Optimal Design of Switched Reluctance Motors

T. J. E. Miller, *Fellow, IEEE*

***Abstract*—The fundamental theory of the switched reluctance motor is presented with a number of new equations. It is used to show how the practical development of a design calculation should proceed, and this leads to a discussion of physical characteristics required to achieve satisfactory performance and to reduce acoustic noise. The paper makes a few generic observations on the characteristics of successful products that use switched reluctance motors. It is written at a basic engineering level and makes no attempt to apply sophisticated optimization theory.**

***Index Terms*—Electric motors, switched reluctance motors.**

β_s	Stator pole arc.
d_r	Rotor slot depth.
ε	Stroke angle.
e	Phase electromotive force (EMF), V.
e_u	End-effect parameter.
f_u	End-effect parameter.
R	Phase resistance, Ω.
i	Phase current, A.
L_{end}	End-winding inductance.
L_u	Unaligned inductance.
L_{u0}	Unaligned inductance (two-dimensional value).
L_{stk}	Stack length, mm.
m	Number of phases.
N_r	Number of rotor poles.
N_s	Number of stator poles.
ρ_a	Absolute overlap ratio.
ρ_e	Effective overlap ratio.
S	Number of strokes per revolution.
t	Time, s.
T_e	Electromagnetic torque.
τ_a	Absolute torque zone.
τ_e	Effective torque zone.
ν	Terminal voltage of the phase winding, V.
ω_m	Angular velocity, rad/s.
ψ	Phase flux-linkage, V·s.
θ	Rotor position, rad.

I. INTRODUCTION

TO A WISE engineer, "optimal design" means a compromise between conflicting factors, often producing an imperfect result from optimistic aspirations. Who would use a title such as "Compromises in the design of switched reluctance motors"? *Optimal* sounds better, particularly if used to describe the production of silk purses from sows' ears. While the switched reluctance motor is not the silk purse of electric machines technology, it fetches more in the market than sows' ears and, therefore, it must be a compromise between these two extremes: in other words, an optimal result arising from less-than-perfect components.

Manuscript received June 5, 2001; revised September 18, 2001. Abstract published on the Internet December 5, 2001.

The author is with the SPEED Laboratory, Department of Electronics and Electrical Engineering, University of Glasgow, Glasgow, G12 8LT, U.K. (e-mail: t.miller@elec.gla.ac.uk).

Publisher Item Identifier S 0278-0046(02)00939-5.

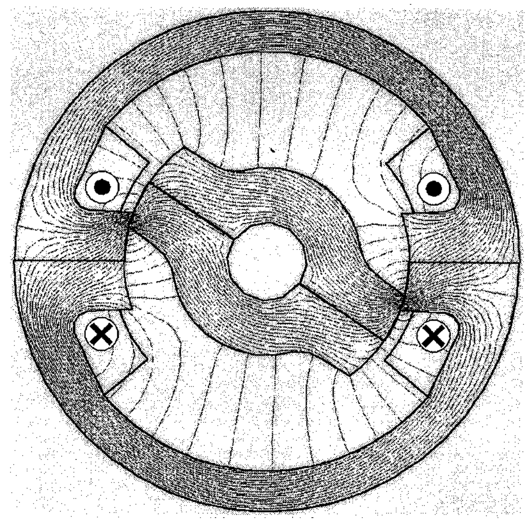

Fig. 1. Simple reluctance machine with one phase and two poles on both the stator and rotor.

The switched reluctance motor turns many of the tenets of classical electric machines technology upside down. This possibly explains why it is popular with academics but rare in the factory. Since the emergence of serious examples of switched reluctance drives in the 1970s, only a few practitioners have made successful businesses with them, while a large number of research papers have had little effect at the factory gates and some of them make claims which are misleading or incorrect. The nature of the switched reluctance motor is discussed in Section II, particularly with a view to understanding its design characteristics and how it compares with other common motor types.

Definition: A *reluctance machine* is one in which torque is produced by the tendency of its moveable part to move to a position where the inductance of the excited winding is maximized, [1], [2]. This definition covers both *switched* and *synchronous* reluctance machines. The switched reluctance motor has salient poles on both the rotor and the stator and operates like a variable-reluctance stepper motor except that the phase current is switched on and off when the rotor is at precise positions, which may vary with speed and torque. It is this switching which gives the switched reluctance motor its name. This type of motor cannot work without its electronic drive or controller.

A primitive example is shown in Fig. 1. This machine is denoted "2/2" because it has two stator poles and two rotor poles. It has only one phase, comprising two coils wound on opposite stator poles. These are excited simultaneously and generate

TABLE I
SELECTED SWITCHED RELUCTANCE PRODUCTS [3]–[5], [7]–[9]

Products	Company
EV drives	Aisin Seiki, Japan
Megatorque direct-drive	NSK Ltd., Japan
Several	Mavrik Motors, USA
Automotive cruise control	DANA Corp., USA
Washer drive	Emerson/SRDL, UK
Pumps, HVAC motion control	Emotron A/b, Sweden
Floorcare	Ametek Lamb Electric, USA
High-speed motors and controllers	AMC NEC/Densei, USA
250 kW low-speed drive	Elektro Magnetix Ltd, UK
General purpose industrial drives 4-43 kW	Oulton, Tasc Drives, part of Laurence, Scott and Electromotors Ltd., UK
Electric doors	Besam A/b, Sweden
Compressors	Compair Broomwade, UK
Mining drives	BJD, UK
Weaving machine servos	Picanol, Belgium
Industrial drives	Sicmemotori, Italy
Train air conditioning	Normalair Garrett, UK

magnetic flux as shown. In the position shown, the resulting torque tends to rotate the rotor in the counterclockwise direction toward the *aligned position*, where the stator and rotor poles are aligned. Torque can be produced over a limited arc of rotation, roughly corresponding to the stator pole arc β_s. This simple machine is the model on which the theory of torque production is based, as in Sections III–V. Methods of starting and the extension to multiple poles and phases are considered in Section VI. Section VII considers acoustic noise and its reduction.

II. Characteristics of the Switched Reluctance Motor

The switched reluctance motor is attractively simple in its mechanical construction and appearance, but it requires a controller that is designed and tuned for each specific application and has little in common with conventional ac drives. At first sight, it appears that the balance between the cost of the motor and the cost of the drive is shifted toward a less expensive motor and a more expensive drive, but this does not obviously produce an overall cost saving.

The enormous investment in tooling and infrastructure for induction motors and ac drives puts the switched reluctance motor at a disadvantage in many large sectors of the motor business. Since the vast majority of induction motors are line-start motors used without electronic drives, the switched reluctance motor has no hope of competing with the induction motor in the bulk of these applications. The infrastructure relates to the design, manufacture, sale, commissioning, maintenance, and control, and in adjustable-speed drives all these are heavily weighted in favor of induction motors. By contrast, the switched reluctance motor and its drive are specials for which very little tooling exists and almost none of the infrastructure. This will limit its role to special applications where the costs of development and support can be absorbed in a larger project—for example, the development of a completely new washing machine [3]. Although the switched reluctance motor can serve important roles like this, the underlying factors will not change in the foreseeable future (see Table I).

Although the switched reluctance motor appears beguilingly simple, it requires a small air gap and better concentricity than an induction motor. To achieve comparable efficiency and power density at full load, it may require more copper and/or a higher winding temperature. For acceptable performance, it requires an accurate shaft position feedback signal, implying the need for a "servo-quality" encoder or resolver or, alternatively, a sophisticated sensorless controller that will, in general, be specific to one application.

The switched reluctance motor has no independent means of excitation. In this respect, it differs from all permanent-magnet machines and from dc or ac machines with separate field windings, such as the classical synchronous machine. The excitation in the switched reluctance machine is in the voltamperes supplied by the drive. It can be regarded as a component or fraction of these voltamperes, the remainder being accounted for by real power, which divides between the useful output and the losses. In this respect, the switched reluctance motor is similar to the induction motor. The fraction of the supplied volt-amperes which is converted to useful output power (i.e., the *power factor*) is similar to that of the induction motor and, therefore, it cannot be said that switched reluctance motors have a problem with excitation, any more than induction motors do. It is not a particularly serious limitation: there are now very few motor drives where separate field windings are employed solely for the purpose of providing independent control of the excitation. One reason for this is that electronic control of the drive is nowadays so cost effective and so sophisticated that such properties as field weakening are readily achieved over a wide speed range at constant power without the need for an independently controlled field winding.

The excitation requirement becomes burdensome in small motors, not so much because of the voltampere requirement *per se*, but because the I^2R losses associated with excitation become disproportionately large as the motor size decreases. Switched reluctance and induction motors behave similarly in this regard and both are at a disadvantage compared with motors that use permanent-magnet excitation, which is essentially lossless.

Larger switched reluctance and induction motors both have high power factors and low excitation losses and, therefore, do not need magnets. In any case, a large permanent-magnet rotor is usually too costly above a certain power level—for example, 25 kW. In high-speed machines, the fixed excitation produced by permanent magnets can produce a high no-load core loss, whereas in switched reluctance and induction motors the flux level can be controlled, even at no load. The only machines which have truly independent means of excitation control are those with field windings.

The provision of 100% of the excitation via the drive is also a key factor in the short-term overload capability of the switched reluctance motor, which can sometimes produce 5–10$\times$ as much torque as would otherwise be limited by its steady-state thermal capacity. In general, this overload capability exceeds that of permanent-magnet machines which are limited by the risk of demagnetization of the magnets. Induction motors also have limited overload capability. Coupled with the low inertia of the switched reluctance motor, this extreme overload capability is a somewhat unique characteristic which helps to make it attractive in actuators that have high pulsed loads in confined spaces, provided that the voltampere sizing of the drive is affordable.

The switched reluctance motor is often claimed to have higher efficiency than an induction motor but these claims often pitch an *optimally designed* switched reluctance motor against an *off-the-shelf* induction motor designed with completely different objectives. In the literature, there is no comprehensive analysis of the relative efficiencies of switched reluctance and induction motors, but only a few anecdotal comparisons. One cannot be too critical of this state of affairs, because the subject is too complex for completely general conclusions. It can be said that switched reluctance motors *should* have better efficiency than comparable induction motors, since the rotor I^2R losses are removed. However, this presupposes that the same torque can be produced with no increase in stator I^2R losses or other losses. Since the switched reluctance motor tends to have a much lower magnetic loading and higher electric loading than an induction motor, this is far from obvious. What is certainly beneficial in the switched reluctance motor is that most of the losses arise on the stator, which makes cooling easier.

Switched reluctance motors are inherently noise prone in the sense that they have pulsed excitation and a mechanical structure that is more susceptible to resonance than that of most "smooth-air-gap" machines. However, a sophisticated drive can alleviate some of the acoustic noise by controlling the excitation in such a way as to exploit the mechanical resonance and snub out the noise to a greater or lesser degree. It has been demonstrated that acceptable results can be achieved without compromising the motor design or making it more expensive, even in applications such as domestic washing machines and automotive electric power steering, both of which require low noise levels, [3]–[5].

Torque ripple is another frequently quoted problem with the switched reluctance motor. Indeed, it can be said that it produces torque by amplifying what is known as cogging torque—a parasitic "nuisance torque"—in other machines. At low speed it can be "controlled out" by shaping the current waveform, but at high speed the current regulator may become "saturated" and a certain amount of torque ripple develops. It must be pointed out that similar phenomena occur in permanent-magnet motors and induction motors driven from electronic inverters. If the current waveform must be controlled to follow a sinusoidal function to eliminate torque ripple in the induction motor, then it can hardly be held against the switched reluctance motor that it too requires current waveshaping for the same purpose. The current waveshapes themselves may be more complex to determine, since they are not sinusoidal; but they are not necessarily more complex to *produce*.

The drive complexity in the switched reluctance motor drive is about the same as in an induction motor drive. What is more significant than the *level* of complexity, is the fact that the theory and architecture of switched reluctance motor controllers are not widely known. Moreover, there are very few established commercial sources of drives for switched reluctance motors, or of the components that go into them. Worldwide, only a handful of engineers understand the art of designing these controllers at an adequate level to make commercially viable products. As is equally the case with field-oriented control of induction motors, much of this art is beyond the scope of a review paper such as this one, which can only set out a few basic principles.

From time to time, it has been claimed that since the torque is independent of the direction of the current, the drive could be arranged to use only one transistor per phase instead of the two used in ac drives. However, the necessary reversal of the phase *voltage* (for commutation and in some cases for current regulation) is much easier to achieve with two transistors per phase than it is with only one. Although many circuit configurations have been invented with one switch per phase, very few (if any) are used in commercial service, because they require auxiliary components, decrease efficiency, limit the control capability, and may even increase the number of winding connections.

In relation to aerospace applications such as starter/generators, fuel pumps, and actuators, the switched reluctance motor has been heralded as "fault tolerant" because of the apparently benign consequences of faults, particularly the obvious ones such as short circuits and open circuits occurring in the motor or its leads or in the drive. The "fault-tolerant" claim is not as disingenuous as it sounds, for who would stand up and say that an aircraft electrical system based on switched reluctance technology could not fail, or could survive all conceivable component failures? Certainly the risk of demagnetization of magnets is not present, but the absence of magnets is not sufficient to ensure that short-circuit faults within the machine can be "tolerated." It is unclear how short-circuit faults in the load supplied by a switched reluctance generating system can be cleared, since the output impedance of such a system is that of a shunt capacitor rather than a series inductor and has no inherent ability to sustain a fault current of sufficient duration and magnitude to trip a standard overcurrent relay. This implies that generating systems based on switched reluctance machines will need a sophisticated system of "smart" protective relays employing solid-state switches and entirely dependent on electronic logic.

Examples: Table I lists a few examples of switched reluctance motor products. These are among the best known: the list is certainly incomplete and there may be other examples whose existence is not publicly known. However, it must be said that the list is extremely short compared to the kind of list that could be generated for the induction motor or brushless permanent-magnet motor, in spite of the extraordinary quantity of published research on switched reluctance motors. One of the interesting features of the list is the wide variety of applications, which bears out the claimed flexibility of the switched reluctance motor and tends to support the idea that its slow commercial progress is due to lack of investment and tooling rather than to any inherent technical limitation.

III. Torque Production

The theory of electromechanical energy conversion in the switched reluctance machine can be expressed in a few short equations as follows. The voltage equation for one phase is

$$v = e + Ri \tag{1}$$

where i is the current, v is the terminal voltage, R is the resistance and the back EMF e is given by Faraday's law as the rate of change of flux linkage ψ

$$e = \frac{\partial \psi}{\partial t}. \tag{2}$$

At constant speed, this can be written

$$e = \frac{\partial \psi}{\partial t} = \omega_m \frac{\partial \psi}{\partial \theta} \tag{3}$$

where ω_m is the angular velocity in radians per second. The relationship between flux linkage and current is nonlinear because of magnetic saturation and it obviously depends on the rotor position as a result of the fact that both the rotor and the stator have salient poles. Assuming for the moment that there is no mutual coupling between phases, we can write this as $\psi(i, \theta)$ and this function can be represented as a set of curves of ψ versus i with θ as a parameter; or as a set of curves of ψ versus θ with i as a parameter; see Fig. 2.

Most analyses of the switched reluctance motor proceed by integrating $(v - Ri)$ with respect to time, step by step. At each time step, the integration produces a new value of ψ, and a new value of i must immediately be recovered from the magnetostatic $\psi(i, \theta)$ data. This is a problem in interpolation and many different methods have been put forward to solve it [6], [7]. As the solution proceeds, the current waveform is developed from a sequence of values and the terminal voltage is switched between the supply voltage $+V_s$, 0, or $-V_s$ depending on the states of the power transistors in the drive. (Additional terms can be included in (1) to represent voltage drops across the power transistors and diodes and in the supply circuit; but they do not change the structure of the solution).

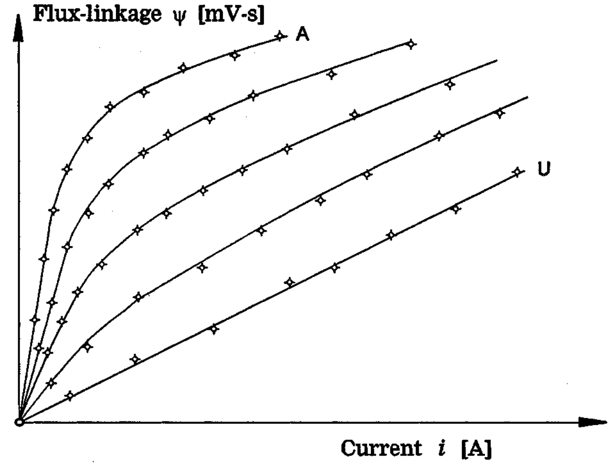

Fig. 2. Magnetization curves, showing phase flux linkage ψ as a function of phase current i, at different rotor positions θ (U = unaligned; A = aligned).

It is not difficult to set up a simulation of this type to calculate the current waveform, provided that a good interpolation procedure is used for the magnetization $\psi(i, \theta)$ data and that these data are accurate. As the calculation proceeds, the operating point (ψ, i) traces out a closed loop corresponding to one "stroke" or conduction interval in each phase (see Fig. 3). The area W enclosed by this loop is the energy converted from electrical to mechanical (or vice-versa) and, therefore, the average torque is given by

$$T_{e[avg]} = \frac{SW}{2\pi} \tag{4}$$

where S is the number of strokes per revolution. S is related to the number of rotor poles N_r and the number of phases m and, in general,

$$S = mN_r. \tag{5}$$

This can be substituted in (4) to give the average torque including all m phases, provided that S and W are the same for all of them.

Instantaneous Torque: The procedure described in the previous section is adequate to design efficient switched reluctance motors for most variable-speed applications. In effect, it parallels the design process used with other types of motor where it has long been sufficient to base designs on the calculation of average torque (over one revolution or cycle). However, the demand for "servo-type" torque control, coupled with concerns

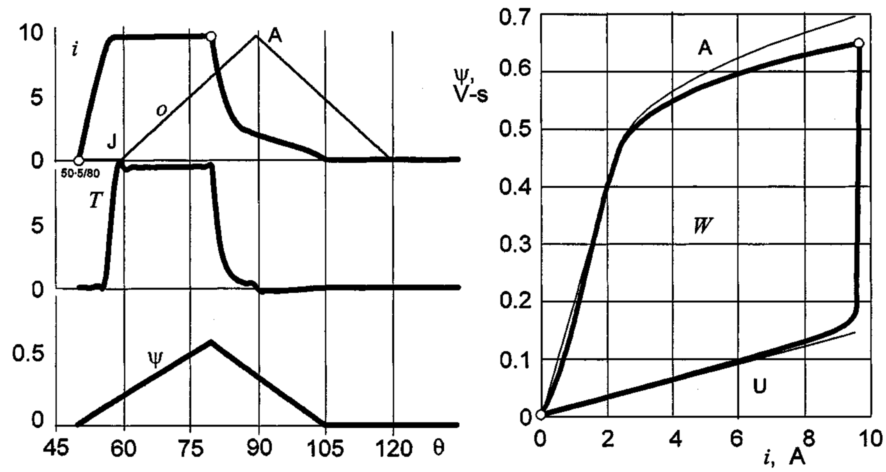

Fig. 3. Current, torque, and flux-linkage waveforms with a naturally determined flat-topped current waveform (i = phase current; T = phase electromagnetic torque; o = per-unit overlap between active stator and rotor poles; ψ = phase flux linkage; W = energy conversion loop area; U = unaligned; A = aligned).

about torque ripple, makes it necessary in some cases to calculate the instantaneous torque waveform.[1]

The instantaneous torque must be computed from a derivative of coenergy W_c or stored field energy W_f, defined as follows:

$$W_f = \int i d\psi;$$
$$W_c = \int \psi di. \tag{6}$$

Then, the instantaneous electromagnetic torque is given by

$$T_e = \frac{\partial W_c(i,\theta)}{\partial \theta} = -\frac{\partial W_f(\psi,\theta)}{\partial \theta}. \tag{7}$$

The flux linkage ψ can also be evaluated from the coenergy using

$$\psi = \frac{\partial W_c(i,\theta)}{\partial i} \tag{8}$$

evaluated at constant θ. If (8) is substituted into (3), the back EMF e can be evaluated using

$$e = \omega_m \frac{\partial^2 W_c}{\partial\theta \partial i}. \tag{9}$$

This suggests that the machine can be represented by a single coenergy function $W_c(i,\theta)$ whose derivatives can be used at any point (i,θ) to determine the back EMF e from (9), the flux linkage ψ from (8), and the electromagnetic torque T_e from (7). The solution proceeds at each time step by calculating

$$i = \frac{v - e}{R} \tag{10}$$

using the current values of v and e. Once the new current is calculated from (10), e is reevaluated using (9) for the next time step.

The coenergy function $W_c(i,\theta)$ can, in principle, be computed directly by the finite-element method using a global integration over the domain of the solution. A practical difficulty with this approach is the determination of an interpolating function whose derivatives in (7)–(9) are sufficiently well behaved. Also, it does not naturally provide data which can be compared with measurements. Furthermore, if the function is computed by two-dimensional finite elements, it is not clear how the result can be modified for end effects. For these reasons, it may be better to construct the simulation as described in the next section, with separate interpolating functions or tables for coenergy (or torque) and current.

IV. Simulation Process Using Separate Interpolating Functions

A. Form and Properties of the Magnetization Curves

The magnetization curves (Fig. 2) may be calculated analytically or imported from measured data or finite-element calculations. Raw data, in the form of arrays of points of flux-linkage versus current, may not necessarily fit on smooth curves, especially if the data are measured. This implies the need for a curve-fitting process. Spurious errors in the raw data can be amplified by contortions of the interpolating functions, especially in the instantaneous torque waveform. The curves may not all extend up to the same maximum current or flux linkage. This means that when the curves are fitted with piecewise sets of polynomials, some extrapolation may be necessary so that the coefficient arrays do not have undefined elements. The spacing $\Delta\theta$ between curves should ideally be small in the neighborhood of the "start-of-overlap" position where the rates of change of back EMF, current, and torque are high. However, if a small $\Delta\theta$ is used over the whole range from unaligned to aligned positions, the number of data points could become very large, requiring an excessive amount of computation or measurement time. In measuring the magnetization curves, it may be difficult to hold the rotor at a fixed position, especially in the neighbor-

[1]In doubly excited machines, the instantaneous torque can usually be computed from the product ei after transforming the variables into a reference frame in which e and i are independent or "orthogonal," as in the well-known field-oriented control of induction motors. In dc machines, e and i are already orthogonal by virtue of the action of the commutator and no transformation is necessary. In the switched reluctance motor, however, no simple transformation exists that will produce the required orthogonality. Physically, the problem is that at any given rotor position it is impossible to know from terminal measurements of v and i, where the instantaneous power is going. In particular, the partition of the product ei between field stored energy and electromechanical energy conversion is not observable and, in fact, it is highly variable as a function of both rotor position and current. By contrast, in a fully compensated dc motor the partition is fixed and normally 100% of ei is accounted for by electromechanical energy conversion.

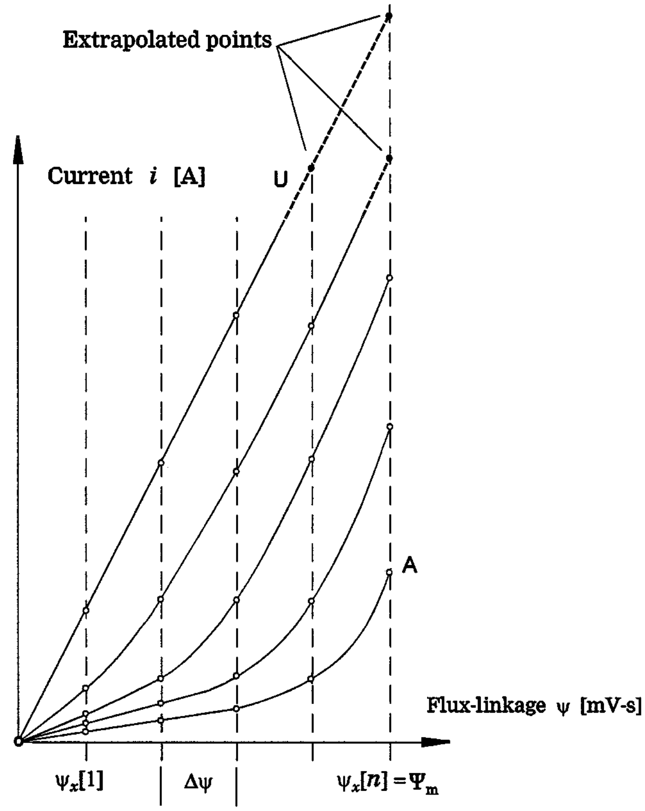

Fig. 4. Mag curves in .psi format of *PC-SRD*.

hood of the "start-of-overlap" position, because of shaft torsion as the current increases. Therefore, a curve which is nominally at one rotor position may actually cover a small *range* of rotor positions.

Each magnetization curve in Fig. 2 is modeled by a piecewise set of cubic splines, with a common base array of knot points $\boldsymbol{\psi}_x[j], j = 1, \ldots n$, where $j = 1$ corresponds to the aligned position and $j = n$ to the unaligned position (Fig. 4). The knot points are evenly spaced by $\Delta\psi$. The subscript x indicates that flux linkage ψ is considered the independent variable, which is convenient for the simulation algorithm but gives rise to a practical difficulty: it is much easier in finite-element analysis or physical measurement to maintain constant *current* rather than constant flux linkage. Therefore, a *realignment* procedure is required to get the curves in the required form, where all the points are on the same base array $\boldsymbol{\psi}_x$. The realignment incorporates a piecewise curve-fitting of the original constant-current curves, followed by an interpolation process. Usually, the original curves extend only up to a certain maximum current, but the realigned curves must extend up to a common maximum flux linkage $\boldsymbol{\psi}_x[n] = \psi_m$, and this usually requires that some of them be extrapolated up to ψ_m; see Fig. 4. The extrapolation is unfortunately in a region of high saturation, even around the unaligned position.

B. Simulation Process

The voltage equation of the phase winding is integrated step by step with respect to time

$$\psi = \int (v - Ri)dt \approx [\psi] + (v - Ri)\frac{\Delta\theta}{\omega_m} \tag{11}$$

where $[\psi]$ is the value of ψ from the previous integration step and $\Delta\theta = \omega_m dt$ is the integration step length.[2] The mechanical equation of motion is integrated at the same time to give a new rotor position θ. At each step, the current i must be updated from the new values of ψ and θ. The procedure is as follows. First, the interval in $\boldsymbol{\psi}_x$ in which the current value ψ lies is identified (Fig. 5). Since the curves all share the same base array $\boldsymbol{\psi}_x$, this interval is the same for all the n curves. Next, an array of currents $i_\square$ is calculated from the n curves, all at the same flux linkage ψ. This calculation requires only a substitution of ψ and θ in the local cubic spline function for each curve. These currents are marked with square markers □ in Fig. 5. A new spline curve of i versus θ is then created at the flux linkage ψ. This curve is shown in the right-hand graph in Fig. 5. The interval in which the current value of θ lies is identified and then i is computed by substituting θ in the local spline formula for this interval.

Instantaneous torque is computed from the rate of change of stored field energy with respect to rotor position, at constant flux linkage, (7). The process is almost identical to the one for determining the current. Before the time-stepping simulation begins, an array of stored field energies is created, represented by a piecewise set of cubic splines corresponding point by point with the phase-current splines in the left-hand graph in Fig. 5. At each time step, a temporary local set of splines is created representing W_f versus θ at constant flux linkage, exactly as in the right-hand graph of Fig. 5, but with W_f replacing i. Suppose the local cubic spline for W_f has the equation

$$W_f = c_0 + c_1\theta + c_2\theta^2 + c_3\theta^3 \tag{12}$$

then the torque is given simply by

$$T_e = -\left(c_1 + 2c_2\theta + 3c_3\theta^2\right). \tag{13}$$

C. Static Torque Curves

The static torque curves are important in switched reluctance motors, as they are in stepper motors. They have the form shown in Fig. 6. They can be calculated by stepping through a range of currents and rotor positions and applying the torque calculation process described above. It may be convenient to precalculate the static torque curves together with a set of interpolating functions before starting the time-stepping simulation, so that the torque at each integration step could be found by interpolation from the static torque curves, instead of using local differentiation of the stored field energy.

V. Obtaining the Magnetization Curves

A. By Calculation

The aligned magnetization curve can be calculated by a straightforward lumped-parameter magnetic circuit analysis, with an allowance for the stator-slot leakage which is especially significant at high flux levels. The unaligned curve is more difficult to calculate because of the complexity of the magnetic flux paths in this position, but quite practical results are reported by Miller and McGilp [7] using a dual-energy method based

[2] Note that this formulation of the integral is invalid at zero speed. At zero and very low speeds the integral should be with respect to time rather than position.

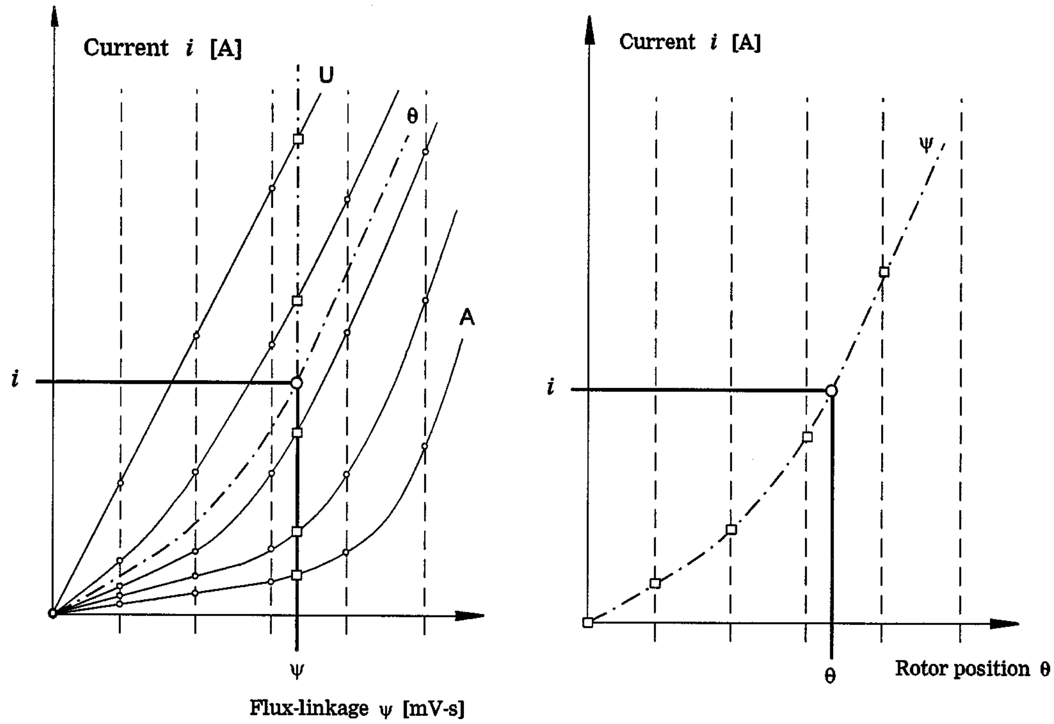

Fig. 5. Solution for current i.

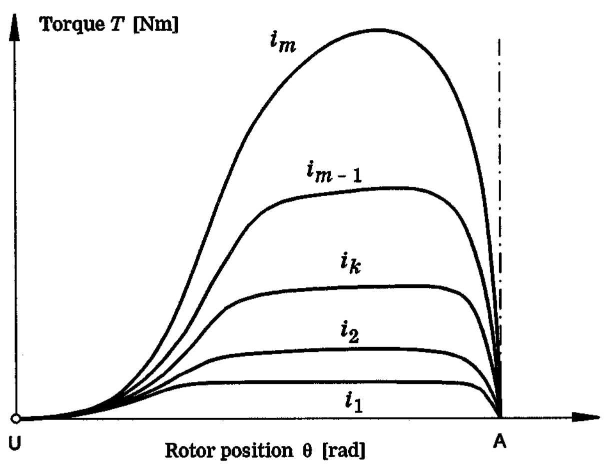

Fig. 6. Static torque curves showing electromagnetic torque versus rotor position for a set of currents $i_1 \ldots i_m$.

on a quadrilateral discretization of the slotted region. Earlier work by Corda and Stephenson [8] also produces adequate results for many practical or preliminary design calculations. The computation should include the "partial linkage" effect, meaning that at certain rotor positions the turns of each coil do not all link the same flux. In a finite-element calculation this implies that the flux *linkage* should be calculated as the integral of $\mathbf{A} \cdot \mathbf{dl}$ along the actual conductors, with the coil sides in the finite-element mesh in their correct positions in the slot.

End effects are important in switched reluctance motors [9], [10]. When the rotor is at or near the unaligned position, the magnetic flux tends to "bulge out" in the axial direction. The associated increase in the magnetic permeance can raise the unaligned inductance L_u by 20%–30%. Since this inductance is critical in the performance calculations, it is important to have a reasonable estimate of it. Unfortunately, two-dimensional finite-element analysis cannot help with this problem and three-dimensional finite-element calculations tend to be expensive and slow. When the rotor is at or near the aligned position, the flux is generally higher and the "bulging" of flux outside the core depends on the flux level in the laminations near the ends of the stack. At or near the aligned position at high flux levels, the stator and rotor poles can be highly saturated and the external flux paths at the ends of the machine can increase the overall flux linkage by a few percent.

In spite of the complexity of the field problem, good results have been obtained with relatively simple end-effect factors for L_u. For example, the *PC-SRD* computer program [11] splits the end-effect calculation into two parts. For L_u,

$$L_u = L_{u0} \times (e_u + f_u) \tag{14}$$

where $e_u = L_{\mathrm{end}}/L_{u0}$ represents the self-inductance L_{end} of the end windings (including any extension), expressed as a fraction of the uncorrected two-dimensional unaligned inductance L_{u0}. L_{end} is the inductance of a circular coil whose circumference is equal to the total end-turn length of one pole coil, including both ends. It is multiplied by the appropriate function of turns/pole and parallel paths before being normalized to L_{u0}. f_u is a factor that accounts for the *axial* fringing in the end region. It is calculated by the approximation

$$f_u = \frac{d_r + L_{\mathrm{stk}}}{L_{\mathrm{stk}}} \tag{15}$$

where d_r is the rotor slot depth; this formula is derived by analogy with the fringing formula for two opposite teeth or poles. For the aligned position, the procedure is similar, with the air-gap length g instead of d_r.

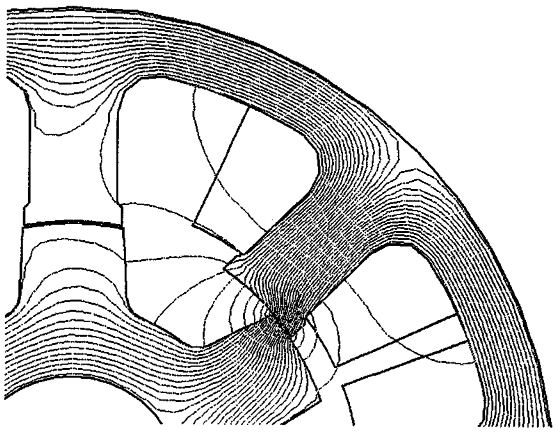

Fig. 7. Finite-element flux plot at a position intermediate between aligned and unaligned.

Although end effects are important in calculating the performance, it is still valid and indeed desirable to rely on two-dimensional finite-element analysis to optimize the shape of the stator and rotor laminations. Thus, if the lamination shapes are adjusted to optimize the torque per ampere on the basis of two-dimensional finite-element analysis, it would not be expected that the addition of end effects would give rise to the need to change these shapes.

At positions intermediate between the aligned and unaligned positions, the calculation of individual magnetization curves is not practical by analytical methods and the finite-element method should be used. Fig. 1 shows a simple example of a finite-element flux plot at a position of partial overlap and Fig. 7 a more complex example. Considerable success has been achieved with interpolating procedures for magnetization curves at intermediate positions, especially in computer programs for rapid design [7], [11], [12].

B. By Measurement

Direct measurement of the magnetization curves is described in [1] and [2]. It is also possible to determine them from the static torque curves, provided that at least one magnetization curve is already known (preferably, the unaligned curve).

C. Summary

In the previous three sections, we have seen that the elegance and brevity of the theoretical equations belie the practical difficulty of solving them in a manner which makes it easy to design and control a switched reluctance motor. For design purposes, a computer simulation is a *sine qua non* because the operation is a series of transients in a highly nonlinear magnetic system, with no discernible steady state that can be expressed by simple algebraic formulas of the type familiar with classical dc and ac machines. The main difficulties in the simulation are in interpolation and in the provision of accurate magnetization curves.

In relation to the control, a "series of nonlinear transients" provides no obvious architecture on which a control strategy can be based. Although the general notion of nested control loops for torque (or current) and speed still applies, the feedforward relationship between current and torque is nonlinear in both current and rotor position; and if more than one phase is simultaneously excited, additional questions of torque sharing between phases may need to be resolved. In this paper, the focus is on the solution of the equations for purposes of designing the motor. The question of control is dealt with in more detail elsewhere [13].

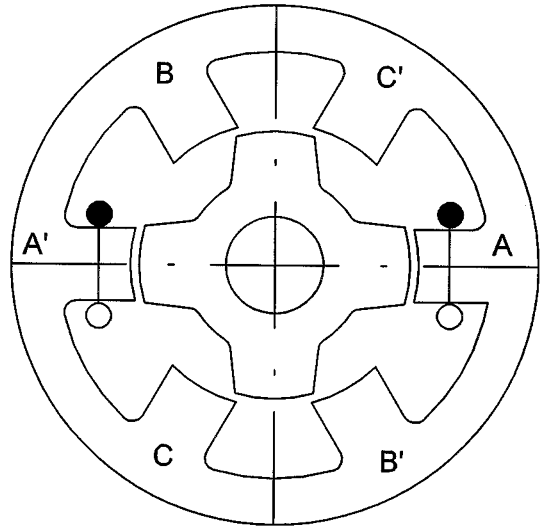

Fig. 8. Three-phase switched reluctance motor with six stator poles and four rotor poles.

VI. Numbers of Poles and Phases

The motor in Fig. 1 is useful for developing the analysis of torque production, and although it can maintain a nonzero average torque when rotating in either direction, this torque is pulsed and discontinuous, which means that continuous rotation depends on the momentum or flywheel effect of the rotating inertia (of motor + load). Moreover it can self-start only from a limited range of rotor positions. At the unaligned and aligned positions, the torque is zero. Unidirectional torque can be produced only over a limited angle where the overlap angle λ between the rotor and stator poles is varying. To provide continuous unidirectional torque, with self-starting capability from any rotor position, the motor is generally provided with additional *phases* which leads to a "multiplicity" of stator and rotor poles, as in Fig. 8.

The motor in Fig. 8 has $m = 3$ and $N_r = 4$, so from (5) the number of strokes per revolution is $S = 12$. The *stroke angle* is $\varepsilon = 360/12 = 30°$. The three phases are labeled AA', BB', and CC', and the ideal current/torque pulses are shown in Fig. 9. The resultant torque is ideally constant and covers 360° of rotation. In practice, the torque waveform may be far from the ideal, because of current-regulator saturation at high speed and the finite time required to commutate the phase currents.

A. Magnetic Frequency

The fundamental frequency f_1 of the current in each phase is evidently equal to the rotor pole passing frequency, i.e.,

$$f_1 = \frac{\text{r/min}}{60} \times N_r \quad \text{Hz}. \tag{16}$$

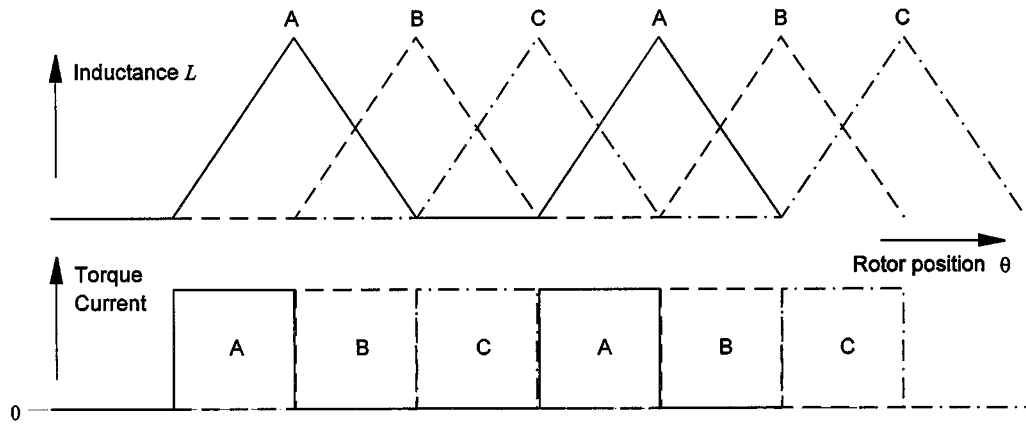

Fig. 9. Idealized torque production in three-phase switched reluctance motor.

The number of strokes per second is given by

$$f = mf_1 \quad \text{Hz.} \tag{17}$$

This frequency and all its harmonics appear in the flux waveforms in various parts of the magnetic circuit. Equation (16) says that the fundamental magnetic frequency is twice as high as in a synchronous ac motor with the same number of rotor poles. The speed of rotation is even less simply related to the number of *stator* poles, since there are generally at least two possible choices of N_s for each value of N_r and further possibilities arise with multiple stator teeth per pole, [14]. The switched reluctance machine is usually classified as a *vernier* machine because its rotational speed is only a fraction of the fundamental electrical frequency.

The flux waveforms in switched reluctance motors are not only at a higher fundamental frequency, but they carry a higher harmonic content than in ac motors and the flux waveforms may be very different in different parts of the magnetic circuit. Although this suggests that the core losses will be higher than in comparable ac motors, in practice, this is not the case because switched reluctance motors are usually designed with a significantly lower magnetic loading (average flux density around the air gap) and higher electric loading. Moreover, the volume of iron is relatively smaller in the switched reluctance motor and both these factors tend to offset the effects of the higher magnetic frequency.

B. Stator/Rotor Pole Numbers

In a *regular* switched reluctance motor, the rotor and stator poles are symmetrical about their center lines and equally spaced. Regular motors usually have the best overall performance and the most sophisticated drive electronics; and usually $m = 3$ or 4 phases. Machines with $m = 1$ or 2 are usually irregular and are designed for specialty applications with limited control requirements. A review of these is given in [2]. There are too many variants to cover in this paper.

The *absolute torque zone* τ_a is defined as the angle through which one phase can produce nonzero torque in one direction. In a regular motor with N_r rotor poles, the maximum torque zone is $\tau_{a(\max)} = \pi/N_r$. The *effective torque zone* τ_e is the angle through which one phase can produce *useful* torque comparable to the rated torque. The effective torque zone is comparable to the lesser pole arc of two overlapping poles. For example, in Fig. 8 the effective torque zone is equal to the stator pole arc: $\tau_e = \beta_s = 30°$. The *stroke angle* ε is given by $2\pi/S$. The *absolute overlap* ratio ρ_a is defined as the ratio of the absolute torque zone to the stroke angle: evidently, this is equal to m/2. A value of at least 1 is necessary if the *regular* motor is to be capable of producing torque at all rotor positions. In practice a value of 1 is not sufficient, because one phase can never provide rated torque throughout the absolute torque zone in both directions. The *effective overlap ratio* ρ_e is defined as the ratio of the effective torque zone to the stroke angle, $\rho_e = \tau_e/\varepsilon$. For regular motors with $\beta_s < \beta_r$, this is approximately equal to β_s/ε. For example, in Fig. 8 the effective overlap ratio is $30°/30° = 1$. Note that $\rho_e < \rho_a$. A value of ρ_e of at least 1 is necessary to achieve good starting torque from all rotor positions with only one phase conducting and it is also a necessary (but not sufficient) condition for avoiding torque dips. Unfortunately, the effective overlap ratio decreases as the current increases, because saturation narrows the effective torque zone; see Fig. 6.

C. Three-Phase Regular Motors

With $m = 3$, $\rho_a = 1.5$, and ρ_e can have values of 1 or more, so *regular* three-phase motors can be made for four-quadrant operation. In the 6/4 motor in Fig. 8, forward rotation corresponds to negative phase sequence. This is characteristic of motors in which the rotor pole pitch is less than π/m. The three-phase 6/4 motor has $S = mN_r = 12$ strokes/revolution, with $\varepsilon = 30°$, giving $\rho_e = \beta_s/\varepsilon = 30/30 = 1$.

With regular vernier motors there is always the choice of having either $N_r = N_s - 2$, as in the 6/4; or $N_r = N_s + 2$, which gives the 6/8 motor shown in Fig. 10; it has $S = 24$ strokes/revolution and $\varepsilon = 15$ and is similar to Konecny's motor used in the Hewlett-Packard Draftmaster plotter [15]. Increasing S reduces torque ripple, but the aligned/unaligned inductance ratio is reduced with the larger number of poles and this may increase the controller voltamperes and decrease the specific output. The core losses may be higher than those of the 6/4 motor because of the higher switching frequency.

The 12/8 three-phase motor is effectively a 6/4 with a "multiplicity" of two. It has $S = 24$ strokes/revolution, with a stroke angle $\varepsilon = 15°$ and $\rho_a = 1.5$. In Fig. 11, $\rho_e = 15/15 = 1$, the same as for the 6/4 motor. A high inductance ratio can be

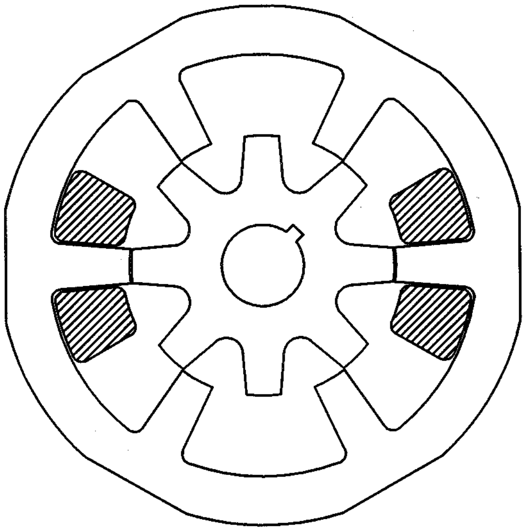

Fig. 10. Three-phase switched reluctance motor with six stator poles and eight rotor poles.

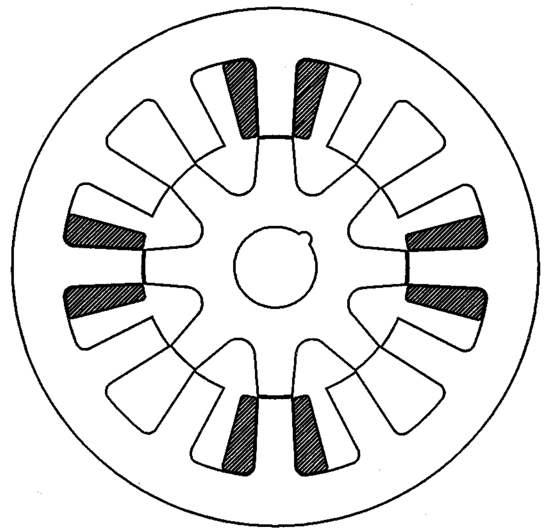

Fig. 11. Three-phase switched reluctance motor with 12 stator poles and eight rotor poles.

maintained and the end windings are short: this minimizes the copper losses, shortens the frame, and decreases the unaligned inductance. Moreover, the magnetic field in this machine has short flux paths because of its four-pole magnetic field configuration, unlike the two-pole configuration in the 6/4 (or the 8/6; see below) and the four-pole magnetic circuit helps to minimize acoustic noise. Although the slot area and, therefore, the slot ampere conductors are reduced, the effects of long flux paths through the stator yoke are alleviated. The 12/8 is possibly the most popular configuration for three-phase machines.

D. Four-Phase Regular Motors

The four-phase regular 8/6 motor in Fig. 12 has 24 strokes/revolution and a stroke angle of 15°, giving $\rho_a = 2$. With $\beta_s = 21°$, $\rho_e = 1.33$, which is sufficient to ensure starting torque from any rotor position and it implies that there will be no problem with torque dips. However, it is generally impossible to achieve the same flux-density waveform in every section of the stator yoke, because of the polarities of the stator poles (NNNNSSSS, NNSSNNSS, or NSNSNSNS). This configuration was one of the first to be produced commercially.[3]

With $N_s = N_r + 2 = 10$, $S = 32$ strokes/revolution and $\varepsilon = 11.25°$. The inductance ratio is inevitably lower than in the 8/6 and the poles are narrower, while the clearance between pole corners in the unaligned position is smaller, increasing the unaligned inductance. This motor is probably on the borderline where these effects cancel each other out; with higher pole numbers, the loss of inductance ratio and energy-conversion area tends to dominate the gain in the number of strokes/revolution. For this reason, higher pole numbers are not considered here.

Table II gives some examples of stator/rotor pole number combinations for motors with up to $m = 7$ phases. The parameter N_{wkPP} is the number of working pole-pairs: that is, the number of pole pairs in the basic magnetic circuit. For example, the four-phase 8/6 has $N_{wkPP} = 1$ (a two-pole flux pattern),

[3]The *OULTON* motor introduced in 1983 by Tasc Drives Ltd., Lowestoft, U.K.

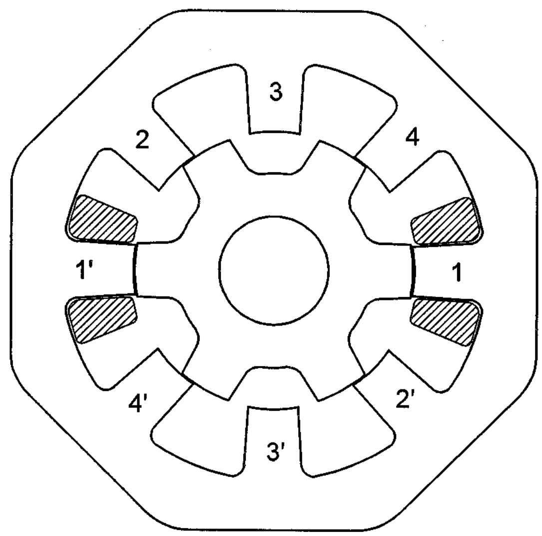

Fig. 12. Four-phase switched reluctance motor with eight stator poles and six rotor poles.

while the three-phase 12/8 has $N_{wkPP} = 2$ (a four-pole flux pattern). The unshaded boxes in Table II are probably the best choices, the others having too many poles to achieve a satisfactory inductance ratio, or too high a magnetic frequency.

VII. Acoustic Noise

Three interesting and contrasting accounts of the development of low-noise switched reluctance motors can be found in [3]–[5]. These publications describe switched reluctance motor drive systems developed for factory production, not merely laboratory machines built for scientific analysis.

To minimize noise, it is advisable to avoid high power density. In particular, the magnetic loading (average air-gap flux density) should be limited, if necessary at the expense of an increase in the electric loading. It is better to use a pole combination with more than one pair of working poles, i.e., with a "multiplicity" greater than 1. For example, a 12/8 machine ought

TABLE II
EXAMPLES OF VALID STATOR/ROTOR POLE NUMBER COMBINATIONS

m	N_s	N_r	N_{wkPP}	$\varepsilon°$	S
2	4	2	1	90.00	4
2	8	4	2	45.00	8
2	4	6	1	30.00	12
2	8	12	2	15.00	24
2	12	18	3	10.00	36
2	16	24	4	7.50	48

m	N_s	N_r	N_{wkPP}	$\varepsilon°$	S
3	6	2	1	60.00	6
3	6	4	1	30.00	12
3	6	8	1	15.00	24
3	12	8	2	15.00	24
3	18	12	3	10.00	36
3	24	16	4	7.50	48

m	N_s	N_r	N_{wkPP}	$\varepsilon°$	S
4	8	6	1	15.00	24
4	16	12	2	7.50	48
4	24	18	3	5.00	72
4	32	24	4	3.75	96
4	8	10	1	9.00	40

m	N_s	N_r	N_{wkPP}	$\varepsilon°$	S
5	10	4	1	20.00	18
5	10	6	1	12.00	30
5	10	8	1	9.00	40
5	10	12	1	6.00	60

m	N_s	N_r	N_{wkPP}	$\varepsilon°$	S
6	12	10	1	6.00	10
6	24	20	2	3.00	120
6	12	14	1	4.29	84

m	N_s	N_r	N_{wkPP}	$\varepsilon°$	S
7	14	10	1	5.14	70
7	14	12	1	4.29	84

to be less noisy than an 8/6 machine. This is because the compliance of the stator core is much less for multipolar excitation than for two-pole "ovalizing", [16]. The stator yoke should be as thick as possible to minimize deflection. Even a small deflection may suffice to cause objectionable acoustic noise. The lamination steel should be truly anisotropic with no trace of grain orientation. It may help in this regard to rotate successive laminations by one stator pole pitch as they are stacked. A relatively low-permeability lamination steel may also help to reduce noise level, as does a larger air gap, although both features reduce the inductance ratio. The rotor and stator poles should be wide to reduce the peak flux densities and increase the mechanical stiffness. The rotor poles should generally be slightly wider than the stator poles. A slight taper on the rotor and stator poles also has a beneficial effect, as does a fillet radius at pole-root corners on the stator and the rotor (where the pole joins the yoke). The edges of the poles can be tapered or shaped slightly to reduce the harmonic content in the instantaneous torque waveform. Notches and holes should be avoided in the laminations. If possible, the laminations should be truly symmetric. It is possible with some production methods to get angular errors in the pole locations: for example, in a six-pole stator two poles might not be exactly opposite, and this will cause unbalance that could contribute to noise. A large shaft diameter is also desirable to help increase the lateral and torsional stiffnesses. The rotor slot depth may need to be made shallower to permit a larger shaft to be accommodated without diminishing the rotor yoke thickness. When the rotor slot depth is reduced, the unaligned inductance increases, but it may be worth losing a little on the inductance ratio to get a stiff shaft. The rotor can be encapsulated to reduce windage noise. Obviously, this must be done in such a way as to support the potting material against centrifugal loads at high speed. The bearings should be located as close as possible to the rotor stack. The end brackets carrying the bearings may sometimes protrude under the end windings to get the bearings close in. The end brackets must be rigidly attached to the frame. A tapered flange or spigot helps this. Sleeve bearings are thought to be unsuitable, although opinions differ on this point, but whatever bearings are used they should maintain the best possible concentricity between the rotor and stator. There should be enough end float to allow centering in the axial direction, but not too much, and one bearing should be spring loaded (use a wavy washer). A small amount of skew may be used on the rotor (or stator). Use an unfinned frame, preferably aluminum (thin rolled steel may resonate and adds little radial stiffness). If possible, mount the machine on flexible antivibration mountings and use a flexible rubber-bush-type coupling to the load. Clamp up the lamination stacks if possible, under pressure. An alternative is to impregnate it with varnish (after winding), preferably by vacuum impregnation. Use nonmetallic slot wedges (top sticks) to close the stator slots after winding and double-thickness slot liners. This adds rigidity and damping to the stator. Wind the coils as tightly as possible and, if possible, encapsulate the stator windings. Tie the end windings with cord before varnishing.

Electrical and magnetic balance is desirable between the coils of each phase; this includes both resistance and inductance and therefore includes leads and internal connections between coils. If possible, use parallel connection of pole coils, not series. This compensates for air-gap inconsistencies by allowing greater current through the poles with the larger gap. Use the highest number of turns consistent with achieving the required torque/speed curve at the available voltage. This reduces the speed at which the control can switch over to single-pulse operation with phase-angle control; this is quieter than low-speed chopping.

The noise "problem" is not simply a motor problem, because many aspects of it concern the drive, and the drive can be used to reduce the noise level. Voltage pulsewidth modulation (PWM) in the drive is known to produce lower noise than hysteresis-band current regulation. With voltage PWM, it may also help to dither the duty cycle and/or the frequency. Even lower noise can be achieved with a variable-voltage dc source, such that the phase transistors are used only for commutation and controlling the firing angles. The current should be chopped with only one transistor, not both ("soft chopping"). The firing angles should be adjusted on test to give the quietest operation at a given load. Even small changes make a marked difference. There may be a tradeoff with efficiency. Current profiling may help, but requires extensive analysis and experimentation and may be complicated. Use the highest possible chopping frequency to minimize ripple current and, if possible, set the chopping frequency above the human audible range (typically, >15–20 kHz).

VIII. Conclusion

As with most engineered products, the "optimal" design of switched reluctance motors is a matter of compromise involving many parameters. The switched reluctance motor is now mature enough to have proved itself in the marketplace in a few different applications. The number and range of these applications remain small compared with those of induction motors or even brushless permanent-magnet motors, but it can be argued that this is partly a consequence of the level of investment and tooling in these technologies, rather than a result of inherent technical deficiencies in the switched reluctance motor itself. Even its widely criticized "noise problem" has not prevented successful commercial applications.

There is, however, a technical impediment to the development of switched reluctance motors in the unfamiliarity of the necessary design procedures, not only for the motor but also for the control. It is self-evident that the technology of induction motor drives, for example, while no less sophisticated than that of the switched reluctance motor drive, is much better established and more widely accepted and is supported by a vast infrastructure of component supplies and all the other factors which go into successful businesses. This paper shows that the mathematical design theory of the switched reluctance motor is simple on paper but difficult to implement in practice, requiring computer methods for even the simplest design calculations. Moreover, the mathematical theory does not suggest a natural architecture for the innermost feedforward control (the relationship between torque and current, the choice of firing angles and the criteria for controlling the sharing of torque between phases).

During the last 30 years, in parallel with the attempt to bring the switched reluctance motor into widespread commercial production, great strides have been made in competing technologies. We have seen spectacular developments in the properties and commercial supply of permanent magnet materials; many detailed improvements in induction motors; and huge strides in the size, reliability, cost, and performance of ac drives. All these set the switched reluctance motor even further behind, making it likely that future successful applications will follow the pattern of those already established: in other words, a highly engineered specialty drive whose development cost must be borne by the application and whose unique features render it the best choice. If you have got one of these, go for it!

What is necessary to develop a successful switched reluctance motor drive is a combination of intensive computation including electromagnetic, mechanical, and electronic design, then, a significant phase of laboratory testing and, finally, a suitable investment in tooling. The controller will be specific for each application. The process requires the abandonment of traditional thinking about sinewaves, space vectors, field orientation, and so on; instead, one must work with computed numerical data whose structure is not easily discernible. The level of acceptability in the market also remains an issue, although the successful pioneering applications show how this can be overcome.

References

[1] T. J. E. Miller, Ed., *Electronic Control of Switched Reluctance Motors*. ser. Newnes Power Engineering Series. Oxford, U.K.: Newnes, 2001.

[2] ——, *Switched Reluctance Motors and Their Control*. Lebanon, OH: Magna Physics/Oxford Univ. Press, 1993.

[3] R. Furmanek, A. French, and G. E. Horst, "Horizontal axis washers," *Appliance Manufacturer*, pp. 52–53, Mar. 1997.

[4] K. McLaughlin, "Torque ripple control in a practical application," in *Electronic Control of Switched Reluctance Motors*. ser. Newnes Power Engineering Series, T. J. E. Miller, Ed. Oxford, U.K.: Newnes, 2001, ch. 8.

[5] W. Pengov and R.L. Weinberg, "Designing for low noise," in *Electronic Control of Switched Reluctance Motors*. ser. Newnes Power Engineering Series, T. J. E. Miller, Ed. Oxford, U.K.: Newnes, 2001, ch. 4.

[6] J. M. Stephenson and J. Corda, "Computation of torque and current in doubly-salient reluctance motors from nonlinear magnetization data," *Proc. Inst. Elect. Eng.*, vol. 126, no. 5, pp. 393–396, 1979.

[7] T. J. E. Miller and M. McGilp, "Nonlinear theory of the switched reluctance motor for rapid computer-aided design," *Proc. Inst. Elect. Eng.*, pt. B, vol. 137, no. 6, pp. 337–347, Nov. 1990.

[8] J. Corda and J. M. Stephenson, "Analytical estimation of the minimum and maximum inductances of a double-salient motor," in *Proc. Leeds Int. Conf. Stepping Motors and Systems*, Leeds, U.K., Sept. 1979, pp. 50–59.

[9] A. M. Michaelides and C. Pollock, "The effect of end core flux on the performance of the switched reluctance motor," *Proc. IEE—Elect. Power Applicat.*, vol. 141, no. 6, pp. 308–316, Nov. 1994.

[10] A. B. J. Reece and T. W. Preston, *Finite Element Methods in Electrical Power Engineering*. London, U.K.: Oxford Univ. Press, 2000.

[11] T. J. E. Miller and M. McGilp, *PC-SRD User's Manual, Version 7.0*. Glasgow, U.K.: SPEED Laboratory, Univ. Glasgow, 1999.

[12] T. J. E. Miller, M. Glinka, C. Cossar, G. Gallegos-Lopez, D. Ionel, and M. Olaru, “Ultra-fast model of the switched reluctance motor,” in *Conf. Rec. IEEE-IAS Annu. Meeting*, St. Louis, MO, Oct. 1998, pp. 319–326.

[13] P. C. Kjaer, J. J. Gribble, and T. J. E. Miller, “High grade control of switched reluctance machines,” *IEEE Trans. Ind. Applicat.*, vol. 33, pp. 1585–1593, Nov./Dec. 1997.

[14] J. W. Finch, M. R. Harris, A. Musoke, and H. M. B. Metwally, “Variable-speed drives using multi-tooth per pole switched reluctance motors,” in *Proc. 13th Incremental Motion Controls Symp.*, Univ. Illinois, Urbana-Champaign, IL, 1984, pp. 293–302.

[15] K. Konecny, “Analysis of variable reluctance motor parameters through magnetic field simulations,” in *Proc. Motor-Con*, 1981, p. 2A.

[16] R. S. Colby, F. Mottier, and T. J. E. Miller, “Vibration modes and acoustic noise in a 4-phase switched reluctance motor,” *IEEE Trans. Ind. Applicat.*, vol. 32, pp. 1357–1364, Nov./Dec. 1996.

T. J. E. Miller (M’74–SM’82–F’96) is a native of Wigan, U.K. He received the B.Sc. degree from the University of Glasgow, Glasgow, U.K., and the Ph.D. degree from the University of Leeds, Leeds, U.K.

From 1979 to 1986, he was an Electrical Engineer and Program Manager with General Electric Corporate Research and Development, Schenectady, NY. His industrial experience includes periods with GEC (U.K.), British Gas, International Research and Development, and a student apprenticeship with Tube Investments Ltd. He is currently a Professor of Electrical Power Engineering and founder and Director of the SPEED Consortium, Department of Electronics and Electrical Engineering, University of Glasgow. He has authored more than 100 publications in the fields of motors, drives, power systems, and power electronics, including eight books.

Prof. Miller is a Fellow of the Institution of Electrical Engineers, U.K.