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Noise Models for Sinusoidal Trajectories Composing Sinogram Data in Positron Emission Tomography

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ABSTRACT

Projection data in Positron Emission Tomography (PET) are acquired as a number of photon counts from different observation angles. Positron decay is a random phenomenon that causes undesirably high variations in measured sinogram appearing as noise. Filtering of the raw data in a stackgram domain is a new technique capable of producing the reliable estimates of underlying activity. However, to facilitate accurate denoising procedure of the emission data, signals constituting the stackgram should be properly modelled. In this study, the choice of appropriate noise model for them is considered. We will demonstrate that the general two-parameter distributions can be employed to properly characterize the source of errors coming from the data measurement process and from the decay process of the positron emitting tracer.

1. INTRODUCTION

Positron Emission Tomography (PET) imaging technique is intended to provide information about biochemical processes taking place in a living tissue over time. It is based upon detection of spatial distribution of a radiating source injected in an organ of interest during the data acquisition period. Raw projection data, known as sinogram are obtained as a number of emitted photon counts registered around the subject with array of detectors repositioned at different angles. Primary goal of PET imaging is to reconstruct the 2-D estimate of the radioactivity distribution in the tissue from projections. In nuclear medicine, such distribution can provide anatomical information or functional behaviour of the tissue [1]. Noise is a very important issue that affects the performance of the reconstructed images. Originating from different sources, mainly from statistical nature of the decay process, it deteriorates the overall quality of the measured data and leads to biases from actual radioactivity concentration values in certain body regions. Diverse noise reduction methods have been developed recently. Most of them employs either sinogram domain filtering or noise removal during image reconstruction. However, these methods often tend to loss in resolution and blurring. To overcome these problems novel stackgram domain approach for emission tomography (ET) data processing [2] allowing representation of 2-D sinogram by its corresponding 3-D stack of projection profiles at different observation angles was suggested. 1-D sinusoidal signals parallel to the angular axis of the stackgram can be effectively separated from noise and transformed back to the sinogram prior to the image reconstruction process.

To grasp an idea about the proper estimator for signals constituting the stackgram data, we should, at first, focus on finding the correct noise model for them. It is a common practice in emission tomography to model count data as an outcome from the Poisson distribution. In this paper, we show that this fact is violated for the sinusoidal curves, which naturally possess the same inherent features as the original sinogram. So, to deal with this problem, we adapt two distributions from the family of dispersion models [3] with positional and dispersion parameter as alternative solutions for PET data modelling. To justify our assumptions we conduct the chi-square goodness-of-fit test in order to produce numerical comparison of observed data with expected values.

2. STACKGRAM DOMAIN

The measured data in tomography are appeared as sinogram. The sinogram function $g(l,\theta)$ is the one-dimensional projection of a function $f(x,y)$ at angle θ and distance l [4]. In other words, each point in (x,y) domain maps into a sinusoid in the (l,θ) plane. The purpose of the stackgram representation is to extract every sinusoidal trajectory and to process it independently. According to [2], mapping from the sinogram domain into the stackgram domain is defined as

$$h(x,y,\theta) = Sg = g(x\cos\theta + y\sin\theta, \theta), \qquad (1)$$

where $x,y \in \Omega \subset \mathbb{R}^2$, $\theta \in [0,\pi)$ and S is the stack-operator, which maps a function of (l,θ) coordinates into a function $h(x,y,\theta)$ called stackgram (see Fig.1). 1-D signals $h_{locus}(\theta)$ are parallel to the vertical axis θ of the stackgram and are called *locus*-signals. The problem of reconstructing the sinogram from the stack of locus-curves is associated with the inverse stack-operator, which can be summarized as

$$g(l,\theta) = S^{-1}h = R_\theta h(x,y,\theta), \qquad (2)$$

where R_θ is the "normalized" Radon transform of function $h(x,y,\theta)$ at one arbitrary angle $\theta \in [0,\pi)$. This can be written as

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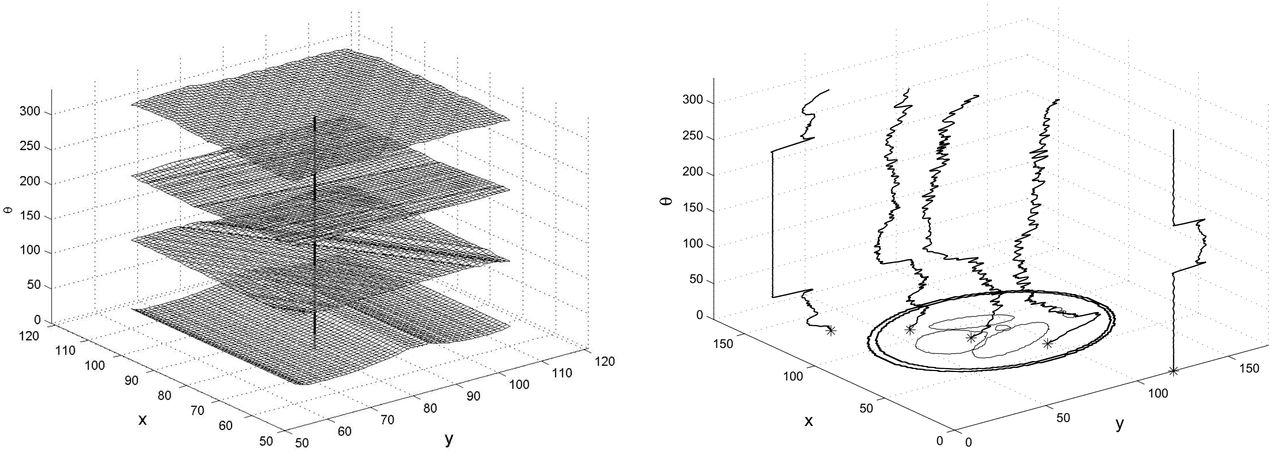

(a) **(b)**

Fig.1. Stackgram visualization. **(a)** Four arbitrary (scaled) layers of the stackgram $h(x, y, \theta)$ obtained from Shepp-Logan phantom. 1-D discrete signals taken parallel to the θ axis as denoted by black vertical line are defined as locus-signals. **(b)** Shepp-Logan phantom contour is superimposed on the first layer of the stackgram. Selected locus-trajectories are superimposed on their corresponding position on the (x, y) image plane marked by asterisk.

$$R_\theta f = \frac{\int\int_\Omega f(x,y)\delta(x\cos\theta + y\sin\theta - l)dxdy}{\int\int_\Omega \delta(x\cos\theta + y\sin\theta - l)dxdy}, \quad (3)$$

if $x\cos\theta + y\sin\theta = l$. Otherwise, right-hand side of equation (3) equals to zero.

In practice, measured 2-D sinogram data are discrete and limited in size. Hence, the transformation from sinogram to stackgram and backwards is performed with the discrete reversible implementation of the stack-operator. Locus-signals in this case are finite vectors with values corresponding to the number of detected photon counts recorded at an equally spaced range of projection angles.

3. COUNT DATA MODELS

The Poisson distribution is conventional and most widely used for modelling of a random number of photon counts per a time interval. If the discrete variable Y is Poisson distributed with parameter λ, its density is given by

$$P(Y = y|\lambda) = \frac{\lambda^y}{y!}e^{-\lambda}, \quad y = 0, 1, 2\ldots, \quad (4)$$

where mean value λ is equal to variance. However, it is a rare case when the data utterly exhibits Poisson nature. Very frequently a sample variance is significantly larger then the mean. This feature is called overdispersion and use of Poisson model in this case can lead to a poor fit and to a loss of efficiency. Very few attempts were undertaken in PET research studies in order to tackle the aforementioned problem. In [5] authors have tried to simulate overdispersed data by adding the Poisson noise plus some supplementary Gaussian part with zero mean and adjusted variance to the original intensity values.

Several more general two-parameter distributions that are able to accommodate overdispersion were considered in numerous count data modelling applications [6],[7],[8]. Probably, the most popular distribution is the *negative binomial* (NBD) that can be specified by its density as in equation (5) on the bottom of the page. Its mean is equal to λ and variance to $\lambda + \alpha\lambda^2$. An alternative model introduced by Consul and Jain [9] in 1973 is based on the *generalized Poisson distribution* (GPD) described by the probability mass function given by equation (6), with mean $\lambda(1-\alpha)^{-1}$ and variance $\lambda(1-\alpha)^{-3}$. For both models estimation can be conducted by maximum likelihood estimation and both reduce to the Poisson case when the dispersion parameter α equals to zero.

$$P(Y = y|\lambda, \alpha) = \frac{\Gamma(y + \alpha^{-1})}{\Gamma(y+1)\Gamma(\alpha^{-1})}\left(\frac{\lambda}{\lambda + \alpha^{-1}}\right)^y \left(\frac{\alpha^{-1}}{\lambda + \alpha^{-1}}\right)^{\alpha^{-1}}, \quad y = 0, 1, 2\ldots \quad (5)$$

$$P(Y = y|\lambda, \alpha) = \frac{\lambda(\lambda + y\alpha)^{y-1}}{y!}e^{(-\lambda - y\alpha)}, \quad \text{for } \lambda > 0,\ y = 0, 1, 2\ldots \quad (6)$$

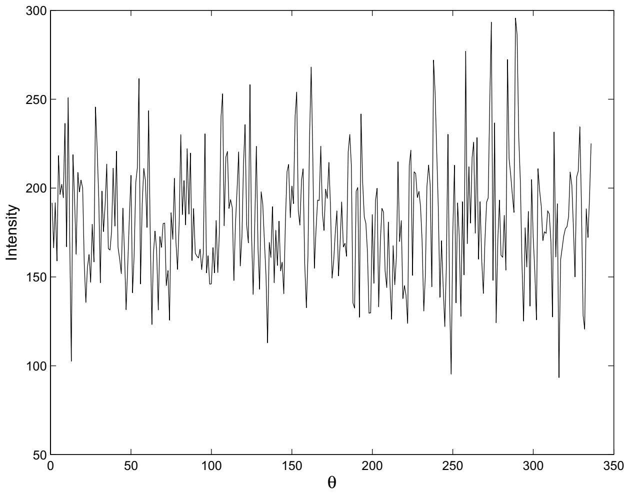

Fig.2. Typical locus-signal taken from the center of the FOV.

4. PRACTICAL EVALUATION OF NOISE MODELS

4.1 Data description

To examine how well the alternative statistical models fit the actual test stackgram data we have used a cylindrical homogeneous phantom of 20 cm in diameter, filled with a uniform solution of fluorine-18 (18F) and water. It was placed in the center of the *field of view*(FOV) and a static emission scan was acquired for 10 minutes. Forward discrete stack-operator was used to transform the sinogram with measured attenuation correction of the size 281 bins by 336 angles to the stackgram domain. Let $\mathbf{G}$ be the sinogram matrix and $\mathbf{H}$ be the respective stackgram of size $375 \times 375 \times 336$. Increased radial bin size is explained by three-pass rotation algorithm used in the stack-operator. As a result, x and y dimensions of the output stackgram are by 1/3 larger then the bin size of respective sinogram.

For simulation purposes we have considered the central part of the cylinder phantom of the to size 30 by 30 pixels, to be referred as the *region of interest* (ROI). Since the area limited by the FOV is supposed to be uniform and flat, locus-signals inside it have approximately the same mean count values without any sharp variations. In such a way they are perfectly suited for analyzing the noise models in PET measurements. A set $\mathbf{H_1} = \{h_1, h_2, \ldots, h_N\}$ of neighbouring sinusoidal trajectories corresponding to the spatial location of ROI was chosen from the stackgram $\mathbf{H}$. Length of each 1-D signal h_i corresponded to the number of projection angles through the observed object. Representation of a typical locus-wave taken from test set is depicted on Fig.2.

4.2. Chi-square goodness-of-fit test

A number of numerical tests exists in order to evaluate whether a statistical model fits well to the empirical data. The chi-square goodness-of-fit test [10] is the one that addresses this issue. It determines how consistent are the observed samples to the expected values, taken from the particular hypothesized distribution. The advantage of the test is that it can be applied to any univariate continuous or discrete distribution for which one can calculate the cumulative distribution function. More formally, the chi-square test statistic for the data divided into k non-empty bins is defined as

$$\chi^2 = \sum_{i=1}^{k} (O_i - E_i)^2 / E_i, \tag{7}$$

where O_i is the observed frequency for bin i and E_i is the expected frequency for bin i. Then, the hypothesis that the data are from a specified distribution is rejected if

$$\chi^2 > \chi^2_{(\alpha, k-c)}. \tag{8}$$

χ^2 is the upper critical value from the chi-square distribution with $k - c$ degrees of freedom and significance level α, where c is the number of estimated parameters plus 1.

4.3. Results of the test simulation

The performance of four noise models considered in this paper was evaluated using the chi-square test. We hypothesized sequentially that the locus-signals obtained from the cylinder phantom are drawn from the Poisson, mixture of Gaussian and Poisson, generalized Poisson and negative binomial distributions, respectively. To test the hypothesis each signal from the test set was grouped into 16 categories, so that the expected bin value was ≥ 5. To calculate the model parameters, we employed the maximum likelihood technique. The number of accepted and rejected original assertions for each distribution at different significant levels is listed in Table 1. As we can see from the table below, none of the test signals taken from the stackgram domain can be approximated with the Poisson distribution. In other words the probability of observing the null hypothesis is below the significance level, or roughly speaking tends to zero. Mixture model of the Poisson and Gaussian distributions also did not give satisfactory results since a significant part of the test dataset was poorly characterized by it. On the contrary, alternative models exhibited equally appropriate fits. About 93-94 % of data were quite consistent with the negative binomial and generalized Poisson distributions.

5. CONCLUSIONS

In PET data analysis the question of great importance is to produce an exact and precise quantitative distributions of radioactivity concentration in the region of interest. However, noise originating from different sources deteriorates significantly the overall image quality. Modelling of data is a fundamental issue helping to develop efficient computational methods for diminishing the drawbacks caused by statistical nature of photon detections in PET imaging. Previously developed stackgram domain method of sinogram

Table 1. Results of chi-square testing for different noise models.

Null hypothesis	Poisson		Generalized Poisson		Negative Binomial		Poisson + Gaussian	
	$\alpha = 0.01$	$\alpha = 0.05$	$\alpha = 0.01$	$\alpha = 0.05$	$\alpha = 0.01$	$\alpha = 0.05$	$\alpha = 0.01$	$\alpha = 0.05$
Number of rejected null hypotheses	900	900	20	65	15	58	81	212
Number of accepted null hypotheses	0	0	880	835	885	842	819	688

filtering is a promising technique for processing and filtering of PET data. In this research work, we have tried to conduct an adequate modelling of signals composing the stackgram, since the customary single-parameter Poisson distribution can hardly approximate them due to the presence of overdispersion. Here, more general family of two-parameter distributions allowing for extra-Poisson variation have been examined and described in order to model the process that might lie behind the formation of 1-D locus-curves. Their characteristics have been studied and the ability to capture the positron emission process has been investigated through the simulation study.

In summary, we point out that results produced by comparative tests suggest that more reasonable and wise way of imitating the noise source in PET is to use the generalized versions of the conventional noise model. Briefly mention here some indications for the future research work. First of all, alternative models proposed in this study should be also tested on dynamic PET phantom, where activity concentration varies with time. Second, based on the derived models it would be beneficial to construct an optimal intensity estimator for photon counts in stackgram domain. Robustness to different statistical noise levels and unbiasedness are those desired features that the sought approach should satisfy.

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REFERENCES

[1] S. Alenius and U. Ruotsalainen, “Bayesian image reconstruction for emission tomography based on median root prior,” in *European Journal of Nuclear Medicine*, vol.24, no.3, pp. 258-265.

[2] A.P. Happonen and S. Alenius, “Sinogram filtering using a stackgram domain,” in *Proceedings of the Second IASTED International Conference: Visualization, Imaging and Image Processing*, Malaga, Spain, pp. 339-343, Sep. 2002.

[3] B.Jørgensen, *The theory of dispersion models*. Chapman&Hall, London, 1997.

[4] A. Jain, *Fundamentals of digital image processing*. Englewood Cliffs, NJ , Prentice-Hall International, 1989.

[5] S.S. Furuie *et al*, “A methodology for testing statistically significant differences between fully 3-D PET reconstruction algorithms,” in *Physics in Medicine and Biology*, vol.39, pp. 341-354.

[6] J.A. Hausman, B.H. Hall, and Z. Griliches, “Econometric models for count data with an application to the patents-R&D relationship,” in *Econometrica*, vol.52, no.4, pp. 909-938, 1984.

[7] A.C. Cameron, P.K. Trevedi, *Regression analysis of count data*. Cambridge, UK: Cambridge University Press, 1998.

[8] R.K. Sheth, “The generalized Poisson distributions and a model of clustering from Poisson initial conditions,” in *Monthly Notices of the Royal Astronomical Society*, vol.299, no.1, pp. 207-217, 1998.

[9] P.C. Consul and G.C. Jain, “A generalization of the Poisson distribution,” in *Technometrics*, vol.15, pp. 791-799, 1973.

[10] G.W. Snedecor and W.G. Cochran, *Statistical methods*. 8th edition, Iowa State University Press, 1989.