VELOCITY MEASUREMENT USING PHASE FITTING OF ANALYTIC SPATIOTEMPORAL IMAGES

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ABSTRACT

In this paper, the problem of one-dimensional (1-D) velocity estimation is addressed, using two-dimensional (2-D) spatiotemporal orientation estimation. A new frequency estimator based on a least square plane fitting of the estimated autocorrelation phase has been previously developed and is applied here to random textured images for the problem of velocity estimation. This algorithm requires the computation of the analytic autocorrelation and of a 2-D phase unwrapping step. Then, a plane fitting applied to the unwrapped phase gives an estimation of the spatiotemporal image orientation. Finally, we show in this work that the velocity can be estimated successfully in the spatiotemporal plane with the use of a 2-D frequency estimator.

1. INTRODUCTION

The general problem of optical flow estimation, one of the fundamental problems in digital video processing, has been widely investigated. The problem is to estimate the two-dimensional (2-D) apparent motion field of a 2-D sequence on a 3-D spatiotemporal lattice. For this purpose several research fields have been conducted, see [1] for a complete survey. Among them, we could mention three principal areas. First of all, gradient-based methods are derived from the use of the well-known optical flow constraint equation (OFCE) defined by Horn and Schunck in [2]. A second class of optical flow estimation methods is based on the block correspondence. In this case, displacements between frames are estimated for a number of image features. Those methods generally require the use of the correlation or block-matching methods with often a hierarchical analysis. Finally, a third class of methods, which is of our interest in this study, is the use of the three-dimensional (3-D) spatiotemporal space (x, y, t) and its associated frequency domain (f_x, f_y, f_t). Actually, the theoretical power spectral density of a translating image, with a constant velocity, belongs to a plane which tilt is related to the velocity. Therefore, in order to estimate the tilt of those planes and then the velocity, particular classes of filters have been developed and called velocity tuned filters. In [3], Adelson and Bergen proposed spatiotemporal filters built on the spatiotemporal impulse response of the eye modelled by a separable filter based on the understanding of the human visual system. Actually this method corresponds to the measurement of local power spectrum in the spatiotemporal plane. More advanced 2-D filters have been proposed for the measurement of local power spectrum, like Gabor filters [3].

In this work, we focus on the problem of velocity estimation along rectilinear trajectories, which corresponds to the problem of 1-D velocity estimation. Thus, assuming for example that the flow is along the x direction, the analysis of the image sequence can be reduced to the problem of texture orientation estimation in the 2-D space : (x, t). For this problem, we propose the use of an estimator previously described in [4] for the general problem of 2-D frequency estimation. This algorithm is applied here to the spatiotemporal plane, thus considering the spatiotemporal oriented texture as a 2-D cosine wave. More precisely, we consider the local spatiotemporal autocorrelation as a centered 2-D cosine wave, which is not necessarily exaggerated, especially around its center.

The use of this algorithm requires the computation of the analytic image. Regarding the studies focused on this problem (e.g., [5]), the computation of the analytic signal in 2-D is not an evident task. Actually, the 1-D Hilbert transform, annealing the spectrum on negative frequencies, cannot be transposed in 2-D since negative frequencies are not immediate. Therefore, other studies have been conducted, introducing the use of the Riesz transform [6] or the quaternionic analytic signal [5]. On the other hand, in the present application, images are supposed to be simple signals i.e. they have only one principal direction [7], thus allowing the use of the partial analytic signal.

The paper is organized as follows. In section 2, we review some general information concerning notations, the velocity measurement principle and the presentation of the experimental setup. The computation of the analytic spatiotemporal image is described in section 3. In section 4 we derive the orientation algorithm and application is given in section 5. Finally, in section 6, we present the main conclusions.

2. PRELIMINARY

2.1. Notations

Through this paper, the spatiotemporal image will be denoted by $f(\mathbf{x})$, where $\mathbf{x} = (x, t)$ and $F(\mathbf{u})$ corresponds to its Fourier transform, where $\mathbf{u} = (f_x, f_t)$. The velocity along the x direction will be denoted v_x, and $\hat{X}$ is the estimate of X.

2.2. Experimental setup

In order to validate the velocity measurement, we propose the use of an experimental setup, which is made of a transparent pipe, where flows water with seeding transparent particles thanks to a peristaltic pump. A rectilinear area of the pipe is observed by a digital camera in order to estimate the particle velocity and therefore the flow rate. The schematic experimental setup is depicted in Figure 1. Figure 2 presents an example of images obtained from this setup.

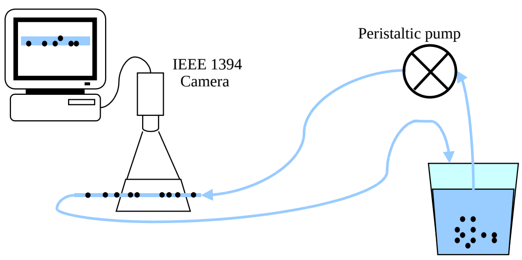

Fig. 1. Experimental setup for flow rate measurements.

2.3. Principle of the velocity measurement

As already mentioned in section 1, the velocity measurement is performed in the spatiotemporal plane. Figure 3 presents an example of such images obtained by the experimental setup above described. Regarding this figure, the spatiotemporal image is made of a texture which orientation is not constant when time varies. This may be due to the fact that the pump flow rate is not constant. Nevertheless, this result proves the need of measuring short-term orientations in order to track those velocity variations.

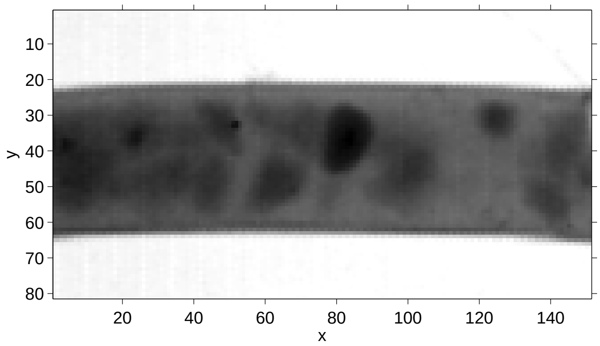

Fig. 2. Experimental image from setup depicted in figure 1.

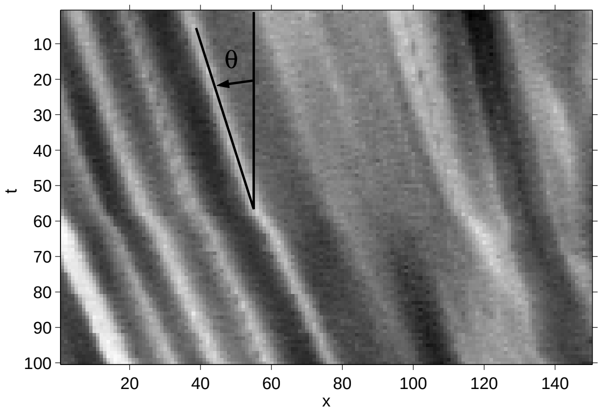

Fig. 3. Spatiotemporal plane from sequence illustrated in figure 2 for $y = 50$.

The velocity v_x of particles is thus estimated in pixel per frames from the estimation of this texture orientation named θ using $\hat{v}_x = \tan(\hat{\theta})$.For this purpose, the orientation is estimated by considering locally the spatiotemporal oriented texture as a 2-D cosine wave defined by $\cos(w_x x + w_t t + \phi_0)$, where ϕ_0 is an initial phase. The pulsations along the x and t directions, denoted w_x and w_t respectively, have to be first estimated, and then used for a direct calculation of $\hat{\theta}$ given by $\hat{\theta} = \text{atan}\left(\frac{\hat{w}_t}{\hat{w}_x}\right)$.Therefore the velocity estimation becomes :

$$\hat{v}_x = \frac{\hat{w}_t}{\hat{w}_x}. \tag{1}$$

3. THE ANALYTIC SPATIOTEMPORAL IMAGE

It is suggested in the previous section that the spatiotemporal plane can be considered locally as a 2-D cosine wave. Moreover, the analytic autocorrelation phase of a 2-D cosine wave is an order 1 polynomial curve, i.e. a plane, which equation is given by $w_x x + w_t t$. Thus 2-D frequency estimator described in [4] can be used in this paper in order to estimate the above mentioned pulsations w_x and w_t, and therefore the velocity. The use of this algorithm requires the computation of the analytic image. For this purpose, we suggest the use of the partial analytic signal, which is defined e.g., in [7]. Actually, for the 1-D Hilbert transform, the spectrum on negative frequencies has to be set to zero. Therefore, a half-plane of the 2-D frequency domain has to be set to zero and can be defined with respect to the orientation of the image. Thus, noting **d** this orientation, the Fourier transform of the analytic spatiotemporal image is defined by :

$$F_A(\mathbf{u}) = F(\mathbf{u})(1 + \text{sign}(< \mathbf{u}, \mathbf{d} >)), \quad (2)$$

where $< \cdot, \cdot >$ denotes the scalar product. The use of this definition is possible in this case in noting that spatiotemporal images are supposed to be simple signals i.e. they have only one principal direction, due to the flow. In order to compute this analytic image, the knowledge of the orientation is not a critical problem. In fact, we propose here the estimation of the image direction in eight equal slices of $45°$ of the 2-D frequency plane. Therefore, the analytic spatiotemporal image, see for example figure 4 (a), can be estimated by the following steps:

1. Computation of the Fourier transform of the spatiotemporal image (figure 4 (b)).
2. Find the pair of opposite 2-D Fourier plane slices with higher power spectral density, giving approximatively the direction **d**.
3. The Fourier transform of the analytic image is estimated by the filter defined in eq. (2) (figure 4 (c)).
4. The analytic spatiotemporal image is then obtained by the inverse Fourier transform. For example, its imaginary part (the Hilbert transform) is depicted in figure 4 (d).

4. ORIENTATION ALGORITHM

Once the analytic spatiotemporal image is calculated, the 2-D frequency estimator can be applied for the problem of orientation estimation. For this purpose, the analytic autocorrelation $\hat{r}(x,t)$ is estimated from a small part of the spatiotemporal plane. An example can be found on figure 4 (e) and (f) presenting the real and the imaginary parts respectively. The use of the algorithm described in [4] consists in considering the analytic spatiotemporal autocorrelation as a 2-D complex exponential surface, which is almost the case near the center of the autocorrelation, regarding to the figures 4 (e) and (f). Thus, the next step of the algorithm is to calculate and to unwrap the analytic spatiotemporal autocorrelation phase denoted $\hat{\phi}(x,t)$. In the present case, several assumptions can be done on the nature of this phase, which

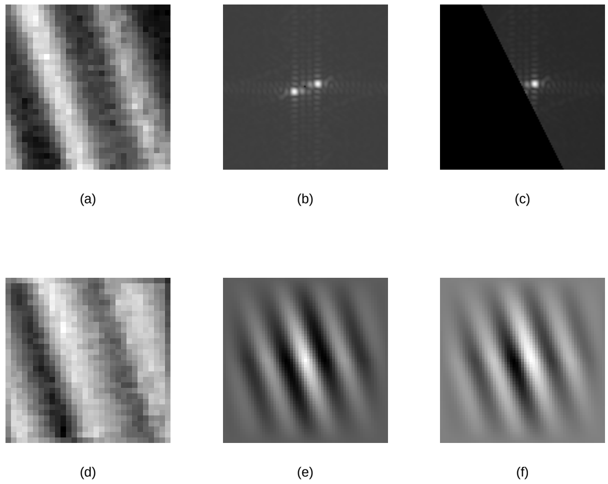

Fig. 4. Estimation of the spatiotemporal analytic image. (a) 30×30 pixels sub-image from figure 3. (b) Fourier transform of (a). (c) Fourier transform of the analytic image using eq. (2). (d) Hilbert transform of (a). Real (e) and Imaginary part (f) of the analytic spatiotemporal autocorrelation.

allows us to unwrap it easily from its center, see [4] for more details. As can be seen in figure 5, the unwrapped phase of the analytic spatiotemporal autocorrelation estimated from figure 4 (a) can be easily modelled by a plane in the neighborhood of its center.

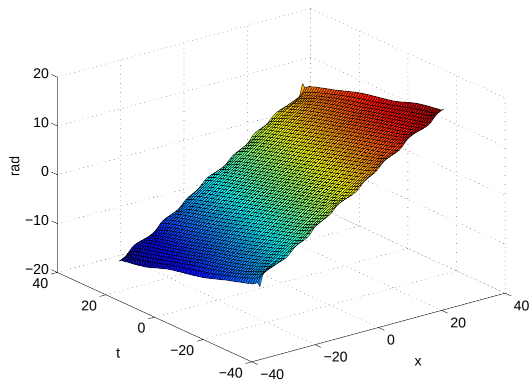

Fig. 5. Unwrapped phase of the analytic autocorrelation estimated from figure 4 (a).

The algorithm described in [4] consists then in a least square plane fitting of this unwrapped phase in order to estimate parameter w_x and w_t. For this purpose, the following equation has to be minimized :

$$J(w_x, w_t) = \sum_{x=-X}^{X} \sum_{t=-T}^{T} \left[w_x x + w_t t - \hat{\phi}(x,t) \right]^2, \quad (3)$$

where $[-X..X] \times [-T..T]$ is the rectangular lattice used for the fitting. It has been previously reported that the variance of the estimated autocorrelation $\hat{r}(x,t)$ increases when $|x|$ and $|t|$ grow, especially when the signal to noise ratio is low. That is the reason why X and T has to be chosen carefully. By differentiating eq. (3) according to w_x and w_l, we find

$$\begin{cases} \sum_{x=-X}^{X} \sum_{t=-T}^{T} x \left[\hat{w}_x x + \hat{w}_t t - \hat{\phi}(x,t)\right] = 0 \\ \sum_{x=-X}^{X} \sum_{t=-T}^{T} t \left[\hat{w}_x x + \hat{w}_t t - \hat{\phi}(x,t)\right] = 0. \end{cases} \quad (4)$$

After simplifications, the estimated parameters become

$$\begin{cases} \hat{w}_x = \frac{6}{X(2X+1)(X+1)(2T+1)} \sum_{x=1}^{X} \sum_{t=-T}^{T} x\hat{\phi}(x,t) \\ \hat{w}_t = \frac{6}{T(2T+1)(T+1)(2X+1)} \sum_{t=1}^{T} \sum_{x=-X}^{X} t\hat{\phi}(x,t). \end{cases} \quad (5)$$

Finally, from eq. (1) and (5) the estimated velocity is given by :

$$\hat{v}_x = \frac{X(X+1)}{T(T+1)} \frac{\sum_{t=1}^{T} \sum_{x=-X}^{X} t\hat{\phi}(x,t)}{\sum_{x=1}^{X} \sum_{t=-T}^{T} x\hat{\phi}(x,t)}. \quad (6)$$

5. APPLICATION AND RESULTS

The algorithm above described is applied in this section to the entire spatiotemporal plane given by the experimental setup. In order to validate the measurements, the orientation estimations are reintroduce in the spatiotemporal plane with the use of oriented vectors. As can be seen in figure 6, the 2-D frequency estimator, applied on 16×16 pixels sub-images, can be used with success for the problem of local velocity estimation. Moreover, synthetic particles can be reintroduced in the video sequence with the estimated velocity. The visual perception of the entire flow with the new particles reveals relevant estimations.

Finally, an estimation of the average flow rate is given according to the spatial and temporal sampling frequencies. A comparison of this estimation with a counter-measure allows the validation of the average measure. Obviously, the proposed algorithm is more accurate in tracking spatiotemporal orientation changes.

6. CONCLUSION

In this work, the general problem of 1-D velocity estimation is treated in the spatiotemporal plane. This problem lies in the fact that the orientation of the spatiotemporal image has to be estimated locally. For this purpose, we have proposed a particular model of the spatiotemporal image in considering it as a 2-D cosine wave. More precisely, the analytic estimated autocorrelation sequence is approximated by a 2-D exponential signal, thus allowing the use of an efficient 2-D frequency estimator previously developed. Finally, results obtained on real data show the good behavior of this algorithm and its ability to estimate short-term velocities.

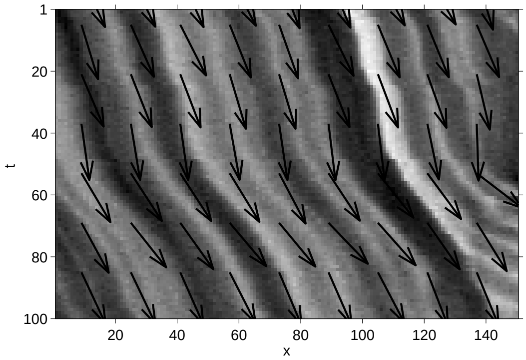

Fig. 6. Spatiotemporal plane with local orientations estimated on 16×16 pixel sub-images.

7. REFERENCES

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