Low Complexity High Resolution Subspace-Based Delay Estimation for DS-CDMA

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Abstract— **In this paper we consider the problem of estimating the propagation delays of a synchronous direct-sequence code division multiple access (DS-CDMA) system operating over a multipath fading channel. In a mobile receiver, the task of delay estimation can be divided into an acquisition of all multipath delays and subsequent tracking of the individual delays e.g. with a RAKE structure. Both tasks are especially challenging in indoor scenarios which commonly exhibit low delay-spread and thus require a high resolution of the estimation algorithms. A novel delay acquisition algorithm is presented in this paper, which is able to resolve multipaths whose delay difference is below one chip duration with high probability of acquisition and low computational complexity. It is based on a decomposition of the time-averaged correlation matrix of the output of a sliding correlator into signal and noise subspaces, with subsequent MUSIC spectrum computation and maximum search. The performance of the algorithm is assessed by means of computer simulations.**

I. Introduction

RAKE receivers employed in current and future DS-CDMA systems require the knowledge of instantaneous channel parameters for detection. A precise estimation of these parameters, the complex-valued path attenuation and the corresponding propagation delay of each channel tap, are especially important in challenging environments such as in indoor scenarios. This is because in such environments, the signal energy is concentrated around the line-of-sight component in the delay domain and a high resolvability of signal components is a prerequisite for the RAKE receiver to efficiently gather signal energy and thus improve its performance. Especially in mobile handsets however, the computational complexity of such algorithms is very critical.

The focus of this work is on the estimation of the propagation delays in the synchronous downlink of a DS-CDMA system. This synchronization task can be subdivided into an initial acquisition of all relevant delays and subsequent tracking of the acquired delays [1]. Previous findings ([2],[3]) have shown that if closely spaced delays (around one chip duration or less for DS-CDMA) can be resolved, SNR gains of several dB can be achieved in indoor scenarios compared to the case when the paths are not resolvable. It was also shown that tracking of closely spaced delays is possible with little complexity if accurate initial estimates are available.

Delay acquisition for DS-CDMA has received much attention in recent literature. Most results however consider either the multiuser problem in the asynchronous uplink ([5],[6]) or static (i.e. not time-varying) channels [7]. In this paper, a novel subspace-based approach to single-user delay acquisition of multipath channels is presented, which assumes knowledge of the user's (aperiodic) spreading sequence and acts as a hierarchical extension to conventional sliding-correlator based acquisition [4]. By exploiting the eigenstructure of the time-averaged sample correlation matrix of a coherent sliding correlator output, the observation space is partitioned into a signal and a noise subspace.

The DS-CDMA signal model assumed for the underlying system is presented in Section II. Section III defines the sliding correlator model and describes the novel acquisition algorithm. The algorithm performance is quantified in Section IV, followed by a conclusion.

II. Signal Model

In DS-CDMA communications, the user data symbols $\{a_n\}$ are oversampled by the spreading factor, multiplied by the spreading sequence and pulse shaped with a wide-band transmit pulse. A baseband-equivalent transmitted CDMA signal for one user can be expressed as

$$s(t) = \sum_{k=-\infty}^{\infty} a_{\lfloor \frac{k}{N_c} \rfloor} d_k g(t - kT_c), \qquad (1)$$

with a_n being the data symbol sent at time instant n, N_c the spreading factor and $\{d_k\}$ the effective spreading sequence. The wideband transmit pulse is denoted by $g(t)$ and T_c is the chip duration. For the third generation wideband CDMA standard draft from 3GPP [10],

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$N_c = 4 \ldots 512, T_c = 260.42$ ns and $g(t)$ is a root-raised cosine pulse with rolloff factor 0.22. The effective spreading sequence is the multiplication of a short spreading sequence with a very long scrambling sequence.

We assume a WSSUS multipath fading channel model with N_p propagation paths and channel impulse response (CIR)

$$h(t;\tau) = \sum_{l=0}^{N_p-1} c_l(t)\,\delta(\tau - \tau_l(t)), \tag{2}$$

with $c_l(t)$ being the complex-valued Rayleigh fading channel phasor and $\tau_l(t)$ the corresponding delay of path l. We shall omit the time-dependency of the delays in the following, since the delay dynamics are usually assumed to be significantly smaller than those of the fading process. The received signal $r(t)$, which is subject to AWGN and other-user interference $n(t)$, is pulse matched-filtered with $g(t)$ and we have

$$\begin{aligned} z(t) &= r(t) * g^*(-t) \qquad (3)\\ &= (s(t) * h(t;\tau) + n(t)) * g^*(-t)\\ &= \sum_{l=0}^{N_p-1} c_l(t) \sum_k a_{\lfloor \frac{k}{N_c} \rfloor}\, d_k R_g(t - kT_c - \tau_l) + n'(t), \end{aligned}$$

where $R_g(t)$ represents the pulse-autocorrelation function of $g(t)$.

III. Propagation Delay Acquisition

A. Conventional Sliding Correlator

The sliding correlator (or equivalently a matched filter) estimates the channel impulse response by correlating the received signal with the known spreading sequence. In order to facilitate the illustration of the proposed algorithm, a signal model in matrix-vector notation will be derived. We will assume the existence of a pilot channel which continuously transmits known symbols. All other data channels pertaining to the same user are assumed to be orthogonal to the pilot channel, such that they can be included in the noise term $n(t)$ in (3). The transmitted signal from equation (1) then reduces to $s(t) = A \sum_{k=-\infty}^{\infty} d_k\, g(t - kT_c)$, with A being the pilot symbol. In the following we shall denote the received signal with $z(t)$ and the received noiseless signal with $\tilde{z}(t) : z(t) = \tilde{z}(t) + n'(t)$. If we sample $\tilde{z}(t)$ at the chip rate, we get

$$\begin{aligned} \tilde{z}(nT_c) &= A \sum_k d_k \sum_{l=0}^{N_p-1} c_l(nT_c)\, R_g((n-k)\,T_c - \tau_l)\\ &= A\, \mathbf{d}^T \mathbf{h}_{eff} \qquad (4) \end{aligned}$$

with

$$\mathbf{d} = [\ldots d_k\ d_{k-1} \ldots]^T \tag{5}$$

$$\mathbf{h}_{eff} = \begin{bmatrix} \vdots \\ \sum_{l=0}^{N_p-1} c_l(nT_c)\, R_g((n-k)\,T_c - \tau_l) \\ \sum_{l=0}^{N_p-1} c_l(nT_c)\, R_g((n-k+1)\,T_c - \tau_l) \\ \vdots \end{bmatrix} \tag{6}$$

The vector $\mathbf{h}_{eff}$ is the *effective CIR*, which is the convolution of the CIR with the effective transmit pulse $R_g(t)$, sampled at the chip rate. It is seen that each received chip contains the contribution of infinitely many chips from the effective spreading sequence. In a practical implementation however, the sliding correlator estimates the effective CIR by correlating with a subsequence of the known effective spreading sequence $\{d_k\}$. Assuming that we want to estimate N_w samples of the effective CIR, starting at chip index n_0, using a sliding correlator of length N, we need to process the incoming signal in blocks of $N + N_w - 1$ chips. The i-th block of data to be processed is then given by

$$\begin{aligned} \tilde{\mathbf{z}}^i &= [\tilde{z}(iNT_c) \ldots \tilde{z}((i+1)N + N_w - 2)T_c)]^T\\ &= A\mathbf{D}^i \mathbf{h}_{eff} \qquad (7) \end{aligned}$$

with

$$\mathbf{D}^i = \begin{bmatrix} \cdots & d_{iN-n_0} & d_{iN-n_0-1} & \cdots \\ \cdots & d_{iN-n_0+1} & d_{iN-n_0} & \cdots \\ & \vdots & \vdots & \\ \cdots & d_{(i+1)N+N_w-n_0-2} & d_{(i+1)N+N_w-n_0-3} & \cdots \end{bmatrix} \tag{8}$$

Note that we assumed in (7) that the channel remains approximately constant for the duration of $N + N_w - 1$ chips. The sliding correlator can be described by a code matrix $\mathbf{C}^i$ of dimension $(N_w \times N + N_w - 1)$ containing shifted versions of a subsequence of length N of the effective spreading sequence, indexed with the desired location n_0 of the effective CIR. At the output of the sliding correlator we then have

$$\begin{aligned} \tilde{\mathbf{h}}_{eff,i} &= \left[\tilde{h}_{i,n_0} \ldots \tilde{h}_{i,n_0+N_w-1}\right]^T\\ &= \mathbf{C}^i A^* \tilde{\mathbf{z}}^i\\ &= |A|\, \mathbf{C}^i \mathbf{D}^i \mathbf{h}_{eff} \qquad (9) \end{aligned}$$

with

$$\mathbf{C}^i = \begin{bmatrix} \mathbf{d}^i & 0 & \ldots & 0 \\ 0 & \mathbf{d}^i & \ldots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & \ldots & 0 & \mathbf{d}^i \end{bmatrix} \tag{10}$$

$$\mathbf{d}^i = \left[d^*_{iN-n_0} \ldots d^*_{(i+1)N-n_0-1} \right] \quad (11)$$

We will denote the correlator output $\hat{\mathbf{h}}_{eff,i} = \tilde{\mathbf{h}}_{eff,i} + \tilde{\mathbf{n}}$ as *channel snapshots*. If the correlation length N is sufficiently long, the product $\mathbf{C}^i \mathbf{D}^i$ will be the $(N_w \times N_w)$ identity matrix. It is noted that the vector $\tilde{\mathbf{n}}$ contains uncorrelated noise samples.

The location of the propagation delays in the delay domain can be estimated by identifying peaks in the *power-delay profile* (PDP). In order to do that, we need to average over the channel fading process, which can be done by averaging over the power of subsequent channel snapshots:

$$\tilde{\mathbf{h}}_{PDP} = \left[\tilde{h}_{PDP,n_0} \ldots \tilde{h}_{PDP,n_0+N_w-1} \right]^T$$
$$\tilde{h}_{PDP,n_0+j} = \sum_{i=1}^{M} |\tilde{h}_{i,n_0+j}|^2, \quad j = 0 \ldots N_w - 1 \quad (12)$$

B. Subspace-Based Delay Estimation

We are interested in resolving closely spaced multipaths. Figure 1 depicts an estimated PDP of a channel with two taps of equal power spaced one chip apart, at T_c and $2T_c$, respectively. In this case, $n_0 = 0$, $N_w = 5$, $N = 256$ and $M = 100$. It is seen that the two paths cannot be resolved by the standard sliding correlator approach.

The MUSIC algorithm ([8] and references therein), originally derived for direction-of-arrival estimation, can be used to estimate the path delays. It requires an eigenvalue decomposition of a sample correlation matrix, which is known to have complexity $O(n^3)$, if n is the dimension of the correlation matrix. For our problem however, the dimensionality of the correlation matrix and thus the overall complexity can be kept low if we process the channel snapshots $\hat{\mathbf{h}}_{eff,i}$ instead of the received data and if we consider only those portions of the PDP which exhibit nonresolvable ”clusters”.

We know from (4) and (9) that the sliding correlator output is an estimate for a sampled version of the effective CIR. The sample correlation matrix can be estimated by averaging over individual snapshot correlation matrices:

$$\hat{\mathbf{R}}_{hh} = \frac{1}{M} \sum_{i=1}^{M} \hat{\mathbf{h}}_{eff,i} \hat{\mathbf{h}}^H_{eff,i} \quad (13)$$

An eigenvector decomposition of $\hat{\mathbf{R}}_{hh}$ yields

$$\hat{\mathbf{R}}_{hh} = \hat{\mathbf{U}} \hat{\Lambda} \hat{\mathbf{U}}^H, \quad (14)$$

with $\hat{\mathbf{U}}$ being a unitary matrix containing the eigenvectors of $\hat{\mathbf{R}}_{hh}$ and $\hat{\Lambda} = \mathrm{diag}(\lambda_0 \ldots \lambda_{N_w-1})$ a diagonal matrix containing the eigenvalues. Notice that the correlation matrix only has size $(N_w \times N_w)$, for the example in Figure 1 (5×5). Assuming that we know the number of signal components, say L, we would then partition the observation space into a *signal-plus-noise subspace*, spanned by the eigenvectors corresponding to the L largest eigenvalues, and its orthogonal complement, the *noise subspace*. If we don't have a-priori knowledge about the number of signal components, this measure must be estimated ([9]). Accordingly, equation (14) can be rewritten as

$$\hat{\mathbf{R}}_{hh} = \hat{\mathbf{U}}_s \hat{\Lambda}_s \hat{\mathbf{U}}_s^H + \hat{\mathbf{U}}_n \hat{\Lambda}_n \hat{\mathbf{U}}_n^H \quad (15)$$

where $\hat{\mathbf{U}}_s$ is $(N_w \times L)$, $\hat{\Lambda}_s$ is $(L \times L)$, $\hat{\mathbf{U}}_n$ is $(N_w \times N_w - L)$ and $\hat{\Lambda}_n$ is $(N_w - L \times N_w - L)$. With $\mathbf{h}_{eff}$ lying in the nullspace of the true $\mathbf{U}_n$, the MUSIC spectrum is

$$\mathbf{P}(\tau) = \frac{1}{\left| \mathbf{h}'(\tau)^H \hat{\mathbf{U}}_n \right|^2}, \quad (16)$$

The vector $\mathbf{h}'(\tau)$ of length N_w is a sampled version of the known effective transmit pulse, delayed by τ: $\mathbf{h}'(\tau) = [R_g(n_0 T_c - \tau) \ldots R_g((n_0 + N_w - 1) T_c - \tau)]^T$.

The MUSIC spectrum will exhibit peaks if the delay τ correponds to one of the channel path delays in the delay region of interest. The division in equation (16) is not required for implementation. Furthermore, the vector $\mathbf{h}'(\tau)$ can also be projected onto the signal subspace, which is favorable if $L < N_w - L$. Figure 1 depicts a MUSIC spectrum which was obtained using the same 100 channel snapshots that were used to average the PDP. The projection grid was chosen to be $T_c/10$. It is clearly seen that the MUSIC spectrum exhibits peaks at the location of the true delays, whereas the conventional PDP fails to resolve the two paths in this environment.

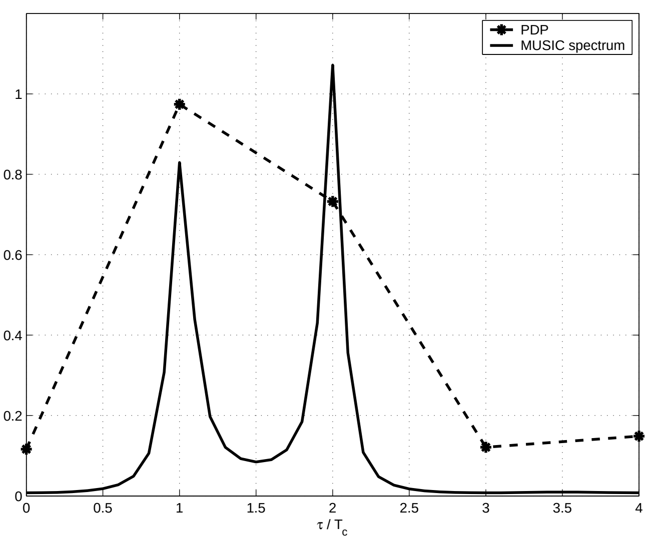

Fig. 1. Comparison of PDP estimate with MUSIC spectrum

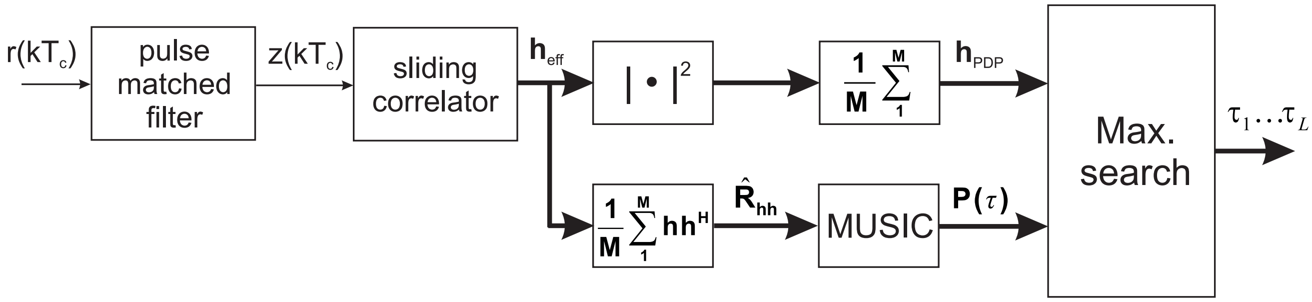

Fig. 2. General acquisition setup

It is noted that a similar application of the MUSIC algorithm is reported in [9], although in a more general context.

The structure of the new delay acquisition is shown in Figure 2 in conjunction with the standard sliding correlator approach. The thin arrows indicate complex-valued signals, whereas the thick arrows stand for vector or matrix signals. In the standard approach in the upper branch, the correlator outputs are magnitude-squared and subsequently averaged to generate the PDP estimate according to (12). In the lower branch, the correlator outputs are processed coherently, with the correlation matrix estimation followed by the subspace decomposition and MUSIC spectrum computation. The maximum search unit can be used for both algorithms.

IV. Performance Analysis

The performance of the proposed delay acquisition algorithm was assessed by means of computer simulations. All simulations were carried out for a channel scenario with two adjacent paths with equal power. The channel snapshot length was chosen to be $N_w = 5$ as for the example in Figure 1. The probability of acquisition, which was estimated by Monte-Carlo simulation of 1000 independent trials, is defined to be the probability that *both* delays were acquired correctly within $0.1\,T_c$.

Figure 3 shows the probability of acquisition as a function of the number of processed uncorrelated channel snapshots M for a delay difference $\Delta_\tau = \tau_1 - \tau_0 = T_c$ and several signal to interference ratios (SIR) measured after correlation. It is seen that already for values of the SIR around 10 dB, corresponding to -14 dB SIR on the channel for $N = 256$, only about 30 channel shapshots are required to acquire both delays with 97% probability. SIR values in this order of magnitude can be expected on the common pilot channel according to 3GPP testcases [11]. The algorithm obviously allows for a certain tradeoff between the SIR and the number of processed snapshots if a fixed acquisition probability is desired. The SIR can be increased by coherent integration of $\mathbf{h}_{eff}$ within the coherence time of the channel if the channel snapshots are correlated i.e. due to slow fading.

If the acquisition time is constrained, one has the possibility to either use less snapshots at a higher SIR or vice-versa. Simulation results have shown that increasing the SIR in general leads to better performance.

The acquisition probability is depicted in Figure 4 for $\Delta_\tau = T_c/2$. Obviously either more channel snapshots ($>$ 400) are required than for $\Delta_\tau = T_c$, or the SIR must be increased (by about 10 dB) in order to achieve the same acquisition probability. Nevertheless it is clearly demonstrated that sub-T_c delay differences can be resolved with the novel algorithm.

The bounds of delay resolvability of the new algorithm are quantified in Figure 5, where the acquisition probability is plotted against the path delay difference Δ_τ for $M = 100$ channel snapshots and several SIR values. It is seen that if an SIR $\geq$ 10 dB is available, 100 snapshots suffice to resolve delay differences even below T_c with high probability.

V. Conclusion

A novel high-resolution delay acquisition algorithm for DS-CDMA was presented and its performance was quantified in a downlink scenario. The algorithm exploits the eigenstructure of the time-averaged sample correlation matrix of a sliding correlator output, decomposing the observation space into a signal-plus-noise subspace and a noise subspace. Projecting delayed versions of the known transmit pulse onto the signal space (or taking the inverse of the projection onto the noise space) yields a MUSIC spectrum which exhibits sharp peaks in the vicinity of the true path delays. Standard maximum search algorithms can be used to determine the path locations.

The algorithm was shown to resolve path delay differences below one chip with high acquisition probability. The complexity of the proposed scheme can be kept low if it is used as a hierarchical extension to a standard sliding correlator, processing only those parts in the delay domain which exhibit non-resolvable clusters in the PDP.

Due to the limited complexity of the algorithm, an implementation in a mobile handset seems feasible. For a realization as a dedicated hardware component or on a fixed-point DSP, the sensitivity of the MUSIC algorithm in general and especially the eigenvector decomposition (EVD) with respect to a fixed-point representation of the data needs to be quantified. Furthermore, alternatives to the EVD such as subspace tracking approaches are under consideration.

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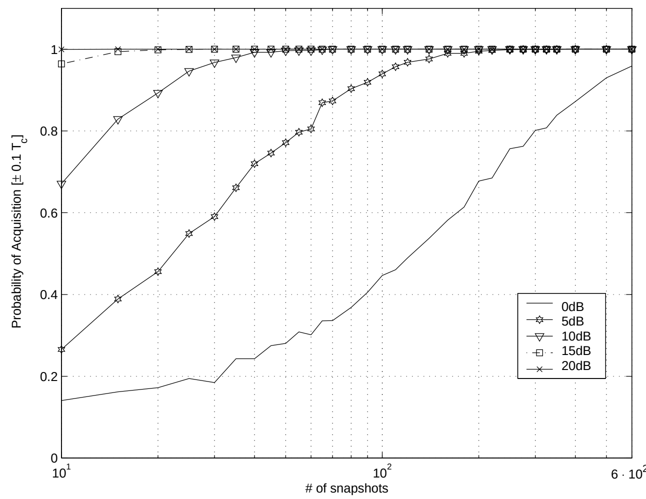

Fig. 3. Acquisition probability, $\Delta_\tau = T_c$

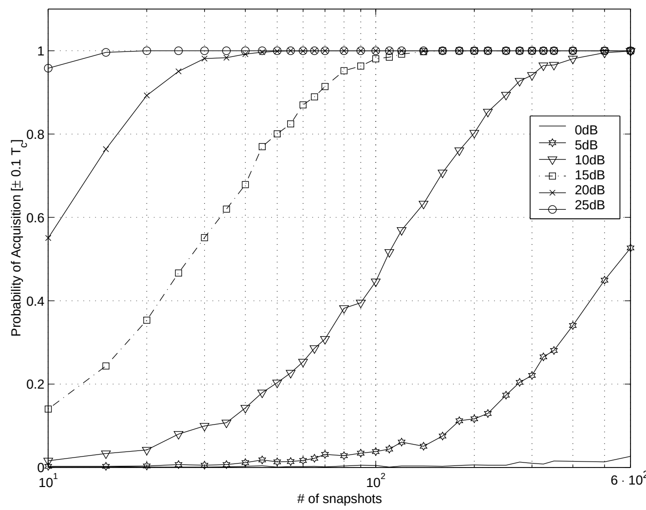

Fig. 4. Acquisition probability, $\Delta_\tau = 0.5\,T_c$

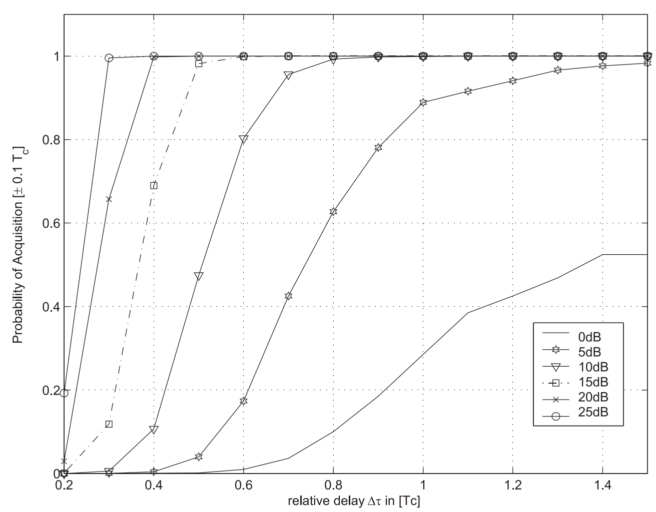

Fig. 5. Acquisition probability, $M = 100$