A Model-Based Approach for Planning and Developing A Family of Technology-Based Products

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Management Department Working Paper

First Draft: December 1996; Revision: April 1998

To appear in INFORMS Journal "Manufacturing & Service Operations Management"

Abstract

In this paper, we address the product-family design problem of a firm in a market in which customers choose products based on some measure of product performance. By developing products as a family, the firm can reduce the cost of developing individual product variants due to the reuse of a common product platform. Such a platform, designed in an aggregate-planning phase that precedes the development of individual product variants, is itself expensive to develop. Hence, its costs must be weighed against the benefits of its reuse in a family. We offer a model for capturing costs of product development when the family consists of variants based on a common platform. It is shown that the model can be converted into a network-optimization problem, and the optimal product-family can be identified under fairly general conditions by determining the shortest path of its network formulation. We also analytically examine the effect of alternative product designs on product-family composition, and discuss the implications of investing in new-product technology. Finally, we illustrate our model and managerial insights with an application from the electronics industry.

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1.1 Introduction

Firms in many industries face an increasing need to offer greater levels of product variety to meet more closely the diverse needs of customers in global markets (MacDuffie, Sethuraman and Fisher 1996). For many firms, especially in the high-technology industry, while the portion of the product lifecycle during which profits can be earned has become progressively shorter, due to rapid technological advances, development costs have risen sharply due to increasing technological and design complexity (Nevens et al. 1990). As a consequence of these trends, insufficient time is available to recoup the significant investments in product development, and the ability to reuse design elements across products has become important for such firms to reduce costs and to benefit from product variety.

In contrast to the "conventional" product-development process in which individual products are developed independently (and often by different design teams), some firms have begun using what we refer to as the *product-family design* approach. In this approach, a firm makes a significant effort in the early stages of product development to design differentiated but related products that are based on a common product platform. With careful planning, similarities among product variants are identified and exploited to specify the platform that eventually leads to a simplification of the individual product development tasks.

The usefulness of the platform-based product-family design approach depends on the ability of the firm to convert the effort invested in developing the platform into reduced cost of developing individual variants. Creating a platform often requires sizable initial investments in time and resources to define the appropriate architecture of the product-family that would lead to a high degree of reusability and the resulting time and cost savings in the individual product-development phase. Returns on these investments depend not only on the market size, but also on the number of products the firm expects to launch, and the similarities among these products. Design of the product-family often involves making joint decisions regarding the architecture and composition of a product family.

1.2 Product-family Design Issues: An Illustration

The following example, taken from our research study at the site of a high-technology firm, serves to illustrate the benefits of and issues facing product-family design. Accu-Data, a market leader in electronic instrumentation, offers data-acquisition (DAQ) products capable of acquiring data (in the form of electrical signals) at rates of several thousand samples per second. (The company name is disguised.) Data thus acquired from the physical world is pre-processed and converted into digital signals that can be processed by a personal computer. These products are used in a variety of industries (industrial automation, telecom, etc) in conjunction with personal computers and often as a substitute for traditional dedicated measurement equipment. Figure 1 shows a block diagram that describes the various components of a DAQ product. DAQ products are designed around a key design parameter called “sampling-rate”, the rate (measured in samples per second) at which a board can sample data reliably. Customer requirements vary widely in terms of the sampling-rate performance required to accurately represent an analog signal in digital form. Product-development efforts at Accu-Data focus on advancing the maximum sampling rate. For a given family, product-development hinges on offering products with different performance characteristics using the design of components MU, A, ADC and TC (Figure 1).

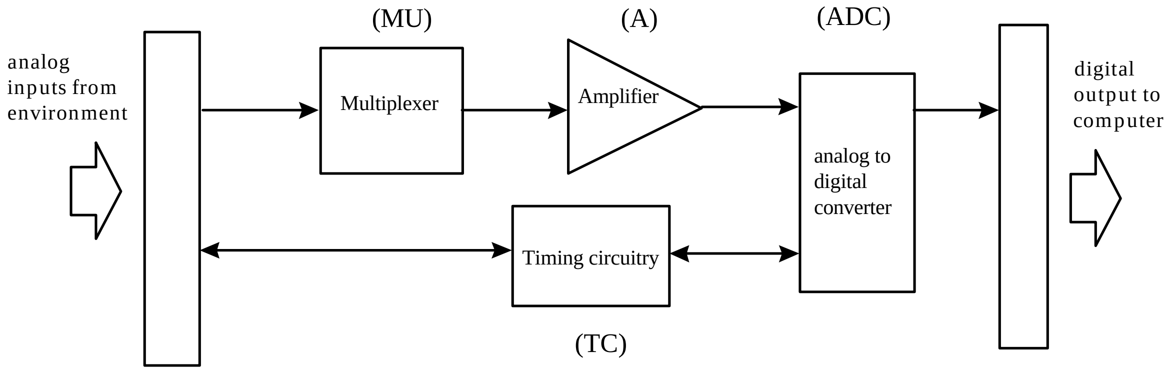


Figure 1: Schematic Diagram of a Data Acquisition Product

Recognizing the rapid growth of a potential market with the need for faster sampling rates and using an emerging computer architecture (the PCI architecture[1]), Accu-Data embarked on an initiative to develop a set of products that meet the range of sample rates desired by customers in this segment. Development costs were expected to be high for individual products in the new family, and investments in developing a platform (comprised of the timing circuitry and other modules) which would be reused in individual products can greatly simplify the task of developing product variants. The up-front investment in time and effort to design the architecture and the platform would now enable the firm to adapt the platform and existing products to create new products. However, developing the platform would cost half a million dollars or more (depending on the extent of the platform), and the benefit of this approach depends on the number of products offered and the savings in developing individual products due to the platform. The number of products itself depends on the customer preferences for different performance levels (sampling rates), and the degree of cannibalization among closely-located products. Thus, the customer-demand information must be integrated with the development-cost information to make platform and product-family planning decisions.

[1] PCI stands for Peripheral Component Interconnect, an emerging computer bus architecture that provides a high-speed data path between a computer’s CPU and peripheral devices (video, disk, network, etc.), and thereby greatly increases data throughput rates over the alternative AT bus architecture.

Personal computer, consumer electronics, and automobile producers are other examples of firms that design and deliver a variety of complex products to the market. As in the example above, high development cost of such products make it useful for firms in these industries to consider developing product variants as a *family*, which is defined as a set of products that share a common platform but differ from each other in the level of performance they deliver to the customer. Attempts to relate the product offerings more closely, however, may result in reduced market coverage due to increased cannibalization among products. Additionally, effort made in the early stages of the product-family design process to create a platform with a number of reusable modules also costs time and money. Also, investments in new technologies may be necessary to increase the range of performance the product family can offer. The firm must weigh the costs and benefits of these investments before expending significant resources in the aggregate development of the product family and in new technology to increase the performance range of the family. In this context, the product-family design issues considered in this paper can be summarized as follows:

(1) What is the optimal product-family composition for a firm in a given market? How many products should the firm offer in its family, and how closely related should these products be?

(2) How does creating a reusable "platform" influence the composition of a family?

(3) What are the relationships between a firm's investments in aggregate development and in acquiring new technology that increases the performance range of the family?

We consider these questions for technology-based products. For many such products, cost and performance are the major factors that influence the evaluation of a product (by the producer as well as the consumer), rather than brand image and other intangibles. We present a model of product-family design that helps (i) identify the optimal set of variants to be offered under sufficiently general conditions, and (ii) obtain insights into the optimal family composition. Our optimization model for product family development combines market factors (demand model) and product development costs (cost model). The model is based on the characterization of the product-family using three factors: the number of products in the family (“population”), maximum performance offered in the family (“span”), and the distance between adjacent products (‘separation”). For the case in which products are selected from a candidate set, we develop a computational procedure to identify the optimal set of variants to be offered under fairly general conditions. This procedure relies on the network structure of the corresponding shortest-path problem. In addition, making some simplifying assumptions, we derive analytical insights into the optimal product-family composition. For example, when the cost of adapting existing products to form new products is a convex function of the separation between products, we find that a *more reusable* platform may make it optimal to offer *fewer* products. We also find that the return on investments in technologies that help increase the performance span is greater when the firm also optimally adjusts its product portfolio in response to these technologies.

The rest of this paper is organized as follows. In Section 2 we review related literature, and in Section 3 discuss our conceptualization of the product-family development process and the formalization of our model. We show in Section 4 how the general version of the product-family design problem can be converted into a network formulation. In Section 5, we analytically explore a number of tradeoffs in product-family design using certain symmetry assumptions to reduce the complexity of the problem. We present an industrial application from one of our research sites in Section 6 to show how our model may be used to plan product-family offerings. We conclude in Section 7 with a discussion of the strengths and limitations of this work and directions for future research.

2. Related Literature

The process of developing new products has attracted growing attention over the last few years (Clark and Wheelwright 1993, Ulrich and Eppinger 1994), yet the notion of developing a family of products using common platforms is relatively new (Meyer, Tertzakian and Utterback 1997). The rationale for developing a family of products–customer demand for product variety and its associated complexity and costs (such as loss of scale economies)–has been studied by researchers in the economics literature (Baumol, Panzar and Willig 1985, Tirole 1988). Based on their empirical study, Kekre and Srinivasan (1990) found that broader product lines were more profitable, despite the increase in production costs. Quelch and Kenny (1993), however, argue that more products may not always mean greater profitability.

Models for optimal product line design have been a topic of considerable attention in the marketing literature (Green and Krieger 1985, Sudharshan, May and Gruca 1988). These models have traditionally focused on the ability of products to attract new customers and their potential to cause demand substitution (cannibalization). Recent literature on product-line design has focused on numerical models and heuristic procedures for identifying optimal products (Kohli and Krishnamurti 1987, Dobson and Kalish 1993, Nair, Thakur and Wen 1995). While the initial work in this stream did not consider development and production costs, subsequent enhancements use a cost structure in which volume-independent fixed costs and volume-dependent variable costs are associated with each product offered in the line. For example, Dobson and Yano (1995) presented a more general model that considers resource sharing among products chosen by combining consumer utility information (obtained using conjoint analysis) with resource usage costs. Our work differs from this stream in that it considers a reusable platform and its effect on the composition of a product family.

The marketing-manufacturing coordination problem examined by de Groote (1994) is similar to the one studied in this paper, but addresses the product-family decision in a different environment. The demand model used by de Groote is similar to ours, and is based on the consumer choice models common in the marketing literature. The focus in de Groote (1994) is, however, on volume-dependent manufacturing costs. Instead, we develop a detailed model of volume-independent product-family development costs that are not considered by de Groote. As we discuss later in section 7, a richer model could include both manufacturing and product-development costs.

The effect of product variety on the production system has attracted considerable attention in recent days. Karmarkar and Pitbladdo (1993) discussed the usefulness of accounting data in supporting decisions regarding the production quantities of a variety of related products under capacity constraints. MacDuffie, Sethuraman and Fisher (1996) empirically studied the costs and benefits of product variety in the automobile industry and found that although parts complexity has a negative effect on product variety, a lean management approach can offer a firm the ability to offer variety at lower cost. Fisher, Ramdas, and Ulrich (1996) developed parts commonization strategies for reducing the cost of variety in automobile braking systems. Our work contributes to this stream of research in that developing a family of products based on a common platform can enable the economic attainment of product variety.

Recently, product design issues have received growing attention in the context of product variety management. It is now well understood that a large fraction of life cycle costs for products is determined at the product design stage itself (Whitney 1988). These costs arise not only from the development effort but also from design choices made in the early stages that impact future manufacturing costs. Product modularity, in particular, has received close examination as an approach to offset development and production costs. Ulrich (1995) discusses several types of modularity and how they contribute to lower costs in a differentiated product line. Clark and Baldwin (1993) also identify a number of benefits of modularity, such as its ability to create economic value when there is uncertainty in the design process. The role of design issues in creating product families has also been empirically investigated by Sanderson and Uzumeri (1996).

The importance of product design decisions made during the early stages of product-family development forms the basic motivation for this paper. Existing approaches for optimal product line design ignore the tradeoffs arising due to design synergies among product variants, the choice of alternative product architectures, and the cost of acquiring technological know-how, and have traditionally targeted consumer products in which market factors such as subjective preferences of consumers and brand image are dominating factors. We focus on products for which design complexity and performance are the driving factors. For such products, the reuse of design elements in a number of product variants within the family is critical to reducing development costs: while higher performance products clearly require a certain amount of extra effort to develop new design elements, a large portion of the design effort may simply involve adaptation of existing design elements already developed for lower performance product variants. These decisions made before resources are committed to design and manufacture a family of new products form the focus of this paper.

3. Model Setting and Formulation

The sequence of events and actions by the firm in our model are as follows: Once the firm identifies a market opportunity for a set of products that meet a range of customer needs, it begins the *aggregate-planning phase* of the product-family. This is the focus of our paper. Two major types of decisions are made during this phase. The first is the selection of a *platform* common to all products in the family. The second is the number of products to be offered in the family and their individual performance levels. The platform, which may not be a physical product itself, is defined here as the set of major components and subsystems shared across all products in the family. (This definition is based on our field studies as well as the discussions of the Special Interest Group on Platform Planning techniques convened by the Product Development and Management Association (Liebe 1996). It should be noted that this usage of platform is different from the term “platform projects” used by Clark and Wheelwright (1993) to refer to next-generation new-product development initiatives.) Once the platform and product offerings have been identified, we assume the firm develops the individual product variants in a *sequence* beginning with the product that has the lowest level of performance, and gradually progressing to more complex variants with greater levels of performance. Note that there may exist alternative ways of developing a family of products, such as simultaneous development of variants or the secondary-wave approach described in Clark and Wheelwright (1993).

The firm targets a performance interval Z in which to introduce products. Let a product variant 0 with performance $f_0 \in Z$ define the lowest performance level of a product in the family (We will henceforth refer to variant 0 as the “base product.”) Further, let the set of products $S = \{1, 2, ..., n\}$ represent variants in the product-family with respective performance levels $\{f_1, f_2, ..., f_n\}$ $(f_i \in Z, i = 1,2,\ldots,n)$ such that $f_0 < f_1 < f_2 < ... < f_n$. The maximum performance f_T in the product-family that the firm can offer (such that $f_n \le f_T$) is determined by the technology T that the firm acquires/develops in house. Let the cost of acquiring/developing this technology be represented by $C_T(f_T, f_P)$, where f_P is due to the level of technology already possessed by the firm. The cost function $C_T(f_T, f_P)$ is assumed to be increasing in f_T and may be discontinuous when technological know-how is acquired / developed in discrete steps.

We now turn our attention to the conceptualization of development costs for a product family. The platform selected in the aggregate-planning phase will be created and reused in all product variants. Each variant consists of components that are either unique to the variant, are a part of the platform, or are adapted from existing variants (see Figure 2). Development cost F_{agg} in the aggregate-planning phase involves the development of the platform and depends on the *scope* g of the platform selected (the number and complexity of functional elements that are reused throughout the family). In our model, the firm selects a platform for development from a set of discrete alternatives with scopes $\{g_1, g_2, ...\}$. Because the measurement of scope / complexity of a platform is largely dependent on the product design in question we leave g as a parameter in our model which is specified by the firm, thereby not making any assumptions about how g is measured. Creating a platform that allows for greater reusability (larger g) may require more effort in the specification of task structures and components, which in the individual development phase would pay off in the simplification of development tasks (Clark and Baldwin 1993). The cost of aggregate development F_{agg} is therefore modeled as an increasing function of the scope of the platform. (Since we will be interested in only one property of a platform - its scope g - we will henceforth refer to a platform g.)

Development costs for an individual variant are incurred in the phase following aggregate development, and consist of (i) *creative design* cost I that involves creation of functional elements which are unique for each variant, and (ii) *adaptation* costs g that involve the reuse and adaptation of components designed for earlier variants. Effort invested in creating a platform g can contribute to the decrease of both the creative design and the adaptation costs. The cost g of adapting components from existing variant i to the design of a new variant j is written as $g(g, f_i, f_j)$. The cost of creating components unique to variant j is $I(h, g, f_j)$, where h is an *efficiency parameter* that is dependent upon the process of design creation: all else being equal, a firm with a more efficient design process, or greater h (perhaps as a result of investments in systems and processes such as CAD / CAM and rapid prototyping), will incur smaller costs in the development of the platform and individual variants. h (like g) is a discrete parameter that is based on the product development process.

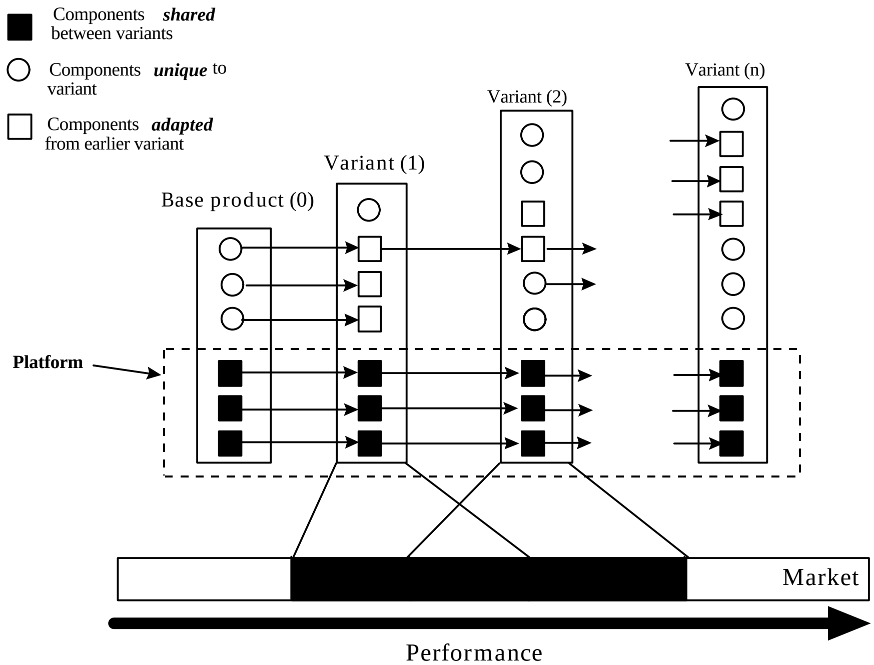


Figure 2: Schematic View of a Product Family

In sequential development, the development of any product i in the set $S = \{1, 2, \ldots, n\}$ involves making changes in the design of product $i-1$, where the number of such changes also depends on the platform. The total cost of developing a product-family with platform g can be written as the sum of aggregate-planning costs and individual product-development costs:

$$F_{agg}(g) + I(h, g, f_0) + \sum_{i=1}^{n} \left[I(h, g, f_i) + g(g, f_{i-1}, f_i) \right] \tag{3.1}$$

We will adopt the convention that the costs in the aggregate-planning phase include the costs of platform development as well as those associated with the development of the base product 0 with performance f_0. We can thus write the total development cost function as

$$\mathrm{G}(g, S) = F_{agg}(g) + \sum_{i=1}^{n} \left[I(h, g, f_i) + g(g, f_{i-1}, f_i) \right] \tag{3.2}$$

Volume-dependent or variable costs of any product i depend on both the platform common to the family and performance f_i of the product and are denoted by $v(g, f_i)$. (We thus allow the variable costs to depend upon the platform, but assume that they are independent of other products in the family. This is done to focus on the product-family design issues.) We do not make any assumptions regarding the functional form of variable cost function $v(g, f_i)$: while material costs may increase for platforms that allow greater reusability (since individual components may have to be over-designed), such platforms may also reduce production costs due to economies of scale resulting from increased number of common elements among products. The exact dependence would be subject to product- and firm-specific factors.

The development cost $G(g, S)$ is a fixed cost with respect to volume, but is a variable cost with respect to the product-family composition. The total cost function for the output (volume) vector $q_S = \{q_1, q_2, ..., q_n\}$ can be written as

$$C(g, q_S) = G(g, S) + \sum_{i \in S} q_i \; v(g, f_i) \quad (3.3)$$

For our analysis in this paper, we assume that the development cost function $G(g, S)$ satisfies the following two properties:

(i) The cost function $G(g, S)$ is subadditive: the cost of developing a higher performance variant j is always less when it is adapted from an intermediate variant i rather than from the base product 0.

(ii) The development-cost function $G(g, S)$ satisfies the *set-monotonicity* property, which states that the cost of developing any set of variants S be no greater than the cost of developing any super-set S' of variants such that $S \subset S'$. Whether a cost function for a product-family possesses the set-monotonicity property can be checked as follows. Consider the case where the firm chooses a set of variants S for market introduction from a candidate set C of variant options (i.e. $S \subseteq C$).

$$S_{ij} \subseteq \{k | \, f_i < f_k < f_j \;\; and \;\; i, j \in S, k \in C - S\} \quad (3.4)$$

whose elements are ordered according to increasing f_k and can be addressed by the labels $l_{(1)}, l_{(2)}, ..., l_{(|S_{ij}|)}$. The set S_{ij} is thus a subset of the set of all candidate products with performance levels between f_i and f_j which are not in S.

Lemma 1: The development cost function for the product-family satisfies the set-monotonicity property if

$$g(g, f_i, f_j) \leq I(h, g, f_{l_{(1)}}) + g(g, f_i, f_{l_{(1)}}) + \sum_{k=2}^{|S_{ij}|} I(h, g, f_{l_{(k)}}) + g(g, f_{l_{(k-1)}}, f_{l_{(k)}}) + g(g, f_{l_{(|S_{ij}|)}}, f_j) \text{ for all } S_{ij}$$

for all choices of $i, j \in S$ *such that* $f_i < f_j$, *where* S_{ij} *is defined in (3.4).*

Proof: The proof proceeds by contradiction. Let a cost function violate the above condition for some set S_{ij}. Then it is possible for the firm to reduce the total development cost by adding products from the set S_{ij} in the product family, thus violating the monotonicity property.

When the firm *extends* an already existing product family, development costs can be described as follows. Let S_e denote the set of performance levels of existing products.

Following our conceptualization of the adaptation cost function, the development cost incurred by the firm to add a new product $j \in S$ to its product-family can be written as
$I(h, g, f_j) + g(g, f_i, f_j)$,

where $f_i = \max\{f_{j-1}, f_e\}$ with $f_e = \max_f\{f \mid f \in S_e \text{ and } f < f_j\}$

If f_e does not exist, then $f_i = f_{j-1}$.

3.1 Market Demand Model For the Product Family

We now discuss the model of market demand for the family. For technology-based industrial products, we find it to be reasonable (from our field studies) to assume that selling price p is a function only of the performance level $z \in Z$. The function $p(z)$ is exogenously determined by the market, and we allow it to be of a general form. For this price function, the firm determines the distribution of customers in the interval Z as the intensity function $f(z)$, where $f(z)dz$ is the number of customers in the performance interval (z, $z+dz$) at the offered price $p(z)$. We refer to $f(z)$ as the "demand intensity function" in this paper. For a product-family offering $S = \{0, 1, 2, \ldots, n\}$, the life-cycle demand d_i for each product i can be obtained from the function $f(z)$ as follows:

$$d_i = \int_0^\infty p_{z,i}\, f(z)\, dz \qquad (3.5)$$

where $p_{z,i}$ is the proportion of customers at performance level z that purchase product i. ($p_{z,i}$ may also be viewed as the probability of purchase of product with performance f_i by a customer at z.) This model permits incorporation of demand uncertainty by allowing the function $f(z)$ to be a random variable at performance level z. When $p_{z,i}$ is independent of $f(z)$, the expected demand $E[d_i]$ is given by:

$$E[d_i] = \int_0^\infty p_{z,i}\, E[f(z)]dz.$$

The demand model described above, although not dynamic, permits a variety of static market behavior. In what follows, we describe briefly its application to a consumer choice model in which consumers choose products "closest" to their ideal points. This demand model besides being commonly used in the marketing literature (Carpenter and Nakamoto 1990) , captures key features of customers in technology-based markets, and hence our choice. However, this is not a limitation of the model presented in this paper as the model can be modified easily to accommodate other demand features.

The ideal-point model of demand is based on the premise that (i) consumers have different preferences for each price-performance combination; and (ii) each consumer prefers one price-performance combination over any other, where product performance defines the ideal product for the consumer. A consumer with an ideal product at performance level z associates a utility value $u_z(f_i, p(f_i))$ with a product of performance f_i, which is a function of the product performance and price (for convenience, we will henceforth simply use the term $u_z(f_i)$ for the utility function). Consumers are performance-sensitive, and do not consider a product for purchase if a product with a performance closer to their ideal point is already available. Thus, the only products considered by the consumer are the two neighboring products i and $i+1$ such that $f_i \le z \le f_{i+1}$. Using a share of utility-based choice rule (see Green and Kreiger 1993) to describe the probability of purchase of a product by any consumer in the market, the purchase probabilities (when the choice is limited to the two products f_i and f_{i+1}) can be described by

$$\begin{pmatrix} p_{z,i} \\ p_{z,i+1} \end{pmatrix} = \begin{pmatrix} \dfrac{u_z(f_i)}{u_z(f_i) + u_z(f_{i+1}) + u_z^0} \\ \dfrac{u_z(f_{i+1})}{u_z(f_i) + u_z(f_{i+1}) + u_z^0} \end{pmatrix} \tag{3.6}$$

where $u_z(f_i)$ and $u_z(f_{i+1})$ are the consumer's utilities for products i and $i+1$, and u_z^0 is the consumer's utility if neither i nor $i+1$ is chosen (“balking”). With the above model to determine $p_{z,i}$, we can write the expected life-cycle demand $E[d_i]$ (when $p_{z,i}$ is independent of $f(z)$) as follows:

$$\begin{aligned} &E[d_i] = \int_{f_{i-1}}^{f_{i+1}} p_{z,i}\, E[f(z)]\, dz \quad \text{for } i = 1,2,\ldots n-1 \\ &E[d_0] = \int_{0}^{f_1} p_{z,i}\, E[f(z)]\, dz; \quad E[d_n] = \int_{f_{n-1}}^{\infty} p_{z,i}\, E[f(z)]\, dz \end{aligned} \tag{3.7}$$

Note that the above formulation does not depend on the developer of products: i.e., (3.6) and (3.7) can be used to describe the demand for product at f_i even if the product belongs to a competitor. Likewise, equation (3.6) can be modified to account for multiple product offerings at the same performance level, perhaps due to competitors.

3.2. Product-family Profitability

The optimal product-family configuration (in terms of the performance levels of the product variants offered in the market) can be obtained by finding the set of product variants that maximize the profit. We begin by specifying the objective function when the target performance range Z does not contain existing products belonging to the firm or a competitor; the case with existing products is considered in detail in Section 4.

When the demand function $f(z)$ is deterministic, the objective function of a firm that maximizes profit can be written as:

GP: Maximize

$$\Pi(g, f_1, f_2, \ldots, f_n, n) = \sum_{i=0}^{n} \left[p(f_i) - v(g, f_i) \right] d_i - F_{agg}(g) - \left[\sum_{i=1}^{n} I(h, g, f_i) + g(g, f_{i-1}, f_i) \right] - C_T(f_n, f_P) \tag{3.8}$$

s.t.

$f_0 \le f_1 \le f_2 \le \ldots \le f_n$

$g \in \{g_1, g_2, \ldots\}$

where f_P is the technology already possessed by the firm.

Observe that **GP** can be generalized to maximize expected profit when the demand is stochastic. Further, when the functions $p(.)$ and $v(.)$ are independent of demand realizations, the objective function is modified by using $E[d_i]$ in place of d_i. Thus, in these situations, we could handle demand uncertainty by using the expected value of the demand intensity function $f(z)$.

Note that in Equation (3.8) profits are maximized not only over the composition of the product family, but also over the set of available platforms $\{g_1, g_2, \ldots\}$. This is a nonlinear optimization problem in which the *number* of variables is also a decision variable. The maximum performance level f_n determines the level of technology to be acquired for the family. The product-family design problem **TP** is a special case of **GP** where technology investments have already been made. The limit on the highest performance level f_n is thus known *a priori* based on the technology available. **TP** differs from **GP** only in the absence of f_n in the list of decision variables and the term $C_T(f_T, f_P)$ from the objective function.

It is evident that both **GP** and **TP**, while representing only a specific class of product-family design problems in which products can be ranked by performance, are still very difficult to solve in the general case. In the following section, we show the network structure of **GP** and present an efficient solution approach by only making an additional assumption that the firm chooses its product-family from a set of candidate products. We also show how the problem can be formulated to account for competitors' products, and how, using the same formulation, optimal product-line extension decisions can be made.

While the underlying network structure is useful as a computational tool, it does not lend itself easily to a study of the various tradeoffs that are inherent in the design of product families. We study such tradeoffs in Section 5 after making certain symmetry assumptions to reduce problem complexity, while preserving the essential features that make the product-family design problem complex: that the optimal family composition depends upon interactions among products through demand as well as cost functions.

4. Network Representation of the Product-family Design Problem

We have thus far considered the case when the product performance can be continuously varied within the range of performance levels. For the purpose of application to most industrial product-family design problems, it is helpful to convert the continuous product-family design problem into a problem of discrete product choice for a given platform g and a pre-specified function that maps performance to price. In this section, we formulate the product-family design problem in its discrete form, and show that it can be represented as a shortest-path problem and solved by finding the minimum-cost path through a network whose nodes represent candidate products. For the formulation discussed here prices are fixed based on performance. Modifications to include budgetary constraints can be made without much difficulty. To bring out the embedded network structure, we first consider the case with no existing products.

Let $\mathbf{C} = \{0,1,2,\ldots,N\}$ represent a set of candidate products with performance levels $\{\Phi_0, \Phi_1, \ldots, \Phi_N\}$, respectively, from which the firm chooses a subset with performance levels $\{f_1, f_2, \ldots, f_n\}$ for its family (for ease of exposition we assume that the base product Φ_0 is always available at the end of the aggregate development phase). Defining $f_{-1} = 0$ we rewrite the objective function (3.8) as follows:

$$\Pi(g, f_1, f_2, \ldots, f_n, n) = \sum_{i=0}^{n} \left[p(f_i) - v(g, f_i) \right] \int_{f_{i-1}}^{f_{i+1}} p_{z,i} \; f(z)\, dz$$

$$- F_{agg}(g) - \sum_{i=1}^{n} \left[I(h, g, f_i) + g(g, f_{i-1}, f_i) \right] - C_T(f_n, f_P)$$

$$= \sum_{i=0}^{n-1} \left(M_{i,(i,i+1)} + M_{i+1,(i,i+1)} - I(h, g, f_{i+1}) - g(g, f_i, f_{i+1}) \right)$$

$$- F_{agg}(g) + \left[p(f_0) - v(g, f_0) \right] \int_{0}^{f_0} p_{z,0} \, f(z) dz \qquad (4.1)$$

$$+ \left[p(f_n) - v(g, f_n) \right] \int_{f_n}^{\infty} p_{z,n} \, f(z) dz - C_T(f_n, f_P)$$

where

$$M_{k,(i,j)} = \left[p(f_k) - v(g, f_k) \right] \int_{f_i}^{f_j} p_{z,k} \; f(z)\, dz$$

The function $M_{k,(i,j)}$ can be interpreted as the net payoff to the firm due to customers that have ideal points in the interval $\lfloor f_i, f_j \rfloor$ and purchase product k. Observe that in (4.1) the objective function is comprised of terms that involve only *adjacent* products: this allows us to construct the network representation for the problem (Figure 3).

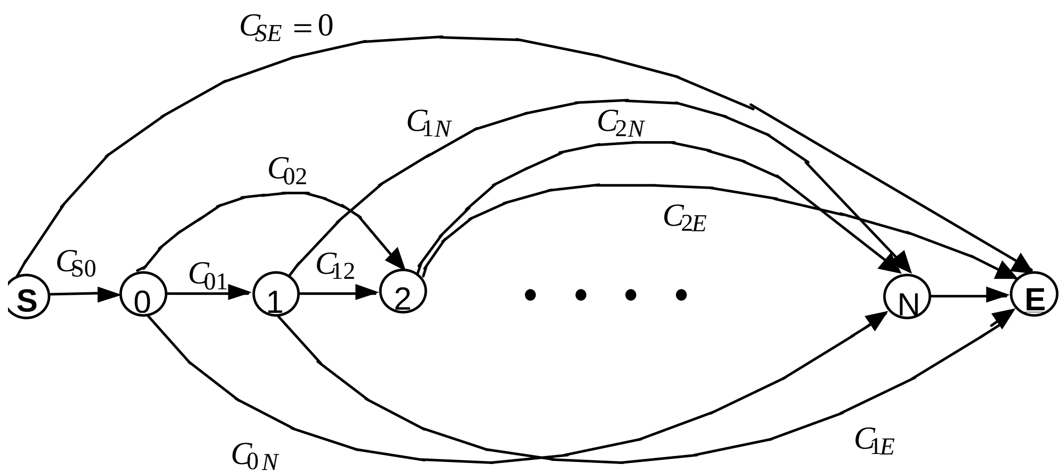


Figure 3: Network Representation of the Product-family Design Problem

In Figure 3, nodes $\{1,2,...,n\}$ represent the candidate products, while node 0 is the base product and nodes S and E respectively denote the source and sink nodes. The cost on arc (i,j), C_{ij}, is (the negative of) the contribution to the firm's profit function with i and j as adjacent products in the family. Following (4.1), C_{ij} is defined as follows:

$$C_{ij} = I(h,g,f_j) + g(g,f_i,f_j) - M_{i,(i,j)} - M_{j,(i,j)} \quad \text{for } 0 < i < j \le N$$

$$C_{SE} = 0$$

$$C_{S0} = F_{agg}(g) - [p(f_0) - v(g,f_0)]\int_0^{f_0} p_{z,0}\, f(z)dz$$

$$C_{0i} = I(h,g,f_i) + g(g,f_0,f_i) - [p(f_i) - v(g,f_i)]\int_{f_0}^{f_i} p_{z,i}\, f(z)dz$$

$$- [p(f_0) - v(g,f_0)]\int_{f_0}^{f_i} p_{z,0}\, f(z)dz \quad \text{for } 0 < i \le N$$

$$C_{iE} = C_T(f_i,f_P) - \left[p(f_i) - v(g,f_i)\right]\int_{f_i}^{\infty} p_{z,i}\, f(z)\,dz \quad \text{for } 0 \le i \le N$$

Note that any path from S to E describes a feasible choice of products for the family, and the cost of the path defines the total cost (negative profit) from offering the products in that path. Hence the shortest path from S to E determines the optimal product-family and their performance levels.

We now show how the above formulation can be adapted for optimal product-family decisions when the firm wishes to expand an existing product family. We also show how such decisions can be made when the performance space is populated by products belonging to passive competitors; i.e., when competitors do not redesign their product families in response to the firm's actions. This can be achieved by modifying the arc costs C_{ij} appropriately. Let two candidate products, i and j, $i < j$, belonging to the candidate set $\mathbf{C}$ ($i, j \in \mathbf{C}$) be adjacent products offered in the family. Define $\mathbf{S_e}$ and $\mathbf{S_C}$ as the set of *performance levels* of existing products belonging to the firm and the competitor, respectively. Let $\mathbf{S_e}(i,j)$ and $\mathbf{S_C}(i,j)$ respectively be the subsets of $\mathbf{S_e}$ and $\mathbf{S_C}$ with products at performance levels in the interval (f_i, f_j).

i.e., $\mathbf{S_e}(i,j) = \left\{ f \mid f \in S_e,\ f_i < f < f_j \right\}$

and $\mathbf{S_C}(i,j) = \left\{ f \mid f \in S_c,\ f_i < f < f_j \right\}$

(i) If $\mathbf{S_e}(i,j) \neq \varnothing$, then $S_{\min} = \min\limits_{f} \left\{ f \mid f \in S_e(i,j) \right\}$ and $S_{\max} = \max\limits_{f} \left\{ f \mid f \in S_e(i,j) \right\}$

else, $S_{\min} = f_j$, and $S_{\max} = f_i$.

(ii) If $\mathbf{S}_c(i,j) \neq \varnothing$, then $C_{\min} = \min\limits_{f} \left\{ f \mid f \in S_c(i,j) \right\}$ and $C_{\max} = \max\limits_{f} \left\{ f \mid f \in S_c(i,j) \right\}$

else, $C_{\min} = f_j$, and $C_{\max} = f_i$.

If the interval (f_i, f_j) does not contain any products belonging to $\mathbf{S_e}$ or $\mathbf{S_C}$, C_{ij} is unaffected by either the firm's existing product-family or by products belonging to competitors, and is as defined above. If the interval (f_i, f_j) contains products belonging either to $\mathbf{S_e}$ or $\mathbf{S_C}$ or both, the costs C_{ij} are redefined as follows:

$$C'_{ij} = \begin{cases} C_{ij} \text{ if } S_e(i,j) = \varnothing \text{ and } S_c(i,j) = \varnothing \\ C_{i.} + C_{.j} \text{ otherwise} \end{cases} \qquad \text{where}$$

$$C_{i.} = \left[p(f_i) - v(g, f_i) \right] \int_{f_i}^{S_{\min}} p_{z,i}\ f(z)\, dz + \left[p(S_{\min}) - v(g, S_{\min}) \right] \int_{f_i}^{S_{\min}} p_{z,S_1}\ f(z)\, dz \text{ if } S_{\min} \le C_{\min}$$

$$C_{i.} = \left[p(f_i) - v(g, f_i) \right] \int_{f_i}^{C_{\min}} p_{z,i}\ f(z)\, dz \ \text{ if } C_{\min} < S_{\min}$$

$$C_{.j} = \left. \begin{array}{l} \left[p(S_{\max}) - v(g, S_{\max}) \right] \int_{S_{\max}}^{f_j} p_{z,S_2}\ f(z)\, dz + \left[p(f_j) - v(g, f_j) \right] \int_{S_{\max}}^{f_j} p_{z,j}\ f(z)\, dz \\ - I(h, g, f_j) - g(g, S_{\max}, f_j) \end{array} \right| \text{if } S_{\max} \ge C_{\max}$$

$$C_{.j} = \left[p(f_j) - v(g, f_j) \right] \int_{C_{\max}}^{f_j} p_{z,j}\ f(z)\, dz - I(h, g, f_j) - g(g, S_{\max}, f_j) \text{ if } S_{\max} < C_{\max}$$

p_{z,S_1} and p_{z,S_2}, respectively, denote the probability of purchase by customers with ideal point z for products at performance level $S_{\min}$ and $S_{\max}$.

It is interesting to note that competitor's products at performance levels f_i and f_j (coinciding with candidate products) do not directly appear in the modification above. Instead, we assume the purchase probability functions $p_{Z,i}$ and $p_{Z,j}$ capture the effect of competitor's products at these levels. The network formulation of the problem is unaffected by the above modification to the calculation of C'_{ij}.

We conclude this section with a few comments. First, shortest-path problems are easy to solve and a number of efficient procedures are available even for large networks, so we do not go into the formulation and solution procedures. Second, the arc SE in our network denotes the consequence of not offering the product family. Third, the only other arc emanating from *S*, *S0*, forces the choice of the base product f_0. This restriction may be relaxed by adding other arcs *Si* with appropriately defined costs. If so, then the shortest path determines the base product, as well as the product-family offerings.

5. Analytical Results

In the previous section, we presented a solution methodology for determining the product-family composition given the platform. Although the approach is very general, it does not offer insights into the relationship between the product platform and the family composition. In this section, we make some assumptions to simplify GP in order to derive some managerial insights.

5.1. Symmetric distribution of products

Problem GP can be simplified greatly if we assume no existing products and that the products offered are distributed symmetrically (located at equal distances from each other) within the performance range $[f_0, f_{max}]$. In addition, when detailed data on costs and demand are not readily available, a symmetric distribution can be used in the aggregate-planning phase by the firm to identify critical tradeoffs that drive the product-family decision. Here, we use a symmetric distribution to obtain a number of results that sharpen our understanding of the joint effects of development cost and demand parameters on the optimal product-family composition. The following result presents the conditions under which symmetric distribution in a product-family is optimal.

Result 1: A symmetric distribution of products across the performance space is optimal if the following conditions are true:

(i) the firm follows a pricing strategy in which profit margins are the same for each product;

(ii) the cost of adapting existing variants to new variants depends only on the relative separation between products, and is a linear or convex function of the separation; and

(iii) demand intensity $f(z)$ is uniform over the given performance range and customers choose products closest to their ideal points; additionally, purchase probability is determined only by the distance from the ideal point.

Proof outline: Consider any variant i that is not symmetrically positioned with respect to its two neighboring variants $i-1$ and $i+1$, i.e., $f_{i+1} - f_i \neq f_i - f_{i-1}$. When conditions (i)–(iii) are satisfied, repositioning of the variant i such that $f_{i+1} - f_i = f_i - f_{i-1}$ always leads to an increase in product-family profitability. Detailed proof can be obtained from the authors.

For the analysis in this section, we assume that the conditions of Result 1 hold and denote by D the demand intensity, $f(z)$. Let the firm's product offering be at performance points $\{f_0, f_1, \ldots, f_n\}$. We refer to n as the *population* of the family. A customer with ideal point z such that $f_i \leq z \leq f_{i+1}$ chooses only between products i and $i+1$, and we can write purchase probabilities (from (3.6)) as:

$$p_i = \{1 - k(z - f_i)\} \text{ for } f_i \leq z < \min\left\{\left(f_i + \frac{1}{k}\right), \left(\frac{f_i + f_{i+1}}{2}\right)\right\};$$

$$p_i = 0 \text{ for } z \geq \min\left\{\left(f_i + \frac{1}{k}\right), \left(\frac{f_i + f_{i+1}}{2}\right)\right\}$$

$$p_{i+1} = \{1 - k(f_{i+1} - z)\} \text{ for } \max\left\{\left(f_{i+1} - \frac{1}{k}\right), \left(\frac{f_i + f_{i+1}}{2}\right)\right\} \leq z \leq f_{i+1};$$

$$p_i = 0 \text{ for } z < \max\left\{\left(f_{i+1} - \frac{1}{k}\right), \left(\frac{f_i + f_{i+1}}{2}\right)\right\} \tag{5.1}$$

The purchase probability is 0 if a product is located farther than $1/k$ from the customer's ideal point, and 1 if the product is located exactly at the customer's ideal point. Balking is thus allowed, and the purchase probability of each consumer for a product decreases linearly with the difference between product performance and the consumer's ideal point[2]. The performance level of product i+1 influences the demand of product i; i.e., adjacent products cannibalize each other. If we denote $d = 1/k$ and the "separation" $f_{i+1} - f_i$ between adjacent products as s, the condition for no cannibalization to be present between products i and $i+1$ is $s \geq 2d$. In this section, we assume throughout that $s \leq 2d$. The formulation for no cannibalization can be derived using similar steps.

Let $d_{i,(i,i+1)}$ denote the demand for product i from customers located in the region $\{f_i, f_{i+1}\}$. Using equations (5.1) and (3.5), we get $d_{i,(i,i+1)}$

$$d_{i,(i,i+1)} = \frac{Ds}{2}\left(1 - \frac{s}{4d}\right), \; i = 1,2,\ldots,n-1 \tag{5.2}$$

[2] Some industrial products (such as the data-acquisition product discussed in this paper) exhibit strict downward substitutability: a consumer with ideal point z would choose only those products i with performance $f_i \geq z$. Analysis of this case proceeds along similar lines to the symmetric case discussed here. See Appendix B.

For products 0 and n, we also have to include customers in the intervals $[0, f_0]$ and $[f_n, +\infty]$ respectively, and we get

$$d_{0,(0,0)} = d_{n,(n,+\infty)} = \frac{Dd}{2}$$

such that the total demand for the product-family is

$$\sum_{i=1}^{n} 2 \cdot \frac{Ds}{2}\left(1 - \frac{s}{4d}\right) + \sum_{i \in \{0,n\}} \frac{Dd}{2} = Df_n\left[1 - \frac{f_n}{4nd}\right] + Dd \qquad (5.3)$$

5.2 Modeling the Product-Family Development Cost

Now we turn to the characterization of the development cost under a symmetric product distribution. Our choice of the cost function is motivated by three considerations: (a) capturing the influence of platform choice on development costs; (b) tractability and sparsity in terms of data requirements; and (c) consistency with data observed in practice (as in the data-acquisition product-family example).

Accordingly, we assume that the creative design cost $I(h, g)$ is equal for all variants and set at I, and model the adaptation cost, $g(\cdot)$ as follows:

$$g(g, f_1, f_2) = g(g, f_2 - f_1) = a_1(g)(f_2 - f_1) + a_2(g)(f_2 - f_1)^{a(g)}$$

Note that $g(\cdot)$ is modeled as a function of the separation between adjacent products. The three parameters a_1, a_2 and a are specified as functions of the platform g. The first term is linear, while the second term captures the non-linearities associated with development costs (due to the exponent $a(g)$). The three cases $a(g) < 1$, $a(g) = 1$ and $a(g) > 1$ describe different forms of the adaptation cost function. When $a(g) = 1$, the adaptation cost function is linear, and both terms can be combined into one. The adaptation cost function is convex (concave) when $a(g) > 1$ (< 1). Convex adaptation costs arise when the adaptation effort for a new product located closer requires only minor modification (updating engineering drawings etc.), while products located farther may require major redesign (changes in component placement and configuration of the product). Concave adaptation costs arise when the opposite occurs. The scaling parameters a_1, a_2 are specified to reflect the relative importance of the linear and non-linear components. For the sake of simplicity, we omit terms independent of f_1 and f_2. These terms, without loss of generality, can be included in I.

To investigate the influence of degree of reusability of a platform on a product family, we analytically examine two cases: (a) platform with $a_1(g) = 1$, $a_2(g) = 0$, and $a(g) = 1$**,** such that $g(\cdot)$ is linear, and (b) platform with $a_1(g) = 1$ and $a(g) > 1$ such that $g(\cdot)$ is convex. (The case when $a(g) < 1$ results in a profit function with possibly multiple local optima, and is not amenable to analysis. However, we examine this case computationally. These results, reported in Appendix C, are consistent with intuition based on the analytical results for the other two cases.) We first consider case (a) in some detail in section 5.2.1, and present the results for case (b) in section 5.2.2.

5.2.1 The case $a(g)=1$: Linear Adaptation Cost Function

For the linear adaptation cost function $g(\cdot)$, the total cost of product development (Equation 3.2) simplifies to

$$\mathrm{G}(g,n,f_n)=F_{agg}(g)+\sum_{i=1,\ldots,n} I(h,g)+a_1(g)[f_i-f_{i-1}] \tag{5.4a}$$

For the case with symmetric distribution of products, (5.4a) reduces to:

$$\mathrm{G}(g,n,f_n)=F_{agg}(g)+nI(h,g)+a_1(g)f_n \tag{5.4b}$$

From (5.3) and (5.4b), the profit function for the family under a symmetric distribution can be written as:

$$\Pi(g,n,f_n)=\begin{cases} mDf_n\left[1-\dfrac{f_n}{4nd}\right]+mDd-\mathrm{G}(g,n,f_n) & \text{for } n\geq 1 \\ mDd-F_{agg}(g) & \text{for } n=0 \end{cases} \tag{5.5}$$

where $f_0 \leq f_n \leq f_T$ and m is the unit margin. (Note that n is the number of *new* variants in the family in addition to the base product 0 offered at f_0).

For a given platform g, the profit function (5.5) is a nonlinear equation in the integer variable n and variable f_n, but offers a major simplification of **GP**. Profit function (5.5) indicates that when the firm commits to offering a family that covers a performance space $[f_0, f_{\max}]$ such that $f_n = f_{\max} \leq f_T$, the product-family decision reduces to the determination of the optimal population n^*. The case $f_{\max}=f_T$ corresponds to "full commitment" on the part of the firm where the product-family offering covers the entire range possible with the technology available to the firm. Alternatively, a "partial commitment" involves offering a family of products that does *not* cover the entire range of available technologies ($f_{\max}<f_T$). We characterize the solution for the full-commitment case and discuss its implications. We discuss the partial-commitment case in Appendix A which also contains conditions under which full-commitment is optimal.

For full commitment, Result 2 below characterizes the product population in the optimal product family, and Corollaries 2a and 2b provide a useful approximation for the product population and separation in the optimal solution. (For ease of exposition, we will omit references to parameters g and h in this section.)

Result 2: Consider a firm committed to offering its products over a performance range $[f_0, f_{\max}]$. When the firm's objective is given by Equation (5.5), the optimal value of the product population n is characterized by all integers that satisfy:

$$\begin{aligned}\left[\frac{1}{n(n+1)}\right]\frac{mDf_{\max}^2}{4d} &\leq I \\ \left[\frac{1}{n(n-1)}\right]\frac{mDf_{\max}^2}{4d} &\geq I\end{aligned} \tag{5.6a}$$

which is equivalent to

$$f_{\max}\sqrt{b\left[1+1/\left(4bf_{\max}^2\right)\right]}-0.5 \le n^* \le f_{\max}\sqrt{b\left[1+1/\left(4bf_{\max}^2\right)\right]}+0.5$$
$$\textit{where} \;\; b=\left(mD/4d\,I\right) \qquad (5.6b)$$

Outline of proof: We have, $f_n = f_{\max}$. Observe that $\Pi(n, f_{\max})$ is concave in n (n takes non-negative integral values). The optimal value of n thus satisfies the condition $\Pi(n-1, f_{\max}) \le \Pi(n, f_{\max}) \ge \Pi(n+1, f_{\max})$. Algebraic simplifications yield the expressions in Equations (5.6a) and (5.6b).

Result 2 reveals several interesting relationships between the span $f_{\max}$, optimal population n^* and the optimal separation s^* in a product-family. First, due to the integrality of product population, we see that there exists a *range* of values of $f_{\max}$ for which product population n^* is optimal. From (5.6b), this range consists of all values of f that satisfy the condition

$$\sqrt{(n^*-0.5)^2/b-1/4b} \;\le\; f \;\le\; \sqrt{(n^*+0.5)^2/b-1/4b}\,. \qquad (5.7)$$

Second, the optimal separation between products in a family $s^* = f_{\max}/n^*$ does not bear a monotone relationship with $f_{\max}$: the optimal separation increases only when $f_{\max}$ increases within the range specified by Equation (5.7) and may decrease otherwise. Third, as seen in the following corollaries which present useful approximations for the optimal value of the product population and separation in a product family, the number of products in the linear case is independent of the adaptation cost.

Corollary 2a: *For large values of* $f_{\max}$*, the optimal value* n^* *can be approximated as* $n^* \approx f_{\max}\sqrt{b}$*, where* b *is defined as in Equation (5.6b).*

Proof: Follows from Equation (5.6b).

Corollary 2b: *For large values of* $f_{\max}$*, the optimal separation between adjacent products,* $s^* = f_{\max}/n^*$ *converges to* $1/\sqrt{b}$.

Proof: Follows from Corollary (2a).

5.2.2 The case $a(g) > 1$: Platforms with Non-linear (Quadratic) Adaptation Costs

Here we present only the results for the case $a(g) > 1$, and highlight the major differences from the $a(g) = 1$ case. For a general $a(g)$, cost function (Equation 5.4a) is

$$G(g, n, f_n) = F_{agg}(g) + nI(h, g) + a_1(g) f_n + a_2(g) n \left[f_n / n\right]^{a(g)}$$

It can be verified that (i) the development cost function $G(g, n, f_n)$ is subadditive in n if $a_1(g) \ge 0$ or $a_2(g) \ge 0$, and (ii) is set-monotone only for values of $a(g) \le a_c(g)$, where $a_c(g)$ is the largest value of $a(g)$ that satisfies the condition $G(g, 1, f_n) \le G(g, 2, f_n)$. We assume for convenience that $a(g) \le a_c(g)$ such that $G(g, n, f_n)$ is both subadditive and set-monotone, but it is easily shown that the characterization of the optimal solution does not require this assumption.

The criteria for the global optima (Equation 5.6a) when $a(g) > 1$ becomes

$$\left[\frac{1}{n(n+1)}\right]\frac{mDf_{\max}^2}{4d}+\left[\frac{1}{n^{a-1}}-\frac{1}{(n+1)^{a-1}}\right]a_2 f_{\max}^a \le I$$
$$\left[\frac{1}{n(n-1)}\right]\frac{mDf_{\max}^2}{4d}-\left[\frac{1}{n^{a-1}}-\frac{1}{(n-1)^{a-1}}\right]a_2 f_{\max}^a \ge I \qquad (5.8)$$

When $a(g)=2$, the optimal value of n is characterized by all integers that satisfy

$$f_{\max}\sqrt{b'\left[1+1/\left(4b'f_{\max}^2\right)\right]}-0.5 \le n^* \le f_{\max}\sqrt{b'\left[1+1/\left(4b'f_{\max}^2\right)\right]}+0.5 \qquad (5.9)$$

where $b'=\left(mD/4d+a_2\right)/I$. The approximations in Corollaries 2a and 2b remain unchanged if b' is used in place of b, and in expanded form we can write them as

$$n^* \approx f_{max}\sqrt{\left(mD/4d+a_2\right)/I}, \text{ and } s^* = 1\Big/\sqrt{\left(mD/4d+a_2\right)/I} \qquad (5.10)$$

which revert to the linear case when $a_2=0$. Thus (5.10) generalizes the results in Corollaries 2a and 2b. All else being equal, $b'>b$, resulting in an optimal product-family with a *larger* number of products and smaller separation.

5.3. Effect of Reusable Product Platforms On Product Variety

The result in (5.10) summarizes the relationship between the span $f_{\max}$, the optimal population n^* and the optimal separation s^* for symmetric product families. Specifically, the relation $n^* \approx f_{\max}\sqrt{\left(mD/4d+a_2\right)/I}$ states that the optimal population (i) grows approximately linearly with the span f_{max}, (ii) increases as consumers become more selective (decreasing d), (iii) is larger for a process with smaller creative design costs (I), and (iv) is *smaller* for a platform with smaller non-linear (quadratic) adaptation cost (smaller a_2). We also note that the approximation for the optimal separation s^* is approximately independent of span f_{max}. It is interesting to observe that expression (5.10) is analogous to the result obtained by de Groote (1994) in that the optimal number of products grows linearly with the performance span.

The above characterization of the optimal population also provides additional insights on the effects of a reusable platform. As we discussed earlier, effort invested in creating a platform contributes to the reduction of both the creative-design costs and the adaptation costs. While the reduction in creative-design costs (I) leads to an increase in the optimal product population, a decrease in adaptation costs due to a reusable platform actually results in a decrease in the optimal product population in the quadratic cost function case. When the effect of the decrease in adaptation cost dominates the effect of decrease in creative design costs, then a more *reusable* platform (with lower adaptation cost a_2) may lead to a *smaller* optimal product population. This is in contrast with the intuition that a firm should always offer more products with a reusable platform, and merits further explanation. When the costs of adaptation are high, a firm is forced to offer product variants at smaller separation in order to reduce its effort in adapting existing design elements to new products. To cover the performance range f_{max} , the firm must therefore offer more products, and in doing so incurs the fixed (creative design) costs of offering these products. When the reusable platform helps reduce the costs of adaptation, the firm can offer product variants at larger separation without incurring significant costs in adapting existing design elements to new products. This enables the firm to cover the performance range f_{max} with fewer products. It is noteworthy that this counter-intuitive effect is observed only when the adaptation cost function is convex. When the adaptation cost function is linear, the number of products is independent of the adaptation cost. For the concave adaptation cost case, computational experiments (presented in Appendix C) suggest that the number of products is a non-increasing function of the adaptation cost.

We conclude this section by noting the effect of two other parameters on the product population. First, cannibalization effects (increasing δ) favor a smaller population and have an effect similar to that of creative design cost parameter *I*. Second, investments in new technology to expand the performance range (f_{max}) lead to increase in product population and may increase the profitability. However, the return on this investment depends on platform choice and product development costs. This issue is examined in more detail in the following section.

5.4. Effect of Technology Investments on Product-family Profitability

To examine the return on investments in new technology , consider the case when the firm uses a platform which results only in linear adaptation costs ($a_2(g) = 0$). Suppose that the firm is exploring investments in *technology* that lead to an increase in the range of performance levels f_T that the firm may offer in the market. Let f_{T_1} and f_{T_2} denote the maximum performance the firm can offer before and after the new product technology is acquired, where $f_{T_2} > f_{T_1}$, and c_T be the cost of acquisition. Further, let n_1 denote the optimal population for $f_T = f_{T_1}$, and n_2 the population when $f_T = f_{T_2}$ determined as in the previous subsections. Consider the benefit of acquiring the new technology in two cases: (a) when the firm's creative-design cost *I* is high, and (b) when the platform is such that the creative-design cost is low ($I' < I$). We examine the effect of technology investments on these two cases in Result 3 when (i) the firm introduces the optimal population in response to the technology acquisition (n_2 is optimally determined), and (ii) when the firm does not adjust its family optimally ($n_2 = n_1$).

Result 3: Investments in acquiring new-product technology provide better returns for the case with lower creative-design costs if the investments are accompanied by optimal changes to the product-family composition. When the firm does not change its product population in response to such investments, the return from the technology investment is the same for both the cases with low and high creative-design costs.

Proof: Denote the net benefit due to new technology acquisition by $B(I)$.

When n_2 is determined optimally, we have

$$B_a(I) = \Pi\left(g, n_2, f_{T_2}\right) - \Pi\left(g, n_1, f_{T_1}\right)$$

$$= \left(f_{T_2} - f_{T_1}\right)\left[mD\left(1 - \frac{1}{4d\sqrt{b}}\right) - I\sqrt{b} - a_1\right] - c_T \quad where \;\; b = \left(mD/4d\right)/I(g)$$

which, upon simplification leads to

$$B_a(I) = \left(f_{T_2} - f_{T_1}\right)\left[mD\left(1 - \frac{\sqrt{I}}{4d\sqrt{mD/4d}}\right) - \frac{I\sqrt{I}}{\sqrt{mD/4d}} - a_1\right] - c_T .$$

Thus, $B_a(I') > B_a(I)$ for $I' < I$, so investments in new product technology provide better benefit for a process with lower creative design costs.

When the firm does not optimally adjust its portfolio in response to the investment ($n_2 = n_1$), we have $B_b(I) = \Pi\left(g, n_1, f_{T_2}\right) - \Pi\left(g, n_1, f_{T_1}\right) - c_T$ independent of I. Thus, the return from the technology investment is the same for both the cases with low and high creative design costs.

Result 3 identifies an important benefit of investments in product-family development. When the firm invests in a platform that reduces the creative-design costs and makes *optimal* decisions regarding the product-family composition, investments in new-product technology provide better returns, thereby motivating the need for optimal decision making about product families.

6. Illustration: Planning the Product Family of a Data Acquisition Product

In this section, we illustrate the model of product-family decision making with an industrial application. The study company ("Accu-Data") is a medium-sized electronics manufacturer, and the product-family in question involved their data-acquisition (DAQ) products introduced in Section 1.2. The company had already developed a platform comprised of the timing-circuitry chip (see Figure 1). We applied the model to identify the optimal product-family composition (separation and population) given their product platform. This application effort was carried out concurrently with the projects underway at Accu-Data to develop products based on the platform. Some of the products from the family have recently been introduced, which helped us compare model results to management's product-family decisions. We present details of the product-family and the data-collection effort in Section 6.1, discuss the results of our study in Section 6.2, and comment on the implementation issues in section 6.3. In addition, we used the model to reexamine the decision to introduce the product family based on the new platform, which involved an investment of $0.5 million. The results of this exercise in hindsight support Accu-Data's decision to adopt a platform-based product-family development strategy - in contrast to its earlier strategy of independent product development. For the sake of brevity, we do not report these results, which can be found in an unpublished dissertation (Singh 1998).

6 .1 The Product Family and the Data Collection Effort

As we noted in section 1.2, our study focused on a new product-family that Accu-Data was developing, which we shall call the PCI family. The PCI family inherited a number of demand, pricing, and cost characteristics from an existing family, which facilitated our data-collection efforts. The PCI family also targeted customers with sampling-rate requirements in the 1 to 1250 kS/s (kilo samples per second) range. Sampling-rate, as mentioned earlier, is a key design specification of a DAQ product, and is inversely related to another design attribute - the “settling time” measured in microseconds. This is the time required for the amplifier (A) to amplify the analog signal and settle to an accurate value that could be used by the analog-to-digital converter (ADC): if the amplifier did not settle quickly for a given sample, the subsequent sample may interfere with the amplification process, and the measurements would be incorrect. For a board to perform reliably, the settling time would have to be less than $1/(\text{sampling rate})$. (Thus, for a 500 kS/s board, the required settling time would be $1/(500 \times 10^3) =$ 2 microseconds.) The design of amplifiers with faster settling times would drive the maximum sampling rate Accu-Data could offer in the market.

Fixed Development Costs: Accu-Data had already adopted a platform, so we treat this investment as sunk cost by setting $F_{agg} = 0$. Fixed cost of designing, marketing and supporting individual products was another important factor. Once the product was introduced, costs of marketing and supporting it in the product-family would be incurred. These costs would limit the number of sampling-rate levels that could be profitably offered in the PCI family. Historical accounting data indicated that marketing and support costs for a product over its life cycle could be estimated as $250,000. This was believed to be a rough estimate, since it ignored the opportunity costs of the marketing and support function, so we perform sensitivity analysis with three levels of this parameter - $250,000, $500,000, and $1 million.

Adaptation Costs: Although the new platform offered the company the potential to reduce subsequent development costs, individual products still required the modification and/or redesign of certain circuit elements, especially the amplifier module (element A in Figure 1), to ensure accurate performance. The amount of modification generally increased with the difference between the performance of the new product and the performance of the product from which it was adapted. The amplifier module, however, introduced step changes in the adaptation cost function. An amplifier designed for a particular product (say 50 KS/sec) required only minor *modifications* within a certain range of performance (till 100 KS/sec). Beyond this range, however, the amplifier had to be completely re-designed. These breakpoints in amplifier design occurred at 100, 250, and 500 KS/sec (corresponding to settlement times of 10, 4, and 2 micro seconds), and introduced four regions in the adaptation cost function (0-100, 100-250, 250-500, 500-1250 KS/sec). The adaptation cost function *g*(.) within a region R is given by:

$$g(f_i, f_j) = 10\,(f_j - f_i) \text{ where } \{f_i, f_j\} \in R$$

When the products f_i, f_j are in different regions R_a and R_b, the adaptation cost function *g*(.) is given by:

$$g(f_i, f_j) = 10{,}000 + 10\,(f_j - f_{Rb}) \text{ where } f_i \in R_a \text{ and } f_j \in R_b$$

where f_{Rb} is the performance level at the lower end of region R_b. Here, the constant (10,000) is the fixed cost of re-designing the amplifier for the new region, while the linear term denotes the cost of continuously modifying the amplifier circuitry to the performance level f_j of the new product.

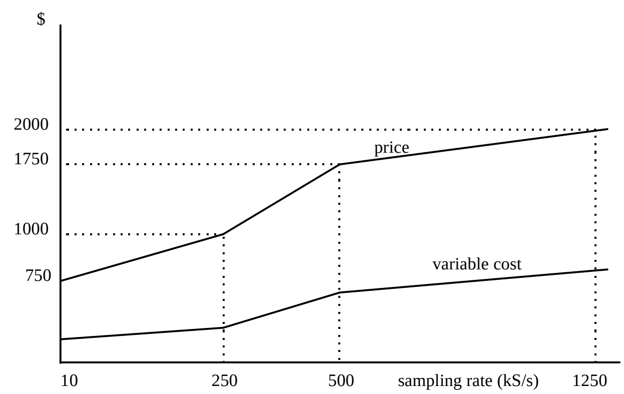


Figure 4: Price and Variable Cost Functions

Price and Unit Variable Cost: Market studies indicated that the PCI product-family should be priced at about the same level as the existing product family, since customers in the high-technology industry increasingly have come to expect improvement in performance over time at a given price. The pricing for the family was decided by the company's marketing department based on the company's gross margin targets and in consultation with key customers; typically, the direct variable costs were about 25% of the price (giving a gross margin of 75%). The price and cost functions are shown in Figure 4. At the planning stages of the PCI family, competitive products were not yet available, and because Accu-Data had a dominant share of the market, competitive factors did not play a significant role in product-family decision making.

Demand Data: Application of our model required: (a) an estimate of the demand intensity function, and (b) cannibalization characteristics. Since the PCI product family was intended to replace the existing family, the company had a good idea of the potential market demand for the PCI family. In addition, using the demand data for the new product offerings (100, 250, and 1250 KS/sec) and the similarity in the demand growth pattern between the existing and the PCI families, the company estimated the lifecycle demand for the new products introduced (without any additional products). But this data gave little or no information about the cannibalization pattern in a typical product family, and good data on cannibalization was unavailable. Interviews with managers at Accu-Data revealed the following pattern of consumer behavior. Customers had well-defined sampling-rate requirements, which determined their "ideal" product. In addition, the choice behavior exhibited downward substitutability, i.e., if the ideal product was unavailable, customers were likely to consider only those products with a higher sampling rate. But the likelihood of purchase decreased as the "performance gap" between the ideal product and the available higher-end substitute increased. The maximum performance gap tolerated by the customers is a function of the performance level (settling time) and the nature of the application. The data used for the performance gap in our computations (shown in Figure 5 (a)) is based on the assessment of the marketing team at Accu-Data.

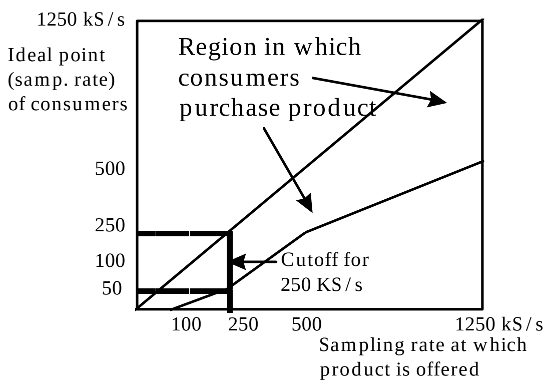
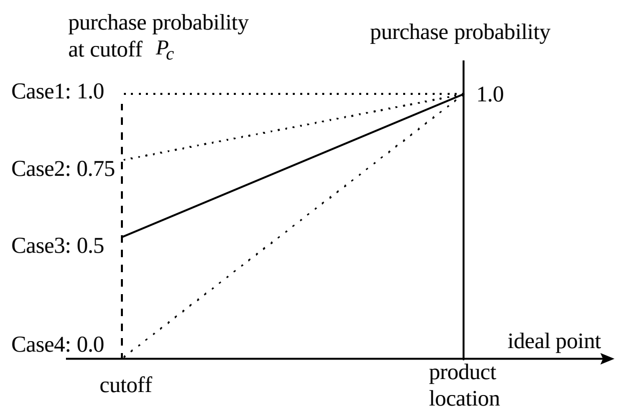


Figure 5(a): Cutoff value



5(b): Purchase probability

This behavior in Figure 5(a) shows for each ideal performance point the range of acceptable product performance. For instance, a product offered at a 250kS/s is likely to be considered for purchase by customers with ideal points in the range 50kS/s to 250kS/s. A second parameter related to cannibalization is the probability of purchase. We assumed that at the ideal point, the probability is 1, and decreases linearly over the acceptable range. Due to the lack of accurate data in this regard, we consider four levels of this behavior defined by the probability of purchase at the cutoff value as shown in Figure 5(b). (It may be noted that the four levels of purchase probability provide indirectly a sensitivity analysis of the performance gap data.)

For our computations, we derived the demand intensity function shown in Figure 6 from the following: (i) lifecycle demand estimates for the three new products (assuming no new additions), and (ii) cannibalization behavior with purchase probability of 0.5 at cutoff. The potential market size estimated from this demand function was in fair agreement with Accu-Data estimates, and provided some validation for the demand model.

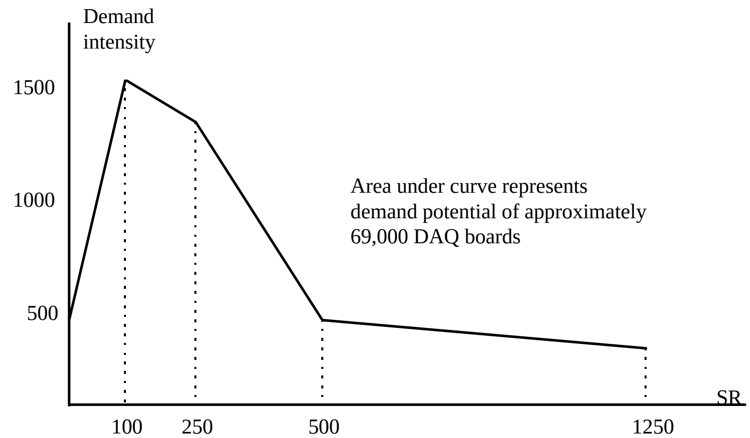


Figure 6: Demand Intensity

6 .2 Analysis and results

As described above, Accu-Data limited its choice of products to those with integral settling times, and had launched in recent months three products at 100, 250 and 1250kS/s performance points. Our study was helpful in answering two categories of questions: (1) How does the optimal product-family compare with the decisions management made, and what is the cost of sub-optimality (if any)? (2) In addition to the products already introduced, which new products should Accu-Data offer? Accu-Data is currently resource-constrained (faces a severe shortage of skilled engineers), and would be interested in the best one or few products it could add to its current offerings.

For the purpose of this study, Accu-Data identified 9 candidate products in the target range of 1 to 1250kS/s: 10, 20, 50, 100, 200, 250, 333, 500, 1250. (In general a much broader candidate set can be used without any problem using our network formulation.) We implemented the computational model outlined in Section 4 as a computer program, and ran the program on the data collected at Accu-Data, the results of which we consider next for discussion.

6.2.1 Optimal Product Family Composition

The optimal product family identified for the above data is presented in Table 1. Because the fixed costs and purchase probabilities were estimates, we chose to do sensitivity analysis with respect to these parameters. While Accu-Data currently estimates its fixed cost of marketing and supporting a product over its lifecycle to be $250,000, it may be argued that these resources can be put to better use given the firm's resource constraints. The opportunity cost of these resources can be as high as one million dollars (per estimates of the vice president of product development), which forms the upper bound in the sensitivity analysis.

Fixed cost of Marketing and Support (CF)	Value of Purchase Probability at Cutoff (P_c)	Optimal product-family Composition	Payoff (in $ million)
250000	**1**	50, 250, 500, 1250	73.1
	0.75	50, 250, 500, 1250	62.8
	0.5	50, 100, 200, 333, 500, 1250	54.2
	0	50, 100, 200, 250, 333, 500, 1250	40.5
500000	**1**	50, 250, 500, 1250	72.1
	0.75	50, 250, 500, 1250	61.8
	0.5	100, 200, 333, 500, 1250	52.8
	0	50, 100, 200, 250, 333, 500, 1250	38.8
1000000	**1**	50, 250, 500, 1250	70.1
	0.75	50, 250, 500, 1250	59.8
	0.5	100, 200, 333, 500, 1250	50.3
	0	100, 200, 250, 333, 500, 1250	35.6

Table 1: Optimal Product-family Composition and Payoff

Table 1 shows the optimal product-family population and composition for different levels of fixed costs of marketing and support and the purchase probability function. As we would expect, for a given slope of the purchase probability function, the optimal product population is decreasing in fixed cost, and for a given fixed cost, the population is increasing in the slope of the purchase probability function. However, two of the performance points (500 and 1250) are optimal for all values of the fixed cost per product (CF) and the purchase probability at cutoff (P_c), indicating these offerings are robust against errors in estimates of fixed cost and cannibalization parameters. As we noted earlier, the company chose to offer 100, 250, and 1250 KS/s products. While the 1250 KS/s figures in the optimal family for all combinations of parameters, the 250 KS/s is optimal at higher levels of the purchase probability function (for $P_c > 0.6$), and the 100 KS/sec is optimal at lower levels of the purchase probability function[3]. Conspicuous by its absence from the company's product offering is the 500 KS/sec product which the model shows to be optimal for all combinations of CF and P_c.

6.2.2 Cost of Suboptimality and Addition of New Products

While the above analysis shows how the optimal family composition compares with the company's offering, it does not quantify the cost of suboptimality. Table 2 presents the results of the analysis for the fixed cost of $250,000. Note that the company's choice results in a substantial loss of profits in comparison with the optimal mix. The penalty ranges from $17.6 million to $28.5million depending upon the purchase

[3] This conclusion is based on the results of sensitivity analysis, the details of which have been omitted for the sake of brevity.

probability at cutoff (P_c). The firm's resource constraints may not allow it to develop, launch, and support the optimal unrestricted family (which in some cases contains as many as seven products), so we examine how the payoffs from the *optimal* 3 product-family compares with the company's existing 3 product offerings (100, 250, and 1250 KS/sec products). The optimal 3 product-family consists of 250, 500, and 1250 KS/sec products for P_c = 1.0 and P_c =0.75. For P_c =0.5, the optimal 3 product-family consists of 200, 500, and 1250 KS/sec products, while for P_c =0.0, optimal 3 product-family contains 333, 500, and 1250 KS/sec products. As shown in Table 2, the increase in profit due to the optimal 3 product-family ranges from 8.6 to 26.7 million dollars depending on the value of P_c.

Purchase probability at cutoff (P_c)	Existing family payoff (million $)	Optimal unrestricted family payoff (million $)	Optimal 3 products payoff (million $)	Optimal 4 products payoff (million $)	Payoff from Optimal addition of 1 product to existing family (million $)
1	44.8	73.1	71.5	72.5	72.2
0.75	39.3	62.8	61.4	62.6	62.5
0.5	33.9	54.2	51.3	52.9	52.8
0	22.9	40.5	31.5	37.2	33.5

Table 2: Payoffs From Adding New products to the PCI Product Family

From Accu-Data's standpoint, it is more useful to identify which new products to add to the company's PCI family. This can be done by treating the development cost of existing products as sunk costs. Given Accu-Data's current family, what products could the firm optimally add to maximize profits? The optimal one product to add to the existing family is the 500 KS/sec product for all values of P_c. We compare the payoff from adding this one product to both the existing family and the optimal 4 product-family (starting from zero products in the family). It is interesting to note that except in the case of P_c =0.0 (when the customers are very selective), adding one product to the family optimally largely closes the gap with the payoff from the optimal 4 product family. This is true for this specific application because the study company's decision was already a good starting point, and adding the 500 KS/sec product makes it nearly optimal. Also, given the company's resource constraints and the fact that the addition of just one product (500 KS/sec) provides near-optimal returns, it may not be necessary for the firm to add any more products. We have reviewed the model results with our study company, and their product managers agree with the model recommendation that the 500 KS/sec would be a good performance point to offer a new product. They also note that the decision to offer the 100 KS/sec product was made to accommodate a special request from a major customer. We next summarize the lessons learned from the application.

6.3 Lessons Learned From the Industrial Application

The above industrial application demonstrates clearly the benefits of the model of product-family development in providing additional managerial insights. Senior managers at Accu-Data, who possessed an intuitive understanding of some of the issues involved in planning and developing a product-family, appreciated the value of integrating demand and development cost information in a model to make product-family decisions. However, we also found that the application area of product development is unlike the more established operational functions such as production planning, scheduling and inventory control in that it is in early stages of application of rigorous optimization models. Development of a full-fledged decision support system (DSS) based on this model requires substantial evolution in a number of areas. In the remainder of this section, we describe some of the key issues in this regard.

The first major barrier to implementing a full-fledged DSS at Accu-Data was the lack of reliable development cost data. We found that the cost-accounting systems were inadequate to provide the information required. This was in part due to the fact that the design engineers' time constitutes a major component of the development cost. Designers often worked on several different projects simultaneously (involving developing new products and sustaining existing products), and the company did not have good mechanisms in place to monitor time spent on different activities. Accu-Data has begun investing in systems to collect more detailed cost data in a systematic and automated fashion. Second, the demand information required for the application was lacking in many respects. While the managerial forecasts at Accu-Data could be adapted to estimate the potential demand, understanding of cannibalization effects was somewhat primitive and more effort was needed to obtain reliable data.

A third major barrier to implementation concerned the mindset of engineers who are potential users and suppliers of cost information required by such a system. While the senior managers appreciated the issues raised and the trade-offs addressed by the model, some new engineers were uncomfortable with the usage of inexact demand/market information and development cost estimates to arrive at product offering decisions. We expect that with better data and more exposure to these issues the engineers will be in a better position to appreciate the role of this model as a complement to the technical issues involved in product design. To facilitate this process, it is preferable to integrate this methodology to the existing design tools. Otherwise, it is likely to be perceived as a distraction from the main design task.

We conclude this section by noting that, partly as a result of the thrust of this research stream to formalize product-family decision making, the company has embarked on modifications of their systems to collect more reliable data and thereby bring more business orientation to product-family decisions. It has also made modifications to its design rules of thumb and component information systems to encourage engineers to carry-over designs from existing products, and thereby reduce adaptation costs. We firmly believe that further work in this area would enable product-family decisions to be made in a more formal and systematic manner.

7. Discussion and Conclusions

We have presented a model-based approach for making product-family decisions that considers development costs and allows for sharing of design elements across products. In our model, a product-family represents a set of products that share a common platform (and thus design elements) but differ from each other in a performance space defined by the level of technology they incorporate in their designs. This setting corresponds closely to a number of firms that develop technology-based products which differ from each other on some fundamental level of performance. The firm makes product-family decisions in an aggregate product-family development phase during which it jointly chooses the product platform and the set of product offerings based on estimated development costs and demand. It then proceeds to develop the individual products sequentially. While developing a product platform that allows for greater reusability may incur larger development costs, it leads to downstream savings in the form of reduced effort in adapting the design of previously developed products (with lower performance) to the design of new products with higher performance. Our first major contribution in this paper is the conceptualization of the product-family development costs and the formulation of the product-family problem (GP) in section 3.

The network representation and shortest-path solution procedure for the general product-family planning problem (GP), presented in section 4, forms our second major contribution in this paper. We showed that under fairly general conditions, the network formulation can be used to choose product variants from a set of candidate products. The shortest path of the network provides both the optimal product-family composition and population. This approach can be extended to incorporate product-line extensions and the presence of products belonging to passive competitors. It is also possible to easily incorporate constraints on the number of products/development budget using a layered network representation, which was not considered in this paper for the sake of brevity.

Our third contribution is to show that combining demand and development cost information leads to a number of conclusions that have not been previously studied in literature. First, due to the integrality of the number of products, there exists a *range* of values of maximum performance for which a particular product population is optimal, and the optimal product population is not strictly monotone increasing in the performance range. Second, when the adaptation cost is convex, a reusable platform may result in a smaller number of products in the optimal product-family. This is because lesser adaptation costs allow for products to be located farther from each other and thereby cover the range of performance needs of the customer with fewer products. Third, the return on investments in a new technology is greater under a platform that lowers the creative design costs, only if the firm optimally adjusts its product family. Fourth, it is not always optimal for the firm to commit to offering a product-family that spans the entire range of technologies available to the firm.

We also show using an industrial application the utility of making product-family decisions by integrating demand and cannibalization data. This exercise shows not only the cost of being suboptimal but also the benefit of adding new products to an existing family based on a formal model. Our effort to illustrate the applicability of the model also gave us a sense of the challenges experienced in implementing a formal model to make decisions in the emerging area of product development.

There is clearly a broad scope for future work in product-family design, especially as the need for technology-intensive products continues to grow in the market. High fixed costs associated with the development, manufacturing and support of new products will continue to force firms to exploit similarities among these products. Being one of the early efforts in integrating demand and development cost information to model product-family decision making, our work has certain limitations which must be addressed in future research. Our cost model is based on one dominant product attribute and focuses on development costs. A model to accommodate more than one attribute would represent a natural extension of this model. The scope of this model could be increased substantially by extending it to include a more elaborate treatment of manufacturing costs. One approach is to combine our model with the manufacturing cost model proposed in de Groote (1994). The demand model presented in this paper is static, and does not consider the timing and sequence of product introductions. Incorporation of these considerations would make the model considerably more complex and need to address the implications of these decisions on market share and demand. Another aspect that merits further research attention is the treatment of price as a decision variable in the model. These could be complemented with a study of best practices in the industry that can help us identify industry- or company-specific issues in designing product families.

Acknowledgements: We gratefully acknowledge insightful comments from two anonymous reviewers, a senior editor, and the editor-in-chief. We are also indebted to several product development and marketing managers at our study company. This work was funded in part by an award from APICS and Marketing Science Institute under the "Harmonizing Marketing and Manufacturing Relationships" research competition.

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APPENDIX A - OPTIMALITY OF FULL COMMITMENT

Let $\hat{n}$ be the population that satisfies the condition (5.7):

$$\sqrt{(\hat{n}-0.5)^2/b - 1/4b} \leq f \leq \sqrt{(\hat{n}+0.5)^2/b - 1/4b}$$

Then, the firm need only consider the values $0,1,...,\hat{n}$ for the optimal product population. Additionally, let

$$x_n = \begin{cases} 0 \; if \; n = 0 \\ \sqrt{(n-0.5)^2/b - 1/4b} \; if \; n = 1,2,...,\hat{n} \\ f_T \;\; if \; n = \hat{n}+1 \end{cases}$$

such that population n is optimal when $f_{\max} \in [x_n, x_{n+1}]$. Further, observe that the value of f_{max} that maximizes profitability for any value of n is given by the relation

$$f_{(n)} = \frac{2d\, n(mD - a_1)}{(mD + 4d\, a_2)}$$

which can be easily obtained from the first and second order optimality conditions for the profitability function for $a = 1$ and 2 in Section 5.2.1 and Section 5.2.2 respectively. Then, we can state the following result:

Result: *The optimal value of* f_{max} *occurs either at some* $f_{(n)}$ *or some* x_n *for* $n \leq n_c \leq \hat{n}$, *and only at some* x_n *for* $n_c < n \leq \hat{n}$. *Here,* n_c *is the minimum value of* n *for which*

$$mD \;\; \geq \;\; a_1 + \left(a_2 + \frac{mD}{2d}\right)\sqrt{\frac{1}{b}\left(1 + \frac{1}{n}\right)}$$

Full commitment is optimal if $\Pi(\hat{n}, f) \geq \Pi(j, x_j)$ *for all* $j = 0, 1, \ldots, \hat{n} - 1$, *and* $\Pi(\hat{n}, f) \geq \Pi(j, f_{(j)})$ *for all ,* $j = 0, 1, \ldots, n_c$.

Proof: The proof is based on the observation that $\Pi(n, f_{\max})$ is concave in each interval $[x_n, x_{n+1}]$, and proceeds in two steps. In the first, we show that we need only consider the values $f_{(n)}$ and x_n for all to find the optimal f_{max}. In the second, we show that if there exists a value for n_c as defined in the above result, then the optimal value for f_{max} occurs only at some x_n.

In the interval $f_{\max} \in [x_n, x_{n+1}]$, since the profitability $\Pi(n, f_{\max})$ is concave in f_{max} it is (uniquely) maximized either at x_n, x_{n+1} or $f_{(n)}$. Thus we need only consider the values $f_{(n)}$ and x_n and x_{n+1}, for all to find the optimal f_{max}.

For any $f_{\max} \in [x_n, x_{n+1}]$, $\Pi(n, f_{\max})$ is non-decreasing in f_{max} if

$$\frac{\P\, \Pi(n, f_{\max})}{\P\, f_{\max}} = mD - a_1 - \left(a_2 + \frac{mD}{2d}\right)\frac{f_{\max}}{n} \geq 0$$

and is non-decreasing throughout if

$$mD \geq a_1 + \left(a_2 + \frac{mD}{2d}\right)\frac{x_{n+1}}{n}$$

$$= a_1 + \left(a_2 + \frac{mD}{2d}\right)\sqrt{\frac{1}{b}\left(1 + \frac{1}{n}\right)}$$

If the above equation is satisfied then the profit maximizing value of $f_{\max} \in [x_n, x_{n+1}]$ occurs at x_{n+1}. It is easily seen that if the above equation is satisfied for any n_c, it is satisfied for $n > n_c$. The optimality condition for full commitment now follows trivially.

The exact value of the technology f_{max} committed to the family can thus be derived in a finite number of steps involving the computation of x_n and $f_{(n)}$ for $n = 1, 2, \ldots, \hat{n}$.

APPENDIX B - ANALYSIS OF DOWNWARD SUBSTITUTABILITY

Downward substitutability refers to the case when the purchase probability of a product is zero if its performance level is below the customer's ideal point. For products whose performance is above the ideal point, the purchase probability gradually drops to zero (at a distance of $1/k$ from the customer's ideal point). Thus, we have:

$$p_i = \{1 - k(f_i - z)\} \text{ for } \max\left\{\left(f_i - \frac{1}{k}\right), \left(\frac{f_i + f_{i-1}}{2}\right)\right\} \leq z \leq f_i;$$

$$p_i = 0 \text{ for } z \leq \max\left\{\left(f_i - \frac{1}{k}\right), \left(\frac{f_i + f_{i-1}}{2}\right)\right\} \text{ and } z > f_i$$

Let $d_{i,(i-1,\, i)}$ denote the demand for product i from customers located in the region $\{f_{i-1},\ f_i\}$. Using the above equations and (3.7), we get $d_{i,(i-1,\, i)}$ as:

$$d_{i,(i-1,i)} = \frac{Ds}{2}\left(1 - \frac{s}{4d}\right),\ \ i = 1,2,\ldots,n \qquad \text{(5.2 revised)}$$

For product 0, we include customers in the intervals $[f_0 - k, f_0]$, so $d_{0,(0,0)} = \frac{Dd}{2}$

The total demand for the product family is

$$\sum_{i=1}^{n}\frac{Ds}{2}\left(1 - \frac{s}{4d}\right) + \frac{Dd}{2} \ = \ \frac{Df_n}{2}\left[1 - \frac{f_n}{4nd}\right] + \frac{Dd}{2} \qquad \text{(5.3 revised)}$$

which is exactly half of the demand experienced in the symmetric case given by (5.3). The rest of the analysis follows easily, and all the remaining arguments for the symmetric case (such as the concavity of the profit function) also hold good for downward substitutability case (with appropriate modifications to the demand term).

APPENDIX C - COMPUTATIONAL TEST OF THE CONCAVE ADAPTATION COST FUNCTION CASE

In this appendix, we provide computational results on the optimal product population when adaptation cost is a concave function of the separation between products ($a < 1$). To perform these tests, we set the problem parameters at values shown in Table A1. The effect of changing the adaptation cost function on the optimal product population is shown in Table A2 for different values of $a < 1$ and for uniform separation. These results show that when the adaptation cost is a concave function of the separation, the optimal number of products is non-increasing in the adaptation cost. This may be explained by the fact that when the adaptation cost is a concave function of the separation between products, the marginal penalty for increasing the separation between products is non-increasing. Offering fewer products is made further attractive by the reduction in cannibalization due to the increase in separation, and the reduction in fixed cost of product development due to fewer products. These observations are based on limited computational tests, and more extensive testing would be necessary to generalize them.

Parameter	Value
m (unit margin)	$200
D (demand intensity)	500
d (cannibalization factor)	25
(f_0, f_{max}) performance range	(0, 100)
I (creative design cost)	$100,000
Number of candidates	80

TABLE A1. INPUT PARAMETER SETTINGS

	a			
a_2	**0**	**0.25**	**0.5**	**0.75**
0	10	10	10	10
1000	10	10	10	10
10000	9	9	9	9
50000	8	8	7	7
100000	7	6	6	4
200000	6	5	3	1
500000	4	3	1	0

Table A2. Number of Products in the Optimal Family as a Function of the Non-Linear Adaptation Cost (a_2), and the Concavity Parameter (a)