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Blind Channel Estimation for Space-Time Coded WCDMA

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Abstract

A new blind channel estimation technique is proposed for space-time coded wideband CDMA systems using aperiodic and possibly multirate spreading codes. Using a decorrelating front end, the received signal is projected onto subspaces from which channel parameters can be estimated up to a rotational ambiguity. Exploiting the subspace structure of the WCDMA signaling and the orthogonality of the unitary space-time codes, the proposed algorithm provides the least squares channel estimate in closed form. A new identifiability condition is established. The mean square error of the estimated channel is compared with the Cramér-Rao bound, and a bit error rate (BER) performance for the proposed algorithm is compared with differential schemes.

Keywords— Space-time coding, Long code CDMA, Least Squares, Blind channel estimation.

I. Introduction

Future wireless systems will require high rate transmissions of multimedia data over time varying fading channels. This is especially the case for the downlink where a mix of voice, low rate data and possibly images are transmitted to mobile users. To increase the capacity and provide reliable communications over a fading channel, diversity techniques in space and time are expected to play a crucial role [1] [2] [3]. A variety of space-time coding schemes have been proposed with multiple transmit antennas and a single or multiple receive antennas (e.g., [4] [5] [6]). Indeed, the third generation wireless standard supports base station transmit diversity using the WCDMA physical layer interface.

Many space-time techniques, the popular Alamouti scheme in particular, are designed for coherent detection where channel estimation is necessary. There is a substantial literature, e.g., [10] [11] [12], addressing the channel estimation issue for (space-time coded) multiple-input multiple-output (MIMO) systems, ranging from standard training based techniques that rely on pilot symbols in the data stream to blind and semiblind estimation where observations corresponding to data and pilots (if they exist) are used jointly. Noncoherent detection schemes for space-time coded systems have also been proposed based on differential or sequential decoding [7] [8] [9]. These methods avoid the need for channel estimation by introducing structures in the transmitted symbol stream. The receiver can demodulate the transmitted symbols directly exploiting the embedded structure. Although these methods increase the bandwidth efficiency by eliminating the necessity of training symbols and is robust to fast fading, they suffer from the performance degradation and error propagation problem.

Several spatial diversity schemes such as orthogonal transmit diversity (OTD) [13], space-time spreading (STS) [14] have been proposed and adopted to WCDMA systems. Such diversity techniques give additional reliability on top of the robustness of CDMA systems against multiuser interference. In this paper, we focus on WCDMA systems with space-time block coding based transmit diversity (STTD). The challenge of channel estimation in such a wideband system is twofold. First, the WCDMA is a multirate system where the delay spread may exceed several symbol intervals causing severe multipath fading and intersymbol interference; the channel is a MIMO system with memory. Second, the increase in the number of channel parameters, due to the use of multiple antennas, makes the conventional training based scheme less reliable and more prone to multiaccess interference. Fortunately, WCDMA also offers signal structures that could be exploited in an estimation scheme.

In [15], the authors proposed a blind channel estimation technique based on Capon receiver or minimum output variance technique for flat fading channel with two spreading codes per user. In this paper, we propose a blind channel estimation technique for frequency-selective fading with a single spreading code per each user. The proposed method requires no more than two pilot symbols per user per slot. (This is the same number of pilot symbols as differential detection schemes.) The proposed algorithm exploits the subspace structure of the long code WCDMA transmission and the orthogonality of the unitary codes which includes the Alamouti code. As a subspace technique, the proposed algorithm can obtain channel estimates quickly using only few symbols, which allows us to deal with rapidly fading channels. Using a RAKE structure, our technique is compatible with standard receiver front-ends that suppress multiaccess interference and perform decoding for each user separately.

The paper is organized as follows. The data model of a space-time coded long code CDMA system is described in Section II. A new blind channel estimation method based on decorrelation and an identifiability condition are proposed and several extensions are discussed in Section III. In Section IV, the detection schemes are briefly discussed. In Section V, the performance of the proposed method is compared with the Cramér-Rao

Bound (CRB) through Monte Carlo simulations and the bit error rate (BER) of the proposed method is compared with that of differential detection schemes.

A. Notation

The notations are standard. Vectors and matrices are written in boldface with matrices in capitals. We reserve $\mathbf{I}_m$ for the identity matrix of size m (the subscript is included only when necessary). For a random vector $\mathbf{x}$, $\mathbb{E}(\mathbf{x})$ is the statistical expectation of $\mathbf{x}$. The notation $\mathbf{x} \sim \mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ means that $\mathbf{x}$ is (complex) Gaussian with mean $\boldsymbol{\mu}$ and covariance $\boldsymbol{\Sigma}$. For a complex quantity α, α^* and $\mathrm{Re}(\alpha)$ denote the complex conjugate and the real part of α, respectively. Operations $(\cdot)^T$ and $(\cdot)^H$ indicate transpose and Hermitian transpose, respectively. $\mathrm{tr}(\cdot)$ denotes the trace of a matrix. $\mathrm{diag}(\mathbf{X}_1, \ldots, \mathbf{X}_N)$ is a block diagonal matrix with $\mathbf{X}_1, \ \ldots, \ \mathbf{X}_N$ as its diagonal blocks. Given a matrix $\mathbf{X}$, $\mathbf{X}^\dagger$ is the Moore-Penrose pseudo inverse, and $\mathbf{X} \otimes \mathbf{Y}$ is the Kronecker product of $\mathbf{X}$ and $\mathbf{Y}$. For a matrix (vector) $\mathbf{X}$, we use $\|\mathbf{X}\|$ for the 2-norm and $\|\mathbf{X}\|_F$ for the Frobenius norm.

II. Data Model

We consider a space-time block coding based transmit diversity (STTD) that requires a single spreading code for each user. Specifically, we consider a WCDMA system with Alamouti coding [4]. We assume two transmit antenna and a single receive antenna, K asynchronous users with aperiodic spreading codes, and slotted transmissions.

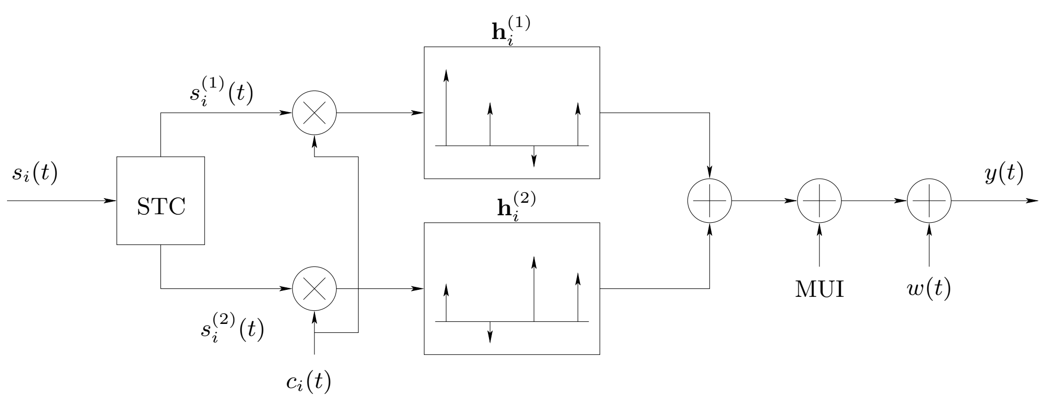

Fig. 1. CDMA system with space-time coding using two transmit antennas (STC: Space-time Encoder).

User i transmits two data sequences $\{s_{im}^{(1)}\}_{m=1}^{M_i}$ and $\{s_{im}^{(2)}\}_{m=1}^{M_i}$, one through each antenna, in every slot. Specifically, the data sequence for user i is space-time encoded as

$$\begin{aligned} s_{im}^{(1)} &= s_{im}, \quad s_{i,m+1}^{(1)} = s_{i,m+1}, \\ s_{im}^{(2)} &= -s_{i,m+1}^{(1)*}, \\ s_{i,m+1}^{(2)} &= s_{im}^{(1)*}, \quad m = 1, 3, \ldots, M_i - 1, \end{aligned} \tag{1}$$

where $s_{im} \triangleq s_i(mT_i)$ is the input data sequence, $s_{im}^{(j)} \triangleq s_i^{(j)}(mT_i),\ j = 1, 2$ the encoded data sequence for transmit antenna j, T_i the symbol interval, and M_i the slot size for user i. Each data sequence is spread by a user-specific long spreading code $c_i(t)$ with spreading gain G_i, followed by a chiprate pulse-shaping filter, and transmitted through the corresponding antenna. Notice that the data sequences for two transmit antennas are spread by the same spreading code here. The separation between two antenna signals is possible with space-time encoding with a single spreading code.[1]

We assume that the channel for each transmitter-receiver pair of each user doesn't change for a single slot period, and model it by a complex finite impulse response (FIR) filter with taps separated by multiples of chip interval. The continuous-time channel impulse response of the path from transmitter j to the single receiver for user i is given by

$$h_i^{(j)}(\tau) = \sum_{l=1}^{L_i^{(j)}} h_{il}^{(j)} \delta(\tau - lT_c - d_i^{(j)} T_c),$$

where $h_{il}^{(j)}$ is the lth path gain for transmitter-receiver pair j for user i and $T_c = T_i/G_i$ is the chip interval. We assume that the channel order $L_i^{(j)}$ and the delay $d_i^{(j)}$ from the frame reference are known. We set L_i as the maximum of $\{L_i^{(j)}\}_{j=1,2}$. When the channel is sparse, it is more efficient to model the channel as separate clusters of multipaths. In that case, we assume that the approximate locations of these clusters are known.

The transmitted signal is also corrupted by the other user interference and additive noise at the channel. At the receiver, the $y(t)$ is passed through a chip-matched filter and sampled at the chip rate. Stacking the chiprate samples, we obtain the discrete-time received signal vector. First, let us consider $\mathbf{y}_{im}$ that corresponds to the noiseless output

[1] When a different spreading code is used for each antenna, this can be considered just as two different CDMA users and the space-time coding is not necessary to achieve the spatial diversity due to the separation capability by spreading codes. However, this method requires twice more spreading codes than the system considered here.

due to the mth symbol interval of user i. Then, $\mathbf{y}_{im}$ is given by

$$\mathbf{y}_{im} = \mathbf{T}_{im}[\mathbf{h}_i^{(1)} s_{im}^{(1)} + \mathbf{h}_i^{(2)} s_{im}^{(2)}], \tag{2}$$

where the discrete-time multipath channel $\mathbf{h}_i^{(j)} \triangleq [h_{i1}^{(j)}, \cdots, h_{iL_i}^{(j)}]^T$, $j = 1, 2$, and $\mathbf{T}_{im}$ is the Toeplitz matrix whose first column is made of $(m-1)G_i + d_i^{(j)}$ zeros followed by the code vector $\mathbf{c}_{im}$—the mth segment of G_i chips of the spreading code of user i—and additional zeros that make the size of $\mathbf{y}_{im}$ the total number of chips of the entire M_i-symbol slot (see Fig. 2).

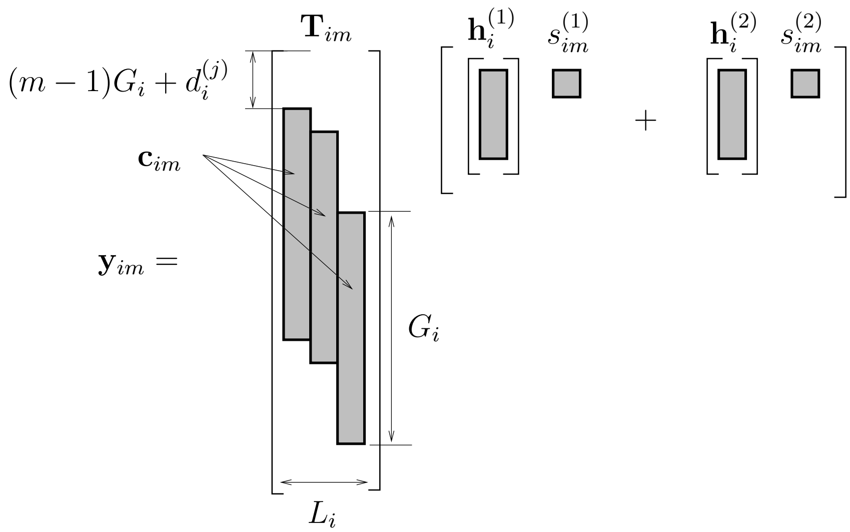

Fig. 2. Noiseless single symbol output $\mathbf{y}_{im}$

Since the channel is linear, the total received noiseless signal for user i is given by the sum of $\mathbf{y}_{im}$,$m = 1, \ldots, M_i$ as

$$\begin{aligned} \mathbf{y}_i &= \sum_{m=1}^{M_i} \mathbf{T}_{im}[\mathbf{h}_i^{(1)} s_{im}^{(1)} + \mathbf{h}_i^{(2)} s_{im}^{(2)}] = \mathbf{T}_i(\mathbf{I}_{M_i} \otimes [\mathbf{h}_i^{(1)} \quad \mathbf{h}_i^{(2)}])\mathbf{s}_i, \\ & \mathbf{s}_i \triangleq [s_{i1}^{(1)}, s_{i1}^{(2)}, s_{i2}^{(1)}, s_{i2}^{(2)}, \cdots, s_{iM_i}^{(2)}]^T, \\ & \mathbf{T}_i \triangleq [\mathbf{T}_{i1}, \mathbf{T}_{i2}, \cdots, \mathbf{T}_{iM_i}], \end{aligned} \tag{3}$$

where $\otimes$ is the Kronecker product and $\mathbf{T}_i$ is the code matrix of user i, and it has a special block shifting structure. Including all users and noise, we have the complete matrix model

$$\begin{aligned} \mathbf{y} &= [\mathbf{T}_1 \cdots \mathbf{T}_K]\mathrm{diag}(\mathbf{I}_{M_1} \otimes \mathbf{H}_1, \cdots, \mathbf{I}_{M_K} \otimes \mathbf{H}_K)\mathbf{s} + \mathbf{w}, \\ &= \mathbf{T}\mathcal{D}(\mathbf{H})\mathbf{s} + \mathbf{w}, \end{aligned} \tag{4}$$

where the overall code matrix $\mathbf{T}$ is composed of all code matrices, $\mathbf{s}$ includes all symbols of both transmitters for all K users, and

$$\mathbf{H}_i \triangleq [\mathbf{h}_i^{(1)} \ \mathbf{h}_i^{(2)}]. \tag{5}$$

$\mathbf{H}_i$ contains the channel for each transmit-receive pair for user i, The matrix $\mathcal{D}(\mathbf{H})$ is block diagonal with $\mathbf{I}_{M_i} \otimes \mathbf{H}_i$ as the block element. (See Fig. 3 for the example of two user case.) The additive noise is denoted by $\mathbf{w}$.

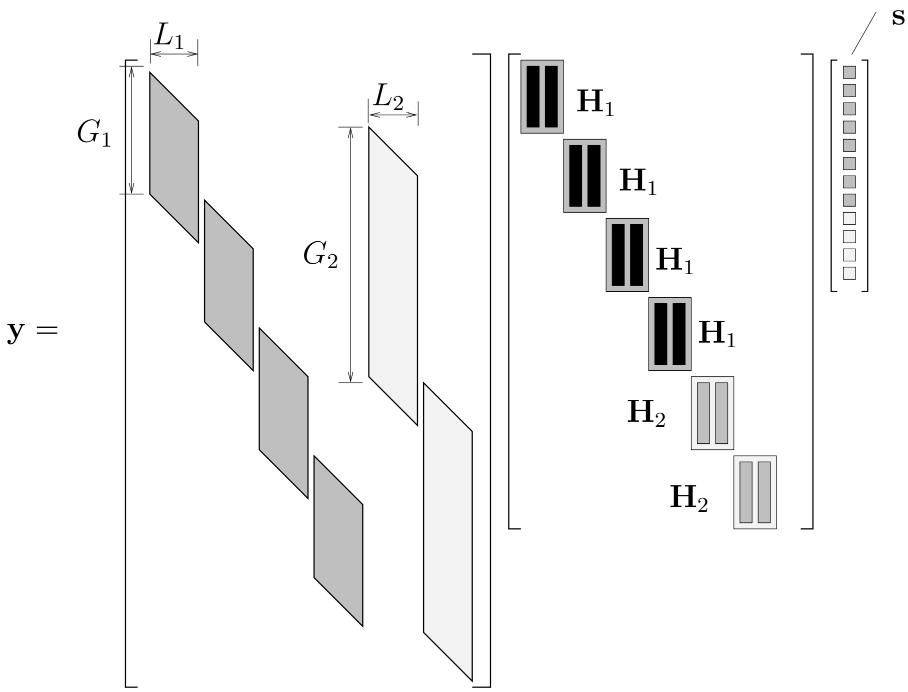

Fig. 3. Multiuser matrix model for the noiseless received signal

We will make the following assumptions.

(A1) The code matrix $\mathbf{T}$ is known.

(A1') The code matrix $\mathbf{T}$ has full column rank.

(A2) The channel matrix $\mathbf{H}_i$ is full column rank.

(A3) The noise vector is complex Gaussian $\mathbf{w} \sim \mathcal{N}(\mathbf{0}, \sigma^2\mathbf{I})$ with possibly unknown variance σ^2.

Assumption (A1) implies that the receiver knows the codes for all users as well as the delay $d_i^{(j)}$ and the maximum channel order L_i. The rough knowledge of the delay $d_i^{(j)}$ is enough since we can over-parameterize the channel to accommodate channel uncertainties. When the knowledge of other users codes is not available, we model other user interference as Gaussian noise. For the case of downlink, the relative delay $d_i^{(j)}$ and the number of multipaths L_i are the same for all the user signals. Since the downlink spreading uses orthogonal codes usually and the orthogonality between user signals is disturbed by multipath only, the other user interference is not severe after equalizing the multipath effect. For case of the multiple spreading codes for a single user, we can model all the codes in the code matrix. Assumption (A1') is sufficient but not necessary for the channel to be identifiable and for the algorithm proposed in Section 3 to produce good estimates. Assumption (A2) requires that the number of multipath is at least two, which is reasonable for typical wireless channels and uncorrelated transmission between two transmit-receive pairs.

III. Blind Channel Estimation

In this section, we propose a blind channel estimator that identifies the channel for both antenna pairs simultaneously up to unitary rotational ambiguity based on the decorrelation of user signals. The proposed method projects the received signal onto a subspace from which the channels of both transmitter-receiver pairs are estimated using a low rank decomposition. Blind estimation is possible due to the unitary property of the space-time codes, especially the Alamouti code. The proposed method combines two consecutive symbols and eliminates the unknown symbols exploiting this unitary property. We assume that the channel and symbols are deterministic parameters.

A. Blind Algorithm

A.1 Front-end processing

We first consider the decorrelator, conventional matched filter, and regularized decorrelator as the front end. The decorrelator is basically used for the algorithm construction. However, other front-ends can be used depending on the situation, which is briefly mentioned in Sec. III-D The decorrelating front end $\mathbf{T}^\dagger$ can be efficiently implemented using a state-space inversion technique that significantly reduces the complexity and storage requirement of the decorrelating receiver by exploiting the structure of the code matrix [17].

The output of the decorrelating matched filter is written in vector form as

$$\begin{aligned} \mathbf{z} &= \mathbf{T}^{\dagger}\mathbf{y} = \mathcal{D}(\mathbf{H})\mathbf{s} + \mathbf{n}, \\ &= \mathrm{diag}(\mathbf{I}_{M_1} \otimes \mathbf{H}_1, \cdots, \mathbf{I}_{M_K} \otimes \mathbf{H}_K)\mathbf{s} + \mathbf{n}, \end{aligned} \tag{6}$$

where $\mathbf{n} = \mathbf{T}^{\dagger}\mathbf{w}$ is now colored. We segment $\mathbf{z}$ to the subvector $\mathbf{z}_{im}$ of size L_i. For example of equal spreading gain and equal channel order, $\mathbf{z}_{im}$ is the $((i-1)M+m)$th L-dimensional subvector of $\mathbf{z}$ where $M_1 = \cdots = M_K = M$, $L_1 = \cdots = L_K = L$. The subvectors corresponding to two consecutive symbols $2n-1$, $2n$ of user i are given by

$$\begin{aligned} \mathbf{z}_{i,2n-1} &= \mathbf{H}_i \begin{bmatrix} s_{i,2n-1} \\ -s^*_{i,2n} \end{bmatrix} + \mathbf{n}_{i,2n-1}, \\ \mathbf{z}_{i,2n} &= \mathbf{H}_i \begin{bmatrix} s_{i,2n} \\ s^*_{i,2n-1} \end{bmatrix} + \mathbf{n}_{i,2n}, \end{aligned} \tag{7}$$

where $n = 1, 2, \ldots, M_i/2$. Rewriting the two vectors in a matrix form gives

$$\mathbf{Z}_{in} \triangleq [\mathbf{z}_{i,2n-1}\ \mathbf{z}_{i,2n}] = \mathbf{H}_i\mathbf{S}_{in} + \mathbf{N}_{in}, \tag{8}$$

where $\mathbf{H}_i$ contains the unknown channel vector for each transmit-receive pair as described in (5), $\mathbf{N}_n \triangleq [\mathbf{n}_{i,2n-1}\ \mathbf{n}_{i,2n}]$, and

$$\mathbf{S}_{in} \triangleq \begin{bmatrix} s_{i,2n-1} & s_{i,2n} \\ -s^*_{i,2n} & s^*_{i,2n-1} \end{bmatrix}. \tag{9}$$

A.2 Low Rank Decomposition

We utilize the orthogonal property of unitary space-time codes that include the Alamouti code to eliminate the unknown symbols. Due to the unitary property of the code, we have

$$\mathbf{S}_{in}\mathbf{S}_{in}^H = \mathbf{S}_{in}^H\mathbf{S}_{in} = \alpha_{in}\mathbf{I}, \tag{10}$$

where $\alpha_{in} = |s_{i,2n-1}|^2 + |s_{i,2n}|^2$. For the noiseless case, it is easily seen that multiplying $\mathbf{Z}_{in}$ by its Hermitian eliminates the unknown symbol which makes blind identification possible. For the noisy case, utilizing all the observations, we can form the least squares estimation for the channel matrix. Let $\mathbf{Z}_i \triangleq [\mathbf{Z}_{i1}, \mathbf{Z}_{i2}, \ldots, \mathbf{Z}_{i,M_i/2}]$. Then, we have

$$\mathbf{Z}_i = \mathbf{H}_i\mathbf{S}_i + \mathbf{N}_i, \tag{11}$$

where

$$\mathbf{S}_i \triangleq [\mathbf{S}_{i1}, \mathbf{S}_{i2}, \ldots, \mathbf{S}_{i,M_i/2}],$$
$$\mathbf{N}_i \triangleq [\mathbf{N}_{i1}, \mathbf{N}_{i2}, \ldots, \mathbf{N}_{i,M_i/2}].$$

The least square estimator for the product $\mathbf{H}_i\mathbf{S}_i$, which ignores the noise color, is given by

$$\arg\min_{\mathbf{H}_i\mathbf{S}_i} ||\mathbf{Z}_i - \mathbf{H}_i\mathbf{S}_i||_F^2. \tag{12}$$

The solution of (12) is obtained by low rank approximation [19] using singular value decomposition (SVD) of $\mathbf{Z}_i$ since $\mathbf{H}_i$ has rank two by its construction. Since $\mathbf{S}_{in}$ is unitary and the estimator for $\mathbf{H}_i$ can be obtained by SVD of a smaller matrix $\mathbf{R}_i$

$$\mathbf{R}_i \triangleq \frac{1}{\alpha_i}\sum_{n=1}^{M_i/2} \mathbf{Z}_{in}\mathbf{Z}_{in}^H, \tag{13}$$

where $\alpha_i = \sum_{n=1}^{M_i/2} \alpha_{in}$ and let the SVD of $\mathbf{R}_i$ be given as

$$\mathbf{R}_i = \mathbf{U}_i\Sigma_i\mathbf{U}_i^H. \tag{14}$$

We obtain the least squares estimator as a low-rank approximation with the minimum Frobenius norm that is given by

$$\hat{\mathbf{H}}_i = \mathbf{U}_i\Sigma_i^{1/2}\mathbf{Q}_i, \tag{15}$$

where $\mathbf{Q}_i$ is an unknown 2×2 unitary matrix. The rotational ambiguity in the above estimate must be removed by either incorporating prior knowledge of the symbol, or by using pilot symbols.

B. Identifiability

We have so far assumed that the overall code matrix $\mathbf{T}$ has full column rank and therefore invertible from the left, i.e., $\mathbf{T}^\dagger\mathbf{T} = \mathbf{I}$. This assumption is usually valid for systems with large spreading gains or small delay spreads (For the equal spreading gain and channel order, the size of the code matrix $\mathbf{T}$ is $GM \times LMK$. We need $G > LK$). Under this assumption, it is clear that each user's channel is identifiable up to a rotational matrix ambiguity. When the spreading gain is small and the system is heavily loaded, $\mathbf{T}$ can be singular. We present a general identifiability condition for the proposed method that is independent of the channel parameters.

Proposition 1: Let $\tilde{\mathbf{T}}_{in} \triangleq [\mathbf{T}_{i,2n-1}\ \mathbf{T}_{i,2n}]$ be the matrix composed of two consecutive code matrices of user i for symbol $2n-1,\ 2n$, and $\check{\mathbf{T}}_{in}$ the submatrix of $\mathbf{T}$ after removing $\tilde{\mathbf{T}}_{in}$. The channel $\mathbf{H}_i$ is identifiable if there exists an n such that

$$\mathcal{C}(\tilde{\mathbf{T}}_{in}) \bigcap \mathcal{C}(\check{\mathbf{T}}_{in}) = \{\mathbf{0}\}. \tag{16}$$

where $\mathcal{C}(\cdot)$ denotes the column space of a matrix.

Proof: If eq.(16) holds for some n, then the range space of $\mathbf{T}$ can be decomposed into the sum of tow subspaces, i.e., there exists a matrix $\mathbf{V}$ with rank($\mathbf{T}$) - rank($\tilde{\mathbf{T}}_{in}$) linearly independent columns such that

$$\mathcal{C}([\tilde{\mathbf{T}}_{in}\ \ \mathbf{V}]) = \mathcal{C}(\mathbf{T}).$$

Let $\mathcal{T} \triangleq [\tilde{\mathbf{T}}_{in}\ \ \mathbf{V}]$. We have, for the noiseless case,

$$\mathcal{T}^{\dagger}\mathbf{y} = \begin{bmatrix} * \\ \mathbf{h}_i^{(1)} s_{i,2n-1} - \mathbf{h}_i^{(2)} s_{i,2n}^{*} \\ \mathbf{h}_i^{(1)} s_{i,2n} + \mathbf{h}_i^{(2)} s_{i,2n-1}^{*} \\ * \end{bmatrix}, \tag{17}$$

which implies that $\mathbf{H}$ is identifiable up to a rotational ambiguity. ■

Since (16) needs to hold only for some n, the use of long codes makes the identifiability condition easier to satisfy. For the downlink case, the condition is easier to satisfy since we have more choices over i.

C. Resolving the Rotational Ambiguity

The unknown unitary matrix $\mathbf{Q}_i$ in (15) and (23) needs to be resolved for coherent detection of symbols. This can be done using only two consecutive pilot symbols. We use least squares estimator for $\mathbf{Q}_i$, given pilot symbols. The problem of estimating $\mathbf{Q}_i$ is formulated, from (8) and (15), as

$$\hat{\mathbf{Q}}_i \ = \ \arg\min_{\mathbf{Q}\in\mathbb{C}^{2\times 2}} ||\mathbf{Z}_{ip} - \mathbf{H}_i \mathbf{S}_{ip}||_F^2,$$

$$
\begin{aligned}
&= \arg\min_{\mathbf{Q}\in\mathbb{C}^{2\times 2}} ||\mathbf{Z}_{ip} - \mathbf{U}_i\Sigma_i^{1/2}\mathbf{Q}\mathbf{S}_{ip}||_F^2, \\
&= \arg\min_{\mathbf{Q}\in\mathbb{C}^{2\times 2}} ||\mathbf{Z}_{ip}\mathbf{S}_{ip}^H - \alpha_{i1}\mathbf{U}_i\Sigma_i^{1/2}\mathbf{Q}||_F^2, \qquad (18)
\end{aligned}
$$

under the constraint

$$\mathbf{Q}\mathbf{Q}^H = \mathbf{I}.$$

Here, for the example of two pilot symbols at slot front, $\alpha_{i1} = (|s_{i1}|^2 + |s_{i2}|^2)$ and the pilot related matrix $\mathbf{Z}_{ip}$ and $\mathbf{S}_{ip}$ are given as

$$\mathbf{Z}_{ip} = [\mathbf{z}_{i1}, \mathbf{z}_{i2}], \quad \mathbf{S}_p = \begin{bmatrix} s_{i1} & s_{i2} \\ -s_{i2}^* & s_{i1} \end{bmatrix}, \qquad (19)$$

where $s_{i1},\ s_{i2}$ are two pilot symbols for user i. The least square solution of (18) is given by the following proposition.

Proposition 2: The least squares estimator for $\mathbf{Q}$ that minimizes the Frobenius norm for additive noise is given by

$$\hat{\mathbf{Q}} = \mathbf{U}_Q\mathbf{V}_Q^H, \qquad (20)$$

where $\mathbf{U}_Q$ and $\mathbf{V}_Q$ are obtained by SVD of the following matrix, i.e.,

$$\alpha_{i1}(\mathbf{U}_i\Sigma_i^{1/2})^H\mathbf{Z}_{ip}\mathbf{S}_{ip}^H = \mathbf{U}_Q\Sigma_Q\mathbf{V}_Q^H. \qquad (21)$$

Proof: See Appendix.

For multiple pilot symbol blocks, we can formulate the least squares problem to incorporate all the pilot symbols similarly to (11).

D. Extensions

Since the noise $\mathbf{n}_{im}$ after decorrelation is colored, a bias is introduced in estimation. We can apply whitening to remove the bias. The expectation of $\mathbf{R}_i$ in (13) is given by

$$
\begin{aligned}
\mathbb{E}\{\mathbf{R}_i\} &= \mathbf{H}_i\mathbf{H}_i^H + \sigma^2\Delta_i, \\
\Delta_i &= \frac{1}{\alpha_i}\sum_{m=1}^{M_i}\Sigma_{im}, \qquad (22)
\end{aligned}
$$

where Σ_{im} is the diagonal block of $\mathbf{T}^\dagger(\mathbf{T}^\dagger)^H$ with size $L_i \times L_i$ corresponding to the mth symbol of user i. The whitened estimator is given as

$$\hat{\mathbf{H}}_i = \Delta_i^{1/2}\tilde{\mathbf{U}}_i\tilde{\Sigma}_i^{1/2}\mathbf{Q}_i, \tag{23}$$

where $\Delta_i^{1/2}$ is the Cholesky factor of Δ_i and

$$\Delta_i^{-1/2}\mathbf{R}_i\Delta_i^{-H/2} = \tilde{\mathbf{U}}_i\tilde{\Sigma}_i\tilde{\mathbf{U}}_i^H. \tag{24}$$

For the downlink case, all the user data experience the same channel $\mathbf{H}_1 = \cdots = \mathbf{H}_K$. We can improve the estimator performance by exploiting this. The least squares approach is to combine the outer product $\mathbf{R}_i$.

$$\begin{aligned} \mathbf{R} &= \frac{1}{K}\sum_{i=1}^{K}\mathbf{R}_i = \frac{1}{K}\sum_{i=1}^{K}\frac{1}{M_i}\sum_{n=1}^{M_i/2}\mathbf{Z}_{in}\mathbf{Z}_{in}^H, \\ \Delta &= \frac{1}{K}\sum_{i=1}^{K}\Delta_i. \end{aligned} \tag{25}$$

This process further improves the performance by averaging out the noise.

Even if the algorithm is derived using the decorrelator as the front end. We can apply the same subspace technique to different front ends depending on the situation. For the case of large spreading factor, the proposed method can be applied with the conventional matched filter $\mathbf{T}^H$ without significant performance loss. When the noise level is high, we can use the regularized decorrelator given by

$$(\mathbf{T}^H\mathbf{T} + \sigma^2\mathbf{I})^{-1}\mathbf{T}^H, \tag{26}$$

to reduce the noise enhancement at the inversion step. As shown in (26), the regularized decorrelating front end requires the estimation of noise power. For the case of conventional matched filter, the algorithm exhibits the well known performance floor due to multiaccess interference. The proposed method with several different front-ends are tested in Section V.

The algorithm is derived for Alamouti coding scheme up to now. However, the proposed method is easily extended to any unitrary square block coding that satisfies (10) when the channel length is no less than the codeblock size.

E. Computational Complexity

The proposed method is described in Fig. 4. The major processing is composed of front end processing, construction of $\mathbf{R}_i$ and SVD of it, and resolving the rotational ambiguity $\mathbf{Q}_i$.

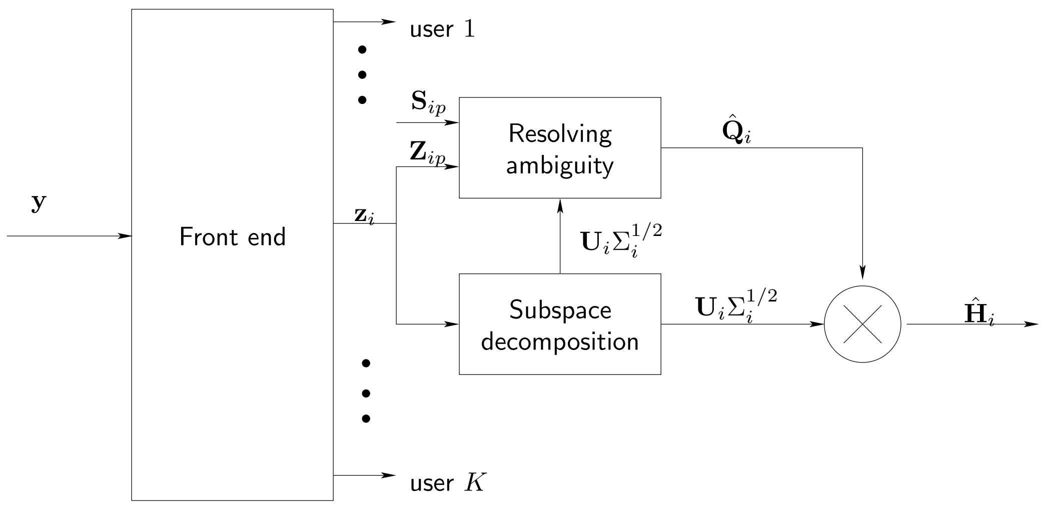

Fig. 4. Overall algorithm for blind channel estimation

The code matrix in (4) is usually very large for K-user long code CDMA systems. For the case of equal spreading gain G and channel order L between users, the size of $\mathbf{T}$ is $GM \times LMK$ where G is the spreading gain, M the number of symbols per slot, and L the channel order.(See Fig. 3.) However, the matrix is very sparse and the number of nonzero elements are approximately $GMLK$ that is also the number of operations required for the conventional matched filter front end. For the decorrelating and regularized decorrelating front ends, the inversion of code matrix $\mathbf{T}$ is necessary. The direct inversion is prohibitive for such a large matrix. However, the required inversion can be implemented in an efficient way utilizing the sparsity via the state-space method described in [17]. The computational complexity of the state-space inversion is in the order of GML^2K^2 that is linear with respect to slot size GM in chips.

Since $\mathbf{Z}_{in}$ is a $L \times 2$ matrix and $\mathbf{Z}_{in}\mathbf{Z}_{in}^H$ is Hermitian, the computation of $\mathbf{Z}_{in}\mathbf{Z}_{in}^H$ requires $\mathrm{O}(L^2)$ operations. Hence, the construction of $\mathbf{R}_i$ in (13) requires $\mathrm{O}(ML^2)$ computations. The SVD of $L \times L$ matrix $\mathbf{R}_i$ can be done with the complexity order of L^3. Similarly, the SVD required to resolve the rotational ambiguity is in order of $\sim 2^3$. Hence, the computational complexity is dominated by the front end processing and the cost for the required subspace decompositions is negligible.

IV. Detections

We consider several possible cases for the symbol detection. First, the coherent detection can be done with the estimated channel. We use the output of the front end processing such as the conventional matched filter, decorrelator, and regularized decorrelator, and perform blockwise maximum likelihood detection to obtain the symbol sequence. Rewriting (7) gives

$$\begin{bmatrix} \mathbf{z}_{i,2n-1} \\ \mathbf{z}^*_{i,2n} \end{bmatrix} = \begin{bmatrix} \mathbf{h}_i^{(1)} & -\mathbf{h}_i^{(2)} \\ \mathbf{h}_i^{(2)*} & \mathbf{h}_i^{(1)*} \end{bmatrix} \begin{bmatrix} s_{i,2n-1} \\ s^*_{i,2n} \end{bmatrix} + \begin{bmatrix} \mathbf{n}_{i,2n-1} \\ \mathbf{n}^*_{i,2n} \end{bmatrix}. \tag{27}$$

Neglecting the color of noise $\mathbf{n}_{i,2n-1}$ and $\mathbf{n}_{i,2n}$, the maximum likelihood estimates for symbol $s_{i,2n-1}$ and $s_{i,2n}$ are given by

$$\begin{bmatrix} \hat{s}_{i,2n-1} \\ \hat{s}^*_{i,2n} \end{bmatrix} = \mathcal{Q}\left(\frac{1}{\beta} \begin{bmatrix} (\hat{\mathbf{h}}_i^{(1)})^H & (\hat{\mathbf{h}}_i^{(2)})^T \\ -(\hat{\mathbf{h}}_i^{(2)})^H & (\hat{\mathbf{h}}_i^{(1)})^T \end{bmatrix} \begin{bmatrix} \mathbf{z}_{i,2n-1} \\ \mathbf{z}^*_{i,2n} \end{bmatrix} \right), \tag{28}$$

where $\beta = (||\mathbf{h}_i^{(1)}||^2 + ||\mathbf{h}_i^{(2)}||^2)$, superscripts T and H denote transpose and conjugate transpose, and $\mathcal{Q}$ is the quantization function which selects the symbol vector with minimum distance. Since the covariance of $\mathbf{n}_{i,2n-1}$ and $\mathbf{n}_{i,2n}$ are available, the whitened matched filter detector can be also used instead of (28) for an improved performance.

Since the proposed blind method requires only one (space-time) codeblock of pilot symbols for resolving of the rotational ambiguity, it is worthwhile to compare the performance with the differential demodulation which also requires the same number of pilot symbols. Several authors proposed noncoherent or differential modulation schemes for space-time coded systems [7] [8]. We consider the differential encoding based on unitary group codes described in [8]. The encoding procedure is given by the following recursion starting with a (unitary) pilot codeblock $\mathbf{S}_{ip}$.

$$\mathbf{S}_{in} = \mathbf{S}_{i,n-1}\mathbf{G}_{in}, \tag{29}$$

where $\mathbf{G}_{in}$ is a unitary matrix belonging to a unitary group $\mathcal{G}$ and carries the information. Even though the encoding and decoding steps for the differential scheme are simple for non-spread systems, the decoding for the CDMA system with multipaths requires additional procedures due to the spreading and intersymbol interference. Similarly to [9], we can use a suboptimal two-step approach. First, we apply the front end processing such the matched filter, decorrelator, or regularized decorrelator to deal with the despreading and multipath interference, and then use the output of the front end for differential decoding. Notice that the front end output (8) in multipath environment has the equivalent signal structure through MIMO channel with a single path for each transmit-receive pair. Neglecting the color of $\mathbf{N}_{in}$, the detected symbols are given, similar to [8] [9], by

$$\hat{\mathbf{G}}_{in} = \arg\max_{\mathbf{G}\in\mathcal{G}} \ \mathrm{tr}(\mathrm{Re}(\mathbf{G}\mathbf{Z}_{i,n}^H \mathbf{Z}_{i,n-1})). \tag{30}$$

Since the front end processing is the dominant factor in complexity in both cases, the complexity of coherent and differential schemes is not significantly different for the space-time coded CDMA systems.

V. Simulation

In this section, we present some simulation results. For channel estimation, the mean square error (MSE) was used for the performance criterion and the proposed estimator was compared with the Cramér-Rao bound (CRB) using Monte Carlo runs. For symbol detection, the bit error rate (BER) was used and the BER of the coherent detector with the proposed channel estimation was compared with that of the differential detection described in Sec. IV.

We considered a downlink CDMA system with two transmit antennas and a single receive antenna. Single ($K = 1$) and four ($K = 4$) synchronous BPSK users with equal power were considered. The spreading codes for users were randomly generated with spreading gain $G = 32$ and fixed throughout the Monte Carlo simulation for MSE and BER. Since our channel model is deterministic, the channel parameter was also fixed during the Monte Carlo runs. For the CRB calculation, the symbol sequence was fixed and for MSE and BER, symbol sequences were generated randomly for each Monte Carlo run.

The block fading channel model was used and the channel for each TX-RX pair had three fingers $L = 3$ (here $LK < G$). The coefficients are given by $\mathbf{h}^{(1)} = [0.0582 + 0.4331i, 0.1112 + 0.1466i, -0.8375 + 0.2715i]$ and $\mathbf{h}^{(2)} = [0.5317 + 0.1396i, -0.1475 +$

$0.2831i, 0.6144 - 0.4673i]$. The slot size was $M = 80$ and two pilot symbols, i.e., one space-time codeblock, were included at the beginning of the slot of each user. These pilot symbols were used to remove the rotational ambiguity of the blind estimator and to serve as the initial reference to differential detection. The signal-to-noise ratio (SNR) is defined by $(||\mathbf{h}^{(1)}||^2 + ||\mathbf{h}^{(2)}||^2) G E_c / \sigma^2$ where E_c is the chip energy and σ^2 is the chip noise variance.

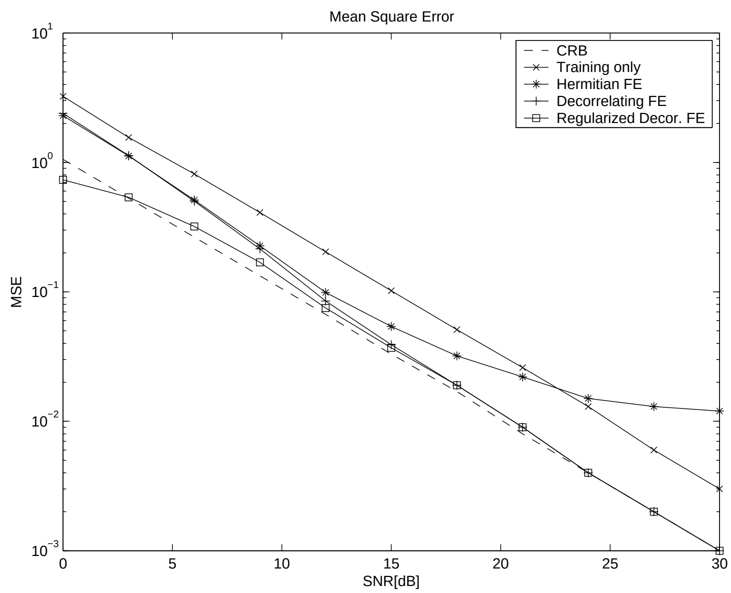

Fig. 5. MSE vs. SNR. - Single User Case

We compared the mean square error performance of the proposed channel estimator using several front ends with the CRB and the conventional training based method. With the availability of the two pilot symbols inserted to resolve the rotational ambiguity, we used the semi-blind CRB with a deterministic assumption on data symbols [18]. For the training based method, a least-squares channel estimate was obtained using data corresponding to the pilot symbols. Fig. 5 shows the MSE performance for the single user case. As shown in the figure, the proposed method with decorrelating and regularized decorrelating front ends closely follows the CRB at high SNR. The semi-blind method with the conventional matched filter deviates from the CRB as SNR increases due to multipath interference. The least square estimator based on only pilot symbols is worse than the proposed method with decorrelating or regularized decorrelating front ends. It doesn't

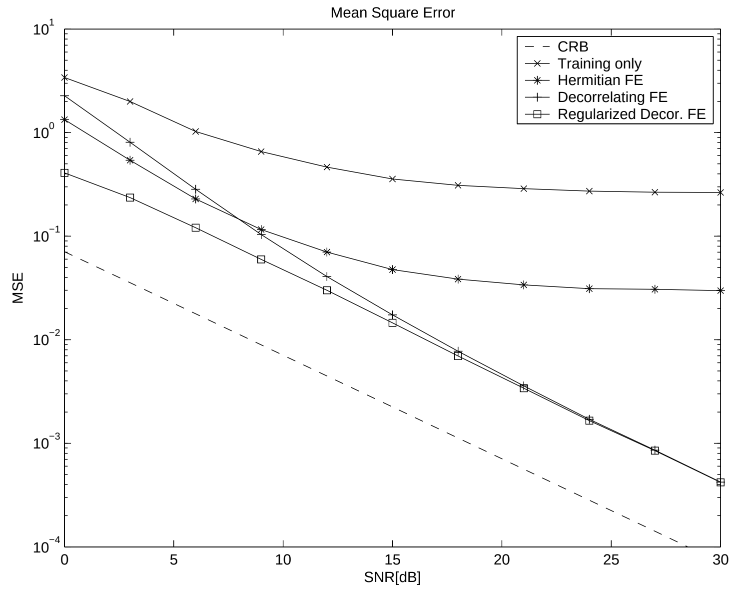

Fig. 6. Channel MSE vs. SNR - Four User Case.

show deviation from CRB since there is no multiuser interference in this case. Notice that the proposed method with regularized decorrelating front end shows an improved performance at low SNR due to the mitigation of noise enhancement by inversion and the MSE is lower than CRB. This is because the proposed estimator with regularized decorrelating front end is not unbiased. Fig. 6 shows the MSE for the four user case where the same channel was used as the single user. In this case, the MSE performance shows the similar behavior with a bigger gap from the CRB.

We evaluated the BER performance for the coherent detector with decorrelating, regularized decorrelating front ends and the differential scheme in in Section IV. For the coherent scheme, we used the whitened version of the ML detector (28). Figure 7 shows the BER performance for a single user case. For the reference, we used the coherent scheme with the regularized decorrelator and true channel.

We observe that the coherent detector with the proposed estimator is slightly better than the differential detector and the difference between different front ends is negligible. Notice that there is about 2.5dB SNR loss at BER of 10^{-3} due to channel estimation errors for the coherent detector. Figure 8 shows the BER performance for four user case. The improvement of the proposed method over the differential scheme is pronounced. In this

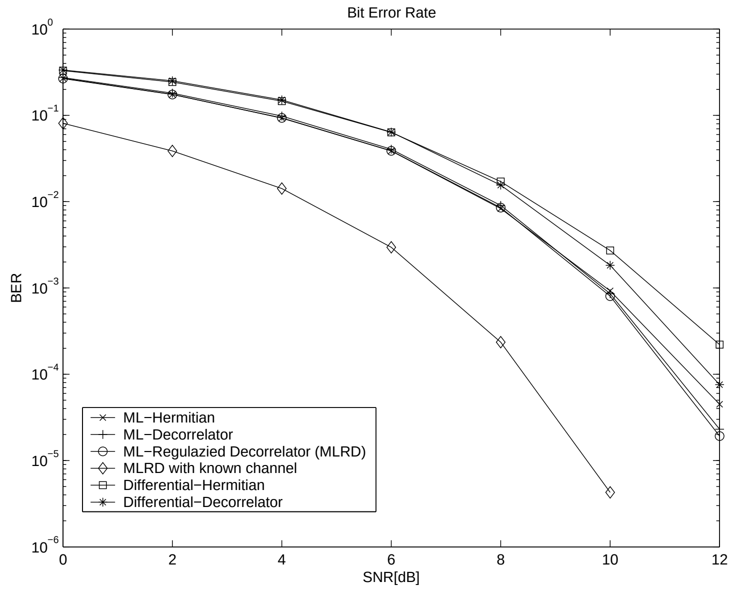

Fig. 7. BER vs. SNR - Single User Case.

case, the difference between the perfect channel knowledge and the proposed estimation is less than 1 dB. This is because the proposed method utilized all the user data constructively to estimate the downlink channel whereas the differential detection is performed individually. The performance of the detector using the conventional matched filter becomes worse as SNR increases due to the multiuser interference as expected. Fig. 8 shows that the coherent detection with the proposed channel estimate performs much better than the differential decoding scheme without significant difference in complexity when both detectors use the same front end and the same number of pilot symbols for block fading channel.

VI. Conclusion

We propose a new semi-blind channel estimation technique for space-time coded CDMA systems. A new identifiability condition is established. The proposed method identifies the channel of each transmit-receive pair simultaneously exploiting the subspace structure of CDMA signals and the orthogonality of space-time codes with a few pilot symbols. The performance of the proposed method is evaluated through the simulation and comparison with differential schemes. The proposed method can also be applied to general unitary space-time coding schemes.

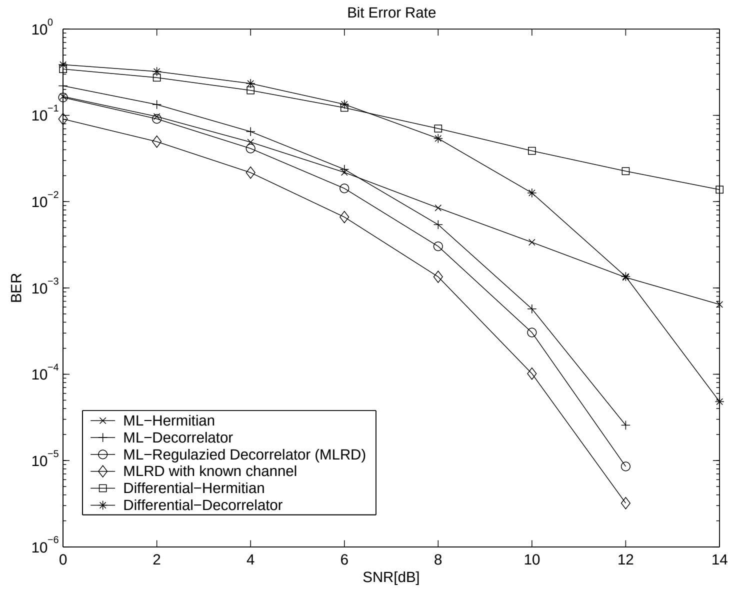

Fig. 8. BER vs. SNR - Four User Case.

Appendix: Proof of Proposition 2

The proof is the complex-valued version of the one in [19]. Let $\mathbf{A} \triangleq \mathbf{Z}_{ip}\mathbf{S}_{ip}^H$ and $\mathbf{B} \triangleq \alpha_{i1}\mathbf{U}_i\Sigma_i^{1/2}$. Then, (18) is written as

$$||\mathbf{Z}_{ip}\mathbf{S}_{ip}^H - \alpha_{i1}\mathbf{U}_i\Sigma_i^{1/2}\mathbf{Q}||_F^2 = ||\mathbf{A} - \mathbf{B}\mathbf{Q}||_F^2.$$

$$\begin{aligned} ||\mathbf{A} - \mathbf{B}\mathbf{Q}||_F^2 &= \mathrm{tr}((\mathbf{A} - \mathbf{B}\mathbf{Q})^H(\mathbf{A} - \mathbf{B}\mathbf{Q})), \\ &= \mathrm{tr}(\mathbf{A}^H\mathbf{A} + \mathbf{B}^H\mathbf{B} - \mathbf{Q}^H\mathbf{B}^H\mathbf{A} - \mathbf{A}^H\mathbf{B}\mathbf{Q}), \\ &= \mathrm{tr}(\mathbf{A}^H\mathbf{A} + \mathbf{B}^H\mathbf{B}) - 2\mathrm{tr}(\mathrm{Re}(\mathbf{Q}^H\mathbf{B}^H\mathbf{A})). \end{aligned}$$

Since $\mathbf{A}$ and $\mathbf{B}$ are given, the optimization problem is equivalent to maximize $\mathrm{tr}(\mathrm{Re}(\mathbf{Q}^H\mathbf{B}^H\mathbf{A}))$. Let the SVD of $\mathbf{B}^H\mathbf{A}$ be given by

$$\mathbf{B}^H\mathbf{A} = \mathbf{U}_Q\Sigma_Q\mathbf{V}_Q^H.$$

Then, we have

$$\begin{aligned} \mathrm{tr}(\mathrm{Re}(\mathbf{Q}^H\mathbf{B}^H\mathbf{A})) &= \mathrm{tr}(\mathrm{Re}(\mathbf{Q}^H\mathbf{U}_Q\Sigma_Q\mathbf{V}_Q^H)), \\ &= \mathrm{tr}(\mathrm{Re}(\mathbf{V}_Q^H\mathbf{Q}^H\mathbf{U}_Q\Sigma_Q)), \end{aligned}$$

$$= \quad \mathrm{tr}(\mathrm{Re}(\mathbf{X}\Sigma_Q)) = \mathrm{Re}\sum_{j=1}^{2} x_{jj}\sigma_j,$$

where $\Sigma_Q = \mathrm{diag}(\sigma_1, \sigma_2)$ and $\mathbf{X} \triangleq \mathbf{V}_Q^H \mathbf{Q}^H \mathbf{U}_Q$. Since $\mathbf{Q}$ is unitary, $\mathbf{X}$ is also unitary. Hence, we have $|x_{jj}| \leq 1$. The maximum of $\mathrm{tr}(\mathrm{Re}(\mathbf{Q}^H\mathbf{B}^H\mathbf{A}))$ occurs when $x_{jj} = 1$ since $\sigma_j \geq 0$ for all j. This implies that $\mathbf{X}$ is an identity matrix, which concludes the proof. ■

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