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ARFIMA Modelling and Persistence of Shocks to the Exchange Rates: Does the Optimal Periodogram Ordinate Matter? [1]

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Abstract: The development of theory of autoregressive fractionally integrated moving average (ARFIMA) process has enabled empirical researchers to observe strong temporal dependence in many financial and economic time series. Time series with a strong temporal dependence are called *long-range-dependent* or *long-memory* series which implies that observations of distance past have powerful influence on the recent ones.

The most common method of modelling the long-memory processes is allowing the differencing parameter (d) to assume non-integer values.

A very popular method, mostly due to its computational simplicity, of estimating the differencing parameter of a long-memory process is provided by Geweke and Porter-Hudak (1983). This is a semi-parametric estimation approach for it only estimates the differencing parameter d. A simulation study by Hurvich, *el. al.* (1998) indicates that the choice of periodogram ordinate m from the sample size T ($m = T^{0.5}$), as originally suggested by GPH, is suboptimal and could lead to inferior performance compared to the asymptotically optimal choice of m.

In this paper we use the plug-in method of Hurvich and Deo(1999) to estimate the differencing parameters for testing the persistence of shocks to a number of daily dollar exchange rates and then compare the results with estimated values of d's of the same series that are based on $\mu = 0.5, 0.55, 0.60$ as suggested in Geweke and Porter-Hudak (1983).
Keywords: Long-memory, differencing parameter, fractional integration, exchange rate, optimal periodogram ordinate

1 Introduction

The development of theory of autoregressive fractionally integrated moving average, the (ARFIMA) process (Granger and Joyeux, 1980; Hosking, 1981), has enabled empirical researchers to observe strong temporal dependence in many financial and economic time series (see for instance Cheung, 1993; Cheung and Lai, 1992, 1993; Diebold and Rudebusch, 1989; Hauser et al., 1994, Lo, 1991, Sowell, 1992; Reschenhofer, 1994; Soofi, 1998). Time series with a strong temporal dependence are called *long-range-dependent*

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or *long-memory* series which implies that observations of distance past have powerful influence on the recent ones. In non-technical terms, powerful influence means that shocks of distance past persist to the present time.

Long-memory processes are characterized by autocorrelation functions which decay at a much slower rate than a *weakly-dependent* or short-memory series.

The most common method of modelling the long-memory processes is allowing the differencing parameter (d) to assume non-integer values, where d is the degree of differencing required for obtaining a stationary series.

Persistence of shocks implies that the series is strongly-dependent and may have important macroeconomic policy implications. For example, establishing that shocks to the daily U.S. dollar exchange rates persist, would give the central bankers an incentive to intervene in the currency markets. These interventions would aim at steering the nominal exchange rates toward their long-run equilibrium paths. For another example, consider the U.S. gross domestic product (GDP). Persistence of shocks in the GDP series, would require corrective monetary and fiscal policies to force the nominal GDP towards its long-run equilibrium path (see Gil-Alana and Robinson, 1977, for a detailed discussion of testing for persistence of shocks and unit root in macroeconomic series). In cases where shocks do not persist, policy activism is not required, since the series will automatically and eventually move towards its long-run equilibrium path.

A very popular method, mostly due to its computational simplicity, of estimating the differencing parameter of a long-memory process is provided by Geweke and Porter-Hudak (1983). This is a semi-parametric estimation approach for it only estimates the differencing parameter d. In the absence of a simultaneous estimation both the orders of autoregressive p, and moving average q components of ARIMA(p,d,q) process, the estimates of d may be inconsistent. Moreover, a simulation study by Hurvich, *el. al.* (1998) indicates that the choice of periodogram ordinate m from the sample size T ($m = T^{0.5}$) as originally suggested by GPH can lead to inferior performance compared to the asymptotically optimal choice of m.

The aim of this paper is to use the plug-in method of Hurvich and Deo(1999) to estimate the differencing parameters for testing the persistence of shocks to a number of daily dollar exchange rates and then compare the results with estimated values of d's of the same series that are based on $\mu = 0.5, 0.55, 0.60$ as suggested in Geweke and Porter-Hudak (1983).

2 Fractionally Integrated Processes

Consider a time series $Z = \{z_1, ..., z_n\}$. It is said to be integrated of order d, signified as $I(d)$, if it has a stationary, invertible autoregressive moving average (ARMA) representation after applying differencing operator $(1-L)^d$, where L is the backward lag operators. The series is fractionally integrated when d is not an integer.

A time series $Z = \{z_1, ..., z_n\}$ follows a fractionally integrated autoregressive moving average (ARFIMA) process if

$$A(L)(1-L)^d Z_t = B(L)\varepsilon_t \tag{1}$$

where $\varepsilon_t \sim iid(0, \sigma^2)$, $A(L) = 1 - a_1 L - ... - a_p L^p$; $B(L) = 1 + b_1 L + ... + b_q L^q$, all roots of $A(L)$ and $B(L)$ are outside of unit circle, and $(1-L)^d$ is the fractional differencing operator defined by

$$(1-L)^d = \sum_{k=0}^{\infty} \frac{\Gamma(k-d)L^k}{\Gamma(k+1)\Gamma(-d)}, \quad (2)$$

with $\Gamma(.)$ being the gamma function. Model (1) extends the standard ARIMA(p,d,q) model to all real values of d. For $0 \leq d < 0.5$, the autocorrelations of Z_t decay at a hyperbolic rate that is proportional to $k^{(2d-1)}$, as compared to a faster, geometric decaying rate of a stationary ARMA process[4].

According to the GPH method, given the periodogram $I(\omega_j)$ of variable Z_t one can estimate d by:

$$\ln(I(\omega_j)) = c - d\ln(4Sin^2(\omega_j/2)) + \eta_j \qquad (J = 1, ..., n), \quad (3)$$

where $\omega_j = \frac{2\pi J}{T}$ for (J=1,...,T-1) denote the harmonic ordinates.

Note that selection of a large sample will contaminate estimation of d, and an inadequate sample size will result in an imprecise estimation. As suggested by GPH (1983), one should use m observations, where $m = g(T) \ll T$.

The critical values for the GPH test are non-standard, and critical evaluation of estimated d requires computation of empirical values[5].

The selection of the harmonic ordinate m is crucial for accurate estimation of the variance, mean, and mean square error of the estimator. It is well known that the optimal value for m is $O(n^{4/5})$, and that the choice of $m = T^{0.5}$ would provide a suboptimal sample size.

Robinson(1995) in addressing this problem proposed a Guassian semiparametric estimator. This estimation method rests on a theory of optimal selection of m yielding a formula which includes both m and d. To deal with this problem Robinson proposed an iterative method of estimating d and m in alternating cycles.

Hurvich and Deo (1999) proposed a plug-in method of selection of m as follows[6].

Suppose that we have n observations $\{z_i\}_{i=1}^{n}$ which is both normal and stationary. Hurvich and Deo(1999) suggests using $\hat{C}$ to construct a regression estimator $\hat{d}_m$ of d where

$$\hat{m} = \hat{C}n^{4/5}. \quad (4)$$

$\hat{C}$ is consistently estimated by

$$\hat{C} = (\frac{27}{128\pi^2})^{1/5}\hat{K}^{-2/5} \quad (5)$$

and,

$$\hat{K} = \sum_{j=1}^{H} b_j \log I_j. \quad (6)$$

[4]For a comparison of the rates of decay of autocorrelations for AR(1) and ARFIMA(0,d,0) models, see Diebold and Rudebusch (1991).

[5]For the quantiles of Monto Carlo distribution of GPH standardized t-statistics which are based on $GPHT = \frac{\hat{d}-d}{SD(d)}$, where $\hat{d}$ is estimate of d, and $SD(d)$ is the asymptotic standard error of $\hat{d}$, see Soofi(1998).

[6]For details of the following derivation, please see Hurvich and Deo (1999).

In this model b_j is the third row of matrix $(X^{'}X)^{-1}X^{'}$, where matrix X has the columns $(\mathbf{1}, \log|2sin(\omega_j/2)|, \omega_j^2/2)$, $(J = 1, 2, ..., H)$, $H = An^{\delta}$, for some arbitary constant A, and $0 < \delta < 1$.To minimize the mean square error we set $\delta = 6/7$. I_j is the periodogram of the jth Fourrier frequency given by

$$I_j = \frac{1}{2\pi n}|\sum_{t=1}^{n} z_t e^{(-i\omega_j)t)}|^2 \tag{7}$$

The regression estimator of d is given by

$$\hat{d}_m = -0.5\frac{\sum_{j=1}^{m}(a_j - \bar{a})logI_j}{\sum_{j=1}^{m}(a_j - \bar{a})^2} \tag{8}$$

where $a_j = \log|2sin(\omega_j/2)|$, $\bar{a} = \frac{1}{m}\sum_{j=1}^{m} a_j$, and m is the Fourier frequencies.

A biased corrected $\hat{d}_{m,u}$ is given by:

$$\hat{d}_{m,u} = \hat{d}_m + \frac{2\pi^2}{9}\tilde{K}\frac{m^2}{n^2} \tag{9}$$

where $\tilde{K}$ is a consistent estimator of K.

3 The Empirical Results

We use five daily dollar exchange rates in this study: Canadian dollar, French franc, German mark, British pound, and Japanese yen. There are 1024 observations starting from 2 January 1997 to 29 May 2001.

Since it is known that these exchange rate series are non-stationary at the level, we convert the series to stationary ones by taking the first difference of the log of the data, which in effect shows the percentage change in the daily exchange rate.

It is well known that [see Hosking (1981)] when $0 < d \leq 0.5$, the series is a long memory process, and when $0.5 < d < 1$ it is short memory. For $-0.5 < d < 0.5$, a fractionally integrated series is stationary and ergodic. For $d \geq 0.5$, the process is non-stationary, however, it can be reduced to case $-0.5 < d \leq 0.5$ by taking appropriate differences.

Table 1 shows the results for the three values of $A = 0.20, 0.25, 0.30$. For each currency, the table shows the estimate $\hat{m}$ of the optimal number of the periodogram ordinates, and two estimates for the differencing parameter: $\hat{d}_m$ computed using equation (8) and its bias-adjusted version $\hat{d}_{m,U}$. The point estimates of the differencing parameters obtained by $\hat{d}_m$ are all very close to zero. The point estimates obtained by the bias-adjusted version $\hat{d}_{m,U}$ range from -0.1070 to 0.1018. Table 1 also shows the asymptotic standard errors of the estimates. Accordingly, all the 95% confidence intervals, based on $\hat{d}_m$ and $\hat{d}_{m,U}$, include zero. These results indicate that the first differences of the log of the exchange rates appear to be white noise with zero correlations. Furthermore, we observe that the point estimates of the differencing parameters and the 95% fall below the interval $0.5 < d < 1$, implying that the series may not be short memory processes.

Table 2 shows the estimates of the differencing parameters obtained by the GPH method for $\mu = 0.50, 0.55, 0.60$, and their asymptotic standard errors. Although, the standard errors of the GPH estimates are larger, the 95% confidence intervals here provide the same results as in the previous case. Again we conclude that the series may be long memory processes.

4 Conclusions

We do a comparative study of two methods in estimating the differencing parameters of five daily dollar exchange rates. The methods show that the differences of the log of the exchange rates may be long-memory processes.

Based on these observations, we conclude that there is no inherent advantage in using the optimal periodogram ordinates, $\hat{m}$ over the m suggested by GPH method. One possible advantage of the plug-in method, however, is that it produces smaller standard errors compared to the GPH method.

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Table 1: Estimates of Differencing Parameter d Based on the Optimal Fourier Frequencies

	Canadian $			Franc			Mark		
a	0.20	0.25	0.30	0.20	0.25	0.30	0.20	0.25	0.30
$m^{opt.}$	73	269	188	83	91	142	71	83	103
$\tilde{d}_m$	-0.00003	-0.00009	-0.00009	0.0060	0.0060	0.0060	0.0060	0.0060	0.0060
$\tilde{d}_a$	-0.1079	- 0.0525	-0.0249	-0.0875	0.0951	0.0783	-0.0960	0.1018	0.0907
d^{UB}	0.1470	0.0765	0.0915	0.1447	0.1384	0.1114	0.1557	0.1439	0.1298
d^{LB}	-0.147	-0.0767	-0.0917	-0.1327	-0.1264	-0.0994	-0.1431	-0.1319	-0.1177
	Pound			Yen					
a	0.20	0.25	0.30	0.20	0.25	0.30			
$m^{opt.}$	65	86	161	119	116	153			
$\tilde{d}_m$	0.00004	0.00004	0.00004	0.0081	0.0081	0.0081			
$\tilde{d}_a$	-0.1075	-0.0917	0.0678	-0.0700	-0.0716	-0.0778			
d^{UB}	0.1559	0.1355	0.0990	0.1233	0.1247	0.1097			
d^{LB}	-0.1558	-0.1354	-0.0989	-0.1070	-0.1085	-0.0934			

$\tilde{d}_a$ is biased corrected estimate of d, d^{UB} and d^{LB} are 95% confidence interval of d, and $\tilde{d}_m$ is the regression estimate of d.

Table 2: Estimates of Differencing Parameter d Based on the GPH Non-parametric Method

	Canadian $			Franc			Mark		
	$\mu = 0.5$	$\mu = 0.55$	$\mu = 0.60$	$\mu = 0.5$	$\mu = 0.55$	$\mu = 0.60$	$\mu = 0.5$	$\mu = 0.55$	$\mu = 0.60$
d	0.1223	-0.0285	-0.0465	0.1230	0.0840	0.0662	0.1209	0.0873	0.0722
σ	(0.1372)	(0.1097)	(0.0902)	(0.1372)	(0.1097)	(0.0902)	(0.1372)	(0.1097)	(0.0902)
	Pound			Yen					
	$\mu = 0.5$	$\mu = 0.55$	$\mu = 0.60$	$\mu = 0.5$	$\mu = 0.55$	$\mu = 0.60$			
d	-0.2018	-0.1443	-0.0801	0.1202	0.0497	0.0239			
σ	(0.1372)	(0.1097)	(0.0902)	(0.1372)	(0.1097)	(0.0902)			

The numbers in the parentheses are asymptotic standard errors.