenditure (yielding immediate utility, at a diminishing marginal rate).

We depart from the assumption of a representative household in the standard model, by supposing that households differ in their preferences. Each household i seeks to maximize a

discounted intertemporal objective of the form

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[u^{\tau_t(i)}(c_t(i); \xi_t) - \int_0^1 v^{\tau_t(i)}(h_t(j; i); \xi_t) dj \right], \quad (2.1)$$

where $\tau_t(i) \in \{b, s\}$ indicates the household's "type" in period t . Here $u^b(c; \xi)$ and $u^s(c; \xi)$ are two different period utility functions, each of which may also be shifted by the vector of aggregate taste shocks ξ_t , and $v^b(h; \xi)$ and $v^s(h; \xi)$ are correspondingly two different functions indicating the period disutility from working. As in the basic NK model, there is assumed to be a continuum of differentiated goods, each produced by a monopolistically competitive supplier; $c_t(i)$ is a Dixit-Stiglitz aggregator of the household's purchases of these differentiated goods. The household similarly supplies a continuum of different types of specialized labor, indexed by j , that are hired by firms in different sectors of the economy; the additively separable disutility of work $v^\tau(h; \xi)$ is the same for each type of labor, though it depends on the household's type and the common taste shock.

Each agent's type $\tau_t(i)$ evolves as an independent two-state Markov chain. Specifically, we assume that each period, with probability $1 - \delta$ (for some $0 \leq \delta < 1$) an event occurs which results in a new type for the household being drawn; otherwise it remains the same as in the previous period. When a new type is drawn, it is b with probability π_b and s with probability π_s , where $0 < \pi_b, \pi_s < 1, \pi_b + \pi_s = 1$. (Hence the population fractions of the two types are constant at all times, and equal to π_τ for each type τ .) We assume moreover that

$$u_c^b(c; \xi) > u_c^s(c; \xi)$$

for all levels of expenditure c in the range that occur in equilibrium. Hence a change in a household's type changes its relative impatience to consume, given the aggregate state ξ_t ; in addition, the current impatience to consume of all households is changed by the aggregate disturbance ξ_t . We also assume that the marginal utility of additional expenditure diminishes at different rates for the two types, as is also illustrated in the figure; type b households (who are borrowers in equilibrium) have a marginal utility that varies less with the current level of expenditure, resulting in a greater degree of intertemporal substitution of their expenditures in response to interest-rate changes. Finally, the two types are also assumed to differ in the marginal disutility of working a given number of hours; this difference is calibrated so

that the two types choose to work the same number of hours in steady state, despite their differing marginal utilities of income. For simplicity, the elasticities of labor supply of the two types are not assumed to differ.

The coexistence of the two types with differing impatience to consume creates a social function for financial intermediation. In the present model, as in the basic New Keynesian model, all output is consumed either by households or by the government; hence intermediation serves an allocative function only to the extent that there are reasons for the intertemporal marginal rates of substitution of households to differ in the absence of financial flows. The present model reduces to the standard representative-household model in the case that one assumes that $u^b(c; \xi) = u^s(c; \xi)$ and $v^b(h; \xi) = v^s(h; \xi)$.

We assume that most of the time, households are able to spend an amount different from their current income *only* by depositing funds with or borrowing from financial intermediaries, that the same nominal interest rate i_t^d is available to all savers, and that a (possibly) different nominal interest i_t^b is available to all borrowers,¹¹ independent of the quantities that a given household chooses to save or to borrow. For simplicity, we also assume that only one-period riskless nominal contracts with the intermediary are possible for either savers or borrowers. The assumption that households cannot engage in financial contracting other than through the intermediary sector represents one of the key financial frictions. We also allow households to hold one-period riskless nominal government debt, but since government debt and deposits with intermediaries are perfect substitutes as investments, they must pay the same interest rate i_t^d in equilibrium, and the decision problem of the households is the same as if they have only a decision about how much to deposit with or borrow from the intermediaries.

Aggregation is simplified by assuming that households are able to sign state-contingent contracts with one another, through which they may insure one another against both aggregate risk and the idiosyncratic risk associated with a household's random draw of its type, but that households are *only intermittently* able to receive transfers from the insurance agency; between the infrequent occasions when a household has access to the insurance agency, it

¹¹Here “savers” and “borrowers” identify households according to whether they choose to save or borrow, and not by their “type”.

can only save or borrow through the financial intermediary sector mentioned in the previous paragraph. The assumption that households are *eventually* able to make transfers to one another in accordance with an insurance contract signed earlier means that they continue to have identical expectations regarding their marginal utilities of income far enough in the future, regardless of their differing type histories.

It then turns out that in equilibrium, the marginal utility of a given household at any point in time depends *only* on its type $\tau_t(i)$ at that time; hence the entire distribution of marginal utilities of income at any time can be summarized by two state variables, λ_t^b and λ_t^s , indicating the marginal utilities of each of the two types. The expenditure level of type τ is similarly the same for all households of that type, and can be obtained by inverting the marginal-utility functions to yield an expenditure demand function $c^\tau(\lambda; \xi_t)$ for each type. Aggregate demand Y_t for the Dixit-Stiglitz composite good can then be written as

$$Y_t = \sum_{\tau} \pi_{\tau} c^{\tau}(\lambda_t^{\tau}; \xi_t) + G_t + \Xi_t, \quad (2.2)$$

where G_t indicates the (exogenous) level of government purchases and Ξ_t indicates resources consumed by intermediaries (the sum of two components, Xi_t^p representing costs of the private intermediaries and Ξ_t^{cb} representing costs of central-bank activities, each discussed further below). Thus the effects of financial conditions on aggregate demand can be summarized by tracking the evolution of the two state variables λ_t^{τ} . The marginal-utility ratio $\Omega_t \equiv \lambda_t^b / \lambda_t^s \geq 1$ provides an important measure of the inefficiency of the allocation of expenditure owing to imperfect financial intermediation, since in the case of frictionless financial markets we would have $\Omega_t = 1$ at all times.

In the presence of heterogeneity, instead of a single Euler equation each period, relating the path of the marginal utility of income of the representative household to “the” interest rate, we instead have *two* Euler equations each period, one for each of the two types, and each involving a different interest rate — i_t^b in the case of the Euler equation for type b (who choose to borrow in equilibrium) and i_t^d in the case of the Euler equation for type s (who choose to save). These are of the form

$$\lambda_t^{\tau} = \beta E_t \left[\frac{1 + i_t^{\tau}}{\Pi_{t+1}} \{ [\delta + (1 - \delta) \pi_{\tau}] \lambda_{t+1}^{\tau} + (1 - \delta) \pi_{-\tau} \lambda_{t+1}^{-\tau} \} \right], \quad (2.3)$$

for each of the two types $\tau = b, s$, where for either type τ , we use the notation $-\tau$ to denote the opposite type, and $\Pi_{t+1} \equiv P_{t+1}/P_t$ (where P_t is the Dixit-Stiglitz price index) is the gross rate of inflation. These two equations determine the two marginal utilities of expenditure — and hence aggregate demand, using (2.2) — as a function of the expected forward paths of the two real interest rates $(1 + i_t^\tau)/\Pi_{t+1}$ and the expected average marginal utility of expenditure far in the future. This generalizes the relation between real interest rates and aggregate demand in the basic New Keynesian model. Note that in the generalized model, the paths of the two different real interest rates (those faced by borrowers and those faced by savers) are both relevant for aggregate-demand determination; alternatively, the forward path of the *credit spread* matters for aggregate demand determination, in addition to the forward path of the general level of interest rates, as in the basic model. (See Cúrdia and Woodford, 2009a, for further discussion.)

Under an assumption of Calvo-style staggered price adjustment, we similarly obtain structural relations linking the dynamics of inflation and real activity that are direct generalizations of those implied by the basic New Keynesian model (as presented, for example, in Benigno and Woodford, 2005). As in the representative-household model, inflation is determined by a relation of the form¹²

$$\Pi_t = \Pi(Z_t), \tag{2.4}$$

where Z_t is a vector of two forward-looking endogenous variables,¹³ determined by a pair of structural relations that can be written in recursive form as

$$Z_t = z(Y_t, \lambda_t^b, \lambda_t^s; \xi_t) + E_t[\Phi(Z_{t+1})] \tag{2.5}$$

where $z(\cdot)$ and $\Phi(\cdot)$ are each vectors of two functions, and the vector of exogenous disturbances ξ_t now includes shocks to technology and tax rates, in addition to preference shocks. (The relations (2.5) reduce to precisely the equations in Benigno and Woodford, 2005, in the case that the two marginal utilities of income λ_t^τ are equated.) This set of structural equations makes inflation a function of the expected future path of output, generalizing the

¹²The definition of the function $\Pi(\cdot)$, and similarly of the functions referred to in the remaining equations of this section, are given in the Appendix.

¹³These are the variables denoted K_t and F_t in Benigno and Woodford (2005) and similarly in Cúrdia and Woodford (2009a).

familiar “New Keynesian Phillips curve”; but in addition to the expected paths of aggregate output and of various exogenous disturbances, the expected future path of the marginal-utility gap $\{\Omega_t\}$ also matters,¹⁴ and hence the expected future path of the credit spread (which determines the marginal-utility ratio). Thus this part of the model is completely standard, except that “cost-push” effects of credit spreads are taken into account. Cúrdia and Woodford (2009a) show that equations (2.4)–(2.5) can be log-linearized to yield a relation identical to the standard “New Keynesian Phillips curve,” except with additional additive terms for the effects of credit spreads.

Finally, our model of the effects of the two interest rates on the optimizing decisions of households of the two types imply an equation for aggregate private borrowing. The effects of interest rates both on expenditure and on labor supply (and hence on labor income) can be summarized by the effects of the expected paths of interest rates on the two marginal utilities of income. In the case of an isoelastic disutility of labor effort function, the degree of asymmetry between the amount by which expenditure exceeds income for type b relative to type s households can be written as a function $B(\lambda_t^b, \lambda_t^s, Y_t, \Delta_t; \xi_t)$, where Δ_t is an index of price dispersion and ξ_t includes disturbances to both technology and preferences. The index of price dispersion is a positive quantity, equal to 1 if and only if all goods prices at that date are identical, and higher than 1 when prices are unequal. Price dispersion matters because total hours worked (and hence the wage income of both types), for any given quantity of demand Y_t for the composite good, is proportional to Δ_t ; greater price dispersion results in a less efficient composition of output and hence an excess demand for inputs relative to the quantity consumed of the composite good.

Real per capita private debt b_t then evolves in accordance with a law of motion of the form

$$\begin{aligned} (1 + \pi_b \omega_t) b_t &= \pi_b \pi_s B(\lambda_t^b, \lambda_t^s, Y_t, \Delta_t; \xi_t) - \pi_b b_t^g \\ &\quad + \delta [b_{t-1} (1 + \omega_{t-1}) + \pi_b b_{t-1}^g] \frac{1 + i_{t-1}^d}{\Pi_t}, \end{aligned} \quad (2.6)$$

¹⁴Note that using (2.2) and the definition of Ω_t , one observes that the values of Y_t, Ω_t and the exogenous disturbances determine the values of λ_t^b, λ_t^s at any point in time.

where ω_t is the short-term *credit spread* defined by

$$1 + \omega_t \equiv \frac{1 + i_t^b}{1 + i_t^d}, \quad (2.7)$$

and b_t^g is real per capita government debt (one of the exogenous disturbance processes in our model). The supply of government debt matters for the evolution of private debt because it is another component (in addition to the deposits with intermediaries that finance their lending) of the financial wealth of type s households; because in our model government debt and deposits are substitutes from the standpoint of type s households (who hold positive quantities of both in equilibrium), in equilibrium government debt must also earn the interest rate i_t^d .¹⁵ Hence the interest rates i_t^d and i_t^b (or alternatively, i_t^d and the spread ω_t) are the only ones that matter for the evolution of private debt. (Note that equation (2.6) does not correspond to any equation of the basic New Keynesian model, as there can be no private debt in a representative-household model.)

Finally, as in Benigno and Woodford (2005), the assumption of Calvo-style price adjustment implies that the index of price dispersion evolves according to a law of motion of the form

$$\Delta_t = h(\Delta_{t-1}, \Pi_t), \quad (2.8)$$

where for a given value of Δ_{t-1} , $h(\Delta_{t-1}, \cdot)$ has an interior minimum at an inflation rate that is near zero ($\Pi_t = 1$) when initial price dispersion is negligible (Δ_{t-1} near 1), and the minimum value of the function is itself near 1 (*i.e.*, price dispersion continues to be minimal). Relative to this minimum, either inflation or deflation results in a greater degree of price dispersion; and once some degree of price dispersion exists, it is not possible to achieve zero price dispersion again immediately, for any possible choice of the current inflation rate.

The system of equations (2.2) consists of eight equations per period, to determine the eight endogenous variables $\{\Pi_t, Y_t, \lambda_t^b, \lambda_t^s, Z_t, b_t, \Delta_t\}$, given two more equations per period to

¹⁵Thus we abstract from any transactions role for the deposits that type s households hold with intermediaries. The model can easily be extended to allow deposits to supply transactions services, at the cost of introducing an additional interest-rate spread into the model. Note, however, that neither our account of the way in which the central bank controls short-term interest rates nor our account of the role of credit in macroeconomic equilibrium depends on any monetary role for the liabilities of private intermediaries.

determine the evolution of the interest rates $\{i_t^d, i_t^b\}$ (and hence of the credit spread). The latter equations follow from the decisions of private intermediaries and of the central bank.

2.2 Financial Intermediaries

We assume an intermediary sector made up of identical, perfectly competitive firms. Intermediaries take deposits, on which they promise to pay a riskless nominal return i_t^d one period later, and make one-period loans on which they demand a nominal interest rate of i_t^b . An intermediary also chooses a quantity of reserves M_t to hold at the central bank, on which it will receive a nominal interest yield of i_t^m . Each intermediary takes as given all three of these interest rates. We assume that arbitrage by intermediaries need not eliminate the spread between i_t^b and i_t^d , for either of two reasons. On the one hand, resources are used in the process of loan origination; and on the other hand, intermediaries may be unable to tell the difference between good borrowers (who will repay their loans the next period) and bad borrowers (who will be able to disappear without having to repay), and as a consequence have to charge a higher interest rate to good and bad borrowers alike.

We suppose that origination of good loans in real quantity L_t requires an intermediary to also originate bad loans in quantity $\chi_t(L_t)$, where $\chi_t', \chi_t'' \geq 0$, and the function $\chi_t(L)$ may shift from period to period for exogenous reasons. (While the intermediary is assumed to be unable to discriminate between good and bad loans, it is able to predict the fraction of loans that will be bad in the case of any given scale of lending activity on its part.) This scale of operations also requires the intermediary to consume real resources $\Xi_t^p(L_t; m_t)$ in the period in which the loans are originated, where $m_t \equiv M_t/P_t$, and $\Xi_t^p(L; m)$ is a convex function of its two arguments, with $\Xi_{L_t}^p \geq 0$, $\Xi_{m_t}^p \leq 0$, $\Xi_{Lm_t}^p \leq 0$. We further suppose that for any scale of operations L , there exists a finite *satiation level* of reserve balances $\bar{m}_t(L)$, defined as the lowest value of m for which $\Xi_{m_t}^p(L; m) = 0$. (Our convexity and sign assumptions then imply that $\Xi_{m_t}^p(L; m) = 0$ for all $m \geq \bar{m}_t(L)$.) We assume the existence of a finite satiation level of reserves in order for an equilibrium to be possible in which the policy rate is driven to zero, a situation of considerable practical relevance at present that raises interesting theoretical issues.

Given an intermediary's choice of its scale of lending operations L_t and reserve balances

m_t to hold, we assume that it acquires real deposits d_t in the maximum quantity that it can repay (with interest at the competitive rate) from the anticipated returns on its assets (taking into account the anticipated losses on bad loans). Thus it chooses d_t such that

$$(1 + i_t^d)d_t = (1 + i_t^b)L_t + (1 + i_t^m)m_t.$$

The deposits that it does not use to finance either loans or the acquisition of reserve balances,

$$d_t - m_t - L_t - \chi_t(L_t) - \Xi_t^p(L_t; m_t),$$

are distributed as earnings to its shareholders. The intermediary chooses L_t and m_t each period so as to maximize these earnings, given i_t^d, i_t^b, i_t^m . This implies that L_t and m_t must satisfy the first-order conditions

$$\Xi_{L_t}^p(L_t; m_t) + \chi_{L_t}(L_t) = \omega_t \equiv \frac{i_t^b - i_t^d}{1 + i_t^d}, \quad (2.9)$$

$$-\Xi_{m_t}^p(L_t; m_t) = \delta_t^m \equiv \frac{i_t^d - i_t^m}{1 + i_t^d}. \quad (2.10)$$

Equation (2.9) can be viewed as determining the equilibrium credit spread ω_t as a function $\omega_t(L_t; m_t)$ of the aggregate volume of private credit and the real supply of reserves. As indicated above, a positive credit spread exists in equilibrium to the extent that $\Xi_t^p(L; m)$, $\chi_t(L)$, or both are increasing in L . Equation (2.10) similarly indicates how the equilibrium differential δ_t^m between the interest paid on deposits and that paid on reserves at the central bank is determined by the same two aggregate quantities.

In addition to these two equilibrium conditions that determine the two interest-rate spreads in the model, the absolute level of (real) interest rates must be such as to equate the supply and demand for credit. Market-clearing in the credit market requires that

$$b_t = L_t + L_t^{cb}, \quad (2.11)$$

where L_t^{cb} represents real lending to the private sector by the central bank, as discussed next. Equations (2.9)–(2.11) then provide three more equilibrium conditions per period, to determine the three additional endogenous variables $\{i_t^d, i_t^b, L_t\}$ along with those discussed in the previous section, given paths (or rules for the determination of) the central-bank policy variables $\{M_t, i_t^m, L_t^{cb}\}$.

2.3 Dimensions of Central-Bank Policy

In our model, the central bank's liabilities consist of the reserves M_t (which also constitute the monetary base in our simple model), on which it pays interest at the rate i_t^m . These liabilities in turn fund the central bank's holdings of government debt, and any lending by the central bank to type b households. We let L_t^{cb} denote the real quantity of lending by the central bank to the private sector; the central bank's holdings of government debt are then given by the residual $m_t - L_t^{cb}$. We can treat m_t (or M_t) and L_t^{cb} as the bank's choice variables, subject to the constraints

$$0 \leq L_t^{cb} \leq m_t. \quad (2.12)$$

It is also necessary that the central bank's choices of these two variables satisfy the bound

$$m_t < L_t^{cb} + b_t^g,$$

where b_t^g is the total outstanding real public debt, so that a positive quantity of public debt remains in the portfolios of households. In the calculations below, however, we shall assume that this last constraint is never binding. (We confirm this in our numerical examples.)

We assume that central-bank extension of credit other than through open-market purchases of Treasury securities consumes real resources, just as in the case of private intermediaries, and represent this resource cost by a function $\Xi^{cb}(L_t^{cb})$, that is increasing and at least weakly convex, with $\Xi^{cb}(0) > 0$, as is discussed further in section 4. The central bank has one further independent choice to make each period, which is the rate of interest i_t^m to pay on reserves. We assume that if the central bank lends to the private sector, it simply chooses the amount that it is willing to lend and auctions these funds, so that in equilibrium it charges the same interest rate i_t^b on its lending that private intermediaries do; this is therefore not an additional choice variable for the central bank. Similarly, the central bank receives the market-determined yield i_t^d on its holdings of government debt.

The interest rate i_t^d at which intermediaries are able to fund themselves is determined each period by the joint inequalities

$$m_t \geq m_t^d(L_t, \delta_t^m), \quad (2.13)$$

$$\delta_t^m \geq 0, \tag{2.14}$$

together with the “complementary slackness” condition that at least one of (2.13) and (2.14) must hold with equality each period. Here $m_t^d(L, \delta^m)$ is the demand for reserves defined by (2.10), and defined to equal the satiation level $\bar{m}_t(L)$ in the case that $\delta^m = 0$. (Condition (2.13) may hold only as an inequality, as intermediaries will be willing to hold reserves beyond the satiation level as long as the opportunity cost δ_t^m is zero.) We identify the rate i_t^d at which intermediaries fund themselves with the central bank’s *policy rate* (e.g., the federal funds rate, in the case of the US).

The central bank can influence the policy rate through two channels, its control of the supply of reserves and its control of the interest rate paid on them. By varying m_t , the central bank can change the equilibrium differential δ_t^m , determined as the solution to (2.13)–(2.14). And by varying i_t^m , it can change the level of the policy rate i_t^d that corresponds to a given differential. Through appropriate adjustment on both margins, the central bank can control i_t^d and i_t^m separately (subject to the constraint that i_t^m cannot exceed i_t^d). We also assume that for institutional reasons, it is not possible for the central bank to pay a negative interest rate on reserves. (We may suppose that intermediaries have the option of holding currency, earning zero interest, as a substitute for reserves, and that the second argument of the resource cost function $\Xi_t^p(b; m)$ is actually the sum of reserve balances at the central bank plus vault cash.) Hence the central bank’s choice of these variables is subject to the constraints

$$0 \leq i_t^m \leq i_t^d. \tag{2.15}$$

There are thus three independent dimensions along which central-bank policy can be varied in our model: variation in the quantity of reserves M_t that are supplied; variation in the interest rate i_t^m paid on those reserves; and variation in the breakdown of central-bank assets between government debt and lending L_t^{cb} to the private sector. Alternatively, we can specify the three independent dimensions as *interest-rate policy*, the central bank’s choice of an operating target for the policy rate i_t^d ; *reserve-supply policy*, the choice of M_t , which in turn implies a unique rate of interest that must be paid on reserves in order for the reserve-supply policy to be consistent with the bank’s target for the policy rate;¹⁶ and *credit policy*,

¹⁶We might choose to call the second dimension variation in the interest rate paid on reserves, which

the central bank’s choice of the quantity of funds L_t^{cb} to lend to the private sector.¹⁷

We prefer this latter identification of the three dimensions of policy because in this case our first dimension (interest-rate policy) corresponds to the sole dimension of policy emphasized in many conventional analyses of optimal monetary policy, while the other two dimensions are additional dimensions of policy introduced by our extension of the basic New Keynesian model.¹⁸ Changes in central-bank policy along each of these dimensions has consequences for the bank’s cash flow, but we abstract from any constraint on the joint choice of the three variables associated with cash-flow concerns. (We assume that seignorage revenues are simply turned over to the Treasury, where their only effect is to change the size of lump-sum transfers to the households.)

Given that central-bank policy can be independently varied along each of these three dimensions, we can independently discuss the criteria for policy to be optimal along each

would correspond to something that the Board of Governors makes an explicit decision about under current US institutional arrangements, as is also true at most other central banks. But description of the second dimension of policy as “reserve-supply policy” allows us to address the question of the value of “quantitative easing” under this heading as well.

¹⁷Here we only consider the kind of credit policy that involves direct lending by the central bank to ultimate borrowers, or (equivalently, in our model, since the loan market is competitive) targeted asset purchases. Thus our “credit policy” is intended to represent, in a stylized way, the kind of programs that became an important part of Fed policy after September 2008, such as the Commercial Paper Funding Facility mentioned in section 1, or the Fed’s purchases of mortgage-backed securities. We do not take up the separate question of what might be accomplished by central-bank lending to intermediaries rather than to ultimate borrowers, as under the “liquidity facilities” that played such an important role in the Fed’s response to the financial crisis up until September 2008. In our model, central-bank lending to intermediaries can also be effective; and as with our analysis of credit policy below, the welfare consequences of such intervention will depend crucially on whether financial disruptions involve increases in the real resource costs of private intermediation or not. Other analyses that distinguish the effects of these two types of credit policy include Reis (2009) and Gertler and Kiyotaki (2010).

¹⁸Goodfriend (2009) similarly describes central-bank policy as involving three independent dimensions, corresponding to our first three dimensions, and calls the first of those dimensions (the quantity of reserves, or base money) “monetary policy.” We believe that this does not correspond to standard usage of the term “monetary policy,” since the main focus of policy deliberations at many central banks prior to the crisis was frequently the choice of an operating target for the policy rate. Reis (2009) also distinguishes among the three dimensions of policy in terms similar to ours.

dimension. Here we restrict our analysis to the two “unconventional” dimensions of policy, reserve-supply policy and credit policy. The consequences of heterogeneity and credit frictions for interest-rate policy (*i.e.*, conventional monetary policy) are addressed in Cúrdia and Woodford (2009a, 2009b). Of course, we have to make some assumption about interest-rate policy when considering adjustments of policy along the other two dimensions; in some of the analysis reported below, we assume that interest-rate policy is optimal (despite not seeking here to characterize optimal interest-rate policy), while in other places we assume a simple conventional specification for interest-rate policy (a “Taylor rule”). It is also true that the changes in reserve-supply policy and credit policy have consequences for optimal interest-rate policy; but these are not the concern of the present study, except to the extent that they influence the optimal use of the unconventional policies themselves.

2.4 The Welfare Objective

In considering optimal policy, we take the objective of policy to be the maximization of average expected utility. Thus we can express the objective as maximization of

$$E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} U_t \quad (2.16)$$

where the welfare contribution U_t each period weights the period utility of each of the two types by their respective population fractions at each point in time. As shown in Cúrdia and Woodford (2009a),¹⁹ this can be written as

$$U_t = U(Y_t, \Omega_t, \Xi_t, \Delta_t; \xi_t), \quad (2.17)$$

where Ω_t is again the marginal-utility gap, $\Omega_t \equiv \lambda_t^b / \lambda_t^s$; Ξ_t is total resources consumed in financial intermediation (also including resources used by the central bank, to the extent

¹⁹Cúrdia and Woodford (2009a) analyze a special case of the present model, in which central-bank lending and the role of central-bank liabilities in reducing the transactions costs of intermediaries are abstracted from. However, the form of the welfare measure (2.17) depends only on the nature of the heterogeneity in our model, and the assumed existence of a credit spread and of resources consumed by the intermediary sector; the functions that determine how Ω_t and Ξ_t are endogenously determined are irrelevant for this calculation, and those are the only parts of the model that are generalized in this paper. Hence the form of the welfare objective in terms of these variables remains the same.

that it lends to the private sector, as discussed further in sections 4 and 5); Δ_t is the index of price dispersion appearing in (2.6)–(2.8); and ξ_t is a vector of exogenous disturbances to preferences, technology, and government purchases.

It may be useful to briefly explain why the arguments in (2.17) suffice, and the way each of them affects welfare. In order to derive the period objective in (2.17), we sum the utility of consumption and disutility of labor for the two types, weighting each type τ by its population fraction π_τ . The average utility of consumption is equal to $\sum_\tau \pi_\tau u^\tau(c_t^\tau; \xi_t)$, which depends on c_t^b, c_t^s , and exogenous shocks to preferences (*i.e.*, to spending opportunities). But it is possible to solve uniquely for c_t^b, c_t^s given values for Y_t, Ω_t, Ξ_t , and the exogenous disturbances (including G_t), using (2.2) and the definition of Ω_t ; hence the arguments of (2.17) suffice to determine this component of average utility. The total disutility of work can be written as a product of factors

$$\Lambda(\Omega_t)\tilde{v}(Y_t; \xi_t)\Delta_t,$$

where $\tilde{v}(Y_t; \xi_t)$ would be the disutility of supplying quantity Y_t of the composite good, if a common quantity $y_t(j) = Y_t$ were produced of each of the individual goods, and if the labor effort involved in producing them were efficiently divided between households of the two types;²⁰ and $\Lambda(\Omega_t)$ is a distortion factor that arises as a result of differing marginal utilities of income of the two types (which means that their relative wages no longer correctly reflect their relative marginal disutilities of work), leading to an inefficient division of equilibrium work effort across the two types of households. Given these other two factors, the total disutility of work is proportional to Δ_t because greater price dispersion results in a less efficient composition of the output that comprises a quantity Y_t of the composite good. (Note that except for the presence of the factor $\Lambda(\Omega_t)$, the total disutility of work is the same as in the representative household model.) Hence the arguments of (2.17) also suffice for the calculation of this term, and so for the calculation of the period t contribution to average utility.

From this discussion, it should be evident how each of the four endogenous variables in (2.17) affects welfare. Given the values of the other variables, increasing Y_t increases the

²⁰Note that this disutility will depend both on the state of productivity and on preferences regarding labor supply, both of which are elements of the vector ξ_t .

average utility of consumption but also the total disutility of work, as in the representative household model. Under standard assumptions about preferences and technology, there is diminishing marginal utility from consumption and increasing marginal disutility of work as Y_t increases, so that there should be an interior maximum for U_t as a function of Y_t (the location of which will depend on preferences, technology, government purchases, etc.). And given the values of the other variables, U_t is monotonically decreasing in Δ_t . Both of these effects are similar to those in the representative household model, where $\{Y_t, \Delta_t\}$ are the *only* endogenous variables that matter for welfare.²¹ With heterogeneity and credit frictions, additional variables become relevant as well. Given values of the other variables, U_t is monotonically decreasing in both Ω_t and Ξ_t . Average utility is reduced by an increase in Ω_t because both the efficiency of the allocation of total private expenditure across the two types and the efficiency of the allocation of total work effort across the two types is reduced; it is reduced by an increase in Ξ_t because, for given values of Y_t and G_t , a higher value of Ξ_t means less total private expenditure and hence lower values of both c_t^b and c_t^s (given a value for Ω_t).

Other variables matter for welfare purely through their effects on the paths of these four endogenous variables. For example, the level of real bank reserves matters in our model, because of its effect on the resources Ξ_t consumed by financial intermediaries. Central-bank credit policy can matter in our model as well, to the extent that it reduces credit spreads and as a consequence the size of the equilibrium marginal-utility gap Ω_t . We turn now to an analysis of the optimal use of these additional dimensions of policy in the light of this objective.

3 Reserve-Supply Policy

We shall first consider optimal policy with regard to the supply of reserves, taking as given (for now) the way in which the central bank chooses its operating target for the policy rate i_t^d ,

²¹Because the evolution of price dispersion is determined entirely by the path of inflation, as indicated by (2.8), we can alternatively state that aggregate output and inflation are the only endogenous variables that matter for welfare in the representative-household model.

and the state-contingent level of central-bank lending L_t^{cb} to the private sector. Under fairly weak assumptions, we obtain a very simple result: *optimal policy requires that intermediaries be satiated in reserves, i.e., that $M_t/P_t \geq m_t(\bar{L}_t)$ at all times.*

For levels of reserves below the satiation point, an increase in the supply of reserves has two effects that are relevant for welfare: on the one hand, the resource cost of financial intermediation Ξ_t^p is reduced (for a given level of lending by the intermediary sector); and on the other hand, the credit spread ω_t is reduced (again, for a given level of lending) as a consequence of (2.9). Each of these effects raises the value of the objective (2.16); note that reductions in credit spreads increase welfare because of their effect on the path of the marginal-utility gap Ω_t .²² Hence an increase in the supply of reserves is unambiguously desirable, in any period in which they remain below the satiation level.²³ Once reserves are at or above the satiation level, however, further increases reduce neither the resource costs of intermediaries nor equilibrium credit spreads (as in this case $\Xi_{mt}^p = \Xi_{Lmt}^p = 0$), so that there would be no further improvement in welfare. Hence policy is optimal along this dimension if and only if $M_t/P_t \geq m_t(\bar{L}_t)$ at all times,²⁴ so that

$$\Xi_{mt}^p(L_t; m_t) = 0. \quad (3.1)$$

This is just another example in which the familiar “Friedman Rule” for “the optimum quantity of money” (Friedman, 1969) applies. Note, however, that our result has no consequences for *interest-rate policy*. While the Friedman rule is sometimes taken to imply a strong result about the optimal control of short-term nominal interest rates — namely, that the nominal interest rate should equal zero at all times — the efficiency condition (3.1), together with the equilibrium relation (2.10), implies only that the interest-rate *differential* δ_t^m should equal zero at all times. With zero interest on reserves, this would also require

²²Log-linearization of equations (2.3) can be used to show that, up to this log-linear approximation, log deviations of the marginal-utility gap should equal a forward-looking moving average of expected deviations of the credit spread from its steady-state level; see Cúrdia and Woodford (2009a).

²³The discussion here assumes that the upper bound in (2.12) is not a binding constraint. But if that constraint does bind, then an increase in the supply of reserves relaxes the constraint, and this too increases welfare, so that the conclusion in the text is unchanged.

²⁴To be more precise, policy is optimal if and only if (3.1) is satisfied *and* the upper bound in (2.12) does not bind. Both conditions will be satisfied by any quantity of reserves above some finite level.

that $i_t^d = 0$ at all times; but given that the central bank is free to set any level of interest on reserves consistent with (2.15), the efficiency condition (3.1) actually implies *no* restriction upon either the average level of the degree of state-contingency of the central bank's target for the policy rate i_t^d .

3.1 Is a Reserve Supply Target Needed?

Our result about the importance of ensuring an adequate supply of reserves might suggest that the question of the correct target level of reserves at each point in time should receive the same degree of attention at meetings of the FOMC as the question of the correct operating target for the federal funds rate. But deliberations of that kind are not needed in order to ensure fulfillment of the optimality criterion (3.1). For the efficiency condition can alternatively be stated (using (2.10)) as requiring that $i_t^d = i_t^m$ at all times. This requires that reserves be supplied to the point at which the policy rate falls to the level of the interest rate paid on reserves, or, in a formulation that is more to the point, *that interest be paid on reserves at the central bank's target for the policy rate*.

Given a rule for setting an operating target for i_t^d (discussed in the next section), i_t^m should be chosen each period in accordance with the simple rule

$$i_t^m = i_t^d. \tag{3.2}$$

To put this rule into practice, when the central bank acts to implement its target for the policy rate through open-market operations, it will automatically have to adjust the supply of reserves so as to satisfy (3.1). But this does not require a central bank's monetary policy committee (the FOMC in the case of the US) to deliberate about an appropriate target for reserves at each meeting; once the target for the policy rate is chosen (and the interest rate to be paid on reserves is determined by that, through condition (3.2), the quantity of reserves that must be supplied to implement the target can be determined by the bank staff in charge of carrying out the necessary interventions (the trading desk at the New York Fed, in the case of the US), on the basis of a more frequent monitoring of market conditions than is possible on the part of the monetary policy committee.

One obvious way to ensure that the efficiency condition (3.2) is satisfied is to adopt a

routine practice of automatically paying interest on reserves at a rate that is tied to the current operating target for the policy rate. This is already the practice of many central banks outside the US. At some of those banks, the fixed spread between the target for the policy rate and the rate paid on overnight balances at the central bank is quite small (for example, 25 basis points in the case of the Bank of Canada); in the case of New Zealand, the interest rate paid on overnight balances is the policy rate itself. There are possible arguments (relating to considerations not reflected in our simple model) according to which the optimal spread might be larger than zero, but it is likely in any event to be desirable to maintain a constant small spread, rather than treating the question of the interest rate to be paid on reserves as a separate, discretionary policy decision to be made at each meeting of the policy committee. Apart from the efficiency gains modeled here, such a system should also help to facilitate the central bank’s control of the policy rate (Goodfriend, 2002; Woodford, 2003, chap. 1, sec. 3).

3.2 Is there a Role for “Quantitative Easing”?

While our analysis implies that it is desirable to ensure that the supply of reserves never falls below a certain lower bound $\bar{m}_t(L_t)$, it also implies that there is no benefit from supplying reserves *beyond* that level. There is, however, one important exception to this assertion: it can be desirable to supply reserves beyond the satiation level if this is necessary in order to make the optimal quantity of central bank lending to the private sector L_t^{cb} consistent with (2.12). This qualification is important in thinking about the desirability of the massive expansion in the supply of reserves by the Fed since September 2008, as shown in Figure 1. As can be seen from Figure 2, the increase in reserves occurred only once the Fed decided to expand the various newly-created liquidity and credit facilities beyond the scale that could be financed simply by reducing its holdings of Treasury securities (as had been its policy over the previous year).²⁵

²⁵Bernanke (2009) distinguishes between the Federal Reserve policy of “credit easing” and the type of “quantitative easing” practiced by the Bank of Japan earlier in the decade, essentially on this ground. Shiratsuka (2009) distinguishes between “pure credit easing” and “pure quantitative easing” in the way that we have proposed, but argues that the actual “unconventional policies” of central banks, including both the

Some have argued, instead, that further expansion of the supply of reserves beyond the level needed to bring the policy rate down to the level of interest paid on reserves is an important additional tool of policy in its own right — one of particular value precisely when a central bank is no longer able to further reduce its operating target for the policy rate, owing to the zero lower bound (as at present in the US and many other countries). It is sometimes proposed that when the zero lower bound is reached, it is desirable for a central bank’s policy committee to shift from deliberations about an interest-rate target to a target for the supply of bank reserves, as under the Bank of Japan’s policy of “quantitative easing” during the period between March 2001 and March 2006.

Our model provides no support for the view that such a policy should be effective in stimulating aggregate demand. Indeed, it is possible to state an *irrelevance proposition* for quantitative easing in the context of our model. Let the three dimensions of central-bank policy be described by functions that specify the operating target for the policy rate, the supply of reserves, the interest rate to be paid on reserves, and the quantity of central-bank credit as functions of macroeconomic conditions.

For the sake of concreteness, we may suppose that each of these variables is to be determined by a Taylor-type rule,

$$\begin{aligned} i_t^d &= \phi^{id}(\pi_t, Y_t, L_t; \xi_t), \\ M_t/P_t &= \phi^m(\pi_t, Y_t, L_t; \xi_t), \\ i_t^m &= \phi^{im}(\pi_t, Y_t, L_t; \xi_t), \\ L_t^{cb} &= \phi^L(\pi_t, Y_t, L_t; \xi_t), \end{aligned}$$

where the functions are such that constraints (2.12)–(2.15) are satisfied for all values of the arguments. (Here the vector of exogenous disturbances ξ_t upon which the reaction functions may depend includes the exogenous factors that shift the function $\Xi_t^p(L; m)$.) Then our result is that given the three functions $\phi^{id}(\cdot)$, $\phi^{im}(\cdot)$, and $\phi^L(\cdot)$, the set of processes $\{\pi_t, Y_t, L_t, b_t, i_t^d, i_t^b, \Omega_t, \Delta_t\}$ that constitute possible rational expectations equilibria is

Bank of Japan in 2001-06 and the Federal Reserve during the current crisis, are mixtures of the two pure types. Ueda (2009) similarly argues that many central banks represent mixed cases.

the same, independently of the choice of the function $\phi^m(\cdot)$, as long as the specification of $\phi^m(\cdot)$ is consistent with the other three functions (in the sense that (2.12) and (2.13) are necessarily satisfied, and that (2.13) holds with equality in all cases where (2.14) is a strict inequality).²⁶

Of course, the stipulation that $\phi^m(\cdot)$ be consistent with the other functions uniquely determines what the function must be for all values of the arguments for which the functions $i^d(\cdot)$ and $i^m(\cdot)$ imply that $\delta_t^m > 0$. However the class of policies considered allows for an arbitrary degree of expansion of reserves beyond the satiation level in the region where those functions imply that $\delta_t^m = 0$, and in particular, for an arbitrary degree of quantitative easing when the zero bound is reached (*i.e.*, when $i_t^d = i_t^m = 0$). The class of policies considered includes the popular proposal under which the quantity of excess reserves should depend on the degree to which a standard Taylor rule (unconstrained by the zero bound) would call for a negative policy rate. Our result implies that there should be no benefits from such policies.

Our result might seem to be contradicted by the analysis of Auerbach and Obstfeld (2004), in which an open market operation that expands the money supply is found to stimulate real activity even when the economy is at the zero bound at the time of the monetary expansion. But their thought experiment does not correspond to pure quantitative easing of the kind contemplated in the above proposition, because they specify monetary policy in terms of a path for the money supply, and the policy change that they consider is one that *permanently* increases the money supply, so that it remains higher after the economy has exited from the “liquidity trap” in which the zero bound is temporarily binding. The contemplated policy change is therefore *not* consistent with an unchanged reaction function $\phi^{id}(\cdot)$ for the policy rate, and the effects of the intervention can be understood to be the consequences of the commitment to a different future interest-rate policy.

Our result only implies that quantitative easing should be irrelevant under two conditions: that the increase in reserves finances an increase in central-bank holdings of Treasury securities, rather than an increase in central-bank lending to the private sector; and that the policy implies no change in the way that people should expect future interest-rate policy to

²⁶This result generalizes the irrelevance result for quantitative easing in Eggertsson and Woodford (2003) to a model with heterogeneity and credit frictions.

be conducted. Our model *does* allow for real effects of an increase in central-bank lending L_t^{cb} financed by an increase in the supply of reserves, in the case that private-sector financial intermediation is inefficient;²⁷ but the real effects of the increased central-bank lending in that case are the same whether the lending is financed by an increase in the supply of reserves or by a reduction in central-bank holdings of Treasury securities. Our model also allows for real effects of an announcement that interest-rate policy in the future will be different, as in the case where a central bank commits itself not to return immediately to its usual Taylor rule as soon as the zero bound ceases to bind, but promises instead to maintain policy accommodation for some time after it would become possible to comply with the Taylor rule (as discussed in the next section). But such a promise (if credible and correctly understood by the private sector) should increase output and prevent deflation to the same extent even if it implies no change in policy during the period when the zero lower bound binds.

While our definition of quantitative easing may seem a narrow one, the policy of the Bank of Japan during the period 2001-2006 fits our definition fairly closely. The BOJ’s policy involved the adoption of a series of progressively higher quantitative targets for the supply of reserves, and the aim of the policy was understood to be to increase the monetary base, rather than to allow the BOJ to acquire any particular type of assets. The assets purchased consisted primarily Japanese government securities and bills issued by commercial banks; while there were also some more “unconventional” asset purchases under the quantitative easing policy — direct purchases of asset-backed securities and of stocks — the size of these operations was quite small relative to the total increase in the supply of reserves shown in Figure 4.²⁸ Finally, there was no suggestion that the targets of policy after the end of the

²⁷This result differs from that obtained in Eggertsson and Woodford (2003), where changes in the composition of the assets on the central bank’s balance sheet are *also* shown to be irrelevant. That stronger result depends on the assumption of a representative household as in the earlier paper, or alternatively, frictionless financial intermediation, as discussed in section 1.

²⁸According to Bank of Japan statistics, these “unconventional” purchases had a value only slightly greater than 2 trillion yen at their maximum, whereas the total increase in the monetary base during the quantitative easing (QE) period was in excess of 45 trillion yen. For more detailed discussion of the different aspects of the BOJ policy during the period, and an attempt to separate the effects of targeted asset purchases from those of quantitative easing, see Ueda (2009). Shiratsuka (2009) provides additional information.

zero-interest-rate period would be any different than before; there was no commitment to maintain the increased quantity of base money in circulation permanently, and indeed, once it was judged time to end the zero-interest-rate policy, the supply of reserves was rapidly contracted again, as also shown in Figure 4.

Our theory suggests that expansion of the supply of reserves under such circumstances should have little effect on aggregate demand, and this seems to have been the case. For example, as is also shown in Figure 4, despite an increase in the monetary base of 60 percent during the first two years of the quantitative easing policy, and an eventual increase of nearly 75 percent, nominal GDP never increased at all (relative to its March 2001 level) during the entire five years of the policy.²⁹ Apart from the absence of the effects on aggregate expenditure that simple quantity-theoretic reasoning would have predicted, there was also little evidence of effects of the policy on longer-term interest rates (the channel through which it might have been expected to ultimately influence aggregate expenditure). Those studies of Japan's experience with quantitative easing that find some reduction in longer-term interest rates attribute this mainly to successful signalling by the BOJ of an intention to maintain the zero-interest-rate policy, and find little effect on bond yields of the increase in the supply of reserves itself (Okina and Shiratsuka, 2004; Oda and Ueda, 2007; Ugai, 2007; Ueda, 2009).³⁰ Of course, it is perfectly consistent with our model that signals about future interest-rate policy can be an effective channel through which a central bank can seek to provide further monetary stimulus even when the current policy rate is constrained by the zero lower bound.³¹

²⁹As indicated in Figure 4, over the first two years of the quantitative easing policy, nominal GDP fell by more than 4 percent, despite extremely rapid growth of base money. While nominal GDP recovered thereafter, it remained below its 2001:Q1 level over the entire period until 2006:Q4, three quarters after the official end of quantitative easing, by which time the monetary base had been reduced again by more than 20 percent. Moreover, even if the growth of nominal GDP after 2003:Q1 is regarded as a delayed effect of the growth in the monetary base two years earlier, this delayed nominal GDP growth was quite modest relative to the size of the expansion in base money.

³⁰Baba *et al.* (2006) find some effects of BOJ purchases of commercial paper on the spreads associated with those particular types of paper. But this is really more evidence of the effectiveness of targeted asset purchases than of effectiveness of quantitative easing as such. See also the discussion by Ueda (2009).

³¹See Eggertsson and Woodford (2003) and Cúrdia and Woodford (2009b) for further discussion of the

4 Central-Bank Asset Purchases: “Treasuries Only”?

We turn now to another of our three independent dimensions of central-bank policy, namely, adjustment of the composition of the asset side of the central bank’s balance sheet, taking as given the overall size of the balance sheet (determined by the reserve-supply decision discussed above). In this section, we take for granted that reserve-supply policy is being conducted in the way recommended in the previous section, *i.e.*, that the rate of interest on reserves will satisfy (3.2) at all times. In this case, we can replace the function $\Xi_t^p(L_t; m_t)$ by

$$\bar{\Xi}_t^p(L_t) \equiv \Xi_t^p(L_t; \bar{m}_t(L_t))$$

and the function $\omega_t(L_t; m_t)$, defined by the left-hand side of (2.9), by

$$\bar{\omega}_t(L_t) \equiv \omega_t(L_t; \bar{m}_t(L_t)),$$

since there will be satiation in reserves at all times.³² Using these functions to specify the equilibrium evolution of Ξ_t^p and ω_t as functions of the evolution of aggregate private credit, we can then write the equilibrium conditions of the model without any reference to the quantity of reserves or to the interest rate paid on reserves. We wish to consider alternative possible state-contingent evolutions for central-bank lending $\{L_t^{cb}\}$, under various maintained assumptions about interest-rate policy (made explicit below).

According to the traditional doctrine of “Treasuries only,” the central bank should *not* vary the composition of its balance sheet as a policy tool; instead, it should avoid both balance-sheet risk and the danger of politicization by only holding (essentially riskless) Treasury securities at all times, while varying the size of its balance sheet to achieve its stabilization goals for the aggregate economy.³³ And even apart from these prudential concerns, if private financial markets can be relied upon to allocate capital efficiently, it is hard to argue that there would be any substantial value to allowing the central bank this additional dimension of policy. Eggertsson and Woodford (2003) present a formal irrelevance proposition in the context of a representative-household general-equilibrium model; in their

effectiveness of and ideal use of this aspect of policy.

³²Even if at some times m_t exceeds $\bar{m}_t(L_t)$, this will not affect the values of Ξ_t^p or ω_t .

³³See Goodfriend (2009) for discussion of this view and a warning about the dangers of departing from it.

model, the assets purchased by the central bank have no consequences for the equilibrium evolution of output, inflation or asset prices — and this is true regardless of whether the central bank purchases long-term or short-term assets, nominal or real assets, riskless or risky assets, and so on. And even in a model with heterogeneity of the kind considered here, the composition of the central bank’s balance sheet would be irrelevant if we were to assume frictionless private financial intermediation, since private intermediaries would be willing to adjust their portfolios to perfectly offset any changes in the portfolio of the central bank, as discussed in section 1.

This irrelevance result does not hold, however, in the presence of credit frictions of the kind assumed in section 2. Here we illustrate the potential use of central-bank credit as an instrument of stabilization policy in a numerical example. After establishing that credit policy *can* be used for macroeconomic stabilization in principle, we then consider the subtler question of whether it is desirable to do so.

4.1 Numerical Calibration

We shall illustrate our main points through an analysis of the quantitative effects of alternative types of purely financial disturbances in a calibrated model. The numerical values for parameters used in our calculations are summarized in Table 1. They are essentially the same as those used in the numerical analysis in Cúrdia and Woodford (2009a), where they are discussed in greater detail. The one important difference is that here we calibrate the model so that the steady-state real return on deposits (identified with the real federal funds rates, for purposes of the empirical interpretation of the model) is 3.0 percent per annum. In Cúrdia and Woodford (2009a), this rate is calibrated to be 4 percent per annum (1 percent per quarter), for the sake of using a round number (when expressed as a quarterly rate). But because we are interested in the consequences of the zero lower bound on the policy rate in some of the calculations reported below, the question of how far the steady-state policy rate is from the lower bound is of non-trivial import. Hence we have here adopted a value for this target that is closer to the average historical level of the real federal funds rate. Several of the numbers in Table 1 are also slightly different from those reported in the appendix to Cúrdia and Woodford (2009a), but all of these are consequences of the change in the

calibration target for the steady-state real policy rate, given unchanged numerical values for the other calibration targets.

Our numerical analysis requires the introduction of functional forms for preferences, production technology, and the functions that define the financial frictions. The utility from expenditure is assumed to be of the form

$$u^\tau (c; \xi_t) \equiv \frac{c^{1-\sigma_\tau^{-1}} (\bar{C}_t^\tau)^{\sigma_\tau^{-1}}}{1 - \sigma_\tau^{-1}},$$

for $\tau = b, s$, where for each type, $\sigma_\tau > 0$ would be the intertemporal elasticity of substitution of expenditure if one's type were expected to remain unchanged, and \bar{C}_t^τ is an exogenous type-specific disturbance, indicating variation in aggregate spending opportunities. The quantity c is a Dixit-Stiglitz aggregate of the household's purchases of the various individual goods, with elasticity of substitution $\theta > 1$, assumed to be the same for households of both types.³⁴ The disutility of work is assumed to be of the form

$$v^\tau (h; \xi_t) \equiv \frac{\psi_\tau}{1 + \nu} h^{1+\nu} \bar{H}_t^{-\nu},$$

for $\tau = b, s$, where $\nu > 0$ is the inverse of the Frisch elasticity of labor supply for both types, and \bar{H}_t is an exogenous disturbance, also common to both types. (Note that the disutility of work is assumed to be the same for both types, except for the multiplicative factors ψ_τ , that differ across types so as to imply identical steady-state labor supplies for the two types, despite their differing steady-state marginal utilities of income.) The production technology for each good j is assumed to be of the form

$$y_t(j) = A_t h_t(j)^{1/\phi},$$

where $\phi > 1$ indicates the degree of diminishing returns, and A_t is an exogenous productivity factor, common to all goods.

Many of the model's parameters are also parameters of the basic New Keynesian model, and in the case of these parameters we assume similar numerical values as in the numerical

³⁴The government is also assumed to purchase an exogenous quantity of the composite good, defined using the same aggregator, so that the government's elasticity of substitution among individual goods is also equal to θ . This makes the elasticity of each firm's demand curve independent of the composition of demand.

Table 1: Numerical parameter values used.

π_b	0.5	s_b	0.798	$(\theta - 1)^{-1}$	0.15	$\bar{\tau}$	0.2
δ	0.975	s_s	0.602	ϕ^{-1}	0.75	$\bar{\Xi}/\bar{Y}$	0.0003
β	0.990	σ_b	13.6	α	0.66	$\bar{\chi}/\bar{L}$	0
ν	0.105	σ_s	2.72	\bar{b}^g/\bar{Y}	0	$1 + \bar{\omega}$	$(1.02)^{1/4}$
\bar{h}^b/\bar{h}^s	1	$\bar{\lambda}^b/\bar{\lambda}^s$	1.22	\bar{b}/\bar{Y}	3.2	η	51.6

analysis of that model in Woodford (2003, Table 5.1.), which in turn are based on the empirical model of Rotemberg and Woodford (1997). Specifically, the values assumed for ν , θ , and ϕ in Table 1 are the same as in Rotemberg and Woodford, and the average of the elasticities σ_τ is chosen so as to imply the same interest-elasticity of aggregate expenditure as in Rotemberg and Woodford.³⁵ The value assumed for α , the fraction of goods prices that remain unchanged from one quarter to the next, is also the same as in Rotemberg and Woodford. The value assumed for β is also the same (to three decimal places) as the one used by Rotemberg and Woodford, though for a different reason: the value reported in the table is required by our calibration target for the steady-state real policy rate, discussed above.³⁶

The new parameters that are needed for the present model are those relating to heterogeneity or to the specification of the credit frictions. The parameters relating to heterogeneity are the fraction π_b of households that are borrowers, the degree of persistence δ of a household’s “type”, the steady-state expenditure level of borrowers relative to savers, and the interest-elasticity of expenditure of borrowers relative to that of savers, σ_b/σ_s .³⁷

³⁵To be precise, these are chosen so that the coefficient $\bar{\sigma}$ defined in the Appendix has the same value as the coefficient denoted σ in Table 5.1 of Woodford (2003).

³⁶Our calibration target for the steady-state real policy rate is slightly lower than the one assumed by Rotemberg and Woodford, but the credit frictions in our model require a rate of time preference that is slightly higher than the steady-state real policy rate, unlike the model of Rotemberg and Woodford; and the consequences of these two changes for the assumed value of β essentially cancel one another.

³⁷Another new parameter that matters as a consequence of heterogeneity is the steady-state level of government debt relative to GDP, \bar{b}^g/\bar{Y} ; here we assume that $\bar{b}^g = 0$.

In the calculations reported here, we assume that $\pi_b = \pi_s = 0.5$, so that there are an equal number of borrowers and savers. We assume that $\delta = 0.975$, so that the expected time until a household has access to the insurance agency (and its type is drawn again) is 10 years. This means that the expected path of the spread between lending and deposit rates for 10 years or so into the future affects current spending decisions, but that expectations regarding the spread several decades in the future are nearly irrelevant.

We calibrate the model so that private expenditure is 0.7 of total output in steady state, and furthermore calibrate the degree of heterogeneity in the steady-state expenditure of the two types so that the implied steady-state debt \bar{b} is equal to 80 percent of annual steady-state output.³⁸ This value matches the median ratio of private (non-financial, non-government, non-mortgage) debt to GDP over the period 1986-2008.³⁹ This requires the values of s_b and s_s shown in the table, where $s_\tau \equiv \bar{c}^\tau/\bar{Y}$ is the steady-state expenditure share for each type τ (using bars to denote the steady-state values of variables).

We assume an average intertemporal elasticity of substitution for the two types that is the same as that of the representative household in the model of Rotemberg and Woodford (1997), as mentioned above, and determine the individual values of σ_τ for the two types on the assumption that σ_b/σ_s is equal to 5. This is an arbitrary choice, though the fact that borrowers are assumed to have a greater willingness to substitute intertemporally is important, as this results in the prediction that an exogenous tightening of monetary policy (a positive intercept shift added to (4.2)) results in a reduction in the equilibrium volume of credit b_t (see Cúrdia and Woodford, 2009a). This is consistent with the VAR evidence on the effects of an identified monetary policy shock presented in Lown and Morgan (1992).⁴⁰

It is also necessary to specify the unperturbed values of the functions $\omega(b)$ and $\Xi(b)$

³⁸In our quarterly model, this means that $\bar{b}/\bar{Y} = 3.2$.

³⁹We exclude mortgage debt when calibrating the degree of heterogeneity of preferences in our model, since mortgage debt is incurred in order to acquire an asset, rather than to consume current produced goods in excess of current income.

⁴⁰It is also consistent with the evidence in Den Haan *et al.* (2004) for the effects of a monetary shock on consumer credit, though commercial and industrial loans are shown to rise. The result for C&I loans may reflect substitution of firms toward bank credit owing to decreased availability of other sources of credit, rather than an actual increase in borrowing; see Bernanke and Gertler (1995) on this point.

that describe the financial frictions, in addition to making clear what kinds of random perturbations of these functions we wish to consider when analyzing the effects of “financial shocks.” In the absence of shocks, we assume that $\chi_t(L) = 0$, so that all loans are expected to be repaid, and the credit spread is due purely to the resource costs of intermediation. We assume an intermediation technology such that when intermediaries hold reserves at or above the satiation level (as occurs in equilibrium under an optimal reserve-supply policy, assumed in all of the numerical exercises reported in this section)

$$\bar{\Xi}^p(L) = \tilde{\Xi}^p L^\eta \tag{4.1}$$

in the absence of shocks. We assume that $\eta > 1$, so that the marginal cost of intermediation is increasing; this corresponds to the idea of a finite lending capacity at a given point in time, due to scarce factors such as intermediary capital and expertise that are here treated as exogenous.⁴¹

In the numerical results reported here, we assume more specifically a value of η that implies that a one percent increase in private lending implies an increase in the marginal cost of intermediation, and hence in the equilibrium credit spread, of one percentage point (per annum). This implies fairly inelastic credit supply by the private sector, but we believe that this is the case of greatest interest for the exercises here, both because private credit supply is often asserted to be quite inelastic during financial crises, and because this is in any event the assumption most favorable to a potential role for central-bank credit policy, as in this case a substitution of central-bank lending for private lending to even a modest degree can lower the marginal cost of private lending and hence the equilibrium credit spread.⁴² We are interested in biasing our results in this direction, not to pre-judge the desirability of central-bank lending, but because our results show in any event that the justification for

⁴¹The assumption that $\eta > 1$ also allows our model to match the prediction of VAR estimates that an unexpected tightening of monetary policy is associated with a slight reduction in credit spreads (see, e.g., Lown and Morgan, 2002, and Gerali *et al.*, 2008). See Cúrdia and Woodford (2009a) for comparison with a model with a linear resource cost function.

⁴²In the case that the private intermediary sector has a constant marginal cost, rather than one increasing with the volume of private lending, then central-bank lending will reduce the equilibrium credit spread only to the extent that it *completely* replaces private lending, by lending at a rate that is too low to be profitable for private intermediaries at any positive scale of operation.

active credit policy is often fairly modest, even in the case of financial disturbances that increase credit spreads by a significant amount. It is most interesting to observe that this is true even under the assumption of quite a large value for η ; in the case of a more modest value of η , then both the shadow value of allowing active credit policy (relaxing the constraint that $L_t^{cb} = 0$) and the optimal scale of central-bank lending in response to shocks will in all cases be substantially smaller than the values reported below.

Finally, we specify the unperturbed value of $\tilde{\Xi}^p$ so that the steady-state credit spread $\bar{\omega}$ equal to 2.0 percentage points per annum, following Mehra *et al.*, (2008). Combined with our assumption that “types” persist for 10 years on average, this implies a steady-state “marginal-utility gap” $\bar{\Omega} \equiv \bar{\lambda}^b / \bar{\lambda}^s = 1.22$, so that there would be a non-trivial welfare gain from transferring further resources from savers to borrowers. Because of the degree of convexity assumed for the intermediation technology, this corresponds to a steady-state resource cost of financial intermediation $\bar{\Xi}$ that is much smaller than 2 percent per year of the steady-state level of private lending, as shown in Table 1;⁴³ hence in our parameterization, the credit spread represents mainly rents earned by private intermediaries, owing to the scarcity of whatever factor allows only particular firms to engage in this activity.

4.2 Credit Policy and Stabilization

We first demonstrate that according to our model, central-bank credit policy can in principle be a highly effective tool for macroeconomic stabilization. In particular, it can be used to offset at least some of the effects of a “purely financial” disturbance, that shifts the relation $\bar{\omega}_t(L)$, by shifting the relationship that would otherwise exist between private borrowing b_t and private lending L_t , so that the relationship between ω_t and b_t need not change. In the numerical examples considered in Figure 5, we assume that $\chi_t(L) = \tilde{\chi}_t L$, and consider the effects of an exogenous increase in the factor χ_t .⁴⁴ Thus we assume that the fraction of loans that are not repaid is independent of the volume of lending, but that it varies over time for

⁴³The value for $\bar{\Xi}$ reported in the table represents the steady-state value of Ξ_t^p , as we assume no central-bank lending to the private sector in the steady state: $\bar{L}^{cb} = 0$.

⁴⁴This is the type of disturbance that is called an “additive χ ” shock in the next section.

exogenous reasons. We assume that

$$\chi_t = \chi_0 \rho^t$$

for all $t \geq 0$, where $\chi_0 > 0$ determines the size of the shock, and in our numerical examples we assume that $\rho = 0.9$.

Let us suppose that interest-rate policy is specified by a ‘‘Taylor rule’’ of the form

$$i_t^d = \max\{\bar{r}^d + \phi_\pi \pi_t + \phi_y \hat{Y}_t, 0\}, \quad (4.2)$$

where our specification assumes that the central bank’s operating target must at all times respect the zero lower bound on short-term nominal interest rates. Here $\pi_t \equiv \log \Pi_t$ is the inflation rate, $\hat{Y}_t \equiv \log(Y_t/\bar{Y})$, and \bar{r}^d is the steady-state real policy rate,⁴⁵ so that the policy rule is consistent with the zero-inflation steady state (discussed above) in the absence of disturbances. In the numerical results shown in Figure 5 and subsequently, the response coefficients are assigned the values $\phi_\pi = 2, \phi_y = 1/4$, in rough accordance with estimates of US monetary policy in recent decades.⁴⁶

As a simple example of endogenous credit policy, let us suppose that the central bank’s lending to the private sector (or purchases of illiquid assets) is given by a rule of the form

$$L_t^{cb} = -\gamma(L_t - \bar{L}), \quad (4.3)$$

where \bar{L} is the level of private lending in the zero-inflation steady state with zero central-bank credit, and $0 \leq \gamma \leq 1$ is a coefficient indicating the degree to which the central bank increases its lending to the private sector to offset an observed decrease in private lending.⁴⁷ The rule is specified so that the central bank adheres to a policy of ‘‘Treasuries only’’ in the steady state, and so central-bank credit policy is eventually phased out again following

⁴⁵As explained above, we calibrate the model so that $1 + \bar{r}^d = (1.03)^{1/4}$.

⁴⁶These are the baseline parameter values used in the numerical analysis of the representative-household New Keynesian model in Woodford (2003, chap. 4). We use these parameter values in our analysis in Cúrdia and Woodford (2009a) of the model’s implications for the effects of disturbances when monetary policy follows a Taylor rule, in order to allow direct comparison with the results shown in Woodford (2003) for the basic New Keynesian model.

⁴⁷Of course, the decrease in private lending in equilibrium will not be independent of the central bank’s intervention; as Figure 5 shows, for a given size of disturbance, more lending by the central bank results in a greater contraction of private lending.

the disturbance. In the case that $\gamma = 0$, it adheres to “Treasury only” even in response to the financial disturbance; in the opposite limiting case, in which $\gamma = 1$, the central bank uses credit policy so aggressively as to completely stabilize private borrowing b_t at the level $\bar{b} = \bar{L}$.⁴⁸

Figure 5 plots the impulse responses of several endogenous variables to a disturbance of the kind proposed, for each of several different values of γ . In the figure, a value of $\tilde{\chi}_0$ is assumed that increases the value of $\bar{\omega}_0(\bar{L})$ by 4 percentage points per annum.⁴⁹ In the case of no credit policy ($\gamma = 0$), and an interest-rate policy that does not respond to the disturbance except to the extent that it affects inflation or output, such a disturbance contracts real activity and creates deflation, resulting in an immediate cut in the policy rate.⁵⁰ The reduction in the policy rate is not as large as the increase in the credit spread ω_t , so the interest rate faced by borrowers increases; given that expected inflation falls at the same time, the increase in the *real* rate faced by borrowers is even greater. In addition to the distortions implied by the effects of the disturbance on aggregate output and inflation, there is also a substantial distortion of the *composition* of expenditure, as type *b* expenditure falls sharply while type *s* expenditure actually increases; since by hypothesis, there has been no change in the marginal utility of each type of expenditure, this shift in composition is inefficient.

The figure shows that active credit policy can mitigate each of the effects of the distur-

⁴⁸In order to determine the general-equilibrium effects of this policy, it is also necessary to make an assumption about the resources (if any) consumed by the central bank’s lending operations. For reasons discussed in the next section, we do assume that central-bank lending to the private sector consumes resources, though central-bank purchases of Treasuries involve no costs. The cost parameter $\tilde{\Xi}^{cb}$ is set in the same way as in Figure 6, discussed below. However, the precise value of this parameter has very little effect on the responses shown in Figure 5.

⁴⁹Even in the absence of active credit policy, this increases the equilibrium credit spread by less than 3 percentage points, because of the endogenous response of the equilibrium credit spread to the reduction in private lending.

⁵⁰Note that in the case of a disturbance of this size, the policy rate is cut by about two percentage points, but not so far as to cause the zero lower bound to become an issue. Hence for any small enough value of $\tilde{\chi}_0$, the responses will look like those in Figure 5, with the size of all responses scaled in proportion to the value of $\tilde{\chi}_0$.

bance just mentioned. If $\gamma > 0$, private borrowing b_t falls by less in equilibrium, while at the same time the credit spread ω_t increases by less (since private lending L_t actually falls *more* if the central bank substitutes for private lenders). As a consequence, the marginal-utility gap Ω_t increases less, there is less distortion of the composition of private expenditure, there is less contraction of aggregate expenditure and of output, and there is less deflation. Moreover, there is less need for interest-rate cuts to head off the output contraction and deflation: output and inflation fall less even though the policy rate is cut less aggressively. In the limiting case in which $\gamma = 1$, there is virtually no change in any of the endogenous variables, except that L_t is falling and L_t^{cb} is increasing (with total lending unchanged).⁵¹ In this case, credit policy succeeds to a large extent in eliminating the instability that would otherwise result from the financial disturbance; moreover, it virtually eliminates the need for a cut in the policy rate.

While it is clear from this example that credit policy can be used for stabilization purposes, one might still wonder whether it is *needed*: could not interest-rate policy already accomplish as much, if we allow interest-rate policy to respond to the shock, rather than assuming mechanical compliance with a simple Taylor rule? While modification of the assumed interest-rate policy (for example, assuming a “spread-adjusted Taylor rule” of the kind considered in Cúrdia and Woodford, 2010, under which the intercept of the Taylor rule is reduced in proportion to any increase in the credit spread) can indeed mitigate the effects of the disturbance that are shown for the case $\gamma = 0$ in the figure, it would not reduce them to the extent that the credit policy with $\gamma = 1$ is able to. The reason is that the disturbance shifts up the schedule $\bar{\omega}_t(L)$; in the absence of central-bank lending, this means that either the credit spread must increase (which will distort the composition of expenditure, regardless of the absolute level of interest rates) or borrowing must decrease (which would also require a reduction of the relative spending of borrowers), or both. Thus while a large enough cut in the policy rate can prevent output from declining, this policy will not eliminate the distortion

⁵¹It is not quite true that there is no change in the various endogenous variables, because the shift from private lending to central-bank lending slightly increases the resources consumed in financial intermediation. However, under our parameterization, this resource use is small enough that the effects on both aggregate output and private expenditure are similar to those that would occur in the absence of the disturbance.

of the composition of expenditure.

The inadequacy of interest-rate policy alone is even more obvious in the case of a financial disturbance large enough to cause the zero lower bound on the policy rate to bind. In the case of a shock of the same type as in Figure 5, but of three times the magnitude,⁵² under the $\gamma = 0$ policy (and again assuming an interest-rate policy determined by (4.2)), the contraction of output and reduction in inflation are sufficiently extreme that the zero lower bound on the policy rate binds for two quarters. The effects of the disturbance on variables such as GDP in the absence of credit policy are as a consequence slightly more than three times as great as those shown in Figure 5, owing to the binding zero lower bound. But even more importantly, it is obvious in this case that it would not be possible to avoid those effects simply by assuming an even more aggressive reduction in the policy rate than that called for by the Taylor rule; for the zero lower bound would in any event prevent deeper interest-rate cuts at the time of the shock. Yet once again, a sufficiently aggressive credit policy can almost completely insulate aggregate output, both types of private expenditure, and inflation from any effects of the disturbance. (In the case of values of γ equal to 0.8 or higher, the zero lower bound never binds, and in these cases the responses are exactly like those shown in Figure 5, except multiplied by three.)

4.3 Credit Policy and Welfare

We have seen from Figure 5 that credit policy can be used to stabilize the economy, most notably in response to purely financial disturbances that shift the $\bar{\omega}_t(L)$ schedule. But is it desirable to do so, and to what extent? Our model can also be used to consider the optimal use of this additional dimension of policy, if we are willing to suppose that the prudential arguments against the central bank's involvement in the allocation of credit should not be determinative, at least in the case of sufficiently severe financial disruptions. In our model, an increase in L_t^{cb} can improve welfare on two grounds: for a given volume of private borrowing b_t , an increase in L_t^{cb} allows the volume of private lending L_t to fall, which should reduce both the resources Ξ_t^p consumed by the intermediary sector and the equilibrium credit spread

⁵²That is, $\bar{\omega}_0(\bar{L})$ is increased by 12 percentage points per annum.

ω_t (due to equilibrium relation (2.9)). Under plausible conditions, our model implies both a positive shadow value $\varphi_{\Xi,t}$ of reductions in Ξ_t (the Lagrange multiplier associated with the resource constraint (2.2)) and a positive shadow value $\varphi_{\omega,t}$ of reductions in ω_t ; hence an increase in L_t^{cb} should be desirable on both grounds.

In the absence of any assumed cost of central-bank credit policy, one can easily obtain the result that it is always optimal for the central bank to lend in amount sufficient to allow an equilibrium with $L_t = 0$, *i.e.*, the central bank should substitute for private credit markets altogether. Of course, we do not regard this as a realistic conclusion. As a simple way of introducing into our calculations the fact that the central bank is unlikely to have a comparative advantage at the activity of credit allocation under normal circumstances, we assume that central-bank lending consumes real resources in a quantity $\Xi^{cb}(L_t^{cb})$, by analogy with our assumption that real resources Ξ_t^p are consumed by private intermediaries. The function $\Xi^{cb}(L)$ is assumed to be increasing and at least weakly convex; in particular, we assume that $\Xi^{cb}(0) > 0$, so that there is a positive marginal resource cost of this activity, even when the central bank starts from a balance sheet made up entirely of Treasury securities.

The first-order conditions for optimal choice of L_t^{cb} then become:

$$\varphi_{\Xi,t}[\bar{\Xi}_t^{p'}(b_t - L_t^{cb}) - \Xi^{cb'}(L_t^{cb})] + \varphi_{\omega,t}[\bar{\Xi}_t^{p''}(b_t - L_t^{cb}) + \chi_t''(b_t - L_t^{cb})] \leq 0, \quad (4.4)$$

$$L_t^{cb} \geq 0, \quad (4.5)$$

together with the complementary slackness condition that at least one of conditions (4.4) or (4.5) must hold with equality in each period. (Here the first expression in square brackets in (4.4) is the partial derivative of Ξ_t with respect to L_t^{cb} , holding constant the value of total borrowing b_t , while the second expression in square brackets is the partial derivative of ω_t with respect to L_t^{cb} under the same assumption.)

A “Treasury only” policy is optimal in the event of a corner solution, in which (4.4) is an inequality, as will be the case if $\Xi^{cb'}(0)$ is large enough. In our view, it is most reasonable to calibrate the model so that this is true in steady state. Then not only will the optimal policy involve “Treasury only” in the steady state, but (assuming that the inequality is strict at the steady state) this will continue to be true in the case of any stochastic disturbances that are small enough. However, it will remain possible for the optimal policy to require $L_t^{cb} > 0$

in the case of certain large enough disturbances. This is especially likely to be true in the case of large enough disruptions of the financial sector, of a type that increase the marginal resource cost of private intermediation (the value of $\bar{\Xi}^p$) and/or the degree to which increases in private credit require a larger credit spread (the value of $\bar{\omega}'$).

However, not all “purely financial” disturbances — by which we mean exogenous shifts in the functions $\bar{\Xi}_t^p(L)$ or $\chi_t(L)$ of a type that increase the equilibrium credit spread $\bar{\omega}_t(L)$ for a given volume of private credit — are equally likely to justify an active central-bank credit policy on the grounds just mentioned. In fact, the optimal credit policy response to a given shift in the function $\bar{\omega}_t(L)$ depends greatly on the source of the shift.⁵³ To illustrate this, we consider the quantitative effects of alternative types of purely financial disturbances in our calibrated model.

4.4 The Value of Relaxing the “Treasuries Only” Restriction

We first assume that a strict policy of “Treasuries only” is maintained, but seek to determine the shadow value of relaxing this constraint, *i.e.*, the marginal increase in the value of the welfare objective (2.16) that is achieved by a marginal increase in L_t^{cb} above zero in some period. This shadow value is given by the left-hand side of (4.4); a positive value would imply that welfare is increased by allowing L_t^{cb} to be positive.⁵⁴ Since the shadow value is obviously reduced if we assume a higher marginal cost $\Xi^{cb'}(0)$ of central-bank lending — and since nothing about the equilibrium associated with the “Treasuries only” constraint is affected

⁵³Our result here is quite different from that in Cúrdia and Woodford (2009a), where the consequence of a “purely financial” disturbance for optimal interest-rate policy, taking as given the path of central-bank lending to the private sector, depends (to a first approximation) only on the size of the shift in $\bar{\omega}_t(\bar{L})$, which is why the paper does not bother to show the optimal responses to more than one type of purely financial disturbance. The same is true of the calculations in the previous section that illustrate the ability of credit policy to stabilize the economy in response to a purely financial disturbance: we show our results for only one type of disturbance because the figures are fairly similar when we consider alternative disturbances that shift the function $\bar{\omega}_t(L)$ to the same extent.

⁵⁴Note that condition (4.4) requires the shadow value to be non-positive under an optimal policy, as there is assumed to be no practical obstacle (apart from those reflected in the central bank’s resource cost function $\Xi^{cb}(L)$) to a positive level of central-bank lending.

by the assumed value of $\Xi^{cb'}(0)$, other than this shadow value — an alternative measure of the value of relaxing the constraint, that is economically meaningful, is to report the value of $\Xi^{cb'}(0)$ that is required in order for the shadow value of relaxation of the constraint to be exactly zero. (The higher the shadow value of relaxing the constraint, in the case of any fixed value of $\Xi^{cb'}(0)$, the higher the value of $\Xi^{cb'}(0)$ that would be required to reduce the shadow value to zero; and indeed one of these quantities is a linear function of the other.)

Thus the quantity that we report in our figures is the value of

$$\Xi_t^{cb',crit} \equiv \bar{\Xi}_t^{p'}(L_t) + \frac{\varphi_{\omega,t}}{\varphi_{\Xi,t}} [\bar{\Xi}_t^{p''}(L_t) + \chi_t''(L_t)] \quad (4.6)$$

in an equilibrium where the “Treasury only” policy is imposed. Note that this quantity exceeds the marginal resource cost of lending by private intermediaries ($\bar{\Xi}^{p'}$) to the extent that either $\bar{\Xi}_t^p(L)$ or $\chi_t(L)$ is a strictly convex function. Under the calibration used here, $\bar{\Xi}^{p'}$ is equal to 2.0 percent per annum in the steady state, while the steady-state value of $\Xi^{cb',crit}$, the minimum marginal cost of central-bank lending required for “Treasury only” to be optimal, is nearly 3.5 percent per annum, as shown in Figure 6. The assumed degree of convexity of the function $\bar{\Xi}^p(L)$ makes a substantial difference.

Under the calibration assumed, “Treasury only” will be optimal in the steady state, if we make the further assumption that $\Xi^{cb'}(0)$ is equal to 3.5 percent per annum or more. But what if a financial disturbance causes a significant increase in the size of credit spreads? The answer depends on the nature of the financial disturbance, and not only on the size of the increase in credit spreads.

To illustrate this point, let us consider four different possible purely financial disturbances, each of which will be assumed to increase the value of $\bar{\omega}_t(\bar{L})$ by the same number of percentage points. By an *additive* shock, we mean one that translates the schedule $\bar{\omega}_t(L)$ vertically by a constant amount; a *multiplicative* shock will instead multiply the entire schedule $\bar{\omega}_t(L)$ by some constant factor greater than 1. We shall also distinguish between disturbances that change the function $\bar{\Xi}_t(L)$ (“ Ξ shocks”) and disturbances that change the function $\chi_t(L)$ (“ χ shocks”). Thus a “multiplicative χ shock” is a change in the function $\chi_t(L)$ as a consequence of which the schedule $\bar{\omega}_t(L)$ is multiplied by a factor greater than 1 for all values of L , and so on.

Figure 6 plots the dynamic response of $\Xi_t^{cb,crit}$ to each of these four types of purely financial disturbance, when the model is calibrated as discussed above. (The quantity defined in (4.6) is multiplied by 400 so that the interest-rate differential is expressed in percentage points per annum.) In these simulations, both interest-rate policy and reserve-supply policy are assumed to be optimal, and each of the four disturbances is of a size that increases the value of $\omega_t(\bar{L})$ by 4 percentage points per annum (*i.e.*, from 2.0 percent to 6.0 percent),⁵⁵ as in Figure 5.⁵⁶ Moreover, the effect of the disturbance is assumed to decay exponentially, so that

$$\bar{\omega}_t(\bar{L}) = \bar{\omega} + [\bar{\omega}_0(\bar{L}) - \bar{\omega}] \rho^t$$

for all $t \geq 0$ for each of the four shocks, and again we assume that $\rho = 0.9$.

The figure clearly shows that the degree to which a financial disturbance provides a justification for active central-bank credit policy depends very much on the reason for the increase in spreads. In fact, the factors that affect the size of $\Xi_t^{cb,crit}$ are not too closely similar to those that affect the size of $\bar{\omega}_t(\bar{L})$. The increase in $\bar{\omega}_t(\bar{L})$ is the sum of the increases in $\bar{\Xi}^{p'}(\bar{L})$ and $\chi'(\bar{L})$. The value of $\Xi_t^{cb,crit}$ is also increased by an increase in $\bar{\Xi}^{p'}$, but not by an increase in χ' as such. Moreover, $\Xi_t^{cb,crit}$ is increased by increases in $\bar{\Xi}^{p''}$ and χ'' , or in the relative shadow price $\varphi_{\omega,t}/\varphi_{\Xi,t}$ (which is generally increased by a financial disturbance, since an increase in credit spreads increases the marginal distortion associated with a given further increase in spreads), whereas these do not change the value of $\omega_t(\bar{L})$.

The figure shows that $\Xi_t^{cb,crit}$ increases the most if the credit spread increases due to a “multiplicative Ξ ” shock, since in this case the increase in the spread is due entirely to an increase in $\bar{\Xi}^{p'}$ and $\bar{\Xi}^{p''}$ (and hence $\bar{\omega}'$) increases in the same proportion as does $\bar{\Xi}^{p'}$. Only one of these two effects (the increase in $\bar{\Xi}^{p'}$ or the increase in $\bar{\omega}'$) is present in the case of the “additive Ξ ” or “multiplicative χ ” shocks, and neither is present in the case of an “additive χ ” shock. In this last case (an increase in the fraction of loans that are expected not to

⁵⁵This is not quite the same thing as defining the shocks so that they all increase equilibrium credit spreads to the same amount, under the assumption of no central-bank lending. Because L_t declines from the value \bar{L} in response to the shocks, $\omega_t = \bar{\omega}_t(L_t)$ does not increase as much as $\bar{\omega}_t(\bar{L})$. We prefer to measure the size of the shock in terms that do not depend on a calculation of the endogenous response to the shock; but each of these shocks does increase the equilibrium credit spread, and to a roughly similar extent.

⁵⁶The disturbance considered in Figure 5 corresponds to the “additive χ ” disturbance of this figure.

be repaid, that is independent of the volume of private lending), neither $\bar{\Xi}'(\bar{L})$ nor $\bar{\omega}'(\bar{L})$ increases due to the shock, while both $\bar{\Xi}'(L_t)$ and $\bar{\omega}'(L_t)$ decrease, owing to the decrease in L_t (as a consequence of the increase in credit spreads).⁵⁷ Hence in the case of an “additive χ ” shock, the marginal social value of central-bank lending actually *decreases* at the time of the shock.

Clearly, the mere fact that a given disturbance is observed to increase credit spreads does not in itself prove that it would increase welfare for the central bank to lend directly to private borrowers. This is not because conventional monetary policy (*i.e.*, interest-rate policy) alone suffices to eliminate the distortions created by such a disturbance (in our model, it cannot, even if it can mitigate the distortions created by the disturbance to some extent); nor is it because central-bank credit policy is unable to influence market credit spreads (in our model, active credit policy would reduce the size of the credit spread). But even granting both of these points, if central-bank lending is not costless (and we believe that it should not be considered to be), then it is necessary to balance the costs of intervention against the benefits expected to be achieved; and our model implies that there is no simple relation between the outcome of this tradeoff and the degree to which credit spreads increase in response to a shock.

Financial disturbances increase the marginal social benefit of central-bank credit policy to a greater extent, however (relative to the size of the disturbance), if the *zero lower bound* on the policy rate prevents the policy rate from being cut in response to the shock as much as is assumed in Figure 6. The distortions created by a binding zero lower bound are even greater under the hypothesis that policy is conducted in a forward-looking way *after* the period in which the zero lower bound constrains the policy rate, so that there is no commitment to subsequent reflation of a kind that would mitigate the extent to which the zero bound results in an undesirably high level of the real policy rate. (An optimal interest-rate policy commitment, that takes account of the occasionally binding zero lower bound, will include a commitment to history-dependent policy of this sort, as discussed in Eggertsson

⁵⁷This last effect, which by itself *reduces* the marginal social value of central-bank lending, is present in the case of all four disturbances, but in the other three cases this effect is outweighed by the effects discussed in the text that increase the value of $\Xi_t^{cbt,crit}$.

and Woodford, 2003, and Cúrdia and Woodford, 2009b. However, such policy requires a type of commitment that actual central bankers seem quite reluctant to contemplate, as discussed for example by Walsh, 2009.) If a binding zero lower bound coincides with this kind of expectations about future monetary policy, the marginal social benefit of credit policy may be much greater than would be suggested by Figure 6.

This is illustrated by Figure 7, where the same four types of financial disturbances are considered, but the disturbances are assumed to be three times as large as in Figure 5. In the figure, the responses of $\Xi_t^{cbt,crit}$ are shown under two different assumptions about interest-rate policy: in the top panels, interest-rate policy is assumed to be optimal, while in the bottom row it is assumed to follow a Taylor rule of the form (4.2), again calibrated as before. The figure also illustrates the consequences of the zero lower bound on interest rates; the panels in the left column show the response of $\Xi_t^{cbt,crit}$ under the assumption that the zero lower bound can be ignored (even though this means that the policy rate is negative in the quarters immediately following the shock, under either assumption about monetary policy), whereas the corresponding panels in the right column show the response when the zero lower bound is imposed as a constraint on monetary policy.

In the upper left panel of Figure 7, interest-rate policy is optimal and the zero lower bound is assumed not to bind, as in Figure 6; hence the responses to all four disturbances are exactly the same as in Figure 6, except that the scale of the departures from the steady state is multiplied by a factor of three. In the lower left panel, we can see the difference that is made by assuming instead that interest-rate policy follows the Taylor rule (4.2).⁵⁸ The increase in $\Xi_t^{cbt,crit}$ is at least somewhat greater in the case of each of the types of financial disturbances; even in the case of the “additive χ ” shock, $\Xi_t^{cbt,crit}$ declines by less than it does under optimal interest-rate policy. Thus optimal interest-rate policy reduces at least slightly the welfare gain from active credit policy.

In both panels of the left column of Figure 7, however, the policy rate is assumed to

⁵⁸Exactly the same difference would be made in the case of disturbances of the size assumed in Figure 6, if we assumed that policy followed the Taylor rule. Note that in the case of the smaller disturbances assumed in Figure 6, there would be no difference between the two columns, as the zero lower bound would not be a binding constraint even when imposed.

become negative in response to the disturbances. The right column shows how the responses are modified when the zero lower bound is respected in our calculations. Because the zero lower bound binds in the quarters immediately following financial disturbances of the magnitude assumed in this figure,⁵⁹ the marginal social benefit of central-bank credit policy is greater than it would be if it were possible for the central bank to lower the policy rate below zero, as assumed in the panels in the left column. Even in the case of an “additive χ ” shock, it can be optimal for the central bank to lend to the private sector when such a shock occurs, even though it was not optimal in the absence of the shock. (In the case shown in the figure, this would be true in the case of a value for $\Xi^{cb}(0)$ between 3.5 and 4.0 percent per annum, for example.) The desirability of active credit policy is even greater if the zero lower bound binds and interest-rate policy is determined as in (4.2). In this case, there is a large increase in $\Xi_t^{cb,crit}$ as a result of a financial disturbance; and interestingly, in this case, the size of the increase is similar regardless of the source of the increase in credit spreads. Under each of the four types of financial disturbance that are considered, it will be optimal for the central bank to lend to private borrowers, at least in the first two quarters, even if the marginal cost of central-bank lending is as high as 10 percentage points per annum.

Thus there is a clearer case for active central-bank credit policy in the case of a financial disturbance that is severe enough to cause the zero lower bound on the policy rate to become a binding constraint; and in this case, it is to a first approximation only the size and persistence of the effects of the disturbance on credit spreads that matters for determining the extent to which credit policy is desirable. But supposing that the value of $\Xi_t^{cb,crit}$ rises above the value of $\Xi^{cb}(0)$ in response to such a disturbance, how much is it optimal for the central bank to lend, and for how long? We turn next to the characterization of optimal credit policy.

⁵⁹In the case of optimal interest-rate policy, the lower bound binds in the first three quarters in the case of the Ξ shocks, and in the first four quarters in the case of the χ shocks. In the case of the Taylor rule, the lower bound binds in the first two quarters under all four shocks. The policy rate remains at the lower bound for a longer time under an optimal policy commitment, for the same reason as in the numerical example of Eggertsson and Woodford (2003).

5 Optimal Credit Policy

We now consider the optimal state-contingent evolution of central-bank lending $\{L_t^{cb}\}$, imposing only the constraint that it be non-negative, and taking account of the resource costs of loan origination and monitoring by the central bank. We continue to assume the existence of a competitive loan market, so that the central bank lends at the same (market-clearing) interest rate i_t^b as the private intermediaries, who continue to lend even when the central bank lends as well. (Our assumption of a highly convex cost function for private intermediaries ensures that they continue to serve part of the market, even if credit policy lowers the equilibrium credit spread by several percentage points.) For simplicity, we assume in this section that the resource costs of central-bank lending are of the form $\Xi^{cb}(L^{cb}) = \tilde{\Xi}^{cb} L^{cb}$ (i.e., that the marginal cost of central-bank lending is independent of the scale of its intervention), but that $\tilde{\Xi}^{cb}$ is at least as large as $\bar{\Xi}^{cb,crit}$, the steady-state value of $\Xi_t^{cb,crit}$, so that “Treasury only” is an optimal policy in the steady state.

5.1 A Simple Case

There are clearly (theoretically possible) circumstances under which it is not only optimal for the central bank to lend directly to the private sector, but under which it is optimal to do this to the extent (or nearly the extent) necessary in order to prevent any increase in credit spreads, or any reduction in private borrowing, in response to a financial disturbance. Here we describe a simple case in which a strong result of that kind can be obtained. Suppose that the only disturbance, relative to the deterministic steady state, is a “multiplicative Ξ ” shock that increases the costs of private intermediation. That is, we continue to assume that $\chi_t(L) = 0$, while $\bar{\Xi}_t^p(L)$ is a function of the form (4.1), with $\{\tilde{\Xi}_t^p\}$ an exogenous process. In the case that we wish to examine, we assume that there are no other disturbances, and that $\tilde{\Xi}_t^p$ is always greater than or equal to its steady-state value.

If this is the only kind of financial disturbance that occurs, (2.9) implies that

$$\omega_t = \eta \tilde{\Xi}_t^p L_t^{\eta-1}. \quad (5.1)$$

We can then write

$$\bar{\Xi}_t^{p''}(L_t) = (\eta - 1) [\eta \tilde{\Xi}_t^p]^{\frac{1}{\eta-1}} \omega_t^{\frac{\eta-2}{\eta-1}},$$

and substituting this into (4.4), the first-order condition for optimal credit policy can be written in the form

$$\varphi_{\Xi,t}[\omega_t - \tilde{\Xi}^{cb}] + (\eta - 1)\varphi_{\omega,t}[\eta\tilde{\Xi}_t^p]^{\frac{1}{\eta-1}}\omega_t^{\frac{\eta-2}{\eta-1}} \leq 0, \quad (5.2)$$

where now ω_t is the only endogenous variable apart from the Lagrange multipliers. This is a condition of the form

$$\omega_t \leq \hat{\omega}(\varphi_{\omega,t}/\varphi_{\Xi,t}; \tilde{\Xi}_t^p), \quad (5.3)$$

and either this condition or (4.5) must hold with equality in any period.

Hence optimal credit policy prevents the credit spread from rising above the ceiling $\hat{\omega}(\varphi_{\omega,t}/\varphi_{\Xi,t})$; it is optimal for the central bank to hold only liquid assets only in the case that ω_t does not exceed $\hat{\omega}_t$ under that policy. We have assumed that this condition is satisfied in the steady state, but it can fail to hold when $\tilde{\Xi}_t$ increases, both because the shock increases ω_t (in the absence of active credit policy, as illustrated in Figures 8 and 9) and because it decreases $\hat{\omega}_t$. A shock of this type decreases $\hat{\omega}_t$ both because of the direct effect of $\tilde{\Xi}_t^p$ on the left-hand side of (5.2) and because it increases $\varphi_{\omega,t}/\varphi_{\Xi,t}$, owing to increased urgency of lowering credit spreads when they are unusually high.

A special case in which we obtain a particularly simple result is that in which we assume that $\tilde{\Xi}^{cb} = \tilde{\Xi}^{cb,crit}$ exactly. In this case, “Treasury only” remains optimal in the steady state, but even an infinitesimal disturbance (of the right sign) can make it optimal to move away from the corner solution. In this case (5.3) holds with equality in the steady state, and if we assume a disturbance under which $\tilde{\Xi}_t^p$ is always greater than or equal to its steady-state value, as proposed above, (5.3) will hold with equality at all times in the perturbed equilibrium as well. In this case, we can replace the pair of conditions (4.4)–(4.5) and the complementary slackness condition by the simple requirement that

$$\omega_t = \hat{\omega}(\varphi_{\omega,t}/\varphi_{\Xi,t}; \tilde{\Xi}_t^p) \quad (5.4)$$

each period.

We can also use (5.1) to write

$$\Xi_t^p = \eta^{-\frac{\eta}{\eta-1}} (\tilde{\Xi}_t^p)^{-\frac{1}{\eta-1}} \omega_t^{\frac{\eta}{\eta-1}}, \quad (5.5)$$

so that

$$\Xi_t = \eta^{-\frac{\eta}{\eta-1}} (\tilde{\Xi}_t^p)^{-\frac{1}{\eta-1}} \omega_t^{\frac{\eta}{\eta-1}} + \tilde{\Xi}^{cb} [b_t - (\eta \tilde{\Xi}_t^p)^{-\frac{1}{\eta-1}} \omega_t^{\frac{1}{\eta-1}}]. \quad (5.6)$$

Substituting this for Ξ_t in (2.2), we then obtain a complete system of equilibrium conditions to determine the endogenous variables $\{\Pi_t, Y_t, \lambda_t^b, \lambda_t^s, Z_t, b_t, \Delta_t, i_t^d, i_t^b, \omega_t\}$ and the Lagrange multipliers, given some specification of interest-rate policy (such as (4.2)), without any reference to the balance-sheet quantities of either private intermediaries or of the central bank. (Of course, once we have solved for the optimal paths of $\{b_t, \omega_t\}$, we can then solve (2.11) and (5.1) for the required paths of $\{L_t, L_t^{cb}\}$ as well.)

In the case of a relatively convex of intermediation technology (so that capacity constraints are significant in the private intermediary sector), optimal credit policy (in the special case just described) will allow relatively little variation in credit spreads in response to financial disturbances. In fact, we show in the Appendix that in the limit as η is made unboundedly large,⁶⁰ equation (5.4) reduces to the limiting form

$$\frac{\omega_t}{\bar{\omega}} = \frac{\varphi_{\Xi,t}/\bar{\varphi}_{\Xi}}{\varphi_{\omega,t}/\bar{\varphi}_{\omega}}, \quad (5.7)$$

while (5.5) reduces to the limiting form $\Xi_t^p = 0$. Note that neither expression depends on $\tilde{\Xi}_t^p$ in the limiting case. Hence in the limit, we have a system of equations to solve for the paths of $\{\omega_t\}$ and the Lagrange multipliers, along with the other endogenous variables listed in the previous paragraph, which are unaffected by any disturbance to the path of $\{\tilde{\Xi}_t^p\}$. In this limiting case, optimal credit policy ensures that $\omega_t = \bar{\omega}$ each period; the central bank should lend to the private sector to the extent necessary to prevent any disturbance to the size of the credit spread.

In fact, this is a good approximation in the case of the degree of convexity assumed in the calibration proposed above. Figure 8 shows the impulse responses to a “multiplicative Ξ ” shock of the magnitude considered in Figure 6, under the assumption that monetary policy follows the Taylor rule (4.2), when the model is calibrated as discussed above and

⁶⁰Here we fix the steady-state credit spread $\bar{\omega}$, and vary the steady-state value of $\tilde{\Xi}^p$ as we vary η , so that the intermediation technology remains consistent with this steady state. We also vary the value of $\tilde{\Xi}^{cb}$ as we vary η , so that it continues to be true that $\tilde{\Xi}^{cb} = \tilde{\Xi}^{cb,crit}$ exactly. The other model parameters are held fixed as we increase η .

$\tilde{\Xi}^{cb} = \bar{\Xi}^{cb,crit}$ exactly. The dashed lines show the impulse responses of several endogenous variables in the case that the “Treasury only” policy is adhered to. The shock increases the equilibrium credit spread by more than 2.5 percentage points,⁶¹ and as a consequence, private borrowing (and likewise private credit) contracts by more than 2 percent. This results not only in a contraction of aggregate output and in deflation, but also in increased inefficiency of the composition of private expenditure: spending by type b households contracts by more than 6 percent, while spending by type s households actually increases (owing to the low returns available on their savings).

Under an optimal credit policy (shown by the solid lines), however, the responses are quite different. Central-bank lending to the private sector should increase, to such an extent that there is barely any increase in credit spreads, and barely any decline in private borrowing. This requires considerable lending by the central bank, since private lending (not shown, but equal to $b_t - L_t^{cb}$) contracts even more (nearly twice as much) as in the absence of credit policy; the reason is that private intermediaries contract credit supply even more when they are unable to increase the interest rate at which they lend. The policy also results in practically no contraction of output, practically no decrease in inflation, practically no change in the composition of private spending, and stabilizes the economy without any need for a cut in the policy rate.

If we assume instead that interest-rate policy is optimal (as in Figure 9, where the disturbance is the same as in Figure 8), the effects of the shock on aggregate output and inflation are much smaller even in the absence of credit policy, and the effects on the relative expenditure by the two types are also somewhat smaller than those shown in Figure 8. Nonetheless, interest-rate policy alone cannot prevent an increase in credit spreads and a contraction of credit supply similar to those in Figure 8 (even though the zero lower bound does not bind for a shock of this size), and there remains a role for credit policy. In fact, the optimal degree of lending by the central bank in response to the shock is essentially the same as in Figure 8, and again this is the amount needed to virtually eliminate any increase in the credit spread or any reduction in private borrowing. The equilibrium under optimal

⁶¹Note that the spread does not increase by the full 4 percent by which $\bar{\omega}_t(\bar{L})$ increases, because of the contraction of L_t .

credit policy is virtually the same as in Figure 8: for while the Taylor rule is not too close an approximation to optimal policy in the absence of credit policy, there is little need for a more sophisticated interest-rate policy if credit policy is used optimally.

5.2 Alternative Cases of Financial Disturbances

The case in which $\tilde{\Xi}^{cb}$ is assumed to be no higher than $\bar{\Xi}^{cb,crit}$ is the one most favorable to an argument for active credit policy, of course. Figure 10 shows the responses under active credit policy (and in the absence of credit policy) for the same disturbance as in Figure 9, and again under the assumption of optimal interest-rate policy, but in the case that $\tilde{\Xi}^{cb}$ is assumed to be 10 basis points higher than $\bar{\Xi}^{cb,crit}$.⁶² It remains optimal for the central bank to begin lending to private borrowers when a disturbance of this kind occurs, as one would expect from Figure 6. (Recall that Figure 6 showed $\Xi_t^{cb,crit}$ rising by much more than 10 basis points in response to a multiplicative Ξ shock.)

However, while the optimal increase in central-bank lending at the time of the shock is not greatly less than in Figure 9, the optimal *duration* of active credit policy is much less in this case (though the disturbance itself has the same persistence in both cases). In Figure 9, optimal credit policy requires the central bank’s lending not to have been completely phased out six years and more after the shock; in Figure 10, central-bank lending should already have been sharply contracted in the second year, and credit policy is phased out altogether after a little more than two years. And while the effects of the disturbance on credit spreads, private borrowing, aggregate output, and the composition of expenditure are all mitigated by optimal credit policy, even relative to what can be achieved by optimal interest-rate policy (again, in a case where interest-rate policy is not constrained by the zero lower bound), the effects are not completely eliminated. If one assumes an event higher value of $\bar{\Xi}^{cb,crit}$, the optimal use of credit policy is further weakened.

The case considered above is also especially favorable to the optimality of active credit policy because the type of financial disturbance considered is a “multiplicative Ξ ” shock, which as shown in Figures 6 and 7 is the type that increases the marginal social benefit of

⁶²Under our calibration, $\bar{\Xi}^{cb,crit}$ is equal to 3.48 percent per annum, as shown in Figures 6 and 7, so this means that $\tilde{\Xi}^{cb} = 3.58$ percent.

credit policy to the greatest extent, for a given size of increase in $\bar{\omega}_t(\bar{L})$. If we were to consider instead an “additive χ ” shock, again of the size assumed in Figure 6 — and if, as in Figure 10, we assume optimal interest-rate policy and a value of $\tilde{\Xi}^{cb}$ 10 basis points higher than $\bar{\Xi}^{cb,crit}$ — then optimal credit policy will instead involve $L_t^{cb} = 0$ in all periods. This can be seen from Figure 6: a disturbance of this kind never raises the value of $\Xi_t^{cb,crit}$ by as much as 10 basis points above the steady-state level.⁶³ Yet this alternative disturbance increases the equilibrium credit spread (under “Treasury only”) by virtually the same amount as the disturbance considered in Figure 10; and in fact, the size of the contraction in credit, the size of the change in the composition of expenditure, and so on, are essentially the same in this case as in the one shown in Figure 10. Hence it matters a great deal what causes an increase in credit spreads, in order to judge the appropriate response of credit policy; and the aspects of the disturbance that matter cannot easily be judged from an observation of the aggregate effects of the disturbance alone.

As still another example (yielding a somewhat intermediate conclusion), Figure 11 displays the optimal policy response in the case of a “multiplicative χ ” shock, also of the size assumed in Figure 6. Under this hypothesis, credit spreads increase owing to a change in the perceived risk of default by borrowers, but the fraction of loans expected to be bad increases with the scale of lending, at such a rate as to leave unchanged the elasticity of the credit-supply curve. It is assumed in this figure, as in Figure 10, that interest-rate policy is optimal and that $\tilde{\Xi}^{cb}$ is 10 basis points higher than $\bar{\Xi}^{cb,crit}$. As Figure 6 indicates, this type of disturbance, like the “multiplicative Ξ ” shock, increases the marginal social value of central-bank lending enough to justify active credit policy. However, in this case, the optimal amount of central-bank lending is both substantially smaller than in Figure 10, and much more transitory: the optimal policy only involves lending to private borrowers in the

⁶³The fact that we do assume a value of $\tilde{\Xi}^{cb}$ slightly above $\bar{\Xi}^{cb,crit}$ is, however, important for this result. If we assume that $\tilde{\Xi}^{cb} = \bar{\Xi}^{cb,crit}$ exactly, as in Figure 9, then in the case of an “additive χ ” shock, it would not be optimal for the central bank to lend at the time that the shock occurs, but it *would* be optimal for it to commit to lend later. The optimal commitment of this kind would actually involve substantial central-bank lending, lasting for many years — something that is not obvious from the path of $\Xi_t^{cb,crit}$ shown in Figure 6. But because this conclusion is radically changed by even a small increase in the assumed cost of lending by the central bank, we do not further discuss this fine point in the theory of optimal credit policy here.

first two quarters. While the credit policy does mitigate the effects of the disturbance to some extent (not too visible in the figure), the economy’s response to the disturbance is not dramatically different than it would be in the absence of credit policy: the credit spread still increases by 2 percentage points, and so on. The size, persistence and effects of optimal credit policy are similarly modest in the case of an “additive Ξ ” shock.

As Figure 7 has already shown, the case for active credit policy is somewhat stronger — and more robust to alternative assumptions about the nature of the financial disturbance — in the case of disturbances large enough to cause the zero lower bound to constrain interest-rate policy, and especially if interest-rate policy is purely forward-looking (as in the case of a Taylor rule). For example, Figure 12 displays the optimal policy response in the case of a “multiplicative Ξ ” shock that is now three times as large as the one considered in Figure 10 (which is to say, of the size assumed in Figure 7). As in Figure 10, $\tilde{\Xi}^{cb}$ is assumed to be 10 basis points higher than $\bar{\Xi}^{cb,crit}$, but interest-rate policy is now assumed to follow the Taylor rule (4.2). As the figure indicates, the zero lower bound on the policy rate does bind in the case of a shock of this size. Because $\tilde{\Xi}^{cb}$ is higher than $\bar{\Xi}^{cb,crit}$, optimal credit policy does not insulate the economy from the effects of the financial disturbance to the extent that was true in Figure 8. But it does prevent the credit spread from rising nearly as much as it would in the absence of credit policy, and averts a large part of the decline in private borrowing as well; this eliminates the declines in output and inflation almost entirely, and greatly reduces the distortion of the composition of private spending.

Figures 13 and 14 show the corresponding optimal responses in the case of “multiplicative χ ” and “additive χ ” shocks respectively, again of the size assumed in Figure 7. While the optimal scale and duration of lending by the central bank is considerably smaller in these two cases than in the case of the “multiplicative Ξ ” shock shown in Figure 12, it is still optimal in these cases for the central bank to intervene in a substantial way for several quarters. And credit policy has important effects in each of these cases, significantly reducing the size of the spike in credit spreads at the time of the shock, and considerably weakening the contraction of spending by type b households. Our conclusions about optimal credit policy are also more robust in this case, not only in the sense that the optimal responses to the two different types of χ shocks are now more similar, but (more importantly) in that our results are no

longer especially sensitive to the precise of $\tilde{\Xi}^{cb}$, even in the case of the “additive χ ” shock.

At the same time, the optimal responses to the χ shocks shown in these last two figures are quite different from the optimal response to a “multiplicative Ξ ” shock. Thus even when the zero lower bound binds and interest-rate policy can be described by a Taylor rule, the size of the increase in credit spreads alone provides insufficient information to judge the optimal credit policy response with much precision.

5.3 Segmented Credit Markets

In the simple model expounded above, there is a single credit market and single borrowing rate i_t^b charged for loans in this market; our discussion of central-bank credit policy has correspondingly simply referred to the optimal quantity of central-bank lending to the private sector overall, as if the allocation of this credit is not an issue. In reality, of course, there are many distinct credit markets, and many different parties to which the central bank might consider lending. Moreover, since there is only a potential case to be made for central-bank credit policy when private financial markets are severely impaired, it does not make sense to assume efficient allocation of credit among different classes of borrowers by the private sector, so that only the total credit extended by the central bank would matter. Our simple discussion here has sought merely to clarify the connection that exists, in principle, between decisions about credit policy and the other dimensions of credit policy. An analysis of credit policy that could actually be used as a basis for credit policy decisions would instead have to allow for multiple credit markets, with imperfect arbitrage between them.

We do not here attempt an extension of our model in that direction. (A simple extension would be to allow for multiple types of “type b ” households, each only able to borrow in a particular market with its own borrowing rate, and market-specific frictions for the intermediaries lending in each of these markets.) We shall simply note that in such an extension, there would be a distinct first-order condition, analogous to conditions (4.4)–(4.5), for each of the segmented credit markets. There would be no reason to assume that the question whether active credit policy is justified should have a single answer at a given point in time: lending might be justified in one or two specific markets while the corner solution remained optimal in the other markets.

Thus the main determinants of whether central-bank credit policy is justified — when it is justifiable to initiate active policy, and when it would be correct to phase out such programs — should not be questions such as whether the zero lower bound on interest-rate policy binds, or whether the central bank continues to undershoot the level of real GDP that it would like to attain. While aggregate conditions will be one factor that affects the shadow value of marginal reductions in the size of credit spreads (represented by the multiplier $\varphi_{\omega,t}$ in (4.4)), the value of this multiplier will likely be different for different markets, and the main determinants of variations in it are likely to be market-specific. This will be even more true of the other variables that enter into the first-order condition (4.4).

6 Conclusions

We have shown that a canonical New Keynesian model of the monetary transmission mechanism can be extended in a fairly simple way to allow analysis of additional dimensions of central bank policy that have been at center stage during the recent global financial crisis: variations in the size and composition of the central-bank balance sheet, and in the interest rate paid on reserves, alongside the traditional monetary policy issue of the choice of an operating target for the federal funds rate (or some similar overnight inter-bank rate elsewhere). But we have found that explicitly modeling the role of the central-bank balance sheet in equilibrium determination need not imply any role for “quantitative easing” as an additional tool of stabilization policy, even when the zero lower bound on the policy rate is reached. While different results might be obtained under alternative theoretical assumptions, our reading of the Bank of Japan’s experience with quantitative easing leads us to suspect that our theoretical irrelevance result is likely close to the truth.

Our analysis indicates that there may, instead, be a role for central-bank credit policy (or for targeted asset purchases), when private financial markets are sufficiently impaired. It is worth stressing that the central bank’s asset holdings should also be irrelevant for macroeconomic equilibrium in the case of well-functioning financial markets that can be accessed at low cost by any economic agents who would benefit from such trades. Hence credit policy is only a relevant additional dimension of central-bank policy to the extent that

private markets are not already effectively eliminating most of the potential gains from trade in financial instruments; and while we recognize that there are circumstances, such as those arising during the recent crisis, in which it is arguable that financial markets fail to fulfill that function, we are inclined to suspect that it is only at times of unusual financial distress that active credit policy will have substantial benefits.

Even when financial markets are seriously disrupted, as indicated by significant increases in interest-rate spreads, one must be cautious in drawing conclusions about the welfare consequences of credit policy. While we have shown that it is possible for disturbances originating in the financial sector to create circumstances under central-bank lending to the private sector can increase welfare, our analysis has also shown that the mere size of the increase in credit spreads does not provide sufficient information about the nature of the disturbance to judge the benefits of active credit policy. Moreover, while we have shown that credit policy is more likely to be justified when a financial disturbance is severe enough to make the zero lower bound a binding constraint on interest-rate policy, the mere fact that the zero bound has been reached is neither necessary nor sufficient for active credit policy to be welfare-improving. In particular, the appropriateness of active credit policy is likely to depend on conditions that are specific to the markets for particular financial instruments, and that therefore cannot be assessed on the basis of macroeconomic conditions alone.

At the same time, when active credit policy is justified, our analysis implies that there is no need to balance the benefits of such policy for the efficiency of financial intermediation against any supposed inflationary threat inherent in the increased size of the central bank's balance sheet. For our analysis shows that decisions about interest-rate policy are not constrained in any direct way by decisions about either the size or composition of the central bank's balance sheet, as long as the central bank is willing to adjust the interest rate paid on reserves appropriately. Payment of interest on reserves can make even a large quantity of excess reserves consistent with high short-term interest rates, and hence with monetary and financial conditions consistent with a central bank's inflation target — and this will be true even when the economy is no longer characterized by any large degree of slack productive capacity.

Thus in considering an appropriate strategy for “exit” from the current unconventional

posture of the Federal Reserve, it is important to recognize that, according to our model, there is no reason that the timing of the Fed's reduction in its holdings of assets other than short-term Treasuries must be tied in some mechanical way to the timing of a decision to raise the federal funds rate above its current historically low level. These are independent dimensions of policy, not only in the sense that they can be varied independently in practice, but in that they have different effects as well, and are appropriately adjusted on the basis of considerations that are fairly different. Just as the primary justification for undertaking non-traditional asset purchases should relate to conditions specific to the markets for those assets, rather than to the central bank's assessment about whether the level of the policy rate is correct, so should decisions about the proper time at which to unwind such purchases.

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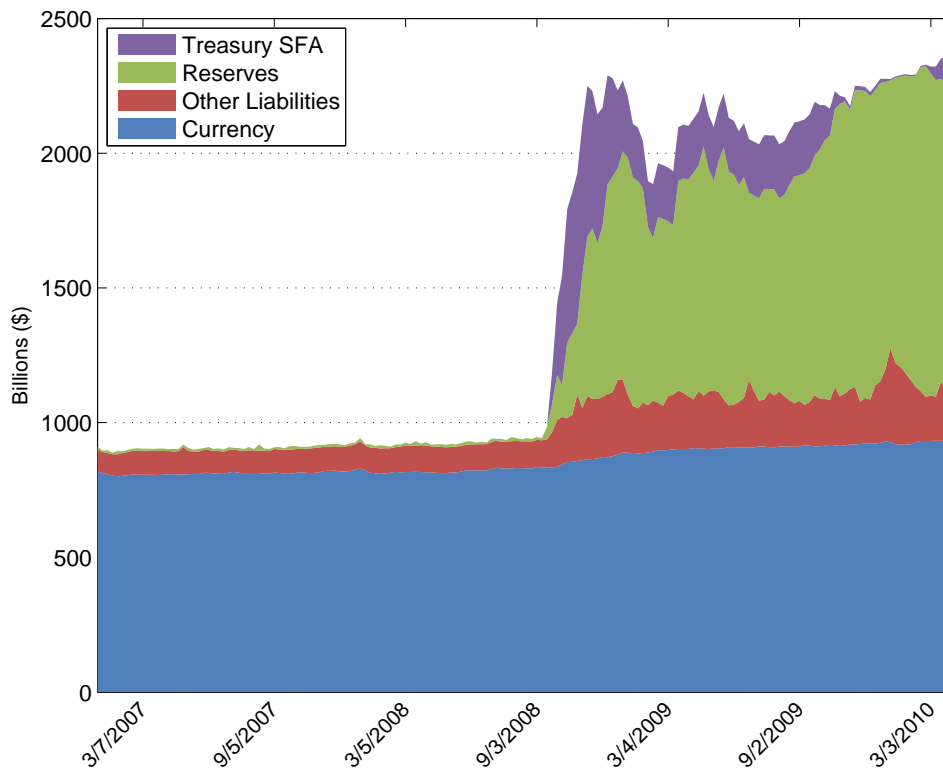


Figure 1: Liabilities of the Federal Reserve. (Source: Federal Reserve Board.)

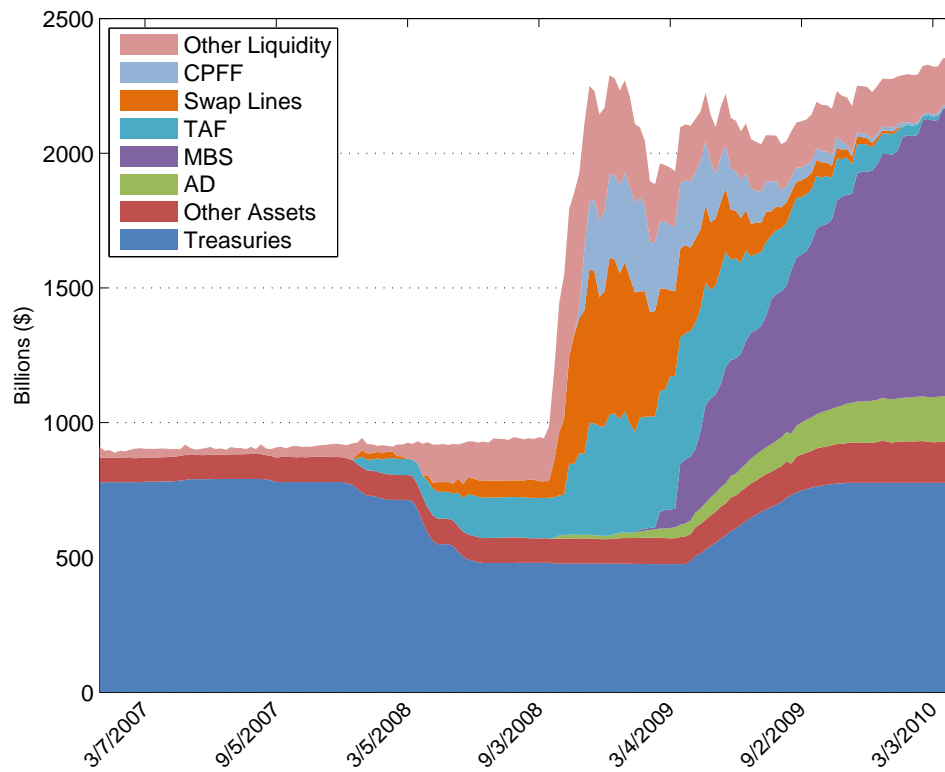


Figure 2: Assets of the Federal Reserve. (Source: Federal Reserve Board.)

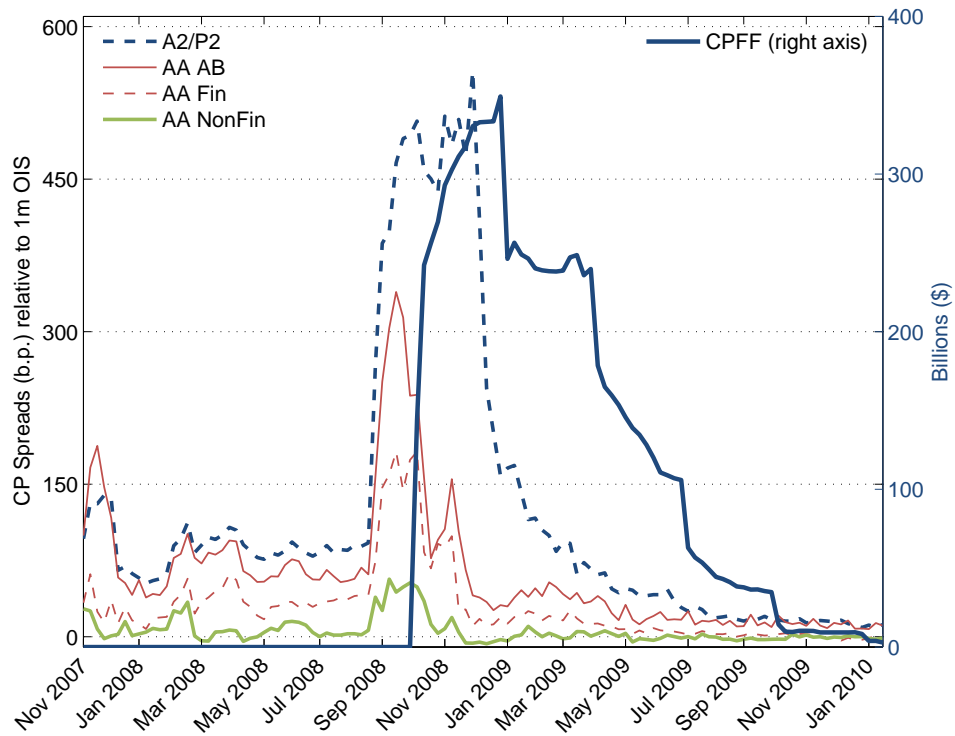


Figure 3: Spreads between yields on four different classes of commercial paper and the 1-month OIS rate, together with the value of paper acquired by the Fed under its Commercial Paper Funding Facility. (Source: Federal Reserve Board.)

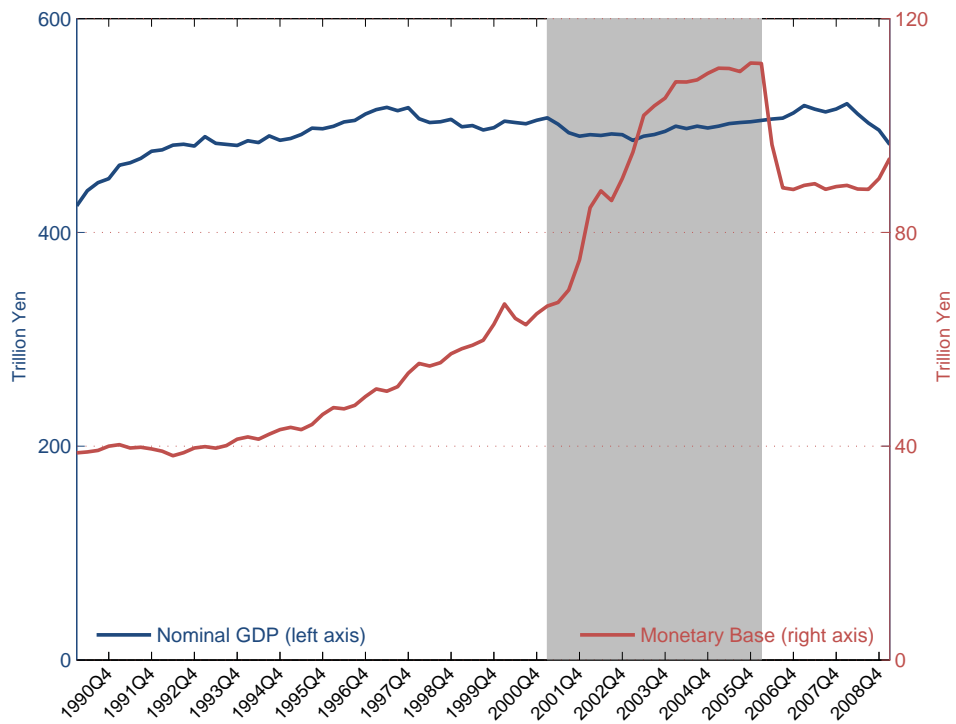


Figure 4: The monetary base and nominal GDP for Japan (both seasonally adjusted), 1990-2009. The shaded region shows the period of “quantitative easing,” from March 2001 through March 2006. (Sources: IMF International Financial Statistics and Bank of Japan.)

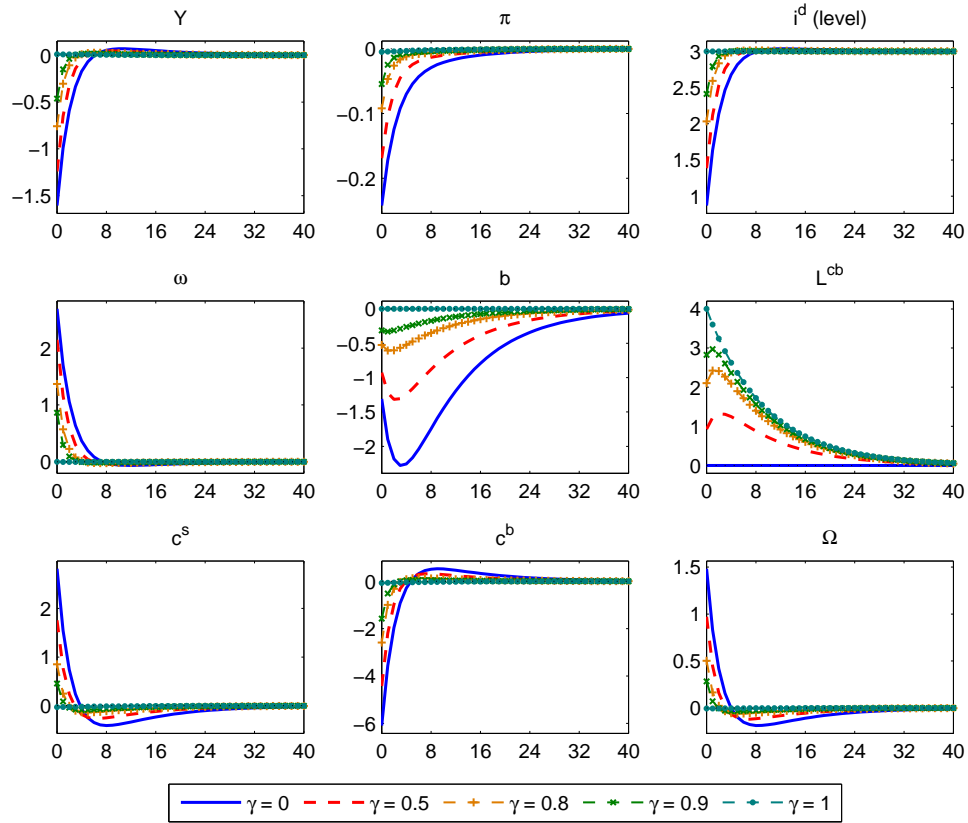


Figure 5: Responses to an increase in $\tilde{\chi}_t$ that increases $\omega_t(\bar{L})$ by 4 percentage points, in the case of credit policies characterized by alternative values of γ . The response of i_t^d is plotted in terms of the absolute level, so that the distance from the zero bound can be observed; all other variables are measured as deviations from their respective steady-state values.

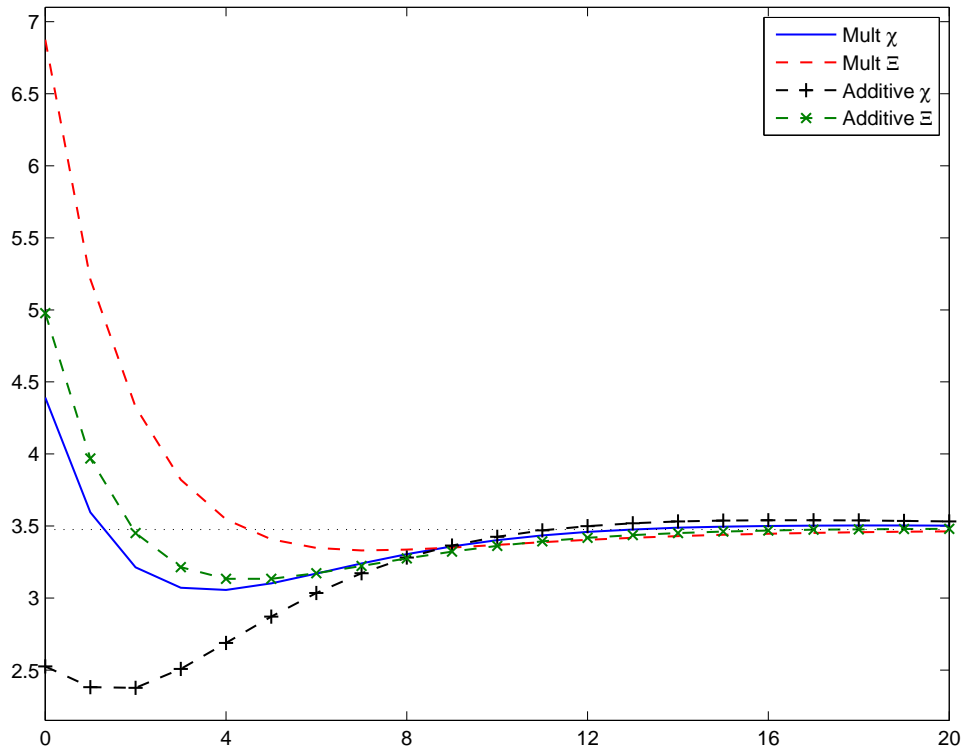


Figure 6: Response of the critical threshold value of $\Xi^{cb'}(0)$ for a corner solution, in the case of four different types of “purely financial” disturbances, each of which increases $\omega_t(\bar{L})$ by 4 percentage points. Interest-rate policy responds optimally in each case.

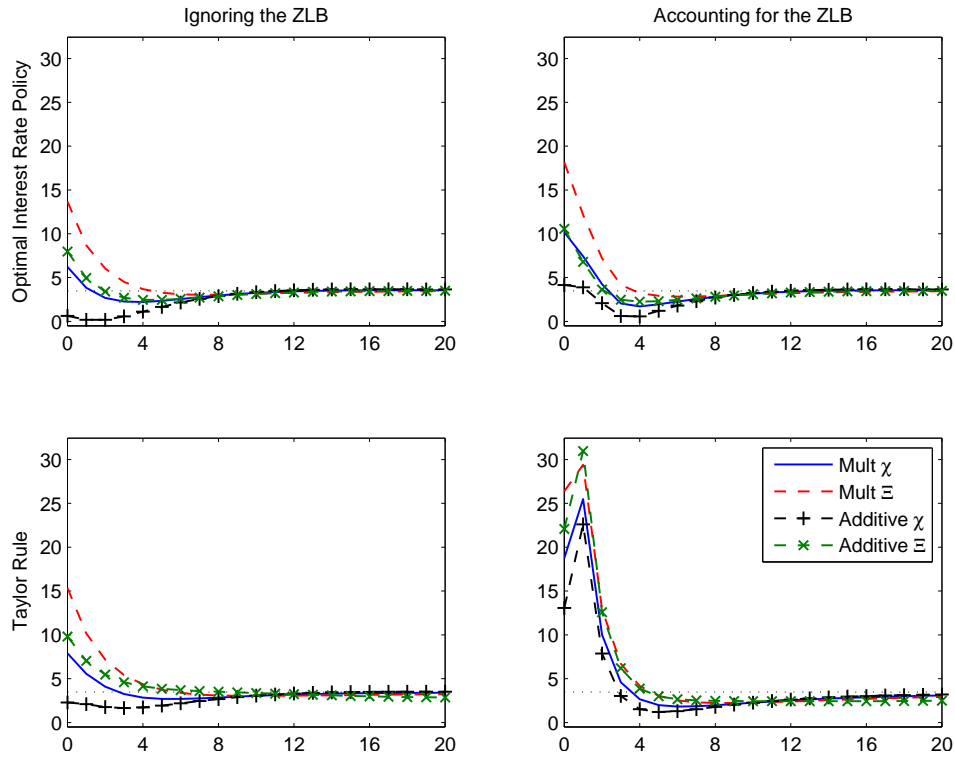


Figure 7: Response of the critical threshold value of $\Xi^{cb}(\bar{L})$ for a corner solution, in the case of financial disturbances that increase $\omega_t(\bar{L})$ by 12 percentage points. Interest-rate policy responds optimally in the panels of the top row, but follows a Taylor rule in the bottom panels. The zero lower bound is assumed not to constrain interest-rate policy in the panels of the left column, while the constraint is imposed in the corresponding panels of the right column.

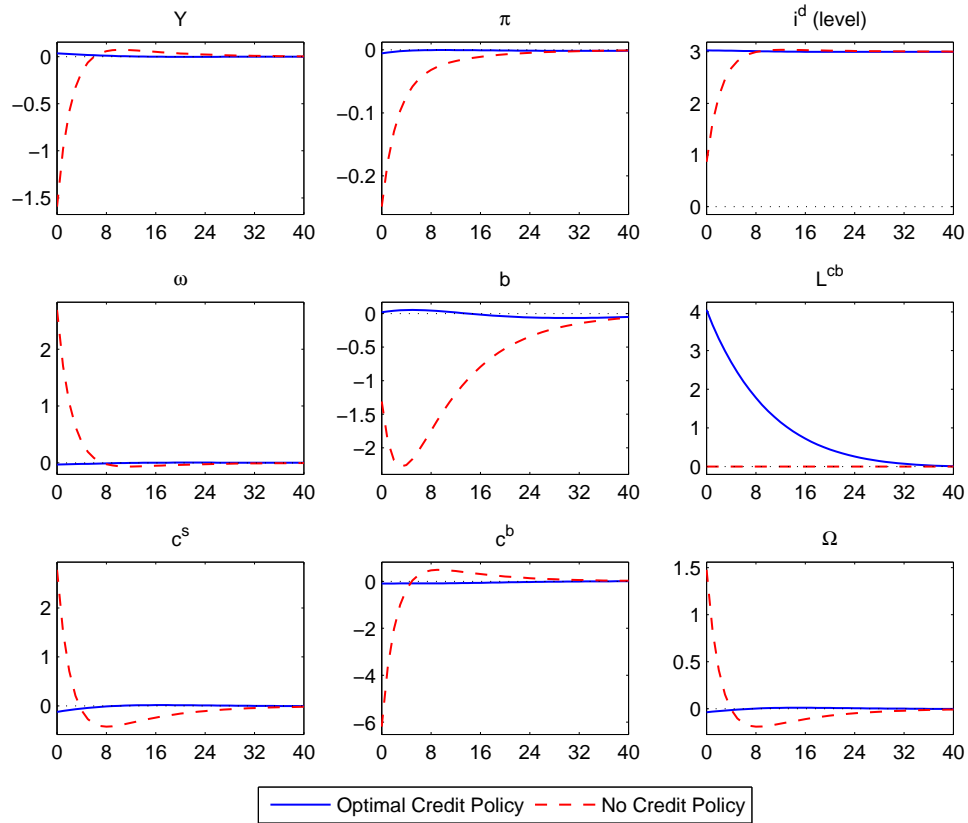


Figure 8: Impulse responses under optimal credit policy compared to those under a policy of “Treasury only,” in the case of a “multiplicative Ξ ” shock of the size considered in Figure 6, if interest-rate policy follows a Taylor rule and $\tilde{\Xi}^{cb}$ is exactly equal to the steady-state critical threshold.

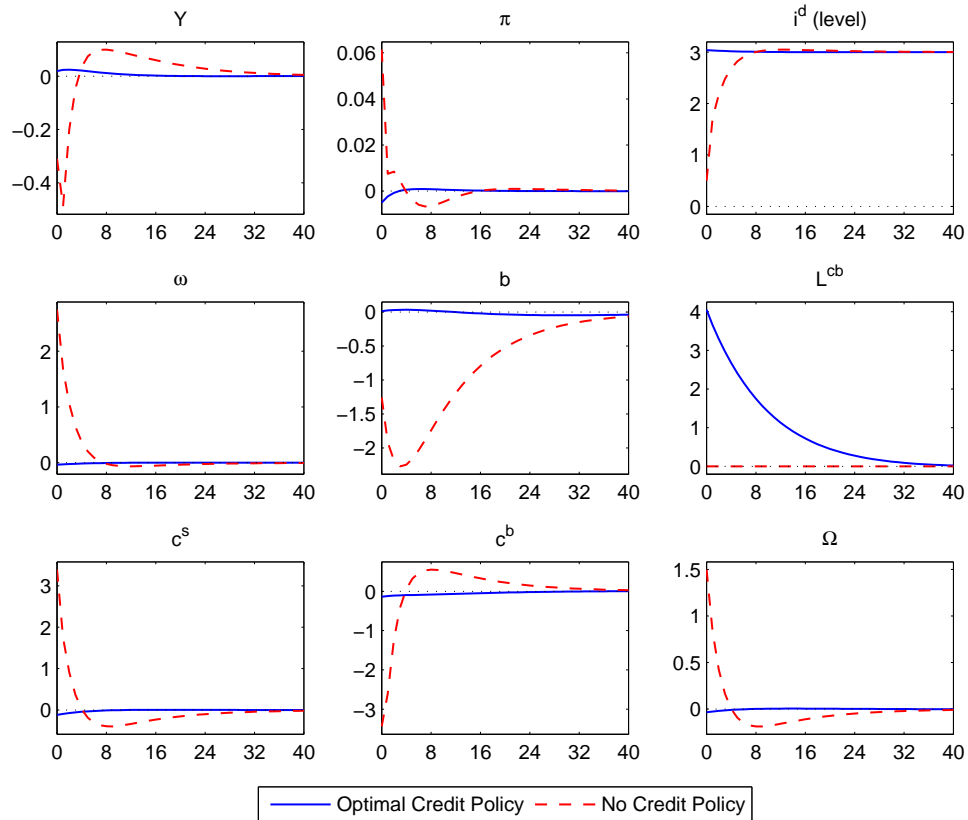


Figure 9: Impulse responses under optimal credit policy compared to those under a policy of “Treasury only,” with the same disturbance as in Figure 8, but under optimal interest-rate policy. Again $\tilde{\Xi}^{cb}$ is exactly equal to the steady-state critical threshold.

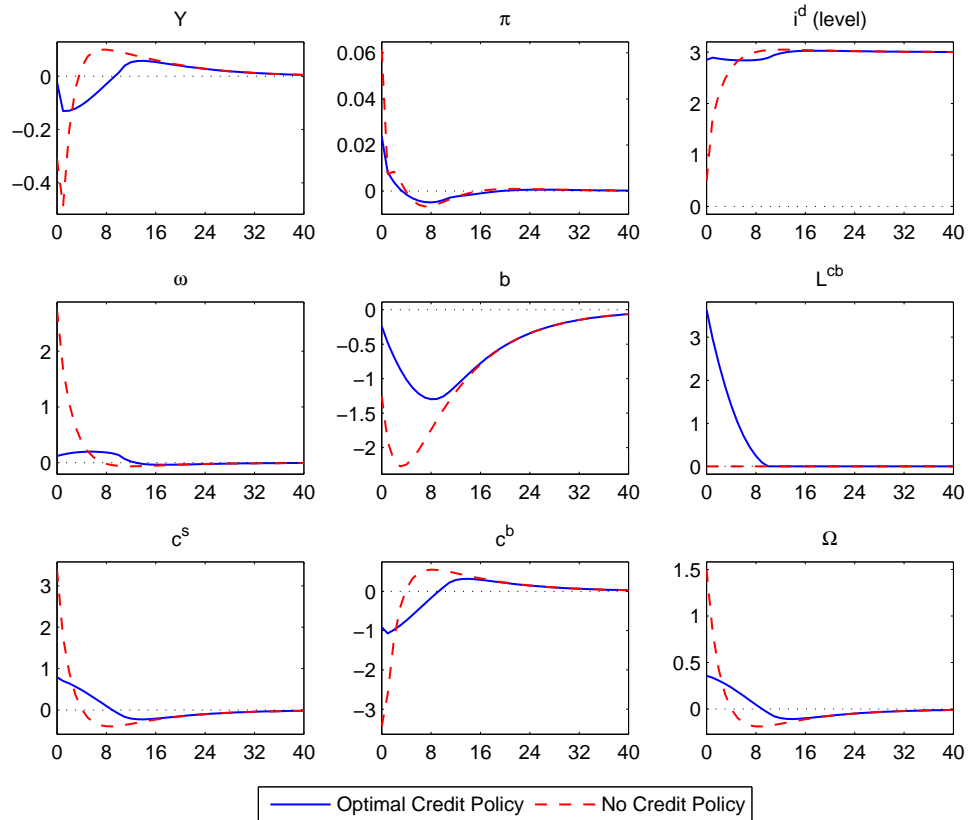


Figure 10: Impulse responses under optimal credit policy and under “Treasury only,” for the same disturbance and interest-rate policy as in Figure 9, but when $\tilde{\Xi}^{cb}$ is 10bp higher than the steady-state critical threshold.

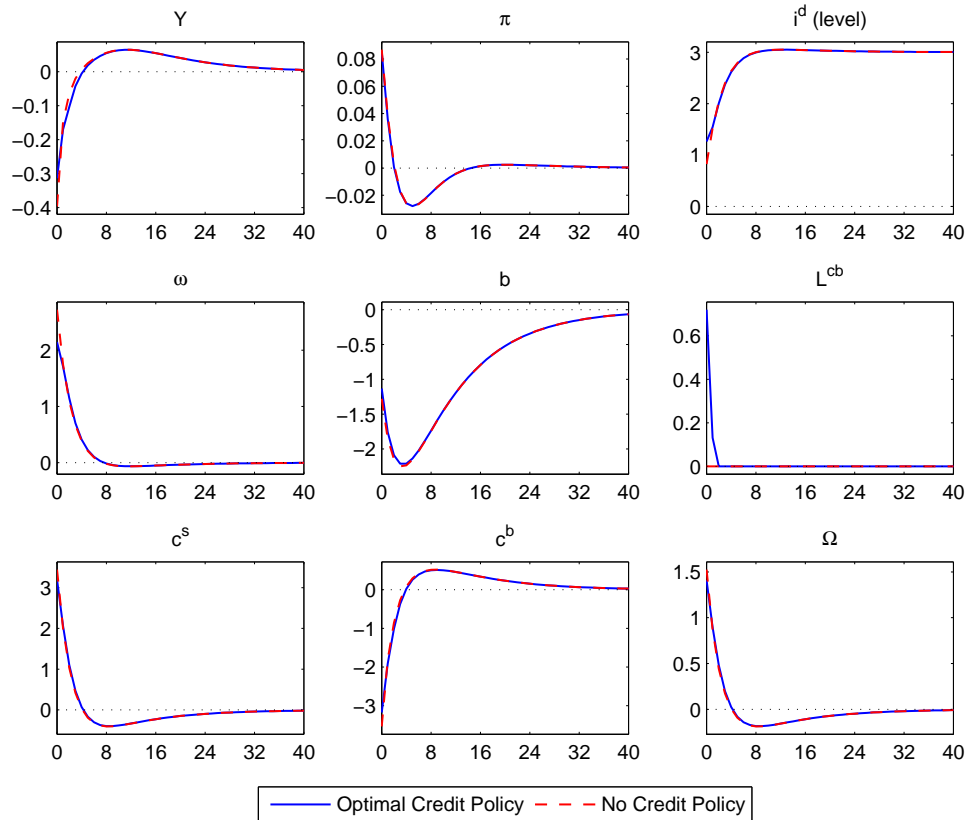


Figure 11: Impulse responses under optimal credit policy and under “Treasury only,” in the case of a “multiplicative χ ” shock of the size considered in Figure 6. As in Figure 10, interest-rate policy is optimal and $\tilde{\Xi}^{cb}$ is 10bp higher than the steady-state critical threshold.

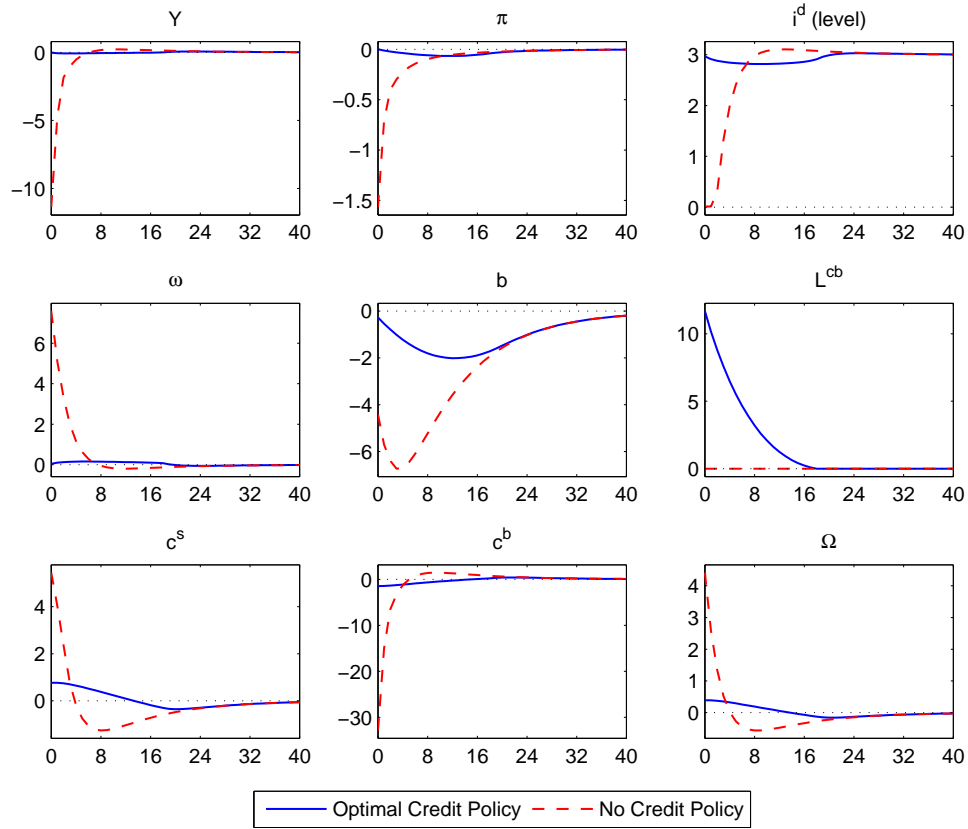


Figure 12: Impulse responses under optimal credit policy and under “Treasury only,” in the case of a “multiplicative Ξ ” shock of the size considered in Figure 7. Interest-rate policy follows a Taylor rule and $\tilde{\Xi}^{cb}$ is 10bp higher than the steady-state critical threshold.

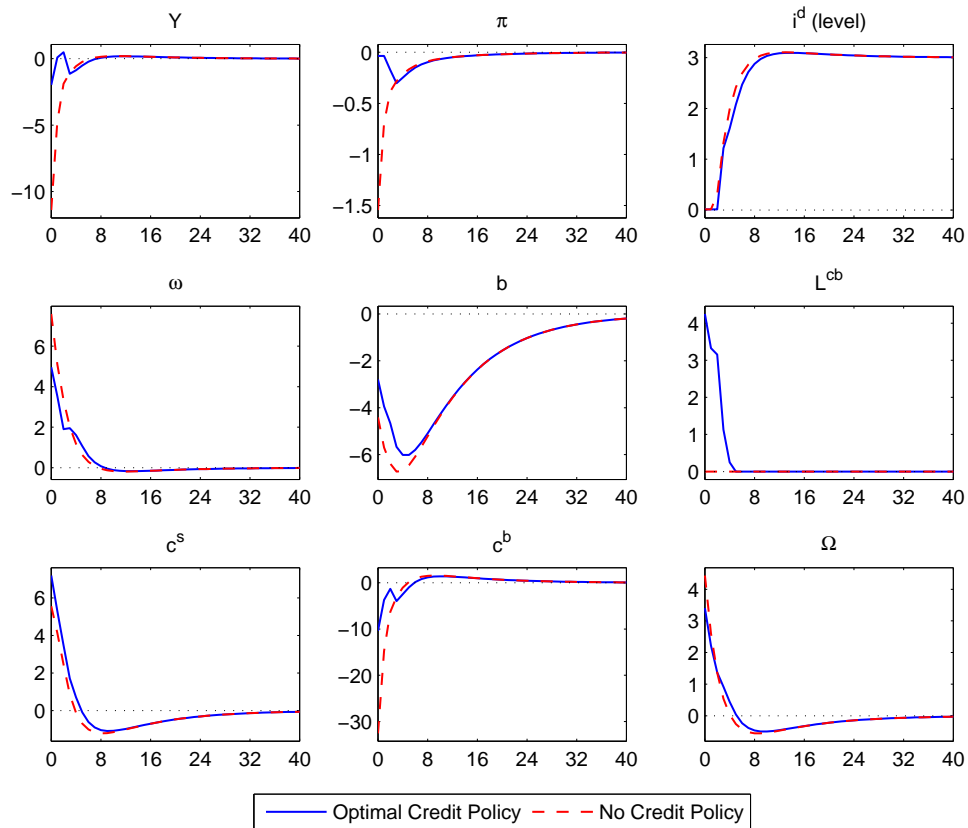


Figure 13: Impulse responses in the case of a “multiplicative χ ” shock of the size considered in Figure 7, under the same assumptions as in Figure 12.

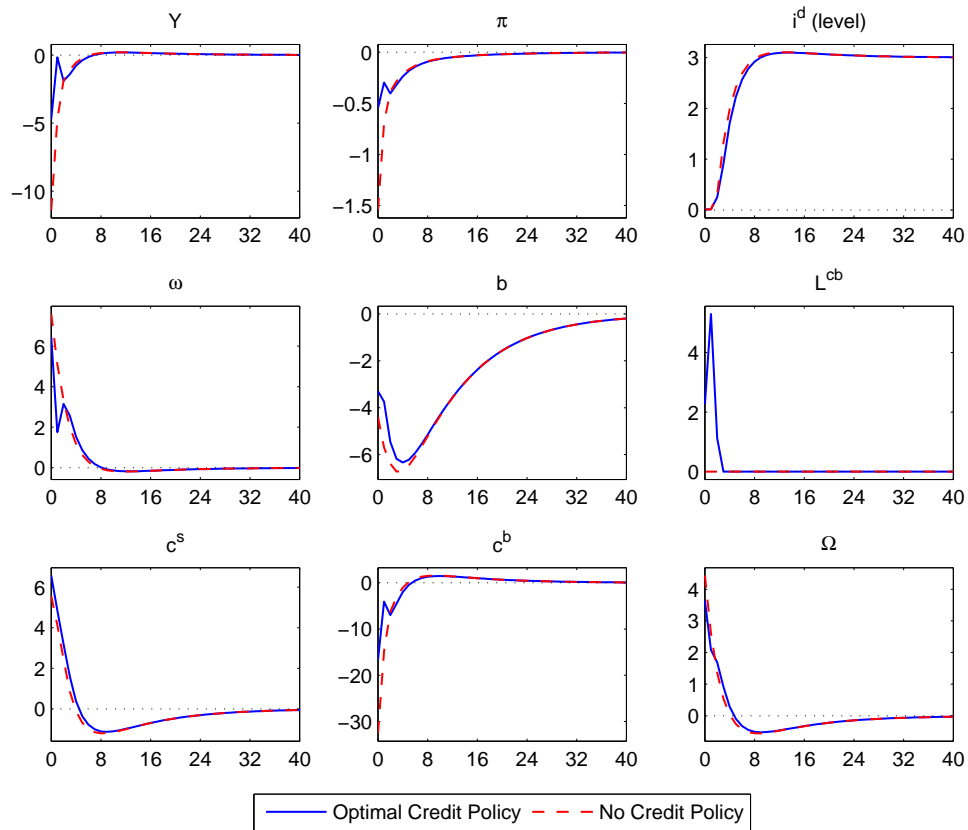


Figure 14: Impulse responses in the case of an “additive χ ” shock of the size considered in Figure 7, under the same assumptions as in Figure 12.