Probabilistic Constraint Nets

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The modeling and analysis of hybrid systems is becoming increasingly important as such models are now widely used to reason about complex physical systems. Hybrid systems are systems that incorporate continuous and discrete state/time structures. A discrete controller for a continuous system represents a typical example of a hybrid system. Traditional approaches to real-time hybrid systems define behaviors purely in terms of nondeterminism. However, many physical systems of interest behave according to a probability distribution, thus a formal framework to model systems with probabilistically unpredictable behaviors is called for. While the modeling of probabilistic systems focuses on the underlying structure of the system, one cannot guarantee that the system will exhibit the desired behavior under all circumstances. One would like to be able to characterize the likelihood of desirable (undesirable) events. Bounds on the probability that certain behaviors do (or do not) occur then replace the concept of unconditional correctness of the system. Probabilistic logics have proved very efficient in the specification of requirements of systems that exhibits uncertainty. However, the verification of such specifications is notoriously hard. Thus, appropriate methods for the verification of probabilistic systems are needed. Ying Zhang and Alan Mackworth [Zhang and Mackworth, 1995] have proposed a unified foundation for hybrid systems. This semantic model for hybrid dynamic systems called Constraint Net (CN) has proved to be a powerful framework for the modeling and analysis of hybrid systems. Unfortunately, although they can handle non-determinism via hidden inputs, CN models cannot model probabilistic systems. Therefore, we propose an extension of the CN framework, which, while profiting from the many advantages of CN, would also strengthen the CN framework by allowing the modeling and analysis of probabilistic systems. We call this new framework the Probabilistic Constraint Nets (PCN) model. We introduce the syntax and semantics of PCN. In addition, we will present formal methods for specifying behavior requirements along with verification techniques, which would quantify the likelihood of the given behavior. Furthermore, we intend to develop learning methods to incorporate with PCN models. This will allow robotics systems to learn and adapt from interaction with their (uncertain) environment. Probabilistic constraint nets, by their modularity and their graphical representation render modeling very easy and extremely powerful. Within the probabilistic constraint net framework, we can represent most probabilistic hybrid systems. Furthermore, it can be shown that for a certain class of PCN, namely a class that we call finite-discrete probabilistic constraint nets, there exists a mapping that can transform any PCN in this family into a Markov Reward Process. This result allows us to take advantage of all the very efficient techniques that have been developed over the years for Markov chains, while still counting on the modeling power of PCN. We are developing probabilistic verification techniques to obtain efficient verification algorithms when analyzing a system modeled as a PCN. We use Algebraic Decision Diagrams (ADDs) (extension of BDDs) as our knowledge representation method. We have recently used ADDs with very promising results in the field of planning with uncertainty [Hoey *et al.*, 1999] while very significant results have arisen in the field of probabilistic verification as well. As mentioned above, we intend to combine learning techniques and the PCN formalism in order to build robot controllers capable of dynamically adjusting their internal parameters as they interact and explore their dynamic environment. We consider approaches from Bayesian Network learning and reinforcement learning, both of which have proved their efficacy within the learning community for several years. Due to its inherent graphical representation, Bayesian Network seem particularly well suited to interface with a PCN model and thus allowing the modeling of complex probabilistic systems combined with non-trivial learning tasks.

References

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