Measuring Interference Temperature

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Abstract

The interference temperature (IT) model offers a novel way to perform spectrum allocation and management. Recent research has proposed schemes that take advantage of the model; however, there are still lingering questions about what exactly IT means, and how it should be measured. The *temperature* characterization may make sense for thermal noise, but interference behaves differently.

This research describes two interpretations of the interference temperature model, and investigates how we can measure IT in each. One of the models lends itself to an interesting analysis of how interference temperature affects power, bandwidth, and capacity, and those relations are investigated.

1 Introduction

In 2003, the Federal Communications Commission (FCC) introduced the concept of *interference temperature* (IT) for "quantifying and managing interference" [2]. The idea is to regulate *received* power rather than *transmitted* power. Using this model, cognitive radios (CRs) operating in licensed frequency bands [3] would be capable of measuring the current interference environment, and adjusting their transmission characteristics in such a way that their transmissions avoid raising the interference temperature over a regulatory limit.

Over the past few years, the interference temperature model has often been recognized as a possible solution to the dynamic spectrum allocation problem [4], but there have been no real schemes for actually using it. Without a concrete technique, it's difficult to say whether or not it will be a practical solution.

This work investigates exactly how one should measure interference temperature. In particular, there are nonlinear interrelations between bandwidth, power, capacity, and interference temperature that make it difficult to compute optimal transmission parameters. We develop some iterative techniques to solve for the parameters in question.

Section 2 introduces the interference temperature model. Section 3 discusses the properties of interference temperature in more detail. Section 4 evaluates the capacity achievable under interference temperature constraints. Sections 5 and 6 describe algorithms for solving the equations proposed in section 4. Section 7 concludes.

2 IT Model

The concept of interference temperature is identical to that of noise temperature. It is a measure of the power and bandwidth occupied by interference. Interference temperature T_I is specified in Kevin and is defined as

$$T_I(f_c, B) = \frac{P_I(f_c, B)}{kB} \tag{1}$$

where $P_I(f_c, B)$ is the average interference power in Watts centered at f_c, covering bandwidth B measured in Hertz. Boltzmann's constant k is $1.38 \cdot 10^{-23}$ Joules per Kelvin degree.

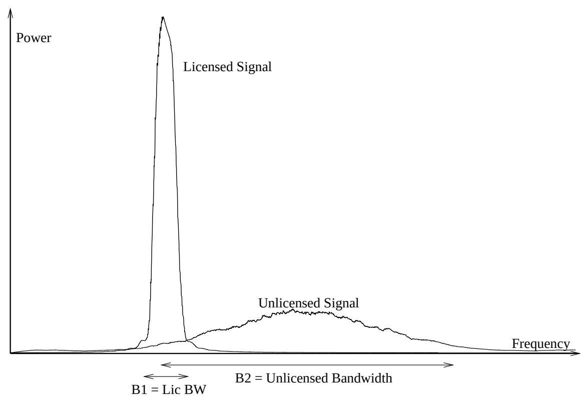

Figure 1: Example PSD for an unlicensed signal partially overlapping a licensed signal

The idea is that by taking a single measurement, a cognitive radio can completely characterize both interference and noise with a single number. Of course, it has been argued that interference and noise behave differently. Interference is typically more deterministic and independent of bandwidth, whereas noise is not.

For a given geographic area, the FCC would establish an *interference temperature limit*, T_L. This value would be a maximum amount of tolerable interference for a given frequency band in a particular location. Any unlicensed transmitter utilizing this band must guarantee that their transmissions added to the existing interference must not exceed the interference temperature limit at a licensed receiver.

While this may seem clear cut, there is ambiguity over which signals are considered interference, and which f_c and B to use. Should they reflect the unlicensed transceiver or the licensed receiver? For example, consider figure 1. Should we use B_1 or B_2 as our bandwidth for our computations? These ambiguities precipitate the need for our two interpretations.

2.1 Ideal Model

In the *ideal interference temperature model* we attempt to limit interference specifically to licensed signals. Assume our unlicensed transmitter is operating with average power P, and frequency f_c, with bandwidth B. Assume also that this band $[f_c - B/2, f_c + B/2]$ overlaps n licensed signals, with respective frequencies and bandwidths of f_i and B_i. Our goal is to then guarantee that

$$T_I(f_i, B_i) + \frac{M_i P}{k B_i} \leq T_L(f_i) \quad \forall\ 1 \leq i \leq n \quad (2)$$

In other words, we guarantee that our transmission does not violate the interference temperature limit at licensed receivers.

Note the introduction of constants M_i. This is a fractional value between 0 and 1, representing a multiplicative attenuation due to fading and path loss between the unlicensed transmitter and the licensed receiver. The idea is that the interference temperature model restricts interference at the licensed receiver, not the unlicensed transmitter, and therefore we must account for attenuation between these two devices. Since we cannot know our distance to all licensed receivers, let us assume that this value is fixed by a regulatory body to a single constant M.

There are two main challenges in implementing the ideal model. The first involves identifying licensed signals. One key question arises: how do you distinguish licensed signals from unlicensed ones? For specific cases, this can be relatively easy. In particular, consider the problems faced by IEEE 802.22 [5], currently under investigation by their spectrum sensing task group. They wish to coexist with DTV signals, and can implement very specialized, matched filter sensors to look for DTV transmitters. If you know exactly with whom you are coexisting, then this problem becomes simpler.

The second problem involves measuring T_I in the presence of a licensed signal. We wish to measure the interference floor *underneath* the licensed signal. Again, this can be relatively easy if we have knowledge of the licensed waveform's structure. For example, with DTV, we can measure during the blanking interval when the signal is not present. Also, if we have precise knowledge of the signal's bandwidth B and center frequency f_c, we can approximate the interference temperature as

$$T_I(f_c, B) \approx \frac{P(f_c - B/2 - \tau) + P(f_c + B/2 + \tau)}{2kB} \quad (3)$$

where $P(f)$ is the sensed signal power at frequency f and τ is a safety margin of a few kHz.

Assuming a specialized environment where we can locate licensed signals and measure interference temperature, our next goal is to determine radio parameters f_c, B, and P that achieve a desired capacity C. This will be a piecewise-continuous optimization problem with constraints defined in (2). If we use a sculptable waveform like OFDM we may be able to more easily meet the various constraints. In depth analysis of this is the subject of future research.

2.2 Generalized Model

The generalized interference temperature model, on the other hand, has a different interpretation to signals and bandwidths. The fundamental premise of the generalized model is that we have no a priori knowledge of our signal environment, and consequently have no way of distinguishing licensed signals from interference and noise.

Under these assumptions, we must apply the interference temperature model to the entire frequency range, and not just where licensed signals are detected. This translates into the following constraint.

$$T_I(f_c, B) + \frac{MP}{kB} \leq T_L(f_c) \quad (4)$$

Notice that the constraint is in terms of the *unlicensed transmitter's* parameters, since the parameters of the licensed receivers are unknown.

One question that immediately comes to mind: under what conditions does the generalized model limit interference as well as the ideal model?

If we rewrite our constraints in terms of P and combine them, we obtain the following requirement:

$$B(T_L - T_I^{gen}(f_c, B)) \leq B_i(T_L - T_I^{id}(f_i, B_i)) \quad \forall\ 1 \leq i \leq n \quad (5)$$

Assuming each licensed signal has power P_i and otherwise the interference floor is defined by the thermal noise temperature T_N, we can transform (5) into the following:

$$kBT_L(f_c)(B - B_i) + kBT_N \sum_{j=1}^{n} B_j \leq \sum_{j=1}^{n} B_j P_j \quad \forall\ 1 \leq i \leq n \quad (6)$$

In general, provided B_i and P_i are sufficiently large, this condition can be easily met.

If we consider only one licensed receiver, the inequality simplifies to

$$\frac{kBT_L}{P_1 - kBT_N} \leq \frac{B_1}{B - B_1} \quad (7)$$

Thus a small T_L, large B_1, or large P_1 will generally satisfy the constraint.

The remainder of this paper describes challenges inherent in selecting transmission bandwidths necessary to meet a particular target capacity in the general interference temperature model. While we can easily measure $T_I(f_c, B)$, inclusion of all signals in its measurement causes a more complex interference environment.

3 IT Properties

One shortcoming in the design of the interference temperature model is its simplicity. The goal was to define a single metric that fully captures both the properties of interference and noise. In the end, a *temperature* approach was used rather than a *power* approach. This accurately models the noise portion of the metric, but not the interference portion. Also, since we have elected to treat all signals other than our own as interference, we now have more signals to accentuate the problem.

Our eventual goal is to determine the difference between the regulatory interference temperature limit and the measured interference temperature. This then defines the *transmission temperature* our cognitive radio can use, where for a given bandwidth we can compute the maximum allowed power.

Let's define things a little more concretely. Thus, the interference temperature T_I can be specified as a function of bandwidth B as

$$\begin{aligned} T_I(f_c, B) &= \frac{1}{Bk} P_I(f_c, B) \\ &= \frac{1}{Bk} \left(\frac{1}{B} \int_{f_c - B/2}^{f_c + B/2} S(f)\, df \right) \\ &= \frac{1}{B^2 k} \int_{f_c - B/2}^{f_c + B/2} S(f)\, df \end{aligned} \tag{8}$$

where $S(f)$ represents power spectral density of our current RF environment.

Next, we must consider how our transmission will affect the received interference temperature $\hat{T}_I(f_c, B)$. As described before, the end goal is to guarantee that for our transmit power P and bandwidth B,

$$\begin{aligned} \hat{T}_I(f_c, B) &\leq T_L(f_c) \\ T_I(f_c, B) + \frac{MP}{Bk} &\leq T_L(f_c) \end{aligned} \tag{9}$$

There are two basic cases to consider. First, B is known, and we wish to compute a valid P. In this case, we can solve the above for P and get

$$P \leq \frac{Bk}{M} T_L(f_c) - \frac{1}{BM} \int_{f_c - B/2}^{f_c + B/2} S(f)\, df \tag{10}$$

If we are trying to compute B in terms of P, we run into some trouble. There is no closed-form solution for a general $S(f)$. However, since $S(f)$ is a real, nondecreasing, continuous function of B, there is a solution $\forall S(f)$, even though it may be outside our radio's dynamic range of $(0, B_{\max}]$.

Consider the degenerate case where $S(f) = c$. This implies some constant level of interference throughout our frequency band of interest. The following solution then arises

$$B \geq \frac{MP + c}{T_L(f_c) k} \tag{11}$$

Interestingly, this now indicates a *minimum* bandwidth required for our transmission, not a maximum. This is because of how the interference temperature limit works in the general model. Our maximum transmit power is $BT_L k$, which increases as a function of bandwidth. This implies that as we use more spectral resources in the frequency domain, we can actually cause *more* interference. This subtlety is counterintuitive.

Since bandwidth and power are so interrelated, we in the next section we consider them jointly in terms of capacity.

4 Capacity and IT

Interference temperature must always be measured at some bandwidth B, due to deterministic interference sources. To measure $T_I(f_c, B)$, down-sample the passband signal such that f_c is at $B/2$. Then quantize the spectrum at rate $2B$, and compute its PSD. This will yield a power spectrum for the frequency range $f_c - B/2$ to $f_c + B/2$, which is $\hat{S}_B(f)$. To compute the interference temperature, integrate as follows.

$$T_I(f_c, B) = \frac{1}{B^2 k} \int_0^B \hat{S}_B(f)\, df \tag{12}$$

Thus, we can now compute our interference temperature as a function of B.

Let's say a minimum capacity of C is necessary for our communication. The next goal is to find a P and B that both meet regulatory requirements and achieve our capacity constraints. In the last section, we showed that choosing a B and solving for a maximum P was a simple approach. However, considering $S(f)$ may have very steep slopes when f_c is close to a powerful licensed signal, this may be problematic.

Thus, we must combine some of our concepts. We must compute a capacity function $C^*(B)$ in terms of B, and then solve $C^*(B) = C$ for B. Let's assume a maximum transmit power is used for our bandwidth selection, or

$$P^*(B) = \frac{Bk}{M} (T_L(f_c) - T_I(f_c, B)) \tag{13}$$

This equation uses $T_I(f_c, B)$, measured using the technique described above.

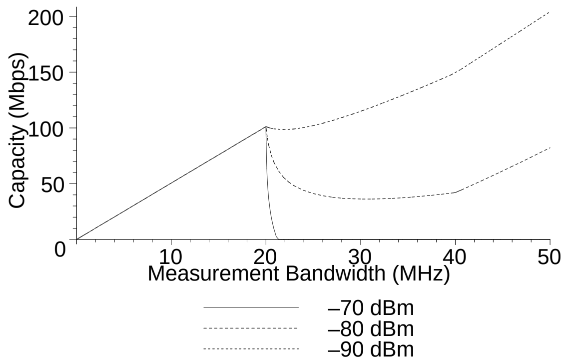

Figure 2: Example capacities as a function of B, assuming a licensed signal of varying strengths located at $[f_c + 10 \text{ MHz}, f_c + 20 \text{ MHz}]$, with $T_L = 10000$ Kelvin and a noise temperature of 300 Kelvin. As the interference power increases, the capacity past 20 MHz falls off more.

Then we can define our capacity as

$$\begin{aligned} C^*(B) &= B \log_2 \left(1 + \frac{LBk(T_L(f_c) - T_I(f_c, B))}{MBkT_I(f_c, B)}\right) \\ &= B \log_2 \left(1 + \frac{L(T_L(f_c) - T_I(f_c, B))}{MT_I(f_c, B)}\right) \end{aligned} \tag{14}$$

Notice the addition of another constant, L. This value is similar to M, except it represents multiplicative path loss between the unlicensed transmitter and unlicensed receiver. We are measuring capacity at the *receiver*, and therefore need knowledge of the bandwidth and power at the receiver.

5 Hill Climbing Approach

Solving $C^*(B) = C$ can be pretty tricky. For a general interference environment, this must be done numerically. Figure 2 shows a simple example of a 10 MHz licensed signal with square power spectral density located 10 MHz from our carrier frequency. We can see that as long as the signal's power is relatively low, e.g. -90 dBm, our capacity function remains relatively linear. However, for -70 dBm, we can see that it will significantly hamper our capacity. We can achieve maximum capacity if we avoid the signal all together.

As the previous example illustrates, the capacity function is not strictly increasing, and therefore there may be multiple bandwidths that give the same capacity. Certainly the best choice is to select the smallest bandwidth possible that will achieve your desired capacity.

One could even take that a step further and add a pricing function. In the previous example, if the interference power is -80 dBm, to go from a capacity of 100 Mbps and a capacity of 105 Mbps requires a tripling in the bandwidth. A pricing function would penalize nodes who use extremely large bandwidths, and therefore select 20 MHz, even if it didn't completely satisfy its capacity constraints, as the payoff for tripling the bandwidth would not be worth the added cost.

Putting the pricing function aside, and assume we have a hard capacity constraint, and we wish to solve $C^*(B) = C$ for B, then we must employ numeric techniques. Using the above equations, for a particular B we can compute $T_I(f_c, B)$ and consequently $C^*(B)$.

One approach is to hill climb, with the object being to minimize the following as a function of B.

$$|C^*(B) - C| \tag{15}$$

This function may have several global minimizers over the bandwidth range of our radio. Our goal is to locate the one corresponding to the smallest bandwidth.

A good approach is to run our hill climbing algorithm several times with

$$B_0 = \left\{ \frac{iB_{\max}}{N} \right\}_{i=1..N} \tag{16}$$

This will yield N, likely non-unique, solutions. Simply select the one with the smallest bandwidth.

A simple pricing scheme can also be used. To find global capacity maximizers, set $C = \infty$ and run the same algorithm. This will yield a set of points $(B_i, C_i)_{i=1..N}$. Let our capacity utility function be $U(C)$ and our pricing function be $P(B)$. We can then select the best point i^* as

$$i^* = \arg\max_{i=1..N} \left(U(C_i) - P(B_i)\right) \tag{17}$$

Lastly, we should address selection of N. The number of local minima will be proportional to the number of interfering signals. This could be computed by the radio by determining the number of local maxima n in $S(f)$ for $f_c - B/2 \leq f \leq f_c + B/2$. If solving for a specific C, let $N > 2n$, since there would likely be a solution on either side of the signal. If searching for global capacity maximizers, then $N > n$ should be sufficient. This operation could be done infrequently, and would provide a good estimate for N, assuming interfering signals are relatively uniformly spaced over the target spectrum band.

While we can use the hill climbing approach to both optimize $C^*(B)$ and solve it for a target capacity, we will see in the next section that fixed-point iteration is a more elegant way to solve $C^*(B)$ for a target capacity. Therefore, hill climbing is most appropriate when trying to maximize capacity.

6 Fixed-Point Iteration

Next, consider a reformulation of the original problem.

Theorem 1 *The sequence* $\{B_i\}_{i=1..n}$ *where*

$$B_{i+1} = \frac{C}{\log_2\left(1 + \frac{L(T_L(f_c) - T_I(f_c, B_i))}{MT_I(f_c, B_i)}\right)} \tag{18}$$

converges linearly to a solution to

$$C = B\log_2\left(1 + \frac{L(T_L(f_c) - T_I(f_c, B_i))}{MT_I(f_c, B_i)}\right) \tag{19}$$

as long as

$$B_0 > \frac{2CT_I^*}{T_N}\log_2\left(1 + \frac{L(T_L(f_c) - T_I^*)}{MT_I^*}\right)^{-2} \tag{20}$$

where

$$T_I^* = \max_{B \in (0, B_{\max}]} T_I(f_c, B) \tag{21}$$

and a solution exists in $B \in (0, B_{\max}]$.

Proof: We're examining our problem in terms of fixed-point approximation. Let

$$g(B) = \frac{C}{\log_2\left(1 + \frac{L(T_L(f_c) - T_I(f_c, B))}{MT_I(f_c, B)}\right)} \tag{22}$$

The theory of fixed-point iteration methods dictates that if $B = g(B)$ has at least one solution in some interval $[a, b]$, $g(B)$ is continuous, and $|g'(B)| < 1$ then any starting point in that interval will converge to a solution [1]. Intersect the interval $[a, b]$ with our feasible interval, $(0, B_{\max}]$. The result is a range for B_0:

$$B_0 \in [a, \min\{b, B_{\max}\}] \tag{23}$$

$T_I(f_c, B)$ is continuous, so consequently $g(B)$ is continuous. The derivative constraint can be expressed as follows:

$$\frac{CLT_L(f_c)|T_I'(f_c, B)|}{T_I(f_c, B)(LT_L(f_c) + (M - L)T_I(f_c, B))} < \log_2\left(1 + \frac{L(T_L(f_c) - T_I(f_c, B))}{MT_I(f_c, B)}\right)^2 \tag{24}$$

Obviously this constraint is not entirely useful, as it is in terms of B, which we do not yet know. In order to simply this further, we need to remove our dependence on B. First, we use the definition of T_I^* provided in the theorem statement, and notice that

$$T_N \leq T_I(f_c, B) \leq T_I^* \tag{25}$$

Next we need to examine the derivative of our interference temperature.

$$T_I'(f_c, B) = \frac{\hat{S}(B)}{B^2 k} - \frac{2}{B}T_I(f_c, B) \tag{26}$$

Thus to maximize $|T_I'(f_c, B)|$, let $\hat{S}(B) = 0$ and $T_I(f_c, B) = T_I^*$. The result is

$$|T_I'(f_c, B)| \leq 2T_I^*/B \tag{27}$$

Substituting, we have

$$\begin{aligned} B_0 &> \frac{CLT_L(f_c)2T_I^*}{T_N(LT_L(f_c) + (M - L)T_N)} \cdot \log_2\left(1 + \frac{L(T_L(f_c) - T_I^*)}{MT_I^*}\right)^{-2} \\ &> \frac{CLT_L(f_c)2T_I^*}{T_N(LT_L(f_c))}\log_2\left(1 + \frac{L(T_L(f_c) - T_I^*)}{MT_I^*}\right)^{-2} \\ &> \frac{2CT_I^*}{T_N}\log_2\left(1 + \frac{L(T_L(f_c) - T_I^*)}{MT_I^*}\right)^{-2} \end{aligned} \tag{28}$$

Thus we have proved our theorem. ■

We now have a viable algorithm for computing the required bandwidth B in terms of desired capacity C. If $B_0 > B_{\max}$, this does not necessarily mean a solution does not exist, since we derived a *sufficient* condition, and not a *necessary* one. If divergence is detected, then the capacity C must be decreased in order to find a solution.

The key point is that fixed-point iteration can find a solution if one exists, but may not always succeed. As a result, it may be useful to implement a hybrid algorithm that first tries fixed-point iteration, and if divergence is detected, switch over to a hill climbing approach. Note that the algorithms can be executed on a PSD snapshot taken with bandwidth $B_{\max}$, and consequently radio sensing resources need not be tied up during algorithm execution.

7 Conclusion

In this paper we have considered how to use both interference temperature and the regulatory interference temperature limit to select an optimal radio bandwidth for a particular interference environment. We've discussed two interpretations of the interference temperature model, and showed the conditions under which the general model yields interference at least as small as the ideal model.

Two main techniques for solving for the desired bandwidth in the general model are presented. The first uses hill climbing and is best suited for scenarios where we wish to maximize capacity for our radio's dynamic bandwidth range. The second uses fixed-point iteration and is designed to find a bandwidth for a specific desired capacity. Future work involves further analysis of bandwidth selection for the ideal model, and also applying pricing functions to bandwidth and examining how this affects network-wide spectral efficiency.

Overall, the interference temperature model offers an exciting new paradigm for dynamic spectrum access.

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