Properties of products with **GL.3**

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The *Cartesian product* $L_1 \times L_2$ of two unimodal Kripke complete logics L_1 and L_2 is the bimodal logic determined by the class of all product frames $\mathfrak{F}_1 \times \mathfrak{F}_2$, where $\mathfrak{F}_i$ is a Kripke frame for L_i, $i = 1, 2$. The *commutator* $[L_1, L_2] \subseteq L_1 \times L_2$ of L_1 and L_2 is the independent join of L_1 and L_2 extended with the Church-Rosser and commutativity axioms. (For more details consult [2] and [1].) In this paper, we present some new results on products with the well-known logic **GL.3** (determined by the class of Noetherian strict linear orders).

Theorem 1. *For every Kripke complete unimodal L, the logics $L \times$* **GL.3** *and $[L,$* **GL.3***$]$ have the same Kripke frames (but do not necessarily coincide).*

Theorem 2. *Let $\mathcal{C}$ be a class of frames such that for at least one frame $\mathfrak{F} = (W, R)$ in $\mathcal{C}$ there is an infinite subset $Y \subset W$ and a point $x \in W$ such that xRy, for all $y \in Y$, $y \neq x$. Then the logic $\mathsf{Log}\mathcal{C} \times$* **GL.3** *lacks the fmp.*

Theorem 3. *Both* $[$**GL.3**, **GL.3**$]$ *and* $[$**Grz.3**, **GL.3**$]$ *are Kripke incomplete.*

Theorem 4. *Let $L \in \{$***K4**, **K4.3**, **S4**, **S4.3**, **GL**, **Grz***$\}$. Then $L \times$* **GL.3** *is undecidable.*

It should be noted that the incompleteness proof in theorem 3 is indirect. It is based on theorem 1 and the fact that the logics $[$**GL.3**, **GL.3**$]$ and $[$**Grz.3**, **GL.3**$]$ are not recursively enumerable (see theorem 7.16 in [1]).

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References

[1] D. Gabbay, A. Kurucz, F. Wolter, and M. Zakharyaschev. *Many-Dimensional Modal Logics: Theory and Applications.* Elsevier, 2003.

[2] D. Gabbay and V. Shehtman. Products of modal logics. Part I. *Journal of the IGPL*, 6:73–146, 1998.