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Modal logic with subjunctive marker: a new perspective on rigid designation.

section: Modal & Epistemic Logic

According to Kripke's *Naming and Necessity* the thesis that proper names are synonymous with certain definite descriptions is false because proper names are rigid designators whereas definite descriptions are (in general) non-rigid designators. It is argued that standard modal logic, the formal tool used by Kripke to deal with these questions, does not reflect adequately the relations between the modal operators, their scopes and the subjunctive mood. Then, by using K. Wehmeier's modal logic with subjunctive marker ($S5^*$), it is shown (against Kripke) that in one sense of rigidity (reflecting Kripke's criterion of rigidity) proper names as well as definite descriptions are rigid designators, but in another sense of rigidity neither proper names nor definite descriptions are rigid designators (apart from definite descriptions that have been called *de facto* rigid by Kripke and alike proper names). Nevertheless Kripke is right in not accepting the synonymy thesis: Proper names are not synonymous with definite descriptions. But the correct reasons for this claim are not Kripke's.

Extended abstract:

1. Kripke on rigid designators

In his very influential *Naming and Necessity* Kripke introduced the distinction between rigid and non-rigid designators. With respect to any possible world rigid designators name always the same object whereas non-rigid designators name different objects with respect to different possible worlds. That means: Inside the scope of a modal operator a rigid designator names always the same object whereas the semantic value of a non-rigid designator depends on the possible world at stake. According to Kripke all proper names are rigid designators but most definite descriptions are non-rigid designators. (There are definite descriptions like "the sum of two plus two" that are rigid because in every possible world always the same object meets the description. Definite descriptions of that kind are called *de facto* rigid by Kripke).

2. Modal logic and scope

Obviously, the standard formal tool to deal with questions of rigid and non-rigid designation is modal logic. As a matter of fact the standard outlook of modal logic nowadays reflects a certain historical development. In the beginning, the modal operators had been designed in such a way that all predications inside their scope had to be evaluated always with respect to the possible world at stake as determined by the modal operator. Thus, standing inside the scope of a modal operator was thought to be a necessary and sufficient condition for every predicative expression in order to get affected by that modal operator. (With nested modal operators things get a little bit more complicated: Then a predicative expression might stand inside the scope of several modal operators but is only affected by one of them).

Later it was found that this strategy is unsatisfactory because there are simple examples that can't be analysed by mere scope distinction (Hazen 1976):

(1) *Under certain counterfactual circumstances, everyone who has flown to the moon would not have flown to the moon.*

The problem arises because on the one hand the expression "everyone who has flown to the moon" must stand inside the scope of the modal operator in order to allow for an adequate analysis but on the other hand this expression refers to the actual world and is thus independent of the modal operator.

To deal with these problems and thus to increase the expressive power of the formal modal language the so-called *actuality* operator had been introduced (Crossley/Humberstone 1977): Expressions standing inside the scope of an *actuality* operator always refer to the actual world, even inside the scope of a modal operator. An *actuality* operator protects against the influence of modal operators, so to say. Now, the example can be rendered as follows:

$$\diamond \forall x(A(Fx) \to \neg Fx) \tag{2}$$

Thus, in modal logic with *actuality* operator it is no longer the case that it is a necessary and sufficient condition to stand inside modal scope for an expression to be evaluated with respect to the possible world at stake because expressions in the scope of an *actuality* operator are always evaluated with respect to the actual world.

But once we have a language in which not all expressions inside modal scope depend on the respective modal operators one can ask whether there is no better way to indicate dependencies and independencies from modal operators than by scope distinctions and supplementary operators as the *actuality* operator.

3. Wehmeier's modal logic with subjunctive marker

(Wehmeier 2002) presents an alternative to standard modal logic, his modal logic with subjunctive marker, $S5^*$ (in (Rückert 2002) Wehmeier's modal logic is used to solve Fitch's paradox of knowability). An expression inside modal scope depending on the respective modal operator needs to be indicated with a subjunctive marker "*" (this corresponds to the subjunctive mood in natural language). Otherwise it refers to the actual world (corresponding to the indicative mood in natural language). In Wehmeier's logic the above mentioned example looks as follows:

$$\diamond \forall x(A(Fx) \to \neg F^* x) \tag{2*}$$

The subjunctive marker "*" functions in analogy to the well-known individual variables, it is nothing else but a variable over possible worlds that needs to be bound up by a modal operator. Thus, expressions that contain subjunctive markers that are not bound by modal operators and thus appear freely are incomplete.

4. Definite descriptions

Now, we apply Wehmeier's modal logic to the problem of rigid designation, starting with an analysis of definite descriptions. An expression of the kind "the teacher of Alexander" is ambigous. Its two readings are:

(a)	*the man who taught Alexander*
(b)	*the man who would have taught Alexander*

According to Kripke this difference has to be reflected by scope distinctions: The subjunctive mood in (b) indicates that the definite description has to stand inside modal scope whereas the indicative mood in (a) indicates that the definite description has to stand outside modal scope (or, as an 'actualised' definite description inside modal scope, respectively). But according to Kripke we are dealing with one and the same definite description in both cases.

Things look different in the framework of Wehmeier's $S5^*$. The two readings (a) and (b) get the following translations:

(a*)	$(\iota x)(Tx)$
(b*)	$(\iota x)(T^*x)$

(a*) is the formal representation of a rigid designator: Even inside modal scope, it always designates the one who taught Alexander in our world, namely Aristotle. On the other hand (b*) is not even the formal representation of a designator at all. It contains a freely occurring subjunctive marker and is thus incomplete: One needs to add information about the circumstances in order to determine a designated object. Thus, if we apply Kripke's rigidity criterion to definite descriptions within the framework of Wehmeier's logic we get the following result: All definite descriptions are rigid designators.

Of course, we can define another concept of rigidity to account for some of Kripke's intuitions. "$(\iota x)(Tx)$" is a rigid$_2$ designator iff the following holds:

$$\Box((\iota x)(Tx) =^* (\iota x)(T^*x)) \tag{3}$$

In this sense Kripke's *de facto* rigid definite descriptions are rigid$_2$ and all other definite descriptions are non-rigid. The idea behind the concept of rigidity$_2$ is the following: One and the same language can be spoken in different possible worlds. When an inhabitant of another possible world uses an expression in indicative mood like "the man who taught Alexander" he refers to his own world. Now, a designator is rigid$_2$ if it names the same object no matter in whatever possible world it is used.

5. Proper names

In an expression like "the teacher of Alexander" we detected an ambiguity earlier on that resulted from a difference between the indicative and the subjunctive mood. The expression has two readings because as a matter of fact it does not contain a verb in the indicative or the subjunctive mood. The ambiguity can be resolved by using other expressions that contain a verb or by using a formal language that makes the difference explicit by help of a subjunctive marker.

Now to proper names. An expression like "Aristotle" does not contain a verb either. But is there also a indicative/subjunctive-ambiguity? In our natural language we have no means to build subjunctive versions of proper names. But that's a contingent fact. Even if in most cases we use an expression like "Aristotle" to refer to the man that was named "Aristotle" in our world, one can imagine contexts in which we are confronted with something like subjunctive versions of proper names: "Imagine a world in which there was a young man that was given the name 'Aristotle'. Aristotle was a very good wrestler and he ... ". I think, most of us understand this story in a way that the word "Aristotle" in the second sentence does not refer to (our) Aristotle but to the young wrestler whoever he is.

Anyway, in a formal language we are free to introduce subjunctive counterparts of proper names reflecting the fact that in other possible worlds the same names might have been given to other objects than in our world. In the possible world of our example, people using the name "Aristotle" name the young wrestler and not (our) Aristotle. Nevertheless, this is no sufficient reason to say that say speak another language. And thus an expression like "Aristotle" (like "the teacher of Alexander") has two readings, too:

(c*) a
(d*) a^*

(c*) is certainly a rigid designator, and (d*) isn't a designator at all because it contains a freely occurring subjunctive marker. In this respect there's no difference between proper names and definite descriptions. Also, most proper names are non-rigid$_2$ too, because

$$\Box(a =^* a^*) \tag{4}$$

is false for most proper names "a". (One might argue that there are rigid$_2$ proper names in our language as for example "2". A language in which "2" does not designate the successor of the number 1 couldn't be the same language as ours.) Again no essential difference between proper names and definite descriptions.

6. Conclusion

I tried to show that accepting Wehmeier's modal logic $S5^*$ as a more appropriate formal tool than standard modal logic leads to new insights concerning the concepts of rigid and non-rigid designation. Against Kripke it is not true that there is a difference in principle between proper names and definite descriptions, the first being rigid designators, the latter non-rigid designators. According to Kripke's conception of rigidity both, proper names and definite descriptions, are (in general) rigid. And according to another conception of rigidity, rigidity$_2$, both are (in general) non-rigid$_2$.

For these reasons, also Kripke's modal argument against the description theory of proper names is not valid. Nevertheless his thesis is correct: Proper names and definite descriptions are not synonymous because (in general) there is for a proper name "a" no definite description "$(\iota x)(Gx)$" such that

$$\Box(a^* =^* (\iota x)(G^* x)) \tag{5}$$

holds. (Maybe a definite description like "the object named 'a' " does the job but this does not yield any substantial form of description theory because the proper name itself appears in the definite description.

References

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