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Precoding for egress reduction in DMT transmitters[1]

Gert Cuypers [2] Koen Vanbleu, Geert Ysebaert and Marc Moonen

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[1]This report is available by anonymous ftp from *ftp.esat.kuleuven.ac.be* in the directory *pub/sista/cuypers/reports/precoding.pdf*

[2]K.U.Leuven, Dept. of Electrical Engineering (ESAT), Research group SISTA, Kasteelpark Arenberg 10, 3001 Leuven, Belgium, Tel. 32/16/32 1927 , Fax 32/16/32 19 70, WWW: *http://www.esat.kuleuven.ac.be/sista*. E-mail: *gert.cuypers@esat.kuleuven.ac.be*. This research work was carried out at the ESAT laboratory of the Katholieke Universiteit Leuven, in the frame of the Belgian Programme on Interuniversity Attraction Poles, initiated by the Belgian Federal Science Policy Office IUAP P5/22 ('Dynamical Systems and Control: Computation, Identification and Modelling') and P5/11 ('Mobile multimedia communication systems and networks'), the Concerted Research Action GOA-MEFISTO-666 and the Research Project FWO nr.G.0196.02.

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Precoding for egress reduction in DMT transmitters

Gert Cuypers* *Member, IEEE,* Geert Ysebaert *Student Member, IEEE,* Koen Vanbleu *Student Member, IEEE,* Marc Moonen *Member, IEEE.*

Abstract

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Index Terms

Digital Subscriber Loop, Discrete Multitone, Multicarrier Modulation, Egress, Windowing

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3-COMM Signal Processing for Communications

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***Correspondence:** Gert Cuypers, Katholieke Universiteit Leuven - ESAT/SCD-SISTA, Kasteelpark Arenberg 10, B-3001 Leuven - Belgium, tel.: 32/16/32 19 27, fax: 32/16/32 19 70, email: *gert.cuypers@esat.kuleuven.ac.be*

I. Introduction

Discrete fourier transform (DFT) based modulation techniques [1] have become increasingly popular for high speed communications systems. In the wireless context, e.g. for the digital transmission of audio and video, this is usually referred to as orthogonal frequency division multiplexing (OFDM). Its wired counterpart has been dubbed discrete multitone (DMT), and is employed e.g. for digital subscriber loop (DSL), such as asynchronous DSL (ADSL) and very-high-bitrate DSL (VDSL).

A high bandwidth efficiency is achieved by dividing the available bandwidth into small frequency bands centered around carriers, often called tones. These carriers are individually modulated in the frequency domain, using the inverse DFT (IDFT). A cyclic prefix (CP) is added to the resulting block of time domain samples by copying the last few samples and putting them in front of the symbol [2]. This extended block is parallel-to-serialized, passed to a digital to analog (DA) convertor and then transmitted over the channel. At the receiver, the signal is sampled and serial-to-parallelized again. The part corresponding to the CP is discarded, and the remainder is demodulated using the DFT.

In case the order of the channel impulse response does not exceed the CP length by more than one, the linear convolution with the channel impulse response can be described as a circular one. Equalization can then be done very easily, using a one-tap frequency domain equalizer (FEQ) for each tone, correcting the phase shift and attenuation at each tone invididually. In case the channel impulse response length exceeds the CP length by more than one, the channel needs to be shortened using a time domain equaliser (TEQ). Alternatively, one can use a per-tone equalizer (PTEQ) [3], which provides an upper bound for the performance of any combination of TEQ+FEQ of the same length. Although the proposed technique in this article is irrespective of the used equalization, for the remainder of the text, a PTEQ is assumed. VDSL systems can also use a cyclic suffix (CS). The difference between the CP and CS is irrelevant for this article, therefore they will be treated as one (larger) CP. More important now, the presence of the CP influences the spectrum of the transmit signal, as will be shown later.

While DMT seems attractive because of its flexibility towards spectrum control, the high sidelobe levels associated with the DFT filter bank form a serious impediment, resulting in an energy transfer between in-band and out-of-band signals. This contributes to the crosstalk, e.g. between different pairs in a binder, especially for next-generation DSL systems using dynamic spectrum management (DSM), where the transmit band is variable [4]. Moreover, because the twisted pair acts as an antenna [5], there exists a coupling with air signals. The narrow band signals from e.g. an AM broadcast station can be picked up and, due to the side lobes, be smeared out over a broad frequency tone range. This problem has been recognised, and various schemes have been developed to tackle it ([6], [7], [8]).

On the other hand, the same poor spectral containment of transmitted signals makes it difficult to meet egress norms, e.g. the ITU-norm [9] specifies that the transmit power of VDSL should be lowered by 20dB in the amateur radio bands. Controlling egress is usually done in the frequency domain by combining neighbouring IDFT-inputs (such as in [10]), or equivalently, by abandoning the DFT altogether and reverting to other filter banks, such as e.g. in [11].

Another approach would be to apply an appropriate time domain window (see [12] for an overview) at the transmitter. Unfortunately, the application of a nonrectangular windows destroys the orthogonality between the tones and scatters the information. In [13], a VDSL system is proposed, where the window is applied to additional cyclic continuations of the DMT symbol to prevent distorting the symbol itself.

The technique proposed in this article avoids the overhead resulting from such symbol extension by applying the window directly to the DMT symbol without adding additional guard bands and which is then observed to correspond to a pre-coding operation at the transmitter. Obviously, this alters the frequency content at each carrier, such that a correction at the receiver is needed. While this compensation is generally nontrivial, we construct a class of windows that can be compensated for with only a minor amount of additional computations at the receiver. The technique can be readily applied to OFDM as well.

When investigating transmit windowing techniques, it is important to have an accurate description of the tansmit spectrum of DMT/OFDM signals. Although DMT and OFDM are commonplace, a lot of misconception and confusion seem to exist with regard to the nature of their transmit signal spectrum. When working on sampled channel data, the continuous character of the line signals is transparent, and usually neglected. However, it is important to realize that the behaviour in between the sample points can be of great importance [14]. The analog signal will generally exceed the sampled points reach, possibly leading to unnoticed clipping, causing out-of-band radiation.

Therefore, section II starts by describing the spectrum of the classical DMT signal. The novel windowing system is then presented in section III. Section IV covers the simulation results. Finally, in section V, conclusions are presented.

II. DMT TRANSMIT SIGNAL SPECTRUM

Consider the DMT system of **figure 1**, with (I)DFT-size N and a CP length ν, resulting in a symbol length $L = N + \nu$. The symbol index is k and $\mathbf{X}^{(k)} = [X_0^{(k)}...X_{N-1}^{(k)}]^T$ holds the complex subsymbols at tones $i, i = 0 : N-1$. In a baseband system, such as ADSL, the time-domain signal contains no imaginary component, requiring that $X_i^{(k)} = X_{N-i}^{(k)*}$. The corresponding discrete time-domain sample vector (at point α in fig.1) is equal to

$$\mathbf{x}^{(k)} = \left[x^{(k)}[0], ..., x^{(k)}[L-1]\right]^T, \tag{1}$$

$$x^{(k)}[n] = \frac{1}{\sqrt{N}} \sum_{i=0}^{N-1} X_i^{(k)} e^{j\frac{2\pi i}{N}(n-\nu)}, \ \ n = 0...L-1. \tag{2}$$

Note that the CP is automatically present, due to the periodicity of the complex exponentials. The total discrete time-domain sample stream $x[n]$ is obtained as a concatenation of the individual symbols $\mathbf{x}^{(k)}$. Interpolation of these samples yields the continuous time-domain signal $s(t)$, given by:

$$s(t) = \int_{\tau=-\infty}^{\infty} v(\tau - t) \left[\sum_{n=-\infty}^{\infty} \delta(t - nT) x[n] \right] d\tau, \tag{3}$$

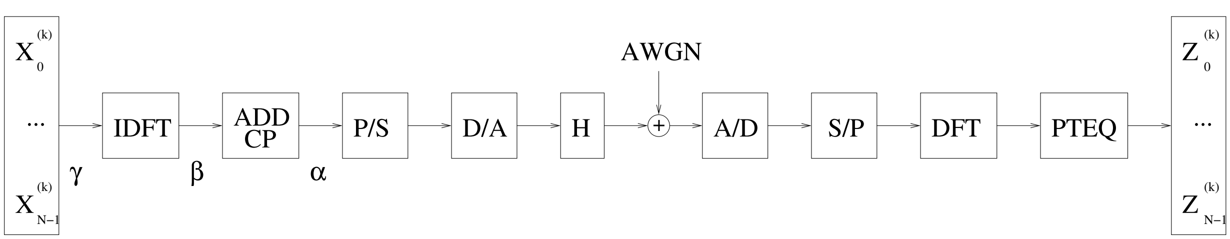

Fig. 1. The basic DMT system (refer to text for α to γ)

$$x[n] = \frac{1}{\sqrt{N}} \sum_{k=-\infty}^{\infty} \sum_{i=0}^{N-1} X_i^{(k)} e^{j\frac{2\pi i}{N}(n-\nu-kL)} w_{r,s}[n-kL], \tag{4}$$

with $\delta(t)$ the *dirac impulse* function, T the sampling period, $w_{r,s}[n]$ a (*rectangular, sampled*) discrete time domain window, $w_{r,s}[n] = 1$ for $0 \leq n \leq L-1$ and zero elsewhere, and $v(t)$ an interpolation function.

The shape of the DMT spectrum will now be derived by construction, starting from a single symbol with only one active carrier at DC. This result will be extended to a succession of symbols with all carriers excited. After this, the influence of time domain windowing will be investigated in section III.

Assume a single DMT symbol, having a duration $L = N + \nu$ in which only the DC component is excited (e.g. with unit value), in other words:

$$X_i^{(k)} = \begin{cases} 1 & i = 0,\, k = 0 \\ 0 & \text{elsewhere.} \end{cases} \tag{5}$$

The corresponding discrete-time domain signal is just a succession of L identical pulses. One can regard it as a multiplication of a rectangular window and an impulse train (**figure 2**). A rectangular window $w_r(t)$ extending from $t = 0$ to $t = L$ has a modulated sinc as its Fourier transform

$$W_r(f) = \frac{\sin(\pi L f)}{\pi f} . \exp(-j\pi L f). \tag{6}$$

The multiplication of $w_r(t)$ with pulses at a distance of T (sampling) results in the spectrum being convolved with a pulse train with period $\frac{2\pi}{T}$. The original sinc spectrum $W_r(f)$ and the

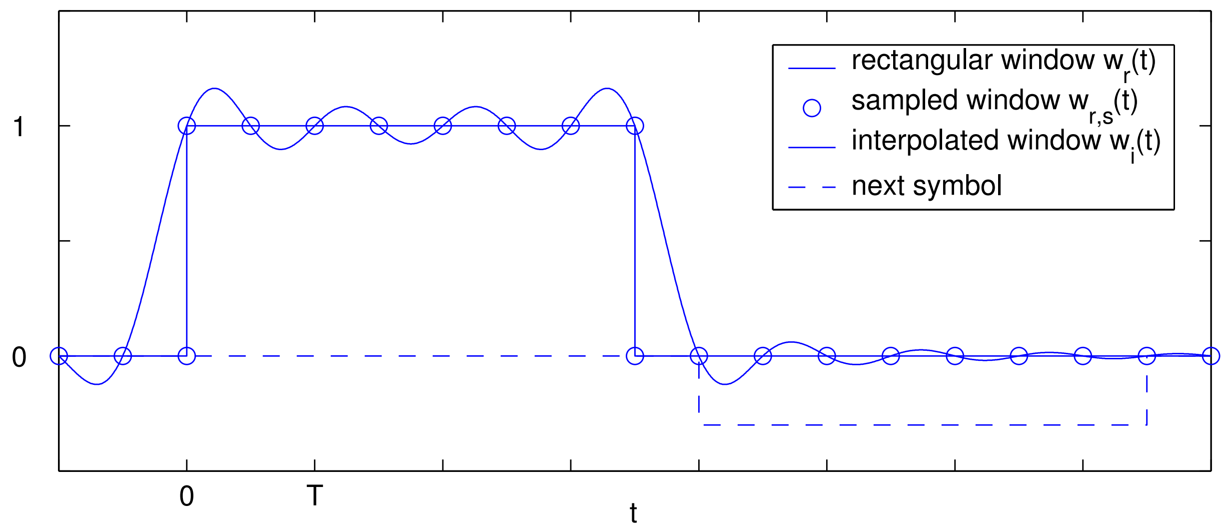

Fig. 2. The first (DC only) symbol as a sampled rectangular window, and a possible next symbol.

convolved one $W_{r,s}(f)$ are respresented in **figure 3**. As $w_{r,s}(t)$ is discrete now, $W_{r,s}(f)$ is periodical with a period $\frac{1}{T}$. Surprisingly, this can be expressed analytically as [15]

$$W_{r,s}(f) = \frac{\sin(\pi LTf)}{\sin(\pi Tf)} \exp(-j\pi Lf). \tag{7}$$

In literature, $W_{r,s}(f)$ is sometimes approximated by a sinc. While this approximation is suitable for most applications, it leads to an underestimate of the (possible egress-) energy in non-excited frequency bands. More specifically, from (7), it is clear that this leads to a maximal error of 3.9dB around $f = \pm\frac{1}{2T}$.

The final DA conversion consists of a lowpass filtering with $v(t)$, such that only the frequencies between $-\frac{1}{T}$ and $\frac{1}{T}$ are withheld. In the case of an ideal lowpass filter, this is equivalent to a time domain interpolation with a sinc function, resulting in $w_i(t)$, as shown in fig.2. Note that the continuous behaviour in between the sampled values is far from constant.

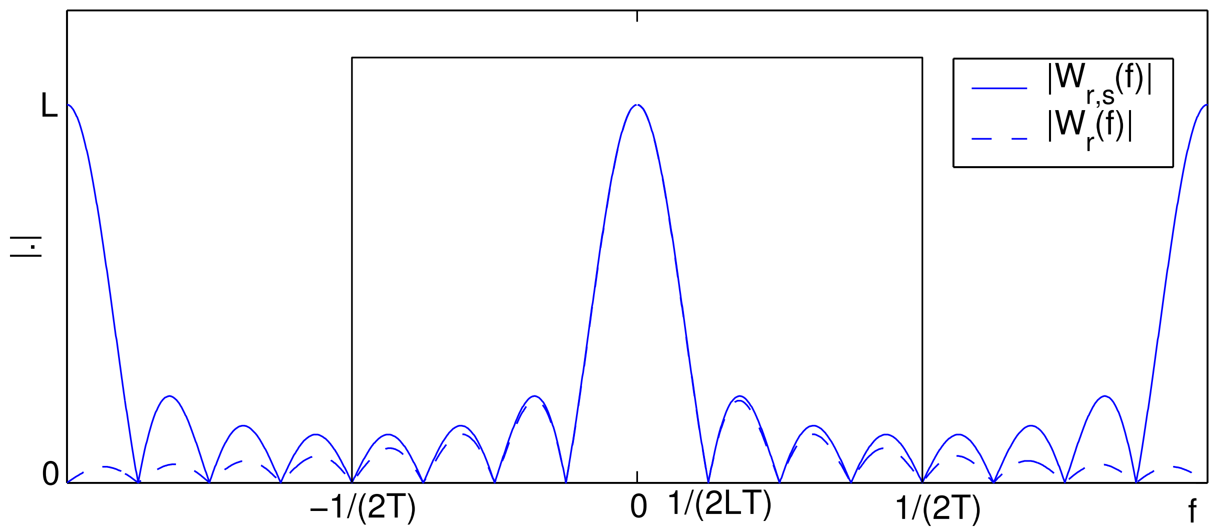

Fig. 3. Spectrum of the continuous and sampled rectangular window

This result can now be extended to describe a succession of multiple symbols ($k = 0, 1, \ldots$), with all tones ($i = 0, 1, \ldots, N-1$) excited. Assume that the $X_i^{(k)}$ have a variance $\mathcal{E}|X_i^{(k)}|^2 = \sigma_i^2$, and are uncorrelated. The power spectral density (PSD) $S(f)$ of $s(t)$ can now be described as

$$S(f) = \sum_{i=0}^{N-1} \sigma_i^2 |W_{r,s}(f - \frac{i}{NT}).V(f)|^2, \tag{8}$$

with $V(f)$ the frequency characteristic of the interpolation filter $v(t)$ (an example of this is shown in section IV).

Only in case the prefix is omitted ($\nu = 0$) and the variances $\sigma_i^2 = \sigma^2$ are equal for all tones (except DC and the nyquist frequency, having only $\frac{\sigma^2}{2}$), this spectrum is flat. In general, the CP results in a serrated spectrum. Indeed, because the symbols are lengthened by the CP, the PSD of the individual tones is narrowed compared to the intertone distance, such that 'valleys' appear in between the tone frequencies. This is demonstrated in **Figure 4**, where a detail of

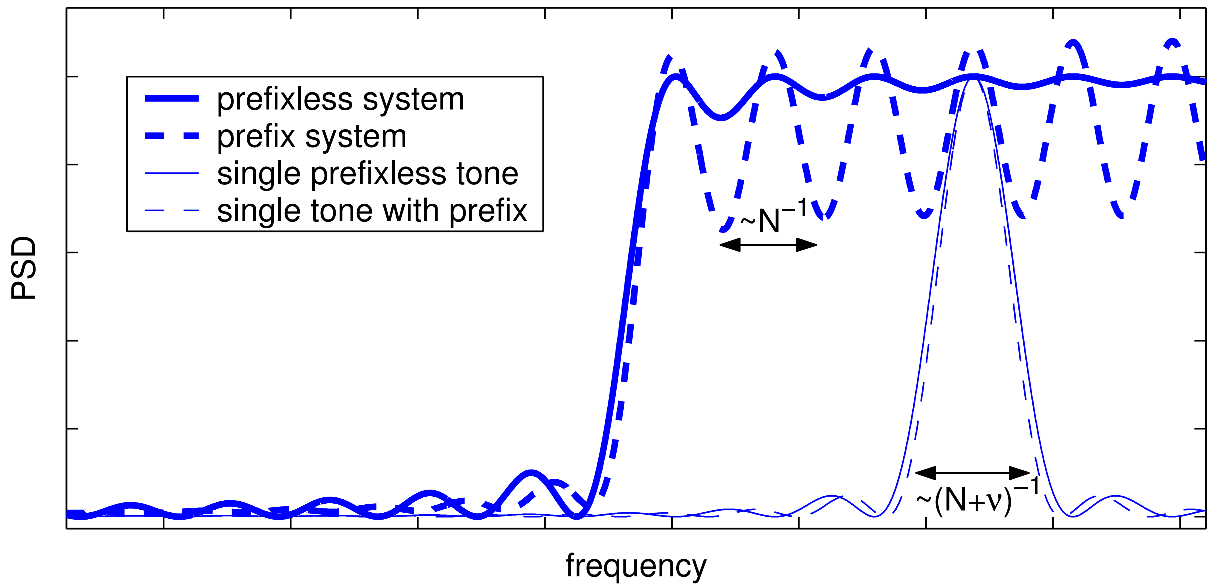

Fig. 4. The cyclic prefix in DMT systems leads to a serrated spectrum exhibiting valleys in between the tones

the spectrum of a prefixless DMT system ($\nu = 0$) is compared to a system using a prefix.

III. Transmitter windowing

Practical lowpass filters are not infinitely steep, such that some small signal components above the Nyquist frequency will remain. The out-of-band performance is largely dependent on the quality of these filters (and possible clipping in further analog stages). On the other hand, the in-band transitions (e.g. for suppression of VDSL in the amateur radio bands) can only be sharpened by the application of a window function on the entire time-domain symbol. To achieve this, the rectangular window $w_{r,s}[n]$ is replaced by another one having faster decaying side lobes.

$$\mathbf{w} = [w(0)...w(N+\nu-1)]^T \tag{9}$$

at point α in fig.1. In the next paragraph, we impose constraints on $\mathbf{w}$, to construct a class of window functions that are easy to compensate for at the receiver.

A. Derivation of the window structure

To ensure the cyclic structure of the transmitted symbols, needed for the easy equalization, we impose the cyclic constraint:

$$w(n) = w(n+N), n = 0, ..., \nu - 1. \tag{10}$$

As a result, instead of applying the window $\mathbf{w}$ at point α (fig.1), one can also apply the window

$$\begin{aligned} \mathbf{g} &= [g(0)...g(N-1)]^T & (11) \\ &= [w(\nu)...w(N+\nu-1)]^T & (12) \end{aligned}$$

at point β. Let $\mathbf{G}$ be a diagonal matrix with $\mathbf{g}$ as its diagonal. After defining $\mathcal{I}_N$ the IDFT-matrix of size N, the vector of windowed samples $\mathbf{x}_w^{(k)}$ at point β (before the application of the CP) can be written as:

$$\mathbf{x}_w^{(k)} = \underbrace{\begin{bmatrix} g(0) & 0 & \dots & 0 \\ 0 & g(1) & \ddots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & \dots & 0 & g(n-1) \end{bmatrix}}_{\mathbf{G}} \mathcal{I}_N \cdot \mathbf{X}^{(k)}. \tag{13}$$

Recalling that the product of a diagonal matrix and the IDFT-matrix can be written as the product of the IDFT-matrix and a circulant matrix, we can rewrite (13) as:

$$\mathbf{x}_w^{(k)} = \mathcal{I}_N \underbrace{\begin{bmatrix} c(0) & c(1) & \dots & c(N-1) \\ c(N-1) & c(0) & \ddots & c(N-2) \\ \ddots & & \ddots & \ddots \\ c(1) & \dots & & c(0) \end{bmatrix}}_{\mathbf{C}} \cdot \mathbf{X}^{(k)}. \tag{14}$$

The circulant matrix $\mathbf{C}$ is fully defined by its first row $\mathbf{c}^T$, with

$$\mathbf{c} = [c(0)...c(N-1)]^T = \mathcal{I}_N \cdot \mathbf{g} \tag{15}$$

i.e. IDFT of $\mathbf{g}$. The transition from (13) to (14) is more than mathematical trickery. Looking at the DMT-scheme incorporating transmitter windowing of **figure 5**, it becomes clear that the weighting operation in the time domain is equivalent to the multiplication of the subsymbol vector with a (pre-)coding matrix $\mathbf{C}$. Compensating for the window at the receiver is now identical to decoding in the frequency domain, which is done by multiplication with the decoding matrix $\mathbf{D} = \mathbf{C}^{-1}$, leaving the rest of the signal path (equalization etc.) unaltered. Thus, appealing windows should not only satisfy the constraint (10), but preferably also give rise to a sparse decoding matrix $\mathbf{D}$. We will now further investigate the nature of such windows.

Being the inverse of a circulant matrix, $\mathbf{D}$ is also circulant. We denote the first row of $\mathbf{D}$ as

$$\mathbf{d}^T = [d(0)...d(N-1)], \tag{16}$$

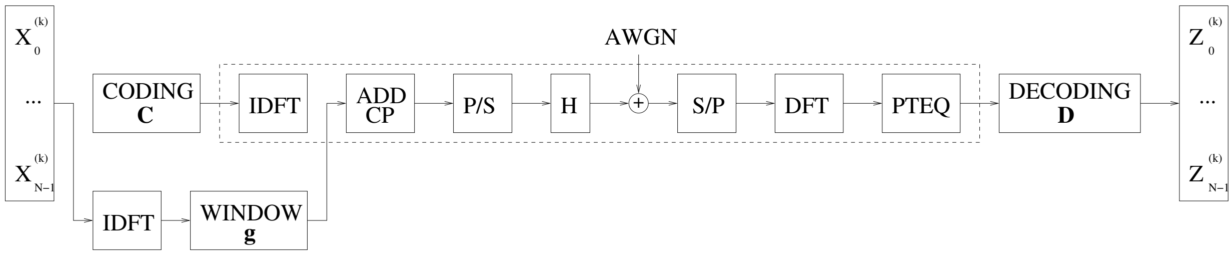

Fig. 5. Transmitter windowing translates to symbol precoding

and define $\mathcal{F}_N$ the DFT-matrix of size N, and

$$\mathbf{f} = [f(0), ...f(N-1)] = \mathcal{F}_N \cdot \mathbf{d}. \tag{17}$$

It is now possible to associate to $\mathbf{D}$ a diagonal matrix $\mathbf{F}$, having on its diagonal the elements of $\mathbf{f}$. The following relations now hold:

- $\mathbf{C}$ and $\mathbf{D}$ are circular, with $\mathbf{C}^{-1} = \mathbf{D}$, and have as a first row $\mathbf{c}^T$ and $\mathbf{d}^T$ respectively.
- $\mathbf{G}$ and $\mathbf{F}$ are diagonal, with diagonals $\mathbf{g}$ and $\mathbf{f}$.
- $\mathbf{c} = \mathcal{I}_N.\mathbf{g}$
- $\mathbf{d} = \mathcal{I}_N.\mathbf{f}$

From this, we can conclude that $\mathbf{F} = \mathcal{I}_N.\mathbf{D}.\mathcal{F}_N = \mathcal{I}_N.\mathbf{C}^{-1}.\mathcal{F}_N = (\mathcal{I}_N.\mathbf{C}.\mathcal{F}_N)^{-1} = \mathbf{G}^{-1}$. In other words,

$$g(n) = f(n)^{-1}, n = 0, ..., N-1. \tag{18}$$

Since $\mathbf{g}$ is real-valued, so is $\mathbf{f}$. Consequently $\mathbf{d}$ is the IDFT of a real-valued vector. Because of the IDFT's symmetry properties, the first and middle element of $\mathbf{d}$ are real-valued, and all other nonzero elements appear in complex pairs.

We can now distinguish three cases:

(i) a general $\mathbf{d}$ (non-sparse)

(ii) a maximally sparse $\mathbf{d}$

A simple choice for $\mathbf{d}$ (with only three non-zero elements) can be as follows

$$d(n) = \begin{cases} a & n=0 \\ b.e^{j\phi} & n=l \\ b.e^{-j\phi} & n=N-l \\ 0 & n \notin \{0, l, N-l\} \end{cases} , \tag{19}$$

with

$$\begin{array}{lll} a, b & \text{real} & \\ \phi & \text{real} \in \begin{bmatrix} -\pi & \pi \end{bmatrix} & , \\ l & \text{integer} \in \begin{bmatrix} 1 & N-1 \end{bmatrix} & \end{array} \tag{20}$$

so that

$$\mathbf{D} = \begin{bmatrix} a & \dots & b.e^{j\phi} & \dots \\ \vdots & \ddots & \ddots & \\ b.e^{-j\phi} & & & \\ & \ddots & & a \end{bmatrix} \tag{21}$$

is a sparse matrix. In practice, this means that $\mathbf{f}$ ($\mathbf{f} = \mathcal{F}_N.\mathbf{d}$) takes the form of a generalized *raised cosine* function. The different parameters influencing $\mathbf{f}$ are the pedestal height a, the frequency and amplitude of the sinusoidal part l and b, and ϕ determining the position of the peak(s).

(iii) intermediate structures

Obviously, multiple complex pairs can be included (hence $5, 7, \ldots$ non-zero elements in $\mathbf{d}$), possibly leading to more powerful windows. A tradeoff should be made between the window quality and the complexity of the decoding.

B. Determining the window parameters

Returning to the original goal of egress reduction, we now need to choose $\mathbf{w}$ such that an improved side-lobe characteristic is obtained. For the rectangular window, the width of the main lobe is equal to $\omega_s = \frac{\pi}{N+\nu}$. Note that this decreases with increasing CP length. As a general design criterion, we specify that the power outside the main lobe $\omega_s = \frac{\pi}{N+\nu}$ should be as low as possible. Assuming that the total energy is kept constant, this is equivalent to *maximising* the energy ρ within the main lobe [16], i.e. maximising

$$\rho = \int_0^{\omega_s} |W(e^{j\omega})|^2 \frac{d\omega}{\pi}, \tag{22}$$

$$\text{with } W(z) = \mathbf{w}^T \mathbf{e}(z), \tag{23}$$

$$\text{and } \mathbf{e}(z) = \begin{bmatrix} 1 & z & \ldots & z^{N+\nu-1} \end{bmatrix}^T \tag{24}$$

(25)

under unit-energy constraint

$$\mathbf{w}^T \cdot \mathbf{w} = 1. \tag{26}$$

Equation (22) can be written as

$$\rho = \mathbf{w}^T \left[\int_0^{\omega_s} \mathbf{e}(e^{j\omega})^* \mathbf{e}(e^{j\omega}) \frac{d\omega}{\pi}\right] \mathbf{w} \tag{27}$$

$$= \mathbf{w}^T \cdot \mathbf{Q} \cdot \mathbf{w}, \tag{28}$$

where $\mathbf{Q}$ has (m,n)th entry

$$q_{\text{m n}} = \int_0^{\omega_s} \cos(m-n)\omega \frac{d\omega}{\pi}, \quad 0 \leq m, n \leq N+\nu-1 \tag{29}$$

$$= \frac{\sin((m-n)\omega_s)}{(m-n)\pi}. \tag{30}$$

$$\tag{31}$$

To enforce the cyclic structure (10), (28) is transformed into a problem in $\mathbf{g}$. After defining

$$\mathbf{P} = \begin{bmatrix} \mathbf{O}_{\nu \mathbf{x} (N-\nu)} & \mathbf{I}_{\nu \mathbf{x} \nu} \\ \mathbf{I}_{N \mathbf{x} N} \end{bmatrix}, \tag{32}$$

with $\mathbf{O}_{m\mathbf{x}n}$ and $\mathbf{I}_{m\mathbf{x}n}$ the all-zero and unity matrices of size $m\mathbf{x}n$, (28) can be written as:

$$\rho = \mathbf{g}^T \cdot \mathbf{P}^T \cdot \mathbf{Q} \cdot \mathbf{P} \cdot \mathbf{g}, \tag{33}$$

and the unit norm constraint becomes:

$$\mathbf{g} \cdot \mathbf{P}^T \cdot \mathbf{P} \cdot \mathbf{g} = 1. \tag{34}$$

We can now again distinguish three cases

(i) a general $\mathbf{d}$ (non-sparse)

The maximization of (33) satisfying (34) can be rewritten as a generalized eigenvalue problem:

$$(\mathbf{P}^T\mathbf{QP})\mathbf{g} = \lambda(\mathbf{P}^T\mathbf{P})\mathbf{g}, \tag{35}$$

and the optimal vector $\mathbf{g}_{opt}$ is equal to the eigenvector corresponding to the largest eigenvalue of $(\mathbf{P}^T\mathbf{P})^{-1}\mathbf{P}^T\mathbf{QP}$. The optimal $\mathbf{w}_{opt}$ is now equal to

$$\mathbf{w}_{opt} = \mathbf{P}.\mathbf{g}_{opt}. \tag{36}$$

Note that $\mathbf{w}_{opt}$ is only dependent on the (chosen) width of the main lobe.

(ii) a maximally sparse $\mathbf{d}$

To obtain the optimal *sparse* decoding matrix $\mathbf{D}$, we have to determine the parameters a, b, ϕ and l from (19) optimizing (28). We propose $l = 1$, and ϕ such that $\mathbf{w}$ is symmetrical (i.e. $\phi = -\frac{\nu\pi}{N}$). Due to the unit-energy constraint, only one of a or b can be chosen freely. This leads to a one-dimensional optimization problen in either a or b. Because only three non-zero coeffi cients are present in $\mathbf{d}$, we denote this optimal (sparse) solution as $\mathbf{w}_{3,opt}$

(ii) intermediate structures

For the intermediate structures, multiple $(5,\ 7,\ \ldots)$ nonzero elements are present in $\mathbf{d}$, leading to $\mathbf{w}_{5,opt}$, $\mathbf{w}_{7,opt}$,. . . . These structures offer a tradeoff between egress reduction and computational complexity. The corresponding optimal windows are found using numerical optimization.

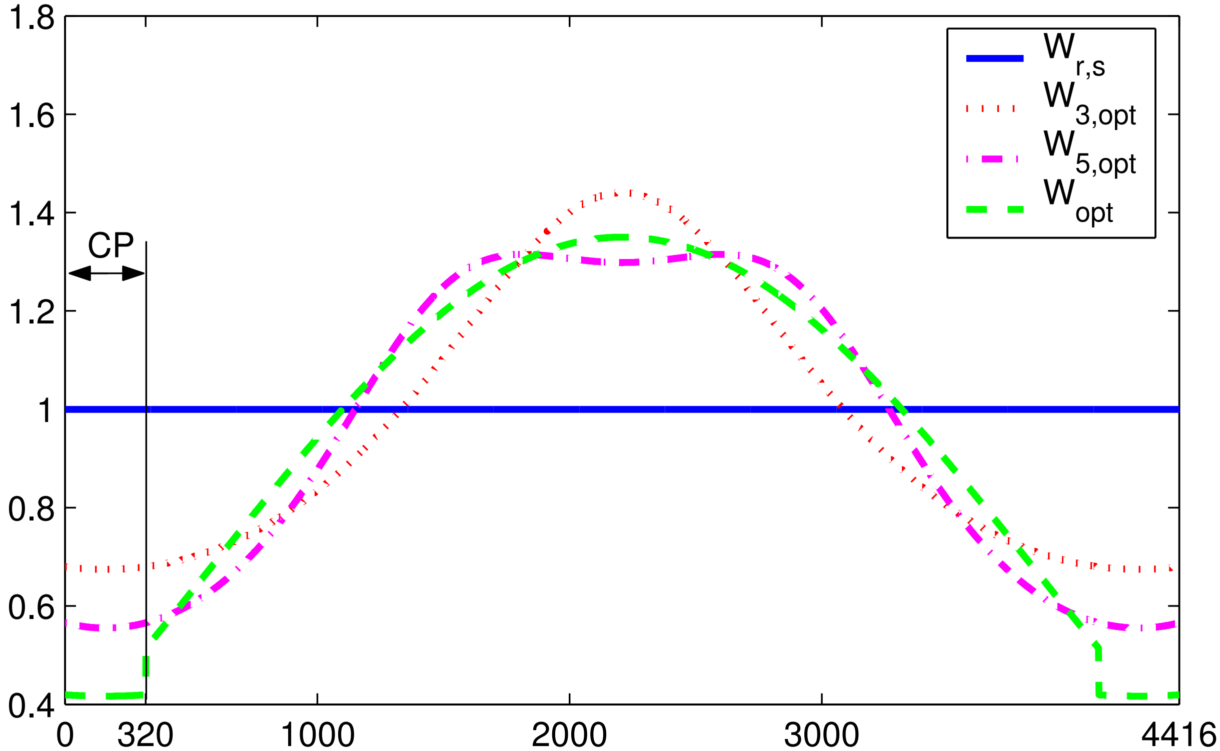

Fig. 6. The rectangular window, $\mathbf{W}_{1,opt}$, $\mathbf{W}_{2,opt}$, $\mathbf{W}_{opt}$

IV. Simulation results

Three windows are presented: the minimal window $\mathbf{w}_{3,opt}$ described by 3 non-zero coefficients in $\mathbf{d}$ (eq. 19), a slightly more complex window $\mathbf{w}_{5,opt}$, for which $\mathbf{d}$ contains 5 non-zero coefficients, and the optimal window $\mathbf{w}_{opt}$ based on (35) and with non-sparse decoding.

The simulations have been done for a VDSL system (some results for ADSL are given in [17]). There are 2048 carriers ($N = 4096$), the prefix length is $CP = 320$ ([18], pp 22). The sampling frequency is 17664 kHz, the tone spacing 4.3125 kHz, such that a symbol rate of 4 kSymb/s is reached.

In **figure 6** the shape of the rectangular window, $\mathbf{w}_{3,opt}$, $\mathbf{w}_{5,opt}$ and $\mathbf{w}_{opt}$ are shown. To

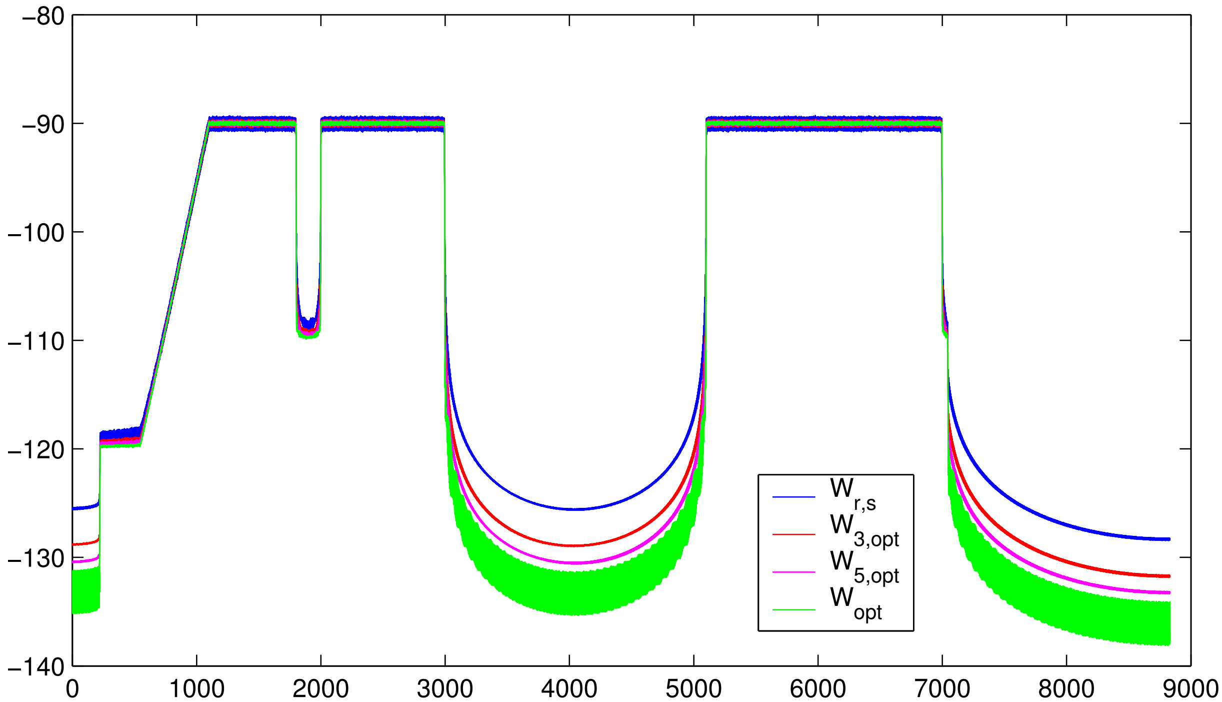

Fig. 7. Spectrum of the rectangular window, $\mathbf{W}_{1,opt}$, $\mathbf{W}_{2,opt}$ and $\mathbf{W}_{opt}$

illustrate the egress-reduction, the spectra are compared for a VDSL upstream based on the power spectral density mask Pcab.P.M1 from [9]. The most important features are that the frequencies between 3000 kHz and 5200 kHz and above 7050 kHz are reserved for upstream communications ([18], pp 17), and the power is lowered by 20 dB in the amateur radio bands, from 1810 kHz to 2000 kHz, and from 7000 kHz to 7100 kHz ([9], pp 35). The results are shown in **figure 7**, and **figure 8**, showing a detail around the first amateur radio band. It is interesting to note that the spectrum is less serrated (the 'valleys' in between the tones are less pronounced). Moreover, there is a significant egress reduction, especially around the band edge (about 5 dB), achieved without adding any additional cyclic extension. Obviously, it would be possible to combine this method with such extensions.

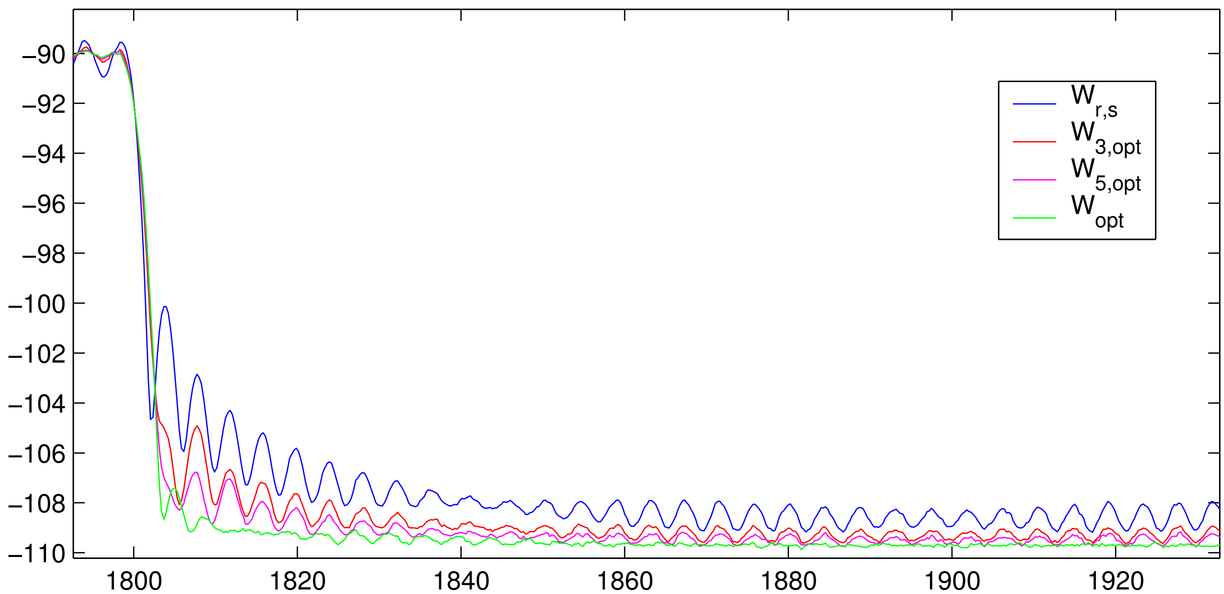

Fig. 8. Spectrum of the rectangular window, $\mathbf{W}_{1,opt}$, $\mathbf{W}_{2,opt}$ and $\mathbf{W}_{opt}$ (detail of amateur radio band)

V. Conclusion and further work

A novel transmitter windowing technique for DMT has been proposed, which does not rely on an additional cyclic extension of the symbol. This inevitably introduces a distortion of the signal. For a special class of windows, this distortion can be described as a pre-coding operation for which the decoding at the receiver can be done easily. In the simplest case, the window function can be described as the pointwise inversion of a raised cosine window. More complex windows can also be described, but the advantage of the easy decoding will gradually diminish. Furthermore, formulas are provided to calculate the optimal window, and this is illustrated for the VDSL case. Future work will focus on a selective windowing of the tones in the vicinity of an unauthorised band, and the combination of the proposed technique with windowing in a cyclic extension of the symbol. Also the tradeoff between decoder complexity and egress should be further studied, as well as the interaction between the transmitter window and the channel equalizer at the receiver.

REFERENCES

[1] S.B. Weinstein and E.M. Ebert, "Data transmission by frequency-division multiplexing using the discrete fourier transform," *IEEE transactions on communications*, vol. 19, no. 5, pp. 628–634, 1971.

[2] A. Peled and A. Ruiz, "Frequency domain data transmission using reduced computational complexity algorithms," in *Proceedings ICASSP-80 (IEEE International Conference on Acoustics, Speech and Signal Processing)*, 1980, pp. 964–967.

[3] K. Van Acker, G. Leus, M. Moonen, O. van de Wiel, and T. Pollet, "Per tone equalization for dmt-based systems," *IEEE transactions on communications*, vol. 49, no. 1, pp. 109 –119, 2001.

[4] K.B Song, S.T. Chung andG. Ginis, and J.M. Cioffi, "Dynamic spectrum management for next-generation dsl systems," *IEEE Communications Magazine*, vol. 40, no. 10, pp. 101–109, 2002.

[5] R. Stolle, "Electromagnetic coupling of twisted pair cables," *IEEE J. Sel. Areas Comm*, vol. 20, no. 5, pp. 883–892, 2002, zegt ook dat vooral de common mode verantwoordelijk is voor egress. Gebruiken in combinatie met ARRL rfi handboek, dat zegt dat je CM kan onderdrukken door het gebruik van ferrietkernen.

[6] A.J. Redfern, "Receiver window design for multicarrier communication systems," *IEEE J. Sel. Areas Comm*, vol. 20, no. 5, pp. 1029–1036, 2002.

[7] S. Kapoor and S. Nedic, "Interference suppression in dmt receivers using windowing," in *Proc. ICC*, 2000, pp. 778–782.

[8] G. Cuypers, G. Ysebaert, M. Moonen, and P. Vandaele, "Combining per tone equalization and windowing in dmt receivers," in *Acoustics, Speech, and Signal Processing, 2002 IEEE International Conference on*, 2002, vol. 3, pp. 2341–2344.

[9] ETSI, "Transmission and multiplexing (tm); access transmission systems on metallic access cables; very high speed digital subscriber line (vdsl); part 1: Functional requirements," TS 101 270-1 V1.2.1 (1999-10), 1999.

[10] K. W. Martin, "Small side-lobe filter design for multitone data-communication applications," *IEEE trans. Circuits Systs.*, vol. CAS-45, no. 8, pp. 1155–1161, 1998.

[11] G. Cherubini, E. Eleftheriou, and S. Ölçer, "Filtered multitone modulation for vdsl," in *IEEE Global Telecommunications Conference, GLOBECOM*, 1999, vol. 2, pp. 1139–1144.

[12] F. J. Harris, "On the use of windows for harmonic analysis with the discrete fourier transform," *Proc. IEEE*, vol. 66, no. 1, pp. 51–83, 1978.

[13] F. Sjöberg, R. Nilsson, M. Isaksson, P. Ödling, and O.O. Börjesson, "Asynchronous zipper," in *Proc. ICC*, 1999, pp. 231–235.

[14] H. Minn, C. Tellambura, and V.K. Bhargava, "On the peak factors of sampled and continuous signals," *IEEE Communications Letters*, vol. 5, no. 4, pp. 129–131, 2001, Gedrag van continue signalen tussen de samples in.

[15] A.D. Poularikas, *The handbook of formulas and tables for signal processing*, p. 3.14, CRC Press/IEEE, 1998.

[16] P.P. Vaidyanathan, *Multirate systems and filter banks*, pp. 50–52, Prentice Hall, first edition, 1993.

[17] G. Cuypers, K. Vanbleu, G. Ysebaert, and M. Moonen, "Egress reduction by intra-symbol windowing in dmt-based transmitters," *Acoustics, Speech, and Signal Processing, 2003 IEEE International Conference on*, vol. 4, pp. IV 532–535, 2003.

[18] ETSI, "Vdsl: Transceiver specification," TS 101 270-2 V1.1.1 (2001-02), 2001.

LIST OF FIGURES