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Using the Concrete–Representational–Abstract Sequence to Teach Subtraction With Regrouping to Students at Risk for Failure

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This study investigated the effects of the concrete–representational–abstract (CRA) instructional sequence on the subtraction performance of students identified as at-risk for mathematics failure. Although the CRA sequence has been demonstrated as effective for teaching a variety of mathematical skills and processes its effects when used to teach subtraction with regrouping have not been studied. This study examined the effects of CRA instruction on elementary students' fluency in computing subtraction problems with regrouping in the tens place as well as regrouping in both the tens and hundreds places. The study also investigated the effects of CRA instruction on students' maintenance of regrouping skills. A multiple probe across students with embedded changing behaviors design was employed. A functional relation between CRA instruction and subtraction with regrouping was demonstrated across all students. The results and their implications are discussed further.

Keywords: *instruction mathematics; general education classroom; strategy instruction; elementary school(s); single-subject research methodology; mathematics*

According to the Individuals With Disabilities Education Improvement Act (2004), states have the option of identifying children with learning disabilities through the use of response to intervention. For any student who struggles academically, this model provides immediate intervention in areas of academic need, and it provides students with increasingly more intensive interventions to remediate academic deficits. The research in the area of math, regarding the interventions for students who are at risk for math failure, is small compared to what is known about other academic areas, such as reading (National Research Council, 2002). With respect to response to intervention, interventions have been studied in the areas of basic mathematics computation (VanDerHeyden & Witt, 2005), strategy instruction for problem solving and the use of the concrete–representational–abstract (CRA)sequence to teach place value, geometry, and fractions (Fuchs, Fuchs, & Hollenbeck, 2007).

The CRA sequence has been shown to be effective for remediating deficits in basic mathematics computation (Harris, Miller, & Mercer, 1995; Mercer & Miller, 1992; Miller & Mercer, 1993; Morin & Miller, 1998), place value (Peterson, Mercer, & O'Shea, 1988), fractions (Butler, Miller, Crehan, Babbitt, & Pierce, 2003), and algebra (Maccini & Ruhl, 2000; Witzel, 2005; Witzel, Mercer, & Miller, 2003). The CRA sequence involves three phases.

First, manipulative objects are used to promote conceptual understanding. Concrete-level instruction proceeds as follows: The instructor demonstrates the mathematical skill/process with manipulatives; the instructor then guides the students by participating in the use of manipulatives with the students, providing prompts and cues; and the students independently use manipulatives to demonstrate the skill/process. Instruction at the representational level follows the same steps, but the manipulative objects are replaced by pictures and/or drawings. After the representational phase, most interventions involving the CRA sequence provide students with a mnemonic strategy for remembering the steps involved in the mathematical skill/process. This serves as a transition from the use of pictures or drawings to the use of numbers only—that is, the abstract phase. During this final phase, students use numbers only in completing a mathematical task, and instruction focuses on fluency.

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CRA Instruction Versus Other Methods

The CRA sequence of instruction differs from other approaches with regard to instructional methods and activities and teacher involvement. The major focus of instruction is on conceptual understanding and students' demonstration of their mastery of this understanding. The use of manipulative objects is not unique to this method. The combination of teacher demonstration, guidance, and student demonstration of mastery over three lessons differentiates this from other methods. The representational level of instruction provides a transition between the concrete and abstract levels. The student uses pictures or drawings to illustrate the mathematical procedure or operation, creating his or her own representation. The use of a mnemonic device is another scaffold between the representational level and the abstract. This type of device aids the student in remembering how to attack a particular problem, and it provides a cue for how to proceed if he or she does not remember a particular fact or procedure. The teacher is directly involved in each level of instruction until the student masters the mnemonic device and increases his or her level of automaticity and independence from the teacher. The direct involvement of the teacher separates CRA from constructivist approaches, but the fading of the teacher's direction differentiates CRA from other remedial approaches.

CRA Mathematics Fact Instruction

Mercer and Miller (1992) taught basic addition, subtraction, multiplication, and division to students with high-incidence disabilities using either the CRA sequence or a traditional curriculum. For example, when teaching a multiplication fact such as 3×2, a number sentence was translated to these words, "Three groups of two objects equals how many?" At the concrete level, three groups of plates were arranged; two objects were placed on each plate; and the total number of objects was counted. At the representational level, three lines were drawn to represent the groups; two tallies were drawn on each line; and the total number of tallies was counted. Multiplication rules such as the commutative property and the zero rule were demonstrated throughout these lessons. Instruction using the CRA sequence included a mnemonic strategy to help students remember the steps for computing basic facts—that is, the DRAW strategy. The DRAW strategy steps were as follows: Discover the sign. Read the problem. Answer or draw and check. Write the answer. Finally, students engaged in fluency activities at the abstract level, using numbers only. Mercer and Miller found that students performed significantly better when instruction involved the CRA sequence and the mnemonic strategy. After CRA instruction, students demonstrated generalization by transferring basic computation knowledge to completion of one-step word problems. Miller and Mercer (1993) replicated their findings with regard to addition fact fluency and generalization to word problems. Through these research studies, Miller and Mercer found that students with learning disabilities needed an average of three experiences at the concrete level before moving to the representational level. This is an important finding because teachers may be less likely to use such a method if they perceive concrete instruction as being time consuming and/or disruptive.

Harris et al. (1995) extended the line of CRA research by using the instructional sequence in inclusive general education classrooms. Initial instruction in multiplication was provided using the CRA sequence and a mnemonic device. As a result of instruction, students with learning disabilities performed at the same level as their peers without disabilities. This was an important finding because it demonstrated a strategy—namely, that CRA could provide students with disabilities greater access to the general education curriculum. Morin and Miller (1998) extended CRA research by teaching multiplication to middle school students with intellectual disabilities. After CRA instruction and use of a mnemonic device, the students demonstrated fluency in basic multiplication. Consistent with results from other research, the students transferred their knowledge to one-step word problems.

CRA Algebra Instruction

Although the CRA sequence has been shown to be effective for teaching basic mathematics computation, it is effective for more complete mathematical processes, which thereby provides students with greater access to the general education curriculum. This sequence has been used to teach algebraic subtraction of integers (Maccini & Ruhl, 2000) and algebraic transformation equations (Witzel et al., 2003). Providing students with greater access to algebra provides students with disabilities the tools to complete high school exit exams and so obtain a high school diploma, as well as advance to higher levels of education. Witzel (2005) compared CRA algebra instruction to traditional methods with students with and without disabilities in general education classrooms.

Maccini and Ruhl (2000) used the CRA sequence to teach algebraic subtraction problems. CRA instruction included the use of a mnemonic device—namely, the STAR strategy. The strategy involved the following steps: Search the problem. Translate the words into an equation. Answer the problem. Review and check. As a result of instruction, high school students with learning disabilities computed algebraic subtraction problems and then generalized those skills to problem-solving situations involving word problems. The students also maintained their performance for 6 weeks.

Witzel et al. (2003) compared the CRA sequence and traditional instruction with regard to teaching algebra to middle school students in general education settings. Students with and without disabilities were matched on disability and algebra performance. Students participated in instruction for solving algebraic equations using either the CRA sequence or traditional instructional methods. Students with and without disabilities who participated in CRA instruction outperformed their peers who received traditional instruction. Witzel (2005) further compared CRA algebra instruction to algebra instruction using numbers only. Middle school students with disabilities and without disabilities participated. Regardless of disability or mathematics achievement, students who participated in CRA instruction outperformed those who received instruction at the abstract level.

CRA Fraction Instruction

Butler et al. (2003) compared the CRA sequence with instruction that involved two phases: representational and abstract (RA). This study investigated the need for a phase involving manipulative objects in the teaching of equivalent fractions. Middle school students with learning disabilities in mathematics participated in either the CRA or the RA sequences of instruction. The researchers found that student performance increased across both CRA and RA instructional groups. However, the students who received CRA instruction demonstrated greater gains than did the students who received RA instruction. This study provided further evidence for using concrete-level instruction. This is especially important for students and teachers at the middle school level, where the use of manipulative objects is presumably not needed for students of a particular age.

CRA Place Value Instruction

Peterson et al. (1988) investigated the use of the CRA sequence to teach place value, comparing it to instruction at the abstract level. Students with learning disabilities at the elementary and middle school levels participated in the study. Consistent with the results of other research studies, the students who received CRA place value instruction outperformed their peers who received abstract-level instruction. These students also maintained their performance over time.

Method

Purpose

Research regarding the CRA sequence has shown it to be an effective method for teaching and remediating deficits in basic computation, fraction skills, algebraic computation, and place value. Place value instruction includes the conceptual understanding of place value as it relates to basic addition and subtraction. However, the role of place value in more complex addition and subtraction, involving regrouping in the tens and hundreds places, has not been the focus of any CRA research. Therefore, the purpose of this study was to investigate the use of the CRA sequence to teach elementary students who were identified as being at risk for failure with regard to subtraction with regrouping in the tens and hundreds place. The research questions were as follows: What are the effects of CRA instruction on students' fluency in computing subtraction problems with regrouping in the tens place and in the tens and hundreds places? What are the effects of CRA instruction on students' maintenance of fluency in computing subtraction problems with regrouping in the tens and hundreds places?

Setting

The study was conducted in an elementary school within a rural district outside a major city in the Southwest. The majority of the enrolled students received free or reduced-price lunch; the cultural background of the students was as follows: < 1% African American, < 1% Asian, 80% Hispanic, and 19% White. Instructional sessions lasted 30 min, scheduled 3 days per week. The students received instruction during a regularly scheduled time, in which math practice and remediation were provided in the general education classroom. Instruction took place outside the general education classroom, in a conference room.

Participants

Table 1 summarizes student characteristics. The criteria for participation were as follows: current failure in mathematics and lack of skill in subtraction with regrouping, defined as fewer than 10 digits written correctly on a curriculum-based measure. Six students in the

Table 1
Participant Information

Student	Age	Grade	Cultural Backgroud	Cognitive Ability[a]	Mathematics Achievement[b]
Ray	8	3	Hispanic	110	105
Al	9	3	Hispanic	118	92
Walt	10	3	African American	112	83
Ron	8	3	Hispanic	96	100
Joe	8	3	Hispanic	103	86
Ann	9	3	Hispanic	120	84

Note: Exceptionality not identified for each student.
a. Standard score on the Wechsler Abbreviated Scale of Intelligence.
b. Standard score on the Operations subtest of the KeyMath–Revised.

third grade participated in the study; all the students had been referred by their teacher to the student support team because of their teachers' concerns regarding mathematics performance. No action with regard to evaluation for special education had been taken. All the students were failing mathematics, according to grades and performance on districtwide benchmark assessments; the purpose of the latter was to predict student performance on state-mandated criterion-referenced assessments administered at the end of the school year. None of the students wrote more than 1 correct digit on a curriculum-based measure of subtraction with regrouping in the tens place. All the students demonstrated mastery of addition and subtraction facts, writing 40 correct digits per minute. The students' mastery of mathematical facts did not influence their subtraction with regrouping performance. None of the students had been identified as students with disabilities, nor had any of the students been evaluated. For several of the students, their classroom failure was not consistent with their assessments. Ray and Ron performed well within the average range on the Operations subtest of the KeyMath–Revised. Although Al's performance was within the average range based on the KeyMath–Revised (standard score = 92), his performance was lower than one might expect, based on his cognitive ability per the Wechsler Abbreviated Scale of Intelligence (standard score = 118). Upon inspection of KeyMath–Revised items missed, none of these students demonstrated regrouping skills. They demonstrated error patterns similar to those on their curriculum-based pretests. It is unknown how these students' mathematics skills, as demonstrated through individualized diagnostic assessment, were not consistent with their performance in the general education classroom. Walt, Joe, and Ann demonstrated discrepancies between their cognitive abilities and their computation performance. However, a comprehensive evaluation, including varied assessment approaches and instruments, would be needed to determine the presence or absence of disabilities.

Materials

For CRA instruction in subtraction with regrouping in the tens place, the researcher used the following materials. The probes that measured student progress consisted of 8.5- × 11-inch sheets of paper with 30 subtraction problems (2-digit from 2-digit) that required regrouping in the tens place. The CRA intervention materials included a contract in which the teacher and students agreed to work rigorously using strategies to master subtraction with regrouping. Each student received an 8.5- × 11-inch sheet with a progress chart, in which the student's probe performance (the number of correct digits) was recorded throughout the intervention. The materials for Lessons 1–3 included base-10 blocks made out of foam and researcher-created 8.5- × 11-inch learning sheets with target subtraction problems (two-digit from two-digit) requiring regrouping in the tens place, written vertically. The learning sheets were divided into three sections: Model (three problems), Guided Practice (three problems), and Independent Practice (six problems). The materials for the representational phase (Lessons 4–6) involved researcher-created learning sheets with target subtraction problems (two-digit from two-digit) requiring regrouping in the tens place, written vertically. The learning sheets were divided into three sections: Model (three problems), Guided Practice (three problems), and Independent Practice (six problems). After instruction at the representational level (Lesson 7), the DRAW strategy was introduced, and the materials consisted of a sheet printed with the strategy (i.e., discover the sign, read the problem, answer or draw and check your answer, write the answer). The last phase of instruction (Lessons 8–10) involved researcher-created learning sheets with target subtraction problems (two-digit from two-digit) requiring regrouping in the tens place, written vertically. The learning sheets were divided into three sections: Model (one problem), Guided Practice (three problems), and Independent Practice (six problems). Any remaining lessons involved fluency activities, and the materials involved sheets with 30 vertically written subtraction problems requiring regrouping in the tens place.

For CRA instruction in subtraction with regrouping in the tens and hundreds places, the researcher used the following materials. The probes that measured student progress consisted of 8.5- × 11-inch sheets of paper with 30 subtraction problems (three-digit from three-digit) that required regrouping in the tens and hundreds places. The problems were equally distributed per level of difficulty; every other problem included a zero in the tens place (e.g., 102 – 89). The materials for Lessons 1–3 were base-10 blocks made out of foam and researcher-created 8.5- × 11-inch learning sheets with target subtraction problems (three-digit from three-digit) requiring regrouping in the tens and hundreds places, written vertically. The learning sheets were divided into three sections: Model (three problems), Guided Practice (three problems), and Independent Practice (six problems). The materials for the representational phase (Lessons 4–6) involved researcher-created learning sheets with target subtraction problems (three-digit from three-digit) requiring regrouping in the tens and hundreds places, written vertically. Problems including zeros in the tens place were also distributed throughout the learning sheets. The learning sheets were divided into three sections: Model (three problems), Guided Practice (three problems), and Independent Practice (six problems). After instruction at the representational level (Lesson 7), the DRAW strategy was reintroduced, and the materials consisted of a sheet printed with the strategy. The last phase of instruction (Lessons 8–10) involved researcher-created learning sheets with target subtraction problems (three-digit from three-digit) requiring regrouping in the tens and hundreds places, written vertically. The learning sheets were divided into three sections: Model (one problem), Guided Practice (three problems), and Independent Practice (six problems). Any remaining lessons involved fluency activities, and the materials involved sheets with 30 vertically written subtraction problems requiring regrouping in the tens and hundreds places.

Instructional Procedures

Before the study began, all students completed probes that included regrouping in the tens place and regrouping in the tens and hundreds places. All the students wrote zero correct digits, demonstrating lack of mastery in both skills. A baseline for all students was established regarding subtraction with regrouping in the tens place. A stable baseline was defined as three consecutive data points in which the points varied no more than 5% from the average rate of responding across all points (Poling, Methot, & LeSage, 1995). When baseline was stable, CRA instruction began with the first student (Ray) while the remaining students continued in baseline. The criterion for phase change was as follows: 20 digits correct on a 2-min probe across three consecutive probes. The criterion of 20 digits was chosen because it is the norm for the end of second grade (Fuchs & Fuchs, 2005) and the participants were beginning the school year as third graders. When the first student reached criterion, he moved into the baseline condition for regrouping in the tens and hundreds places, during which no instruction or practice opportunities were provided. When baseline was stable, the first student began CRA instruction for subtraction with regrouping in the tens and hundreds places. The criterion for mastery was three consecutive 2-min probes with 20 correct digits written. After reaching criterion, the student proceeded to the maintenance phase, in which no instruction or practice opportunities were provided for 6 weeks. At the end of 6 weeks, maintenance of regrouping in the tens and hundreds places was measured using a 2-min probe. When the first student (Ray) began baseline for tens and hundreds regrouping, the second student (Al) began CRA instruction. These procedures were used for the third student (Walt) and replicated with three other students (Ron, Joe, Ann).

CRA instruction was implemented according to the structure of Miller and Mercer's *Strategic Math Series: Place Value* (1992). First, the teacher described CRA and its rationale based on pretest performance and so obtained a commitment from the student in the form of a contract. Figure 1 provides a graphic representation of CRA instruction for regrouping in the tens place at the concrete level. CRA instruction for regrouping in the tens place (Lessons 1–3) involved manipulative objects. The following activities were modeled and guided by the teacher and practiced independently by the students: (a) inserting and labeling the ones column and the tens column and translating the subtraction problems, such as "32 – 15 = means 3 tens and 2 ones minus 1 ten and 5 ones equals how many"; (b) using base-10 manipulative blocks to represent the minuend (a problem such as "32 – 15" was represented with 3 tens blocks and 2 ones blocks); and (c) solving the problem by regrouping the manipulatives in the tens place (removing 1 tens block from the minuend

Figure 1
Concrete-Level Instruction for Regrouping in the Tens Place

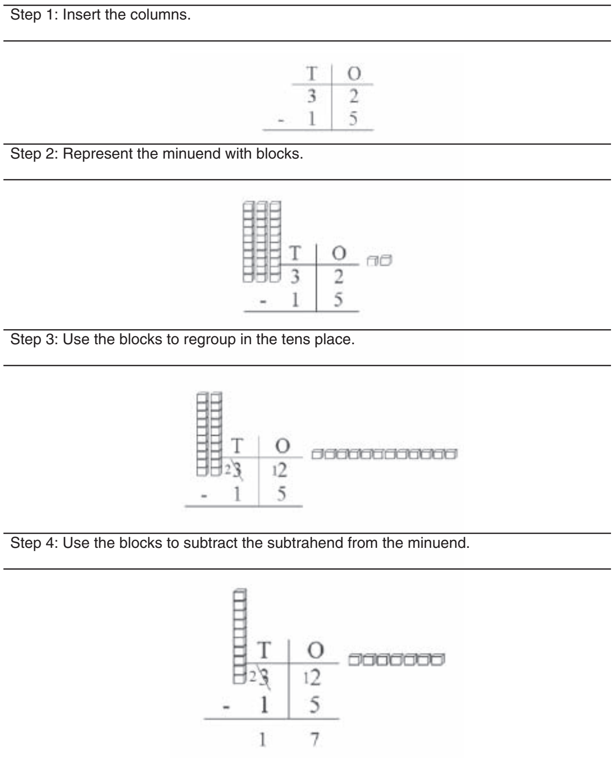
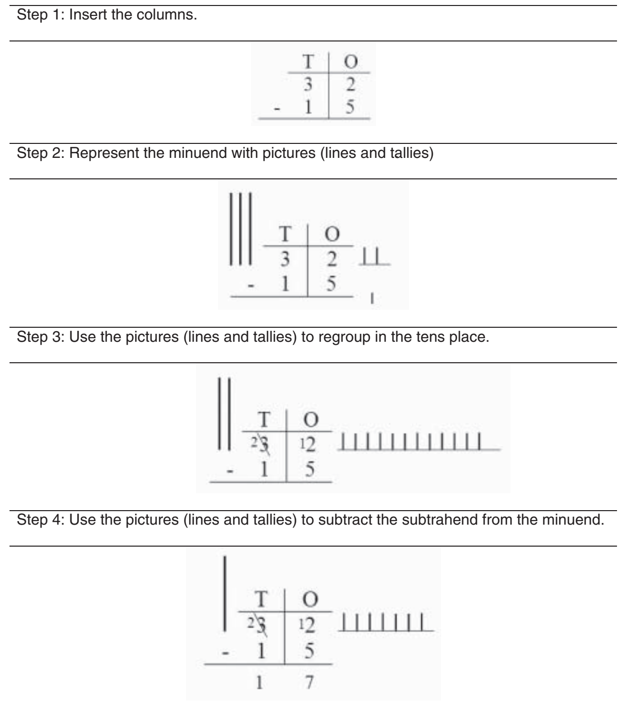


Figure 2
Representational-Level Instruction for Regrouping in the Tens Place



and adding 10 ones blocks to the existing 2 ones blocks within the minuend) and subtracting the appropriate number of blocks based on the subtrahend.

After three lessons with the student's achieving 80% accuracy or better, instruction progressed to the representational level. The same teaching procedures were used. Figure 2 includes a graphic of the steps involved in subtraction at the representational level. However, drawings were used instead of three-dimensional objects. The ones were represented as small vertical tallies, written on a horizontal line, and the tens were represented using long vertical lines. Regrouping in the tens place was represented by circling one of the long vertical lines and adding 10 small tallies to the horizontal line. Subtraction in the ones place was represented by circling the appropriate number of lines or tallies, based on the subtrahend. After the student achieved at least three lessons with 80% accuracy on independent lesson tasks, the DRAW strategy was introduced. The teacher modeled and guided the student through each step of the strategy: Discover the sign. Read the problem. Answer or draw and check. Write your answer. The student independently used the strategy to solve problems and so moved to the next phase of instruction when he or she solved problems with at least 80% accuracy and could recite the steps accurately. The next phase of instruction was the abstract level, in which the student was encouraged to answer problems from memory, rather than use drawings, but he or she could use the DRAW strategy. The last phase of instruction involved fluency activities in which the student was given 2 min to complete a sheet containing subtraction problems. CRA instruction for subtraction in the tens and hundreds places followed the same procedures. Concrete-level instruction included base-10 blocks representing ones, tens, and hundreds. Figure 3 includes a graphic representation of CRA instruction for regrouping in the tens and hundreds places. Representational instruction included the use of squares to represent the hundreds. Figure 4 includes a graphic depiction of representational-level instruction. Progress was measured using curriculum-based measures, which were administered at the beginning of each instructional session. Throughout each intervention, measures consisted of 2-min probes based on Beck, Conrad, and Anderson's *Basic Skill Builders* (1999). Maintenance measures were 2-min probes that were administered 6 weeks after instruction ended.

Figure 3
Concrete-Level Instruction for Regrouping in the Tens and Hundreds Places

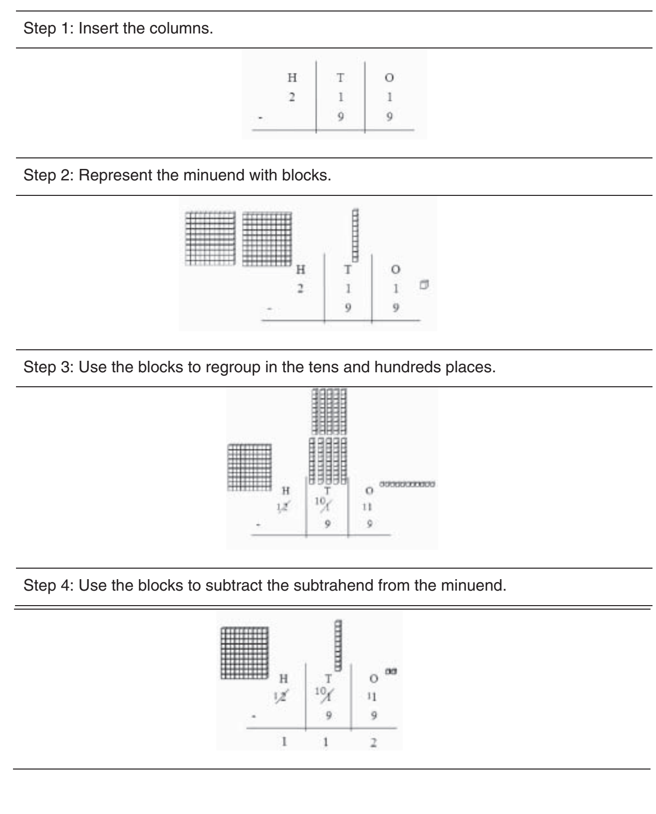
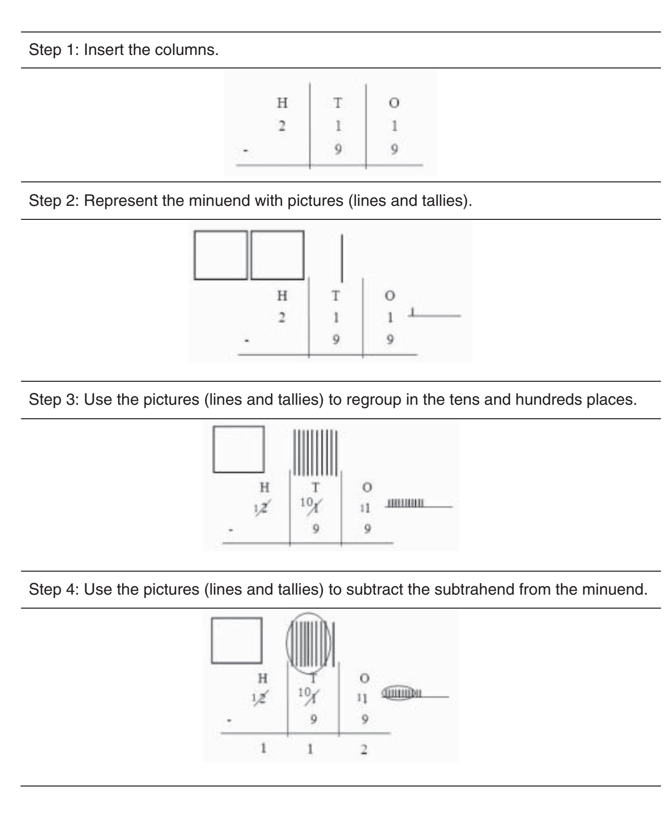


Figure 4
Representational-Level Instruction for Regrouping in the Tens and Hundreds Places



Treatment Integrity and Interobserver Agreement

Treatment integrity was conducted during 30% of the lessons (Poling et al., 1995). These instructional lessons were recorded using digital video. A treatment checklist for the intervention was used to ensure that procedures were carried out correctly. A graduate assistant was trained in using the treatment integrity checklists through demonstration and practice. When the graduate assistant completed a checklist with 100% accuracy, treatment integrity checks began. Treatment integrity was calculated at 100% for the study.

Interrater reliability was conducted for 80% of the subtraction probes administered. A graduate assistant was trained through demonstration and practice in the scoring procedures, and reliability checks began after the graduate assistant had scored a probe with 100% accuracy. The primary researcher collected data before each instructional lesson and scored fluency probes. The trained graduate assistant scored the same multiplication probes. To calculate interrater reliability, the total number of agreements between the graduate assistant and the primary researcher were divided by the total number of observations, and this answer was multiplied by 100 (Poling et al., 1995).

Social Validity

Social validity was assessed through interviews conducted before and after the study. The students and their teachers answered questions regarding the need for the intervention, the efficacy of the intervention, and recommendations for other students and teachers. The students who participated were placed into three classrooms, and each classroom teacher participated in the interviews. All teachers reported that there was a need for the subtraction-with-regrouping intervention, and all teachers reported that the intervention was effective, as measured by their observations of their students' computation within the general education classroom. The teachers reported increases in the students' regrouping performance on district-mandated mathematics benchmark testing. All teachers reported that they would recommend the strategy to other teachers. All students stated that subtraction

Figure 5
Number of Correct Digits Written on Probes by Ray, Al, and Walt

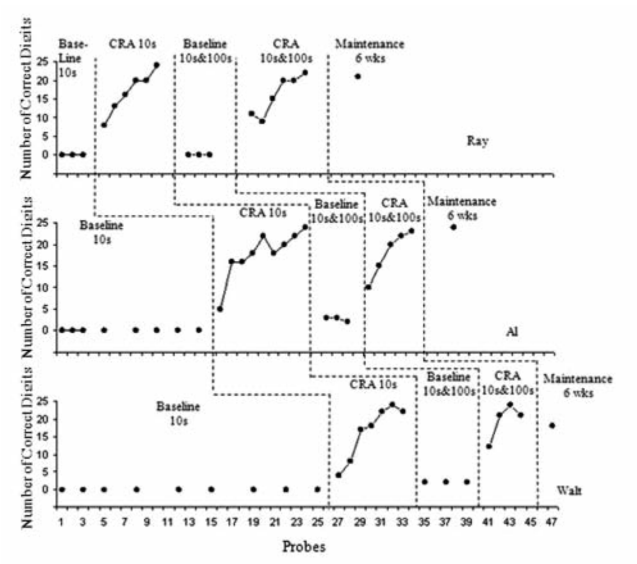
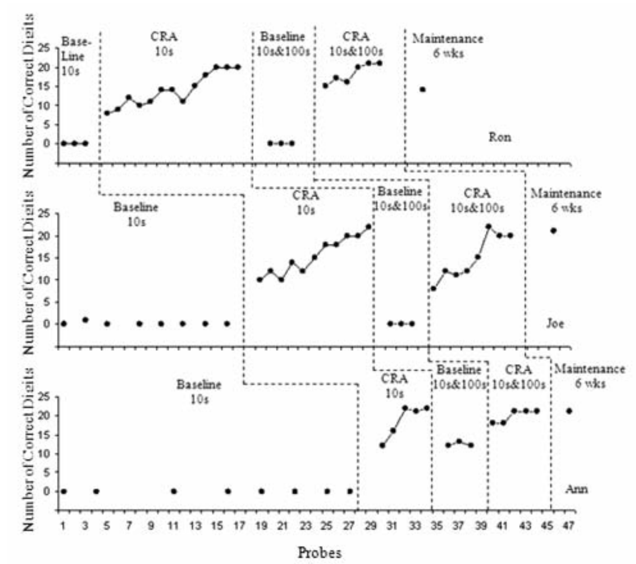


Figure 6
Number of Correct Digits Written on Probes by Ron, Joe, and Ann



with regrouping was difficult, and they expressed interest in participating in the intervention. The students made a commitment to participate and to put effort into learning a new way to complete subtraction problems. The students reported that they liked the strategy and that it made subtraction easier. All the students said that they would recommend the strategy to other students.

Research Design

The study employed a multiple probe (Tawney & Gast, 1984) across students, with embedded changing-behaviors design to evaluate the efficacy of CRA instruction for teaching subtraction with regrouping. The multiple-probe design was replicated one time with an additional three students. The multiple probe across students, with embedded changing-behaviors design, was employed to show a functional relation between the CRA intervention and a behavior that could not be reversed or unlearned. The data were interpreted by visual inspection, and the following characteristics were noted: overlap between each baseline and treatment, the slope of each treatment data path, and the number of data points from the beginning of each treatment to criterion.

Results

Figure 5 presents the number of correct digits written for Ray, Al, and Walt; Figure 6 presents the number of correct digits written for Ron, Joe, and Ann. The *x*-axis presents the subtraction-with-regrouping probes; the *y*-axis represents the number of correct digits written for each probe.

Baseline

Ray's tens regrouping baseline performance was stable, writing zero correct digits on all probes. His tens and hundreds regrouping baseline was stable, writing zero correct digits on all probes. Al's tens regrouping baseline performance was stable, writing zero correct digits on all probes. His tens and hundreds regrouping baseline was stable, writing between two and three correct digits on the probes. Walt's tens regrouping baseline performance was stable, writing zero correct digits on each probe. His tens and hundreds regrouping baseline was stable, writing two correct digits on all probes. Ron's tens regrouping baseline performance was stable, writing zero correct digits on each probe. His tens and hundreds regrouping baseline was stable, writing zero correct digits on all probes. Joe's tens regrouping baseline performance was stable, writing zero correct digits on each probe.

His tens and hundreds regrouping baseline was stable, writing zero correct digits on the last six probes. Ann's tens regrouping baseline performance was stable, writing zero correct digits on each probe. Her tens and hundreds regrouping baseline was stable, writing between 12 and 13 correct digits on the probes.

CRA Performance

Ray. After beginning CRA tens instruction, Ray reached criterion after six probes, ranging from 8 to 24 correct digits. There was an immediate change in the level of performance between baseline and CRA, and there were no overlapping data points between baseline and CRA tens phases. Ray's average performance during the CRA tens phase was approximately 17 correct digits. The data points of the CRA tens phase show an upward path, indicating steady improvement.

After reaching criterion for the CRA tens phase and establishing a stable baseline for the CRA tens and hundreds phase, Ray began CRA instruction for regrouping in the tens and hundreds places. He reached criterion after six probes, ranging from 9 to 22 correct digits. There was an immediate change in the level of performance between baseline and CRA, and there were no overlapping data points between baseline and CRA tens and hundreds phases. Ray's average performance during the CRA tens and hundreds phase was approximately 16 correct digits. The data points of the CRA tens and hundreds phase show an upward path, indicating steady improvement.

Al. After beginning CRA tens instruction, Al reached criterion after nine probes, ranging from 5 to 24 correct digits. There was an immediate change in the level of performance between baseline and CRA tens, and there were no overlapping data points between these phases. Al's average performance during the CRA tens phase was approximately 18 correct digits. The data points of the CRA tens phase show an upward path, indicating steady improvement.

After reaching criterion for the CRA tens phase and establishing a stable baseline for the CRA tens and hundreds phase, Al began CRA instruction for regrouping in the tens and hundreds places. He reached criterion after five probes, ranging from 10 to 23 correct digits. There was an immediate change in the level of performance between baseline and CRA and there were no overlapping data points between baseline and CRA tens and hundreds phases. Al's average performance during the CRA tens and hundreds phase was approximately 18 correct digits. The data points of the CRA tens and hundreds phase show an upward path, indicating steady improvement.

Walt. After beginning CRA tens instruction, Walt reached criterion after seven probes, ranging from 4 to 22 correct digits. There was an immediate change in the level of performance between baseline and CRA, and there were no overlapping data points between baseline and CRA tens phases. Walt's average performance during the CRA tens phase was approximately 16 correct digits. The data points of the CRA tens phase show an upward path, indicating steady improvement.

After reaching criterion for the CRA tens phase and establishing a stable baseline for the CRA tens and hundreds phase, Walt began CRA instruction for regrouping in the tens and hundreds places. He reached criterion after four probes, ranging from 12 to 24 correct digits. There was an immediate change in the level of performance between baseline and CRA, and there were no overlapping data points between baseline and CRA tens and hundreds phases. Walt's average performance during the CRA tens and hundreds phase was approximately 20 correct digits. The data points of the CRA tens and hundreds phase show an upward path, indicating steady improvement.

Ron. After beginning CRA tens instruction, Ron reached criterion after 13 probes, ranging from 8 to 20 correct digits. There was an immediate change in the level of performance between baseline and CRA, and there were no overlapping data points between baseline and CRA tens phases. Ron's average performance during the CRA tens phase was approximately 14 correct digits. The data points of the CRA tens phase show an upward path, indicating steady improvement.

After reaching criterion for the CRA tens phase and establishing a stable baseline for the CRA tens and hundreds phase, Ron began CRA instruction for regrouping in the tens and hundreds places. He reached criterion after six probes, ranging from 15 to 21 correct digits. There was an immediate change in the level of performance between baseline and CRA, and there were no overlapping data points between baseline and CRA tens and hundreds phases. Ron's average performance during the CRA tens and hundreds phase was approximately 18 correct digits. The data points of the CRA tens and hundreds phase show an upward path, indicating steady improvement.

Joe. After beginning CRA tens instruction, Joe reached criterion after 11 probes, ranging from 10 to 22 correct digits. There was an immediate change in the level of performance between baseline and CRA, and there were no overlapping data points between baseline and CRA tens phases. Joe's average performance during the CRA tens phase was approximately 16 correct digits. The data points of the CRA tens phase show an upward path, indicating steady improvement.

After reaching criterion for the CRA tens phase and establishing a stable baseline for the CRA tens and hundreds phase, Joe began CRA instruction for regrouping in the tens and hundreds places. He reached criterion after eight probes, ranging from 8 to 22 correct digits. There was an immediate change in the level of performance between baseline and CRA, and there were no overlapping data points between baseline and CRA tens and hundreds phases. Joe's average performance during the CRA tens and hundreds phase was approximately 16 correct digits. The data points of the CRA tens and hundreds phase show an upward path, indicating steady improvement.

Ann. After beginning CRA tens instruction, Ann reached criterion after five probes, ranging from 12 to 22 correct digits. There was an immediate change in the level of performance between baseline and CRA, and there were no overlapping data points between baseline and CRA tens phases. Ann's average performance during the CRA tens phase was approximately 19 correct digits. The data points of the CRA tens phase show an upward path, indicating steady improvement.

After reaching criterion for the CRA tens phase and establishing a stable baseline for the CRA tens and hundreds phase, Ann began CRA instruction for regrouping in the tens and hundreds places. She reached criterion after five probes, ranging from 18 to 21 correct digits. There was an immediate change in the level of performance between baseline and CRA, and there were no overlapping data points between baseline and CRA tens and hundreds phases. Ann's average performance during the CRA tens and hundreds phase was approximately 20 correct digits. The data points of the CRA tens and hundreds phase show an upward path, indicating steady improvement.

Maintenance

Four of the six students maintained their performance at or above the criterion level after 6 weeks of no instruction or practice opportunities. Ray's maintenance performance was 21 correct digits. Al's maintenance performance was 24 correct digits, a slight increase in performance. Walt's maintenance performance was 18 correct digits, a 3-digit decrease in performance. Ron's maintenance performance was 14 correct digits, a 6-digit decrease in performance. Joe's maintenance performance was 21 correct digits. Ann's maintenance performance was 21 correct digits.

Interrater Reliability

Interrater reliability for the probe scoring was calculated by dividing the number of agreements by the total number of agreements and disagreements. Approximately 80% of the students' probes were checked for interrater reliability. The total number of probes checked was 119. There were 116 agreements and 3 disagreements, for an interrater reliability of 97%. Interrater reliability was 100% for probes completed by Al (20/20), Joe (22/22), and Ann (20/20). Interrater reliability was 95% for Ray's (18/19), Walt's (18/19), and Ron's (18/19) probes.

Discussion

The purpose of this study was to investigate the effectiveness of CRA instruction in teaching subtraction with regrouping in the tens place and in the tens and hundreds places to students who are struggling in mathematics. Functional relations were demonstrated between CRA and the students' performance on subtraction problems with regrouping in the tens place and between CRA and the students' performance on subtraction problems with regrouping in the tens and hundreds places. All the students met criteria in the two regrouping phases—the tens place and the tens and hundreds place—writing at least 20 digits correct. There were no overlapping data points between baseline and CRA phases, and all students' data paths were increasing. Six weeks following the end of instruction, four students maintained their performance, completing subtraction problems with regrouping in the tens and hundreds place at the same rate or higher than criterion.

Before CRA instruction began, none of the students wrote more than one correct digit on the baseline probes. The students demonstrated declarative knowledge associated with place value, as demonstrated by the following: first, inserting columns and labeling the ones and tens columns when completing problems; second, verbally identifying the number of ones, tens, and hundreds within a given number during one-on-one achievement testing. However, the students also appeared to have a lack of conceptual understanding of place value and its role in computation. A common error lay in subtracting the ones in the minuend from the ones in the subtrahend (e.g., 31 – 19 = 28). Another error involved regrouping without taking from the tens place (e.g., 31 – 19 = 22). After CRA instruction for regrouping in the tens place began, students no longer made these errors. Ray, Walt, and Ann reached criterion for regrouping in the tens place before all instructional lessons were implemented. Ray reached criterion after the first abstract-level lesson. Walt reached criterion after the second abstract-level lesson. Ann reached criterion after the last representational-level lesson. Ron, Al, and Joe completed the entire lesson sequence before reaching criterion. All the students met the criterion, which was based on norms for children of the similar grade level (Fuchs & Fuchs, 2005).

It was expected that the students' learning during the CRA tens phase would transfer to more complex subtraction to some extent, thereby resulting in increased performance during the second baseline, when compared to the first. However, only three students demonstrated increased performances during the second baseline, and two of the increases were small: Walt's baseline performance increased to an average of 2 digits; Al's baseline performance increased to an average of 3 digits; and Ann's baseline performance increased to an average of 12 digits. The other four students appeared frustrated and bewildered by these baseline probes. When given a problem such as "201 – 99," students reverted to their old error patterns and solved the problem in ways such as the following: "201 – 99 = 298." Solving problems with a zero in the tens place is particularly difficult. After CRA instruction for regrouping in the tens and hundreds places, the students' performance improved. Five of the students reached criterion for the CRA tens and hundreds phase before all the instructional lessons were implemented. Ray and Ron reached criterion after the first abstract-level lesson. Al and Ann reached criterion after the last representational-level lesson. Walt reached criterion after the second representational-level lesson. The transfer effects from the first CRA phases may have resulted in more efficient learning during the second CRA phase.

Six weeks after instruction, four of the six students maintained their fluency in computing subtraction with regrouping in the tens and hundreds places. Although their rate decreased, Ron (14 correct digits) and Walt (18 correct digits) demonstrated mastery by accurately computing all problems attempted on the maintenance probe. All the students' confidence in their mathematics ability appeared to increase after the CRA interventions as demonstrated by (a) teachers' reports of increased volunteering during math class, (b) an increase in positive comments made during intervention sessions with the researcher, (c) an increase in their willingness to actively participate in sessions with the researcher.

Limitations

The research design presents a limitation to the study because CRA was not compared to another subtraction intervention. Therefore, there may be other interventions that are equally effective or more so, and further research using other designs is needed. Another limitation involves external validity, the degree to which the results can be generalized beyond the experimental conditions (Kazdin, 1982). A small group of students who varied somewhat in their levels of mathematics achievement demonstrated success with the program. However, more students with varied levels of functioning are needed in order to generalize the results to the larger populations. A second limitation lies in the generalization of the results across settings. This study was conducted in a small group setting, and it is not known whether the same effects would occur if implemented in a general education setting. Another limitation is the generality of the behavior change agent (Kazdin, 1982). Instruction was implemented by the researcher, not the students' teachers. The researcher was trained in this methodology as a classroom teacher and had experience using it with students with disabilities, which may have increased the treatment integrity. Yet, the researcher's presence may have influenced the students' performance by increasing their motivation to perform well for a new person. Furthermore, the results may be less practical or applicable because the classroom teachers did not implement the program. The teachers' implementation in a typical classroom would strengthen the results. To bridge the gap between research and practice, appropriately trained teachers should implement research procedures in a typical classroom. Another limitation is that students were not asked to discriminate between situations in which regrouping was required and when it was not. Future research should include discrimination measures in which students discriminate between addition and subtraction problems with and without regrouping.

Implications and Future Research

All the students had previous instruction in the area of subtraction with regrouping. The school used a reform-based mathematics curriculum that included an emphasis on conceptual understanding. CRA instruction differed from the approach of this curriculum on the basis of the level of teacher direction, practice opportunities until mastery, use of pictures and student-created drawings, and a mnemonic device. Further research might investigate (a) which aspect of CRA instruction had the greatest effect on student performance, (b) whether the combination of these strategies caused the change in student performance, and (c) whether the different individual needs of the students interacted with particular aspects of the instructional program

The findings also have implications for remediation with the general education setting. All the students were failing mathematics, and given the students' benchmark assessment performance, their teachers were concerned about their ability to pass the state-mandated high-stakes test at the end of the school year. The students' teachers had referred them to the student support team, and they were concerned about the potential presence of disabilities. Based on the results of the standardized individual assessments administered by the researcher, it is unlikely that some of the students would have met the state criteria for special education services using the discrepancy model for identification of learning disabilities (response to intervention is an option in the state, but procedures for its use have not been articulated). Even though some of the students demonstrated a discrepancy between their cognitive ability and computation performance, it is not known whether their mathematics achievement could be changed through classroom interventions. Participation in the CRA interventions provided all the students with an immediate and more intensive intervention than what was provided in the general education classroom. All the students responded to the intervention regardless of their standardized assessment scores and the presence of discrepant scores. Although the CRA interventions were provided outside the general education classroom, a pull-out model is not necessary for their implementation. The students were pulled out of a period devoted to practice and remediation. A restructuring of this period could have been devoted to more intensive instruction, perhaps using differentiated instructional procedures. The use of CRA interventions implemented within the general education classroom would provide students with an immediate and more intensive intervention, thereby preventing further mathematics failure. The materials were accessible items available in most classrooms, thus making this an inexpensive intervention. Future research is needed to investigate how interventions such as CRA might be included in a differentiated model of mathematics instruction. As response-to-intervention procedures are implemented per the Individuals With Disabilities Education Improvement Act (2004), research is needed in the area of mathematics to demonstrate the efficacy of interventions such as CRA across concepts and grade levels.

Further research is needed to investigate how long the treatment effects are maintained, given that maintenance was measured at only the 6-week mark. Also, further research may be needed to investigate whether practice is needed to maintain the treatment effects over time and, if so, how much. Further investigation of generalization is also needed given that this study did not involve programmed instruction for generalization, a component of other instructional programs (Deshler, Schumaker, Harris, & Graham, 1999; Schmidt, Deshler, Schumaker, & Alley, 1989). The measures in this study did not extend to problem solving and real-world application of skills, as called for by the National Council of Teachers of Mathematics (2000). Future research should involve instruction and measurement in this area.

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