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Quantitative geography: representations, practices, and possibilities

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Abstract. Representations of quantitative geography, both by practitioners and by others, have tended to associate quantification with empiricism, positivism, and the social and academic status quo. Qualitative geography, by contrast is represented as nonempiricist or postempiricist, sensitive to complexity, contextual, and capable of empowering nonmainstream academic approaches and social groups. Attempts to engage in debate between these positions rarely challenge this dualism, reproducing the representation of quantitative geography as logical positivism, and a dualism separating quantitative and qualitative geography. I argue that this dualism can be broken down, by deconstructing the underlying representation. I discuss why this representation came into existence and how it was stabilized; how close attention to the practices of quantitative geographers, and particularly to the evolution of these practices, reveals its inadequacies; and what new possibilities for quantitative practices emerge from this deconstruction. GIS, one of the recent manifestations around which representations of quantitative geography polarize, is used as a case study to illustrate these arguments. I pay particular attention to the question of the relevance of quantitative practices for an emancipatory human geography.

Geographers who see their work as contributing to quantitative geography and those who emphasize a qualitative approach seemingly have little in common, but they agree on one thing.(1) Quantitative geography generally is taken to be embedded within an empiricist, (logical) positivist and 'normal science' approach to the field.(2) This representation of quantitative geography has experienced remarkable stability and persistence over the last twenty-five years, notwithstanding occasional prominent and passionate attempts to deconstruct it. In this paper, I seek to argue that it is a misrepresentation to claim that quantitative geography must be positivist.

My own thinking, as a radical geographer socialized within some of the principal centers of calculation of the quantitative revolution (Bristol and Toronto in the 1970s), has consistently struggled with, and against, this representation. As a geographer who uses mathematical methods in order to critique neoclassical economic geography and develop a Marxian political economic alternative, who has supported the growth of feminist, anti-essentialist and poststructuralist human geographies, and who works

(1) The distinction drawn here between quantitative and qualitative geography has developed since the late 1970s, after humanistic and radical human geographers began to develop critiques of the philosophical and methodological foundations of the geography of the quantitative revolution. Those advancing hermeneutic, feminist, realist, and dialectical methods as an alternative to logical positivist methods have increasingly seen themselves as pursuing a qualitative rather than a quantitative human geography. This is reflected in the recent growth of qualitative methods courses in Anglo-American geography departments, as an alternative to quantitative methods. For an overview, see Eyles and Smith (1988), Eyles (1993), Lawson (1995), Johnston (1997), Jones et al (1997), and Fotheringham et al (2000).

(2) In this paper, I will use positivist as a shorthand for logical positivism; the Vienna Circle brand of positivism that became the norm for 'normal' science in geography, as in most other disciplines. It is important to recall that, in its original Comtean form, positivism was conceived as a humanist enlightenment project, notwithstanding the ways in which, as logical positivism, much of this emancipatory concern has been set aside (Gregory, 1978).

with marginal communities struggling to use GIS to better understand and improve their environments, I have long been convinced that progressive human geography can take advantage of quantitative practices. This seems to be an increasingly unpopular position. The rediscovery of the importance of qualitative and reflexive methods, their refinement under the influence of feminist geographers, and their establishment for the first time as a legitimate alternative to quantitative methods in many geography departments has been wonderful to see. But too often qualitative approaches are placed in opposition to quantitative/positivist geography (for a more constructive exception, see Dixon and Jones, 1998). At times this dualism has been stark indeed, as when the authors of a paper presented at an IBG meeting in the early 1990s began by apologizing to their audience for presenting some quantitative data in table form as part of their paper (David Rigby, personal communication).

Reproducing this qualitative–quantitative dualism seems particularly paradoxical given that feminist and poststructural theory seeks to escape dualistic thinking. I argue here that this dualism should and can be deconstructed. Through tracing the evolution, reproduction, and contradictions of this representation, I suggest that it caricatures quantitative practices in geography in an overly narrow way, and that, in fact, the potential of quantitative geography for postpositivist geographical practices is significant. The argument is organized as follows. First, I trace how this representation has become hegemonic, arguing that its persistence is a result of how it serves the contrasting interests and identities of quantitative and qualitative geographers. I summarize periodic attempts to break down the dualism, and discuss why these have failed. Second, I briefly review the diverse practices of quantitative geography and contrast these with the dominant representation, drawing implications for the unrealized potential of quantitative practices. Third, I examine the case of GIS, around which the association of quantification with positivism has been resurrected. Finally, I discuss the possibilities for quantitative practices as one component of a progressive human geography in what I will argue is an increasingly quantified world.

Prior to engaging in the argument, however, some preliminary remarks about mathematics as a language of theory are in order. I adopt the view that mathematics is a humanly constructed language for describing and conceptualizing the world (compare Barnes, 1998; Gould, 1999).[3] As with any language, it has been developed by social actors to carry out certain tasks, and continues to evolve in ways framed by our cultural context, our ability to use it, and the tasks to which mathematical communication is applied. Two consequences follow from this. First, mathematics is not a hermetically sealed logical system with its own internal rules that enable the truth of all mathematical statements to be determined (Gödel, 1931), nor is it a mirror of the natural order of the world. While mathematics is a better language than others for certain purposes, that is true of all languages, and we cannot conclude that mathematics is superior to other languages in general. But if claims that mathematics can be elevated above other languages because of its 'rigor'—its ability to determine the truthfulness of statements—are questionable, how is it that mathematics nevertheless has achieved the 'structural effect' (Mitchell, 1999) of being elevated in this way, above and separated from other languages? I take seriously the arguments of Bruno Latour and others, that the answer to this question comes from examining the dynamics through which certain claims come to be taken as truthful and natural, as well as the processes and networks through which agreements about naturalness and truthfulness

[3] It is important to recognize that mathematics is not simply numbers. In fact, it frequently has little to do with numbers per se. Its key features are not the objects manipulated in mathematical reasoning, whether these be numbers or letters, but the mathematical grammar through which relationships between these objects are expressed.

are reproduced (compare Latour, 1987; 1996).[4] I will suggest that it is exactly through such networks that the status of quantitative geography has been contested.

Second, again like other languages, mathematics is a 'live' language, continually evolving as both practical problems and new ways of thinking challenge our ability to express ourselves with current linguistic tools. This is often forgotten, particularly by those proponents content to use the mechanistic Newtonian mathematics of the 19th century to describe the world of the 21st century (Mirowski, 1984; Gould, 1999). But it is also neglected by critics of quantitative geography, whose own vision of what can be done with mathematics often is similarly frozen in the past (Marchand, 1974). Proper attention must be paid both to the contested status of quantitative geography and to the evolution of quantitative practices, in order to deconstruct the identification of quantitative with positivist geography and the dualism between quantitative and qualitative geography, and to unpack the potential of quantitative practices for an emancipatory postpositivist human geography.

Representations of quantitative geography

Quantitative practices in geography date back at least to Greek attempts to measure the circumference of the earth, but the term 'quantitative geography' was coined in the 1960s. The terminology was deliberate, an attempt to promote the 'quantitative revolution' as a new and better scientific geography over an older regional geography. (For a variety of situated participants' accounts, compare Burton, 1963; Taylor, 1976; Berry, 1993; Hanson, 1994; King, 1994.) The power of this moment in Anglo-American geography, both in its own terms and also in generating a concern for theory and for philosophical foundations that remains with us today, is no doubt one reason why the term quantitative geography still resonates with what was practiced then.

Elsewhere I have suggested that a variety of processes underlay both the development of a network of quantitative geographers and their rapid rise to prominence in the face of considerable intradisciplinary opposition between 1957 and 1963 (Sheppard, 1995a). These include external forces which offered external legitimation for geography as a discipline if it became more quantitative: the prevalence in the academy of positivism as the model of good science; the recognition brought to geography in the National Academy of Sciences as it adopted quantitative approaches; the rapid diffusion of this model of knowledge production into the social sciences, after its successful application as operations research during World War 2; and the desire to apply this knowledge to make society better through social engineering (an important part of securing support for capitalism during the Cold War), in areas ranging from transportation planning to the war on poverty. But there were important forces working within the discipline as well. Quantitative geography became the rallying point around which a new postwar generation of geographers, and geography departments (particularly the University of Washington), could promote their influence and careers by promulgating a new and distinctive approach, presented as both critical of and superior to the preceding focus on areal differentiation and regional synthesis. Fred Schaeffer (1953), trained by the logical positivist philosophers of science at the Universities of Minnesota and Iowa, aggressively made the case for difference and superiority; Fred Lukermann (1989) and John Agnew (1989) show that the case was greatly overstated. In the context of the ongoing postwar expansion of universities, and the growing demand

[4] I also share the reservations of those critics who argue that Latour's radically anti-essentialist stance, what Laurier and Philo (1999, page 1064) refer to as the 'flattened spatial imaginary' of Latour's actor networks and Haraway (1997, pages 33–35) terms the memetic nature of his argument, make it difficult to account for why certain voices are consistently reinforced, and others marginalized, by the 'centers of calculation' that emerge.

for graduate students to occupy the new teaching positions, the success of the new approach in capturing the intellectual high ground of geography resulted in a rapid growth to prominence of quantitative geography as the center of calculation for postwar human and physical geography.

The quantitative revolution brought a change not only in disciplinary language, but also in worldview, each reinforcing the other. The philosophy drawn on to promote quantitative geography over Richard Hartshorne's exceptionalist approach was positivism. Interestingly, the term itself was not widely used by quantitative geographers (Hill, 1981), until they came under attack from postpositivist geographers, and quantitative geographers have paid little attention to the finer points of distinction between empiricism, positivism, logical positivism, and critical rationalism. Nevertheless, David Harvey's (1969) *Explanation in Geography* canonized this approach and clearly was influenced by the same logical positivist philosophers as Schaeffer. The result, by the end of the 1960s, was a hegemonic representation of modern geographic practice, in which quantitative practices are fused with scientific geography, made possible by the myriad of processes and actors, external and internal to geography, marshaled in support of this representation. A useful shorthand for this is Latour's term, technoscience.

Representations are always contested and contradictory, and the cluster of activities associated with the quantitative revolution was fraught with contradictions. The continued willingness of individual geographers to identify themselves with this approach diminished. Many quantitative geographers wished to make society more equitable, and became disenchanted as they began to realize that the particular practices that came to dominate quantitative geography made these goals difficult to realize (Sheppard, 1995a). At the level of philosophical debate, it became clearer that logical positivism is not the foolproof method for making sense of the world that it had been thought to be, but (like all philosophies) has its own inconsistencies and contradictions. At the level of society, even the so-called golden age of Fordist First World economic growth brought persistent and enhanced inequality, which the application of techno-social science in the public and private sectors failed to ameliorate. This resulted in widespread social protests against states and corporations, and parallel criticisms of positivist social science within the academy. Such contestations have been present with varying modalities since the 1960s, and over time the influence of positivist views has waned. Yet proponents and opponents have both shared the view that quantitative geography meant positivist geography (as I detail below). In addition, both sides have focused on one particular quantitative practice as canonical of positivist/quantitative geography: spatial statistical analysis.(5)

For proponents, spatial statistical analysis was the hallmark of scientific geographic practice, because it provided tools to test hypotheses against 'just the facts'. It typified the path to new knowledge within a positivist epistemology. It was also an area where geographers developed new statistical methods for dealing with the interdependencies of spatiotemporal data; brilliant technical innovations that made both statisticians and nonspatial scientists sit up and take notice of the discipline. Spatial statistical analysis successfully forced changes in the basic theories of statistics in a way that has been unparalleled by geographers' forays into mathematics and computer science. Thus, spatial statistical analysis provided internal and external validation for a technoscience agenda, and became the focus of the majority of research developing and applying quantitative methods in geography. Quantitative approaches in general, and spatial

(5) I use the term spatial statistical analysis to refer to all attempts to use statistical tools to describe spatial patterns and make statistical inferences about the causes of those patterns: from point pattern analysis to spatiotemporal regression models.

statistical analysis in particular, continually have been cited by their proponents as both making the practice of geography more rigorous (in a positivist sense), and as enhancing the respectability of geography among the other 'sciences'. This last point is an issue of particular sensitivity in the USA, where geography remains a peculiarly marginal part of the academic and societal enterprise.

The critics of quantitative/positivist geography have made their case on similar grounds. For critics, characterizing quantitative geography as positivist, and its quantitative practice as spatial statistical analysis, was instrumental in driving home criticisms of this hegemonic paradigm and gradually undermining its dominance. Although many issues were at stake, the most effective mechanism has been to characterize quantitative geography as positivist, and to highlight the weak spots in this philosophical position. As radical, humanist, and feminist critics took their turn, they highlighted such shortcomings as: a tendency to Cartesian reductionism, an essentialist treatment of human agency, a self-imposed restriction to analyzing the empirical status quo, an inability to identify the processes and structures driving observed relationships, claims of objectivity, and a masculinist 'view from nowhere' which treats scientists as a privileged source of expertise and producer of knowledge (Harvey, 1972; Gregory, 1978; Ley, 1978; Rose, 1993; Barnes, 1996). Each of these criticisms was carefully honed, and together they have effectively deconstructed any agreement among human geographers that geography must be positivist—a position that some physical geographers have also echoed (Rhoads and Thorn, 1996; Richards et al, 1997; Harrison and Dunham, 1998).

Early criticisms equating quantitative geography with positivism, developed by Andrew Sayer (1976) and Derek Gregory (1978), did discuss both statistical and mathematical geography. They saw spatial statistical analysis as the hypothesis-testing arm of a positivist geography, guilty of assuming that observable relationships could be reduced to general recursive causal systems (dubbed functionalism by Sayer), that data are not theory laden, and that reliable predictive models can be treated as explanatory models (instrumentalism). Their critique of mathematical geographical theory was on different grounds—its link with neoclassical economics.[(6)] Ron Johnston (1981) narrowed the focus to empirical statistical models, and even though he suggested that such approaches need not be associated with positivism alone, his equation of quantitative practices with spatial statistical analysis was adopted by others. Sayer's subsequent book (1984) developed a distinction between empirical statistical models, critiqued as an extensive methodology unable to reveal contingent causal processes, and intensive multimethod case-study analysis which is sensitive to context and better able to tease out the operation of real causal mechanisms (see also Massey, 1985; Massey and Meegan, 1985). Feminist geographers, in arguing for reflexive qualitative feminist geographic methodologies sensitive to the complexities of identity construction and discursive practices and to the voices of all participants, counterpose qualitative empirical methods to spatial statistical analysis, highlighting the inadequacies of the latter (McDowell, 1992; Staehili and Lawson, 1995; Jones et al, 1997).

The persistence of this representation of quantitative geography as statistical spatial analysis, notwithstanding the much more complex set of practices pursued by quantitative geographers, is a product of the ways in which this representation has served the interests and identities of both sides. Both those claiming to be positivists and those working to displace positivism found this representation to be an effective foil in arguing their contradictory positions. This mutual reinforcement has helped reproduce

[(6)] Some radical economists, reacting similarly to the equation in their discipline of mathematics with marginalist neoclassical theory, concluded that the use of mathematics and statistics is a bourgeois ideological practice.

and give truth value to this representation; an example of what Bruno Latour would refer to as the stabilization of meaning through the operations of the situated actor networks of academic geography.

Some prominent researchers have attempted to destabilize this representation, also focusing on the practice of statistical spatial analysis, although their arguments have fallen on deaf ears. Robert Bennett argued vigorously that quantitative geography is not necessarily positivist (Bennett and Wrigley, 1981; Bennett, 1985). Peter Gould has both termed the equation of quantitative and positivist geography as "bizarre" (1999, page 258) and has been particularly critical of the limitations of spatial statistical analysis as a branch of quantitative geography (1976; 1984). In a special journal issue on feminist methods titled "Should women count?", Sarah McClafferty and Vicky Lawson both sought to break down the dualism between qualitative and quantitative methods in geography, which they saw as unproductive for feminist research (Lawson, 1995; McClafferty, 1995). For Lawson and McClafferty practicing quantitative geography means numerical descriptions of women's lives and gender inequalities, and empirical spatial associations. Recently, even Stan Openshaw (1998) has shifted his position as the bête noire of qualitative and critical geography, arguing that there is space for both quantitative and 'soft' geography—while still maintaining that 'computational' geography's strength is that it engages in "a systematic, rigorous, and self-critical search for explanation by testing theories against facts" (page 331).

It is possible to conclude, therefore, that proponents and opponents of quantitative geography share a conventional view, or stabilized representation, of the nature of quantitative practices. Quantitative geography is a form of 'science and technology': geography as science combined with all the technologies incorporated into its practices (most prominently, spatial statistical analysis), where the trials of responsibility (for good or ill) have been settled on the shoulders of its proponents (Latour, 1987, page 174). Yet the contradictions between this particular representation of quantitative geography and its far more complex practices provide a basis to deconstruct both the all too neat equation: quantitative—positivist—scientific geography, and the dualism between quantitative and qualitative geography.

Practices of quantitative geography

There are three broad ways in which practices of quantitative geography differ from the representation summarized above. First, its emphasis on statistics has eliminated from view the significant mathematical practices of geographers. Second, quantitative practices of both a mathematical and a statistical nature cannot be equated with positivism. Third, the evolution of practices in all areas has been neglected. If we take seriously the evolutionary nature of the language of mathematics, critics of quantitative geography cannot be satisfied with analyzing past practices—as if quantitative geography stopped evolving in the early 1970s (only to rise again from its coffin, in the view of such critics, reincarnated through GIS like a bad Dracula movie). Just as critics think they have pinned mathematics down and identified some fundamental flaws, the language evolves to address some limits to its representational capabilities, while retaining or creating others (Marchand, 1974). In this section, I primarily focus on mathematical geography and on its evolution, to make the point that significant nonpositivist quantitative practices have been overlooked. But I will also argue that a similar diversity can be uncovered in the area of spatial statistical analysis.

A major component of quantitative geography in the 1970s, as even a cursory perusal of *Geographical Analysis* reveals, was the use of mathematics to construct purely theoretical arguments about human geographic processes; mathematical models of movement, location, and diffusion. In many cases, no attempt at empirical analysis was envisaged. Authors of such models realized that they were based on assumptions that would make them virtually impossible to test, recognized that the observational or data requirements for testing were too great, and/or were content to engage in theoretical reflection. Such activities were central to the practices of quantitative geography at that time. They can be seen as focusing on the analytical statement side of logical positivism, but with little desire to link these with synthetic statements in order to create empirical laws—thus departing from the overall positivist program. Some practitioners were willing to test empirical hypotheses even if the mathematical models generating these hypotheses were patently unrealistic. They adopted the questionable methodological prescription of Milton Friedman, that it does not matter how simple the assumptions are as long as the predictions look plausible (Friedman, 1953; Dacey, 1975; Hanson, 1994). Whether or not there was an interest in hypothesis testing, almost all mathematical human geographers adopted (in addition to the uniform plane metaphor) assumptions prioritizing the methodological individualism, rational choice, competitive market equilibrium, and economism characteristic of the voluntaristic social theories of neoclassical economics (Barnes and Sheppard, 1992; Barnes, 1996).

This self-imposed alignment with neoclassical theory was a contingent choice —reflecting a lack of awareness of alternative economic paradigms, and/or a willingness to ride on the intellectual coattails of the seemingly scientific neoclassical economic paradigm whose proponents have not been averse to claiming the mantle of positivism to legitimize their abstract theoretical exercises. Yet the choice to align with neoclassical economics shaped the acceptability of quantitative practices in general by postpositivist human geographers. Before the neoclassical research program had gained a great deal of momentum in geography, Doreen Massey and Andrew Sayer provided a political economic critique which made visible and effectively critiqued its neoclassical presumptions (Massey, 1973; Sayer, 1976). A natural conclusion, therefore, was to reject the messenger along with the message; if mathematical theories are associated with neoclassical economic geography, then both should be rejected. Mathematical theorizing in geography thus became associated not only with positivism but with neoclassical economics.[7]

In principle, any set of assumptions could have been used for mathematical theoretical reflection in economic geography, including those of Marxist economics (Morishima, 1973). Indeed, a few mathematically inclined economic geographers subsequently pursued such a path of regional political economy. They developed mathematical models of a Marxian capitalist space economy which take seriously the geographically extensive nature of economic systems, call into question some of the fundamental principles of neoclassical, and Marxist, aspatial economic theory, and make Marxian empirical analysis possible (for a review, see Sheppard, 2000). By the time this research began to be published, however, a consensus equating mathematical with neoclassical and positivist human geography already dominated discourse. Gregory (1993) did recognize that this research program raised new possibilities for a postpositivist mathematical geography, but most looked at this research as an anachronism, standing in the way of the task of erasing positivist, and quantitative, geography.

The possibility of a nonpositivist mathematical geography is reinforced, once attention is paid to new developments in mathematics. Even as the quantitative revolutionaries

[7] At the other extreme from rational choice models, as David Sibley (1998) has noted, are approaches, such as the random spatial economy and random walk theories of Leslie Curry (1962; 1998), that are so minimalist in their assumptions about underlying processes that they also have been regarded with some suspicion by spatial scientists.

were learning to use Newtonian calculus in the 1970s, some mathematicians had become very dissatisfied with the representational limits of mainstream mathematics, and proposed new mathematics to overcome these problems. I will focus on two developments, each of which geographers began to take up a decade or more later: challenging Cartesian representations, and challenging equilibrium representations.

A common criticism of positivism is that it propagates a Cartesian view of the world; one in which theoretical analysis is founded on (reducible to) a natural set of ontologically prior, homogeneous objects of fixed definition.[8] In short, ambiguity is eliminated; objects have pregiven identities inside fixed boundaries, and are precisely related to one another. Mathematics, whose key operators are equality and inequality, is easily categorized as Cartesian. Yet considerable effort has been put into transcending the inability of mathematics to deal with ambiguity. One way is to rethink the algebra of mathematics. For example, Gunnar Olsson (1991) proposes replacing the = operator with an alternative operator (/) that better captures ambiguity: "a symbol of relations, of the unity between identity and difference" (page 98). See also Olsson (2000). A similar, if less radical approach can be found in fuzzy set theory (Zadeh, 1965). In this branch of mathematics, objects are conceptualized as simultaneously belonging to various degrees to several predefined sets. For example, a person may be measured as 45% male and 55% female; a hybrid identity. Fuzzy set theory is simply an algebra of categorization. There is concern neither for how such identities are constructed or might change, nor with the coherence and construction of the sets themselves. Yet it does capture some ambiguity by making the boundaries between categories fuzzy.

A third form which attempts to express the fuzziness of the boundaries between objects is the mathematics of fractals (Mandelbrot, 1977). Fractals describe objects whose boundaries are inherently vague. The classic example is a coastline, for which finer resolutions do not reduce the complexity of the boundary between land and water. A variety of analysts see fractals as potentially quite profound. The cultural theorist Ira Livingston (1997, pages 122–123) argues that the liminal spaces of fractals are "where everything (interesting) happens .... This 'between' is not a simple compromise or equilibrium nor a static middle ground". For the mathematician Roger Penrose (1989), the existence of fractals calls into question the research program of artificial intelligence, and its presumptions that computers can mimic the human mind.[9] As an exercise in pattern analysis, fractals themselves again provide no insights into how ambiguity is created. Yet, taken together, the ideas of Olsson, Zadeh, and Mandelbrot show that mathematics need not be Cartesian, and thus not positivist.

A variety of developments in nonlinear dynamic modeling have added to the language of mathematics a set of tools for addressing the complexity of disequilibrium dynamics. Prior to this, mathematical theorizing in urban and economic geography (and in climatology, geomorphology, and biogeography) either focused on equilibrium

(8) As used here, Cartesian thinking makes four ontological commitments: (1) any system can be reduced to a set of natural units; (2) all such units can be treated as internally homogeneous; (3) these units have intrinsic properties which are independent of the systems that they constitute; and (4) causes are separate from effects in such systems (Harvey, 1996). Cartesianism is not peculiar to logical positivism, but logical positivism—like much natural science—adopts a Cartesian worldview.

(9) Penrose suggests that the Mandelbrot set, the building block of fractal patterns, is not a recursively enumerable set. This would mean that no computable algorithm can be constructed to determine, in a finite number of steps, whether a point in space lies within this set (that is, whether it is inside the fuzzy fractal boundary). He argues that the prospect of artificial intelligence is predicated on the ability to represent thinking about reality as computable algorithms.

patterns or assumed that the future could be smoothly predicted from the past. This perspective fits closely with the logical positivist worldview, which values universal laws and general deductive conclusions, and which treats prediction and explanation as symmetric [every scientific explanation takes the form of a prediction, and every prediction is also an explanation (Kellert, 1993, page 97)]. Thus to criticize approaches prioritizing general laws, stability, and predictability, as Sayer's realist account effectively did, was to criticize both positivist and mathematical geography (see also Jones and Hanham, 1995). The ongoing critique of neoclassical economic geography also picked up on these issues; attacking its presumption that the economic landscape can be described as a market-clearing equilibrium, reflecting the harmonious operation of Adam Smith's invisible hand [a theme still pervasive today in economists' revived interest in economic geography (compare Sheppard, 2000)].

Until recently, it was very difficult to express disequilibrium dynamics using the vocabulary and grammar of mathematics. Disequilibrium dynamics were a puzzle twenty years ago: mathematical theorists insisted on using mathematical tools that gave general results, and most disequilibrium dynamics were beyond the capability of mathematicians to analyze them in this way. This was one reason why neoclassical theory, for example, focused on equilibrium outcomes. As Philip Mirowski (1984) has pointed out, economists were using 19th-century mathematics as the foundation for their theoretical approach. New possibilities were created by a path of mathematical thinking beginning with catastrophe theory (Thom, 1975), a topological approach which attempts to categorize the types of possible qualitative changes that a system can undergo. This was followed by chaos theory, the idea that deterministic systems could behave in an apparently random and completely unpredictable manner, and now complexity theory. Complexity theory refers to the development of mathematical languages to describe the path-dependent and complex evolution of systems of nonlinearly related phenomena; systems that frequently are expected to be far from equilibrium and 'on the edge of chaos'. Complexity theory has made it possible to mathematically analyze disequilibrium dynamics, evolutionary systems, and morphogenesis (the creation of structure and form). This is happening in a range of disciplines—although economists have found it harder than most to adapt to this shift in thinking (Anderson et al, 1988; Krugman, 1996).

Complexity theory poses a series of puzzles for both positivist and Bhaskarian realist philosophy, and is catalyzing a revolution in the kinds of mathematical practices that are considered intellectually respectable. Prediction becomes much more difficult in complexity theory. Exogenous disturbances can have unpredictable effects on system dynamics, and small differences in parameter values can have major impacts on projected development paths. Although the proverbial butterfly in the Amazon may not force global climate change, the symmetry between prediction and explanation that is a hallmark of logical positivism breaks down in complexity theory (Kellert, 1993). By the same token, a central distinction in Sayer's Bhaskarian realism, between closed theoretical systems exhibiting predictable necessary relations and open empirical systems in which geographical context shapes the variable contingent outcomes of these relations, also breaks down. Even small, closed nonlinear systems, such as the population models of Robert May (1976), can exhibit variable and unpredictable 'necessary' relations (Sheppard, 1996).

In addition, the traditional emphasis in mathematics (and positivism) on deriving general theorems and laws as the source of deductive knowledge, has been called into question by complexity theorists. A revolution in mathematical practices has begun, in which insistence on only using mathematical reasoning that results in general theorems supported by a deductive proof has been displaced by new practices that value computational and simulation strategies as highly as general proofs. This change is contested, of course, particularly in mainstream economics, and may reflect only a temporary uncertainty among mathematicians about how to apply their theorem-proof approach in this new and complex context. Yet such a shift in practices again underlines the variety of ways that mathematics can be practiced. Indeed, complexity theory has been pointed to by social and cultural theorists as being of importance well outside the philosophies and accepted practices of normal science. Immanuel Wallerstein (1991, chapter 8) argues that chaos theory is creating a crisis in normal science, making space for alternative knowledges and conceptions of science (see also Wallerstein, 1996). Harvey (1996, pages 58–59) notes parallels between dialectical reasoning, chaos theory, and fractals, and Massey (1999) notes parallels with Gilles Deleuze.(10) Although some might dismiss these as the thoughts of unreconstructed modernists, Ira Livingston (1997), Isabelle Stengers (1997), and other cultural theorists (Rasch and Wolfe, 2000) are similarly intrigued by the possibilities of the mathematical languages of complexity.

The use of the mathematics of fuzzy sets, fractals, catastrophe, chaos, and complexity theory by geographers has been sporadic (Wilson, 1981; Batty and Longley, 1994; Phillips, 1999), their impact on human geography remains to be determined, and their applicability in the human realm continually must be critically scrutinized (Gould, 1999). Yet they do demonstrate not only that quantitative practices are a moving target for critics, but also that they need not be associated with positivist geography. Mathematical postpositivist geography is possible.

Even within spatial statistical analysis, the variety of practices can diverge significantly from the conventional image of positivist hypothesis testing. As Bennett (1985) has noted, Bayesian analysis in statistics incorporates the best guesses of the investigator into the analysis from the beginning, through specification of prior probabilities. Bayesian work has not been common in geography (perhaps the most widespread application being for the interpretation of entropy models of spatial interaction in the 1970s). Indeed, it has been controversial more generally in statistics, perhaps because of a reluctance to countenance a subjective approach to hypothesis testing. [The critical rationalist Karl Popper (1959) proposed falsification because hypothesis testing is plagued by investigator bias, but most variations of positivism emphasize that hypothesis testing must be objective.] Bayesian analysis departs from positivist hypothesis testing, because it explicitly recognizes that investigators cannot be separated from objects of analysis. Indeed, it could even be seen as a reflexive approach to investigation, more generally associated with hermeneutics and feminism than with statistics, because it invites the investigator to make his or her situated knowledge part of, and a subject for, analysis.

Statistical analysis also has turned away from the deductive and general to the inductive and computational. Long-standing concerns and debates exist in spatial statistics about the robustness of these techniques in the presence of nonstationary autocorrelated spatiotemporal data (whose mean, variance, and correlograms may vary across space and time); about the absence of sufficient data for reliable estimates;

(10) It is worth noting that Harvey also finds the mathematical philosopher Alfred Whitehead to be a central figure in dialectical thought, because of his concern for the constantly changing relational construction and mutual constitution of objects, and with possible worlds. Although much mathematics is Cartesian in conception, there are other branches which focus on relationships rather than objects (Gunnar Olsson, personal communication, 1989). Indeed this is how I thought we were using the language of mathematics in order to examine certain aspects of the capitalist space economy (Sheppard and Barnes, 1990); unpacking, rather than departing from, Harvey's (1982) *The Limits to Capital.*

about the validity of assumptions that variables are normally or log–normally distributed; about sampling methods; and about the distinction between samples and populations. These debates have led to uncertainty about the robustness and accuracy of frequency-based approaches to spatial statistical inference. At the same time, increased computational capacity has made it easier to replace deductive methods based on maximum likelihood theorems with exploratory spatial data analysis, and with a variety of brute-force computational strategies such as distribution-free approaches, computing a million or so correlation coefficients, or 'geographical analysis machines' (Getis and Ord, 1992; Openshaw and Openshaw, 1997; Openshaw, 1998). David Sibley (1998, page 243) describes exploratory data analysis as follows: "there are no critical points at which decisions have to be made .... The data are held in fond embrace rather than being forced into some a priori scheme, a characteristic which should encourage dialectical thinking." Perhaps Sibley's interpretation of exploratory data analysis is optimistic, but it does show that there is an increasing willingness also in statistics to depart from a general deductive and objective approach to truth, in favor of approaches which emphasize the ingenuity and interpretive skills of the investigator and his or her ability to tell persuasive stories with spatial data.

This analysis of some quantitative practices that have been overlooked in dominant representations of quantitative geography reveals that the taken-for-granted bracketing of quantitative and positivist geography is not a necessary relationship, but rather a social product of disciplinary rivalries and debates. These rivalries and debates have resurfaced around the currently dominant quantitative practice, geographic information systems (GIS). GIS is an influential set of computational geographic practices, which has been used to revive the spatial science research program in geography. As a result, GIS has recently become a focus of debates about quantitative and positivist geography.

Practicing GIS

Both proponents and critics have represented GIS as an arm of spatial analysis in the service of positivist geography.[11] Three closely associated attributes of GIS practices are highlighted in this representation: that they are quantitative, empiricist, and positivist. It is seemingly self-evident that GIS is quantitative and empiricist, given its computational roots in Boolean mathematics and its use for manipulating empirical spatial databases. Mathematical precision and empirical accuracy are valued aspects of GIS, even though its capabilities for mathematical and statistical analysis remain limited by comparison with those of quantitative geographic practices more generally. Attached to this image is that of GIS as a tool of positivist geographic science. As John Pickles (1995) has documented, proponents of GIS have stressed its parallels to positivist spatial analysis, and its links to hypothesis testing, deductive logic, and value neutrality (see also Dobson, 1983; Openshaw, 1991). Critical social theorists agree, representing GIS as a positivist Trojan horse—smuggling *geographicus quantifactus* back in under the cover of darkness, just when Helen seemed to have been saved from the positivist seige (Taylor, 1990; Lake, 1993; Pickles, 1995). The representational limits of GIS itself also have been a common target of postpositivist critiques: its preoccupation with Newtonian concepts of space, its Cartesian 'view from nowhere' approach to mapping, and the presumption that the world can be accurately captured using Boolean algorithms combined with translation rules linking abstract variables to real-world objects (an artificial intelligence perspective) (Sheppard, 1995b; Curry, 1998). These arguments parallel postpositivist critiques of quantitative geographic practices more generally.

[11] As in the case of quantitative geography, there have been dissenting voices, to little effect. Of particular note are Daniel Sui (1994) and Michael Curry (1998).

From a postpositivist perspective, the dangers of positivist GIS as an epistemological program for geography are associated with an equally dangerous disregard for the social and ethical implications of GIS: its use for surveillance; its association with the military; its inaccessibility to disadvantaged communities; and its ability to shape forms of life and the places where these are lived (Pickles, 1991; Smith, 1992; Goss, 1995; Curry, 1997; Harris and Weiner, 1998). Notwithstanding a considerable rapprochement between certain GIS practitioners and social theoretic critics (Pickles, 1999; Sheppard et al, 1999), many GIS researchers have neither engaged with critiques of GIS as a positivist practice, nor taken much interest in the social consequences of GIS practices (Pickles, 1997). Such attitudes are, in this view, consistent with positivist conceptions of the role of science; other epistemologies are a distraction from positivism's foolproof methods for seeking the truth, and furthermore the practice of science should not be sullied by political considerations.

The link between GIS and science has been made more explicit with recent attempts to rename this research program geographic information science (GISci) (Goodchild, 1992; Couclelis, 1997; Goodchild et al, 1999). Replacement of 'systems' with 'science' conveys quite conventional images of science, which themselves perform important strategic roles of legitimating GIS-related practices in the scientific community. As a linguistic strategy, GISci has been successful both among practitioners, whose journals have been renamed (*Cartography and GIS* has become *Cartography and GISci*, *The International Journal of GIS* is now *The International Journal of GISci*) and whose US association is the University Consortium for Geographic Information Science, and in the larger research community, where GISci is now the favored term in such places as the National Science Foundation. Once science replaces systems, the implicit adjective 'positivist' is usually not far behind, because that is the conventional sense of what is meant by science.

Interestingly, however, although the desire to legitimate GIS practices within the larger academic community is an important reason for the change to GISci, those coining and propagating the term are also attempting to broaden the epistemological remit of science. Dawn Wright et al (1997, page 353) are willing to include much of the postpositivist pantheon within their view of science. Whether this ecumenical definition of GISci is accepted by practitioners (and critics), or whether the continued successful propagation of GISci will be tied to representing GISci as positivist—even if this is contrary to its authors' wishes—remain open questions.

Once again, however, tendencies to equate GIS (and GISci) with positivist geography require a critical reexamination. Michael Curry (1998) in particular has shown that things are much more complex than this. GIS is, of course, itself a social construct. It is not a tool with inherent properties that make it universally useful, but a social technology shaped by the societal and geographic context within which it developed—by its inventors and practitioners, and by the evolution of information technologies more generally. In short, GIS as we know it is the end result of a particular evolutionary path that may or may not be the best possible path (Chrisman, 1987; Dosi et al, 1990; Arthur, 1994; Sheppard, 1995b). Yet, like all representations, GIS as we know it is taken for granted and becomes simply GIS. Critical scholars must remain open, however, to the contingent and changing nature of GIS practices, as for quantitative practices more generally. This means that any assessment of GIS must ask not only whether GIS as we know it is positivist, but also whether GIS as it might have become, or might become in the future, must be positivist. In the remainder of this section, I consider whether GIS as we know it is necessarily quantitative, empiricist, and positivist, before turning to the question of alternative GISs.

GIS practices rely heavily on the processing and analysis of databases categorized according to conventional measurement scales. These may include qualitative (nominal and ordinal) data, but modes of analysis are essentially quantitative. Ambiguity is typically eliminated (with the exception of some use of fuzzy sets and fractals), and the outputs from GIS analysis are typically quantitative even when the data inputs are not (for example, the percentage of land cover classified as urban). Yet, Weiner et al (1995) have experimented with ways of superimposing sketch maps incorporating local narratives of land-use alienation, gathered through ethnographic fieldwork, on conventional GIS maps. In addition, ESRI Inc., makers of ARC/INFO are beginning development of sketch-mapping software, to be linked to GIS as we know it, which local residents can use to include their situated accounts of neighborhoods as a part of community-based planning. In addition, the multimedia capacity of computer software is making it increasingly easy to conceive of including, as extra layers in a GIS, not only 'objective' information about, for example, the chemicals located at a site, but also photographs, videos, and narratives about the objects depicted in a GIS layer. Such situated accounts are not databases, even though they can be recorded in digital form, and their existence within a GIS does not mean that they can or should be subject to quantitative analysis.(12) Thus GIS as we know it can incorporate situated knowledge and ethnographic material, encouraging users to employ it in an interpretive manner.

GIS practices also are heavily empiricist, because GIS analyses depend on secondary databases containing empirical representations of the world. Heavy reliance on secondary data can be very problematic, and indeed can undermine the canons of positivist research that data should be accurate and appropriate variables should be collected (Sheppard, 1993). First, the provenance of spatial data is not always clear, notwithstanding the extensive effort put into data-quality analysis and into attaching metadata, explaining how the data were collected, to databases. Second, positivist research canons stress the importance of using data collected for the purpose at hand, because secondary data are typically collected for purposes which differ from, and generally do not meet the measurement criteria of, the variables specified by the theory being tested. Some of those close to positivism recognize that even primary data custom collected for the purpose at hand, are generally theory laden, compromising the positivist principle that data are the independent arbiter of the truth of a hypothesis (Popper, 1959). For a variety of reasons, therefore, empirical GIS practices may depart from positivist ideals.

In addition, GIS as we know it need not be restricted to empirical analysis. Hypothetical databases, representing other possible worlds, could be analyzed using standard GIS algorithms, and the evolution of unobserved processes could be simulated. Exactly this kind of hypothetical reasoning has been pursued using cellular automata, for example. Even when such models are theoretically implausible, they represent nonempiricist uses of GIS that are analogous to hypothetical mathematical modeling in quantitative geography more generally. It is even increasingly difficult to know when empirical analysis is actually being carried out, because digital technologies are making it harder to distinguish 'real' from virtual reality. The proliferation of databases over the Internet, often with few or no metadata, not only create headaches for those trying to catalog data, but also for those wishing to carry out empirical analysis. Census data copied from a 1950 census volume and transcribed accurately by the analyst into his or her computer meet conventional definitions of reliable empirical information, but what about data found on a noncensus website

(12) On the difference between situated knowledge and that of empiricism, see Donna Haraway (1991).

that purports to originate from the Census? As in other areas of electronic communication, it becomes increasingly difficult in an Internet world of increasingly intermediated information to know the reliability of information—to determine that it is what it claims to be. By the same token, the capacity to generate realistic but hypothetical landscapes using fractal algorithms, and to create realistic but hypothetical photographs using Photoshop™, means that realistic images and maps may not be what they seem. Once GIS can both produce realistic but hypothetical maps and also incorporate false but real-looking images, without the user being able to distinguish the empirical from the hypothetical, then GIS practices have moved well beyond the domain of empiricism and positivism.

If GIS as we know it can be used for empirical representations that are interpretive and qualitative in intent, and for spatial analysis of hypothetical data, then it follows that GIS practices are not necessarily positivist. Even though the vast majority of GIS practices are probably positivist in intent, that may have less to do with the nature of GIS as we know it than with the context in which it was developed; to facilitate empirical analysis, but also for normative purposes—creating landscapes as well as accounting for them. Norms of scientific practice and technologies developed to advance such practices often go hand in hand, as technoscience, but this does not mean that they cannot be separated. GIS as we know it can be used for other purposes (epistemological and political).

The prospect of other kinds of GIS, GISs that we do not yet know, extends the bounds of GIS practices, and further weakens their putative association with positivism. This is a possibility that has been broached, although its implications to date remain unclear. Alternative visions have been discussed, variously dubbed AltGIS, GIS_2, place information systems, and public participation GIS, but with little consensus about the form they might take or their potential significance (Curry, 1998; Obermeyer, 1998; Sheppard et al, 1999). One example, however, is suggestive of what is possible, and why critics must be attentive to changing possibilities and not fixate on current practices and limitations. Consider a series of websites maintained by different residents of the same neighborhood on which they post their own narratives, representations, desires, and dreams about that place, using this as a vehicle for exchange and debate, and for political discussion about how to change that place. This would be based on computer software, possibly incorporating capabilities for sophisticated map-making (including sketch maps), multimedia, and collaborative use (whereby different residents can work on the same project simultaneously from different computers). It would display and represent geographic information and knowledge, but in a form more consistent with the communicative rationality of everyday life (Habermas, 1984) than the instrumental rationality of GIS. It shares much with GIS as we know it, but is not data driven, makes no claims to objectivity, and does not seek to produce correct understandings of the world or optimal solutions to problems.

Possibilities for quantitative geography

The practices of quantitative geography have been misrepresented both as a branch of positivist geography and as dominated by spatial statistical analysis. This seems quite natural, a stabilized representation of geographic technoscience that is familiar to and consonant with the beliefs of both neopositivist geographers and their postpositivist critics, but is nonetheless problematic. One consequence of the dominance of this misrepresentation over discourse about quantitative geography is that the practice of critical geography has become increasingly antiquantitative in tone. Yet once this representation is called into question, we begin to notice a variety of quantitative practices that in fact have little to do with positivism, and connect in interesting ways with a variety of postpositivist approaches to social theory. It remains to be determined what the long-term influence of the different quantitative practices discussed here might be for postpositivist research, but they are just examples of a wider set of already existing or potential practices whose possibilities for a postpositivist human geography must continue to be assessed with a critical but open mind. The philosophical status of all quantitative practices in geography can and should be contested, of course. Both new applications of old practices and new practices will be continually drawn into positivist and conservative geographical projects, particularly given the status of mathematics within such projects. But they can also be harnessed for other uses.

Why should postpositivist geographers pay any attention to quantitative practices?[13] We live in an increasingly quantitative world; one in which the languages and practices of mathematical reasoning increasingly penetrate everyday life. More numbers are presented to more people every day, more aspects of the world are being commodified, and more lives are affected by the quantitative models used to plan public and private sector behavior. This ranges from the instrumental use of mathematics to shape investment strategies, election campaigns, consumption behavior, the Internet, and bombing campaigns; to its prevalence in daily discourse, on television (in game shows and stock market tickers), in the supermarket, and in techno and new age music. Day trading personifies this influence, as does the haggling over prices that any informal street trader must be proficient in if he or she is to survive. Rejecting the Aristotelian logic of instrumental rationality may be an understandable reaction to what Habermas has termed the colonization of the lifeworld by the system, and could be justified as strategic essentialism—promoting qualitative and denigrating quantitative geography, in order to legitimize postpositivist geography in the face of its more powerful positivist alternative. But it is now time to reassess this strategy. In the final analysis, a critical and dissenting tradition of geography that remains antiquantitative is unsustainable.

Becoming skilled in the practice of quantitative as well as qualitative geography can only strengthen the influence of critical thought, within the academy and without. First, developing effective critiques of hegemonic research programs, such as those of neoclassical economics which are so central to the logic of globalization, requires tackling such approaches on their own territory and showing that they are inadequate in their own terms. "Genuine refutation must penetrate the power of the opponent and meet him on the ground of his strength; the case is not won by attacking him somewhere else and defeating him where he is not" (Adorno, 1982, page 5). Effective critique is possible, but only once one has become skilled in the assumptions, logic, and mathematical language favored in such research programs (Sheppard, 2000).

Second, becoming skilled in documenting, analyzing, and representing the inequalities produced by dominant social practices, and by the research programs used to justify them, is an essential step in questioning the adequacy of such practices. For example, GIS can be used also in a very conventional way to document social inequalities in exposure to environmental hazards, and thereby to catalyze social resistance and public concern, as the history of the environmental justice movement indicates. Third, as many social movements have learned, the ability to make an effective case for

[13] Of course, there is no reason that postpositivist geography is necessarily progressive, and progressive geography need not be postpositivist or grounded in participatory local politics. Because current debates in geography pit self-consciously postpositivist, qualitative and progressive human geographers against positivist geographers inclined to neoclassical and rational choice theoretic approaches, however, I focus here on questioning the claim that postpositivist emancipatory geography cannot be quantitative. See also Plummer and Sheppard (2001).

change often requires insurgent quantitative practices; using the tools of industry and the state to beat them at their own game. GIS can be used, for example, to construct neighborhood environmental inventories or to document public health hazards, as part of effective strategies to persuade companies to reduce pollution. Finally, it is hard to imagine how large-scale social movements can be organized effectively on the basis of qualitative practices alone, as the sheer logistical challenge requires an ability to plan schedules and budgets, develop strategies, and master communications technologies.

This does not imply simply a return to quantitative geography as we knew it. The theoretical and philosophical problems underlying how quantitative geography was used in the era of spatial analysis have been persuasively documented, and critical quantitative practices must be developed with this in mind. In addition, even though the boundaries and limits of quantitative practices are always shifting, important boundaries will remain. As Bob Rundstrom (1995) and Michael Curry (1998) have argued for GIS, and as Ken Hillis (1999) has pointed out for virtual reality, some of these limitations remain intransigent (particularly the difficulties posed by the contextual and communicative nature of human communication). Thus we must remain attentive to their limits.

An important part of critically reengaging with quantitative geography is rethinking the teaching of quantitative methods. Quantitative education has been strongly gendered and raced (inside and outside geography), and remains inaccessible to those marginalized groups who have had to contend with educational systems unable or unwilling to provide them with the necessary equipment or training. Even public education has reinforced such inequality, with quantitative techniques favored by those on the professional career ladder, and inaccessible to those deemed to be outside the centers of calculation of contemporary society. Yet these are challenges to be overcome, not a reason for abandoning quantitative education. A progressive geographic education system would reject such bias; would expose the delusion that mathematics is more difficult than, say, deconstruction; would take seriously the complex relationships between methods, philosophy, and social theory; would break down the dualism between quantitative and qualitative geography (while maintaining a creative tension between the two); and would seek to make students familiar enough with the full gamut of languages and methods to know the strengths and limitations of each.

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