Solving Two-Player Mixed-Motive Games in which One Player's Payoff is Simulated Due to Concealment of Information

A.R. Rajesh[1] and Badri Toppur[2]

The US airways memo case deals with identifying the best strategy in a two person mixed motive game. The case narrates the price war between the arch rivals US airways and SW airlines in the US domestic airlines market, narrating the example of Nashville- Los Angeles route. Our plan is to consider the reality that given an honest consultant, only one airline's pricing and passenger load information would be available for secondary data research. We found the "best-fit" polynomial functions for the fall in average capacity utilization of US Airways, for increasing ticket prices, at fixed ticket price levels of the competitor. We then hypothesize about how this function can be used to simulate the passenger load of the other carrier given the same conditions. We test this hypothesis using market survey data provided in the case about the Southwest airline. Then the best strategy for the players is identified which would ensure them maximum payoff with minimal risk.

INTRODUCTION

The "two-player mixed motive games" can be solved by minimax criteria or by the best payoff method. The solution algorithm looks fairly simple as the player simply chooses that strategy which maximizes his pay off or minimizes the risk. However the methods can be applied only if the payoffs for both the players at various instances of the competitor's decision are available. This is a difficult proposition considering the direct rivalry that happen in the market place between the competitors. The payoffs are generally concealed and even at times manipulated to mislead the competitor. This usually lead to the situation where the players try different alternatives which they might find appropriate and move towards equilibrium point in the long run.

In this paper, we attempt to "simulate" the payoff of the competitor based on the payoffs of the player and check the accuracy of the choice of strategy with the simulated payoff values.

CASE FACTS

We have elaborated upon the US airways case by Shor (Shor, 2002). The case deals with the operation of Nashville- Los Angeles route where US Airways is directly competing with Southwest Airlines. The price war between the two carriers has been continuing over years and the players had not reached an equilibrium point. According to Steven Elkin, Sr Director of Marketing Systems Development for NW Airlines; "Pricing is like an atomic bomb, so dangerous

[1] PGP II Student, TAPMI
[2] Faculty, TAPMI

that no carrier can permit the competitor to wield it unchallenged" (Nomani, 2001). The SW being a low cost carrier has greater contribution margin due to its no frills approach. US Airways provide better quality service and operates larger aircrafts. It was observed by David M Davis, Vice President of Financial Planning and Analysis of US Airways that the passenger load in the carrier depends mostly on the price charged by the competitor and the price charged by the US Airways (Shor, 2002). It is also observed that the Nashville market is quite similar to other airline routes and a strategy developed for Nashville route can be generalized.

The effort by US Airways to increase the price in 2002 was not retaliated by SW airlines leading the US airways to run carriers with less passenger load. The market studies were carried out to find the consumer perception about increasing quality of service, such as more leg room. It was learned that though consumers appreciated greater service qualities, price remained the paramount factor in selecting the carrier for travel.

David in his letter addressing the consultant enquires about three issues;

1. Are the present prices stable at current capacities in long run?
2. Is it possible to raise the prices and increase the net contribution, in light of the threat of SW lowering the price?
3. Can the service level be improved by increasing the leg room, at the cost of losing on capacity, without affecting the contribution?

David could collect the average passenger load for US Airways and SW Airlines at various price points through a market survey. However since SW never charged a price above $150, further passenger load data is based on the conjoint analysis carried out by the market research agency.

RESEARCH OBJECTIVE

1. Find the relationship between price of US Airways, Price of SW airways and the average capacity utilization of the US Airways.
2. Use the observed relation to simulate the average capacity utilization of the SW Airlines and validate it with the data collected through conjoint analysis (Case Facts)
3. Find the best pricing point for US Airways through the two-player mixed motive game analysis, using the predicted passenger loads.

ASSUMPTIONS

1. Both the aircrafts are identical in terms of service and differences, if any, do not influence the consumer preference for one airline over the other.
2. The capacity does not play any role in determining the choice of carrier, ie consumer is indifferent between 137-seater aircraft and 179-seater aircraft.

3. The market conditions remain stable and the demand is influenced only by the price.
4. The fixed cost for US Airways/ Seat = $50 and that for SW Airlines is $20, as furnished in the case.

POLYNOMIAL INTERPOLANTS FOR AVERAGE CAPACITY UTILIZATION

The capacity utilization ratio, which is the average passenger load divided by the maximum number of seats was selected as dependent variable. Use of capacity utilization as the variable makes both the airlines comparable. The dependent variables are, price charged by US airways and Price charged by SW airlines.

It is observed from the plot between the passenger load of US Airways at various price points against a fixed price level of SW airlines, that the variables form a non linear relationship (Appendix A). The best fit polynomial for this family of curves is a third degree polynomial, as compare to exponential, logarithmic and quadratic functions (Appendix B). The equations for the individual curves are plotted and the variation of the coefficients of the equations with respect to the price charged by SW airlines is estimated. The best-fit in this case is observed to be a fourth degree polynomial. The final best-fit polynomial for the average capacity utilization is a third degree polynomial of the form:

$$C_{us} = f_1(P_{sw}) \text{ x } P_{us}^3 + f_2(P_{sw}) \text{ x } P_{us}^2 + f_3(P_{sw}) \text{ x } P_{us} + f_4(P_{sw}) \quad (1)$$

Here:
C_{us}: the capacity utilization of US Airways
P_{us}: Price charged by US Airways
P_{sw}: The price charged by SW airlines
f_{1-} f_4: fourth degree polynomials as per the best fit curve for the coefficient of equations The detailed process is enclosed as Appendix C.

The capacity utilization predictor equation estimated for US Airways can be interchangeably used for SW airlines due to the equality assumption made earlier. Hence the predictor equation for SW airlines is deduced from Equation 1 by interchanging the price values. The equation is then validated using the market survey data furnished in the case. The standard error observed for the predicted variable is 0.50 and the predicted data amounted to a standard deviation of 0.204. Appendix D shows the variation between the predicted and the market survey results against the price levels.

It can be seen that the predictor shows a great deal of volatility at the higher prices. This is due to the huge multipliers which enlarge the small tolerances to higher proportions. For example the predictor error varied in a particular case (P_{sw} = P_{us}= 500) to as high as 54%. Also an effort is made to logically limit the capacity utilization values exceeding 100% (Airlines does not permit passengers to travel standing!) or less than 0%. This occurs in 13% of cases due to the third degree approximation in the best-fit curve.

The pay off matrix for the various competitor strategies is now formulated and the best strategy can be chosen (Appendix E & F).

OPTIMAL STRATEGY

The optimal price point of US airways is found out with the pay offs of both the players listed at various price points. The best pay off method is selected under the assumption that both the players are rational and tries to maximize their pay off. A best strategy set $\{S_i\}$ (price level) for each of the competitor strategy is found out. Similarly the strategy set for SW airways is also found out. Those price levels which are not part of the best strategies for both the players are eliminated. The process is repeated with new best pay off identified after the elimination and a new best strategy set $\{S_j\}$ is formed. This process is continued until an equilibrium point is reached (McCain, 2003). (Appendix G)

It is observed that the strategy for US Airways in both the cases is to adopt the price level of $200. This does not vary in the case of market survey data. However the predicted values for SW airlines passenger load shows that the best strategy for SW Airlines is $100 while the market survey shows that SW airlines should increase the price to $150.

REFERENCES

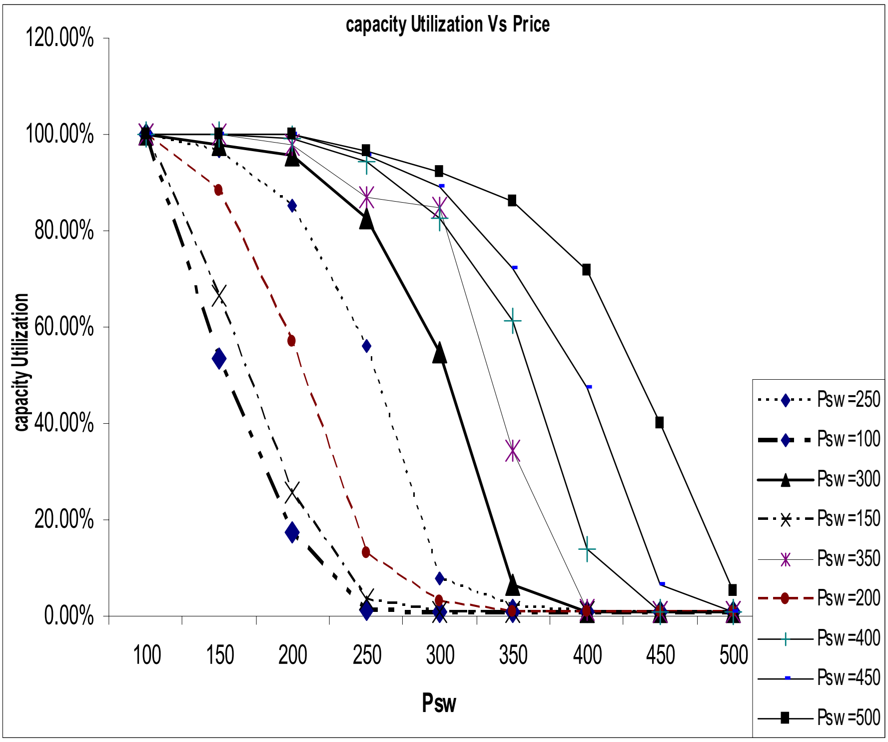
Shor, M., 2002. Case of US Airways Internal Memo.

McCain, R. A. 2003, ***Game Theory, A Non Technical Introduction to the Analysis of Strategy***. London: Thomson Press

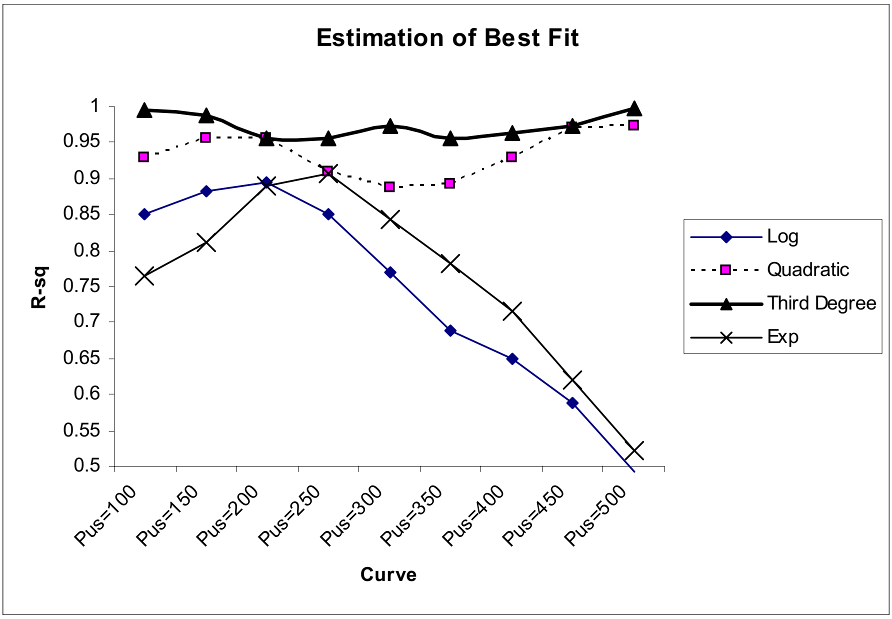
Rasmusen, E. 2001. ***An Introduction to Game Theory.*** Blackwell Publications.

Nomani, Asra Q. 2001. Fare Warning: How Airlines Trade Price Plans. In Rasmusen, E (Ed.) ***Reading in Games and Information.*** Boston: Blackwell Publications.

APPENDIX A
Capacity Utilization of US Airways with Price points



APPENDIX B
Plot of R-squared values of various best fit curves



APPENDIX C
Estimation of C_{us}- Method-I

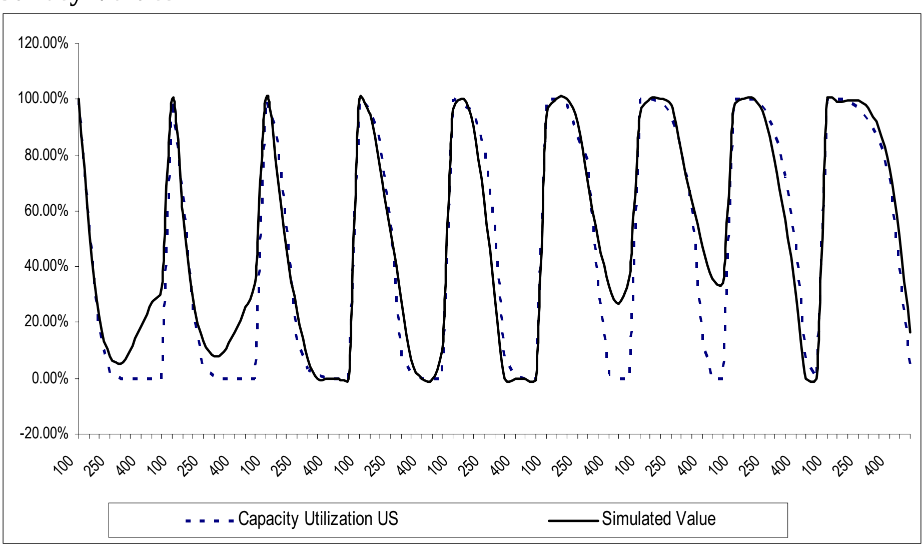
$Y=C_{us}$, $X=P_{us}$	x^3	x^2	x	C	Equation
P_{sw}=100	-7.00E-06	0.0084	-3.1053	371.45	$y = -7E\text{-}06x^3 + 0.0084x^2 - 3.1053x + 371.45$
P_{sw}=150	-5.00E-06	0.0066	-2.6489	346.32	$y = -5E\text{-}06x^3 + 0.0066x^2 - 2.6489x + 346.32$
P_{sw}=200	6.00E-07	0.001	-1.1692	253.64	$y = 6E\text{-}07x^3 + 0.001x^2 - 1.1692x + 253.64$
P_{sw}=250	8.00E-06	-0.0062	0.9182	104.65	$y = 8E\text{-}06x^3 - 0.0062x^2 + 0.9182x + 104.65$
P_{sw}=300	1.00E-05	-0.0099	2.1949	1.2381	$y = 1E\text{-}05x^3 - 0.0099x^2 + 2.1949x + 1.2381$
P_{sw}=350	1.00E-05	-0.0092	2.2211	-11.111	$y = 1E\text{-}05x^3 - 0.0092x^2 + 2.2211x - 11.111$
P_{sw}=400	7.00E-06	-0.0069	1.7622	15.437	$y = 7E\text{-}06x^3 - 0.0069x^2 + 1.7622x + 15.437$
P_{sw}=450	1.00E-06	-0.0026	0.7818	79.333	$y = 1E\text{-}06x^3 - 0.0026x^2 + 0.7818x + 79.333$
P_{sw}=500	-4.00E-06	0.0023	-0.4282	162.01	$y = -4E\text{-}06x^3 + 0.0023x^2 - 0.4284x + 162.01$

Y= Coeff; z=P_{sw}	P_{sw}^4	P_{sw}^3	P_{sw}^2	P_{sw}	P_{sw}^0	RHS of Equation
Coeff of x^3	9.00E-15	-1.00E-11	5.00E-09	-6.00E-07	2.00E-05	$9E\text{-}15z^4 - 1E\text{-}11z^3 + 5E\text{-}09z^2 - 6E\text{-}07z + 2E\text{-}05$
Coeff of x^2	-9.00E-12	1.00E-08	-5.00E-06	0.0007	-0.0214	$-9E\text{-}12z^4 + 1E\text{-}08z^3 - 5E\text{-}06z^2 + 0.0007z - 0.0214$
Coeff of x	2.00E-09	-3.00E-06	0.0013	-0.192	5.8169	$2E\text{-}09z^4 - 3E\text{-}06z + 0.0013z - 0.192z + 5.8169$
Constant term	-2.00E-07	0.0002	-0.0955	14.42	-320.74	$-2E\text{-}07z^4 + 0.0002z^3 - 0.0955z^2 + 14.42z - 320.74$

$C_{us}=$	$(9E\text{-}15P_{sw}^4 - 1E\text{-}11P_{sw}^3 + 5E\text{-}09P_{sw}^2 - 6E\text{-}07P_{sw} + 2E\text{-}05)P_{us}^3 + (-9E\text{-}12P_{sw}^4 + 1E\text{-}08P_{sw}^3 - 5E\text{-}06P_{sw}^2 + 0.0007P_{sw} - 0.0214)P_{us}^2 + (2E\text{-}09P_{sw}^4 - 3E\text{-}06P_{sw}^3 + 0.0013P_{sw}^2 - 0.192P_{sw} + 5.8169)P_{us} + (-2E\text{-}07P_{sw}^4 + 0.0002P_{sw}^3 - 0.0955P_{sw}^2 + 14.42P_{sw} - 320.74)$

APPENDIX D

Comparison between the Predicted values of US Airways with Market Survey Values



APPENDIX E

Compendium of Research Papers

The Pay off Matrix with US Airways passenger load and the predicted southwest airlines passenger load at various price points

Passenger Load (SW, US)		US Airways Price																	
		100		150		200		250		300		350		400		450		500	
SW Airways Price	100	137	*179*	137	*155*	137	*128*	110	*87*	129	*60*	137	*15*	137	*13*	137	*12*	137	*10*
	150	70	*179*	99	*173*	124	*148*	123	*99*	128	*84*	128	*15*	136	*13*	137	*12*	137	*10*
	200	18	*179*	33	*179*	73	*172*	117	*115*	135	*96*	114	*16*	120	*14*	127	*12*	133	*10*
	250	8	*179*	25	*179*	36	*179*	82	*169*	110	*114*	104	*17*	110	*15*	116	*13*	122	*10*
	300	0	*179*	18	*179*	29	*179*	37	*179*	47	*167*	95	*25*	103	*16*	108	*14*	113	*11*
	350	0	*179*	11	*179*	23	*179*	32	*179*	38	*178*	48	*166*	96	*19*	102	*14*	107	*12*
	400	0	*179*	4	*179*	18	*179*	27	*179*	6	*179*	0	*177*	14	*140*	70	*17*	101	*13*
	450	0	*179*	0	*179*	12	*179*	22	*179*	29	*179*	0	*179*	0	*175*	2	*89*	57	*15*
	500	0	*179*	0	*179*	8	*179*	18	*179*	25	*179*	0	*179*	0	*179*	0	*135*	12	*39*

APPENDIX F

The Pay off Matrix with US Airways passenger load and the southwest airlines passenger load observed from market survey (Case facts) at various price points

Passenger Load (SW, US)		PRICE CHARGED BY US AIRWAYS																	
		100		150		200		250		300		350		400		450		500	
PRICE CHARGED BY SOUTHWEST	100	137	*179*	137	*155*	137	*128*	137	*87*	137	*60*	137	*15*	137	*13*	137	*12*	137	*10*
	150	73	*179*	90	*173*	120	*148*	132	*99*	134	*84*	137	*15*	137	*13*	137	*12*	137	*10*
	200	24	*179*	35	*179*	78	*172*	117	*115*	131	*96*	134	*16*	136	*14*	137	*12*	137	*10*
	250	2	*179*	5	*179*	18	*179*	77	*169*	113	*114*	119	*17*	129	*15*	131	*13*	132	*10*
	300	0	*179*	1	*179*	4	*179*	11	*179*	75	*167*	106	*25*	113	*16*	122	*14*	126	*11*
	350	0	*179*	0	*179*	1	*179*	3	*179*	9	*178*	47	*166*	84	*19*	99	*14*	118	*12*
	400	0	*179*	0	*179*	0	*179*	0	*179*	1	*179*	2	*177*	19	*140*	65	*17*	98	*13*
	450	0	*179*	0	*179*	0	*179*	0	*179*	0	*179*	0	*179*	0	*175*	9	*89*	55	*15*
	500	0	*179*	0	*179*	0	*179*	0	*179*	0	*179*	0	*179*	0	*179*	0	*135*	7	*39*

APPENDIX G

Elimination of strategies based on the best contribution for a given competitor strategy; with predicted pay off values for SW Passenger load

	US Airways	**SW Airlines**
Iteration 1	100, 150, 450, 500	250, 300, 350, 400,450, 500
Iteration 2	250, 300, 350, 400	None
Iteration 3	None	150, 200
Optimal Strategy	**200**	**100**

Elimination of strategies based on the best contribution for a given competitor strategy; with pay offs obtained trough market survey for SW Passenger load

	US Airways	**SW Airlines**
Iteration 1	100, 150, 450, 500	400, 450, 500
Iteration 2	400	100, 350
iteration 3	350	None
Iteration 4	None	300
Iteration 5	300	None
Iteration 6	None	250
Iteration 7	250	None
Iteration 8	None	200
Optimal Strategy	**200**	**150**