

# Coarse Time Delay Estimation for Pre-Correction of High Power Amplifiers in OFDM Communications

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*Abstract*— Orthogonal Frequency Division Multiplexing (OFDM) modulations used in DAB, DVB-T and future 4G communications standards are highly sensitive to the presence of non-linear distortion and synchronization errors between the transmitted and received signals. Digital pre-correction schemes can be applied for the compensation of the AM/AM and AM/PM distortion introduced by High Power Amplifiers (HPA). However, this linearization process cannot be realized successfully unless the path delay introduced by the analog chains, required for the observation of the HPA output, is previously estimated. A coarse time delay estimation (TDE) algorithm robust to the presence of unknown non-linearities is presented in this paper. The proposed technique provides the basis for a fast time alignment of base-band signals within a range of several samples with a discrete resolution. Any residual timing offset can be then estimated and compensated by defining some analysis criteria over the obtained TDE spectrum.

*Keywords*— Synchronization, communication systems, nonlinear distortion, orthogonal frequency division multiplexing, multicarrier systems.

## I. INTRODUCTION

NON-CONSTANT Amplitude modulations have become extensive since OFDM format has been designed as a standard transmission technique in the framework of a variety of communications services. Indeed OFDM systems are already operational for DAB (Digital Audio Broadcasting [1]) and DVB-T (Terrestrial Digital Video Broadcasting [2]) communications, while it has been adopted for the new high-bit-rate Wireless Local/Metropolitan Area Networks (W-LAN/MAN) standards, as IEEE802.11a-1999 and HIPERLAN II ETSI specifications [3]. Besides, its application is also being considered for future standards for transmitter diversity such as OFDM-MIMO. In contrast to its advantageous properties in lessening the effects of multipath fading and its implementation facilities, the OFDM signal is characterized by a non-constant envelope with a large Peak-to-average power ratio (PAP), so that these systems are extremely sensitive and vulnerable to non-linear distortion typically introduced by High Power Amplifiers (HPAs) [4]. The RF non-linear HPAs used in transmitters and in repeaters for this kind of broadcasting networks introduce two types of distortion: AM/AM and AM/PM on the modulus and phase of the transmitted signal, respectively. This type of distortion is solely dependent on the

modulus of the input base-band signal and appears at the receiver as a warped symbol constellation (in-band distortion) thereby degrading the bit error rate (BER), while in frequency domain, the distorted signal undergoes spectral regrowth (out-of-band distortion) which generates intermodulation products, and adjacent channel interference (ACI). The performance detriment due to non-linear distortion has been widely dealt with in the literature, and its effects have been well characterized for OFDM systems [5] [6] [7]. To overcome the linearization challenges, several digital pre-distortion (PD) schemes have been proposed [8] [9] assuming ideal synchronization conditions. In PD, a non-linear complex function is applied to the modulus and phase of the modulator base-band signal on its in-phase and in-quadrature components. As this non-linear function constitutes the inverse to the HPA distortion, the linearity of the link is preserved. Adaptive schemes are used to estimate recursively the digital pre-correction function. Nevertheless, the time delay introduced by the analog chains responsible of frequency up-conversion to the HPA input and frequency down-conversion from the HPA output to obtain and observable of the amplifier distortion must be compensated for before estimation of the pre-correction coefficients. Hence, a time delay estimation module is necessary for time-alignment before any adaptive pre-correction scheme is initiated. This module must also be immune to the amplifier distortion characteristic, to guarantee correct alignment when the adaptive pre-correction algorithm is activated. The delay estimation algorithm is formulated in the digital domain, assuming suitable conditions of sampling frequency, which is also evaluated with regard to its influence over the proposed estimation parameter.

## II. NON-LINEAR SYSTEM MODEL

Let us consider an equivalent base-band model of a generic OFDM signal with  $N$  carriers and an associated information symbol rate of  $f_{sym} = 1/\Delta t$ , which gives an OFDM symbol period of  $T = N\Delta t$ . The complex envelope of the OFDM signal driving the input of the HPA can be written for a time interval  $\tau_i \triangleq [iT, iT + T]$  as follows,

$$b_x(t) = \sum_{k=0}^{N-1} a_k(i) \exp(j2\pi f_k t) \quad (1)$$

where  $a_k(i)$  is the data symbol from a M-QAM alphabet that modulates the  $k$ -th carrier of frequency  $f_k$  during the  $i$ -th OFDM symbol interval. Let  $b_x(t)$  be the equivalent

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analog base-band HPA input signal and let  $b_y(t)$  be the corresponding distorted output signal. In the system shown in figure (1), these signals provide information to the pre-distortion algorithms for the identification of HPA transfer function. Then, the simplified input signal model will be given by

$$b_x(t) = u_x(t) \exp^{j \alpha_x(t)} \quad (2)$$

with  $u_x(t) = |b_x(t)|$ . Then, the non-linear distortion of the HPA can be modeled in base-band as a multiplicative complex gain according to the following relationship,

$$b_y(t) = b_x(t) \text{HPA}(u_x(t)) \quad (3)$$

where both the amplitude and phase distortions are jointly declared as input-modulus-dependent functions.

Besides complex non-linear distortion, real HPAs introduce an important memory effect. Such effect is also implicitly assigned to those elements within the dash-lined block in figure (1). The up and down-conversion chains therein included, encompass analog pass-band filters that introduce an important group delay at the signal which, in addition to the HPA's memory, contributes to the time misalignment between  $b_x(t)$  and  $b_y(t)$ . Therefore an effective time delay term  $\Delta$  between the input and output base-band signals (sampled at the input and the output of DAC and ADC, respectively), must be considered.

The continuous time base-band signals are sampled at sufficiently high sampling frequency  $f_s = 1/T_s$ , with  $T_s \ll \Delta t$ , to consider the aliasing related to the (theoretically) infinite bandwidth expansion of the signal due to non-linear distortion and the DAC-ADC conversion branch, negligible compared to the effect of nonlinear distortion itself. Thus, the final non-linear distortion base-band model, in discrete time, can be approximated as follows,

$$\begin{aligned} b_y(n) &= b_x(n - \Delta) \cdot \text{HPA}[u_x(n - \Delta)] \\ &= u_x(n - \Delta) A[u_x(n - \Delta)] e^{j[\alpha_x(n - \Delta) + \Phi[u_x(n - \Delta)]]} \\ &= u_y(n) e^{j \alpha_y(n)} \end{aligned} \quad (4)$$

where  $A[\cdot]$  and  $\Phi[\cdot]$ , represent AM/AM (amplitude modulation/amplitude modulation) and AM/PM (amplitude modulation/phase modulation) distortion respectively. With digital pre-distortion, linearization is obtained estimating a discrete inverse multiplicative function  $\text{HPA}^{-1}[\cdot]$  such that,  $b_x(n - \Delta) = b_y(n) \cdot \text{HPA}^{-1}[u_y(n)]$ . This is only possible if the time delay  $\Delta$  is estimated. Hence, our purpose is to obtain a coarse estimation of  $\Delta$  for synchronization between  $b_x(n)$  and  $b_y(n)$ , necessary to obtain the inverse non-linear characteristic of the HPA. The AM/AM pre-distortion estimation is an inversion problem, and is applied as a multiplicative inverse function,

$$u_{xPD}(n) = u_x(n) A^{-1}(u_x(n)) \quad (5)$$

The AM/PM PD is in turn applied as an additive pre-correction term over the input phase  $\alpha_x(n)$ . The modulus-dependent phase distortion as defined in (4) can be compensated with the pre-corrected phase of the signal driving the HPA input when the PD block is activated,

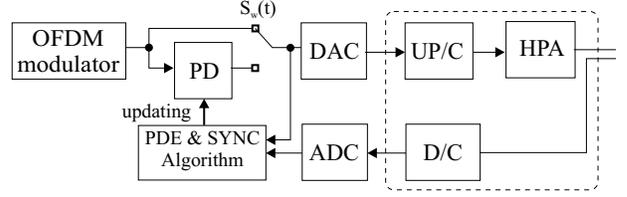


Fig. 1. Block diagram of the pass-band OFDM system model.

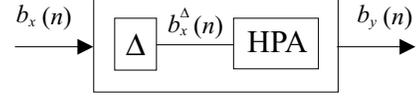


Fig. 2. Baseband model of the HPA including memory effect

$$\alpha_{xPD}(n) = \alpha_x(n) - \Phi[u_{xPD}(n)] \quad (6)$$

where PD can be performed by interpolation of the input signal with respect to a set of coordinated points that define the AM/AM and AM/PM curves.

### III. TIME DELAY ESTIMATION (TDE) ALGORITHM.

The TDE algorithm described in this section is based on the definition of an intelligent cross-correlation between the input and output of the HPA base-band signals. Usual cross-correlation algorithms perform poorly due to the presence of AM/PM distortion and non-uniform probability distribution of the sampled signals. In figure (2) the joint memory effect is modeled as an independent delaying block whose output  $b_x^\Delta(n)$  is unavailable for sampling, and is defined according with the definitions below,

$$b_x(n) = u_x(n) e^{j \alpha_x(n)} \quad (7)$$

$$b_x^\Delta(n) = b_x(n - \Delta) = u_x(n - \Delta) e^{j \alpha_x(n - \Delta)} \quad (8)$$

$$b_y(n) = b_x^\Delta(n) \cdot \text{HPA}(u_x(n - \Delta)) \quad (9)$$

Therefore, this delayed input signal must be estimated to obtain the characterization of the inverse AM/AM and AM/PM curves of the memoryless HPA block. Basically, the goal is to define a parameter proportional to the difference  $|\Delta - \hat{\Delta}|$ , thus obtaining the so-called "Time Delay Spectrum" (TDS), which is a function of the variable testing delay  $\hat{\Delta}$ , with an absolute minimum in  $\hat{\Delta} = \Delta$ . The coarse TDE algorithm will compute a best integer  $\hat{\Delta}$  (expressed as an integer number of samples) in the near vicinity of the real delay  $\Delta$ . To estimate  $\Delta$  we define a modulus-selective cross-correlation between the input and output signals. The output modulus range, defined by the HPA model, is first uniformly divided into  $N_b$  non-overlapping bins. Then, parallel cross-correlations are carried out over the bin array with regard to the selective activation of the intervals so defined. The bin activation function is a vector of the modulus  $u : \mathbf{f}(u)$ . When the  $i$ -th bin is activated by the modulus of a signal sample, the corresponding  $i$ -th

component is set to one and the rest  $N_b - 1$  are set to zero. Hence, the fourth-order cross-correlation function (parameter  $C$ ) can be formerly defined over a pair of input and output sample vectors of length  $N$  according to,

$$C(\Delta) = \mathbf{c}^H(\Delta)\mathbf{c}(\Delta) \quad (10)$$

$$\mathbf{c}(\Delta) = \sum_{n=1}^N b_y^*(n)b_x(n+\Delta)\mathbf{f}(u_y(n)) \quad (11)$$

Then, the computation of the TDS by means of the  $C$  parameter can be performed efficiently by defining first two sample vectors  $\mathbf{b}_x$  and  $\mathbf{b}_y$  sampled respectively from the input and output discrete base-band signals as described in (eq.4). The estimation range of the TDS is limited to  $R$  sample periods. Then for any  $i$ -th time interval where  $n = \{L(i-1)+1, \dots, iL\}$  two  $L$ -length sampled data sets are acquired and subdivided as follows,

$$\mathbf{b}_x(i) = [ b_{x(1)} \cdots b_{x(R/2+1)} \cdots b_{x(R/2+N)} \cdots b_{x(L)} ] \quad (12)$$

$$\mathbf{b}_y(i) = [ b_{y(1)} \cdots b_{y(L)} ] \quad (13)$$

Where  $L$  is the total length of the whole available data sampled at the  $i$ -th interval from the input and output of the HPA+ $\Delta$  combination. Two sub-vectors with a reduced length of  $N = L - R$  are then defined from (12) and (13). A fixed input sub-vector  $\mathbf{b}_{x[N]}(i)$ , and a windowed output sub-vector  $\mathbf{b}_{y[N]}^{\hat{\Delta}}(i)$ , so that it will be possible to displace the window that defines the output subset making  $\hat{\Delta} = \{0, 1, \dots, R\}$  up to entering  $(R+1)$  shift steps in order to obtain the TDS. The set of  $(R+1)$  windowed-testing pairs of sub-vectors will be given by,

$$\mathbf{b}_{x[N]}(i) = [ b_{x(R/2+1)} \cdots b_{x(R/2+N)} ] \quad (14)$$

$$\mathbf{b}_{y[N]}^{\hat{\Delta}}(i) = [ b_{y(1+\hat{\Delta})} \cdots b_{y(N+\hat{\Delta})} ] \quad (15)$$

Hence, if  $\hat{\Delta} \neq \Delta \implies \mathbf{b}_{y[N]}^{\hat{\Delta}} \neq \text{HPA}[b_{x(\Delta+n)}]$  and the vectors lack synchronization between them. An auxiliary vector  $\mathbf{p}$  containing the limits of the  $N_b$  intervals division is also defined,

$$\mathbf{p} = [p_{(1)}p_{(2)} \cdots p_{(N_b+1)}] \quad (16)$$

Then, for each pair of sub-vectors from (14) and (15), the correlation vector  $\mathbf{c}(\hat{\Delta})$  defined in (11) can be easily computed as,

$$\mathbf{c}(\hat{\Delta}) = \mathbf{F}^{\hat{\Delta}}(\mathbf{b}_{y[N]}^{\hat{\Delta}*} \odot \mathbf{b}_{x[N]}) \quad (17)$$

Where  $\odot$  stands for the Schur-Hadamard vectorial product (element-by-element), and  $()^*$  indicates that the terms in the vector are complex conjugated. In this expression,  $\mathbf{F}^{\hat{\Delta}}$  is applied instead of the activation vector  $\mathbf{f}(u_y(n))$  defined in (11).  $\mathbf{F}^{\hat{\Delta}}$  corresponds to the information distribution matrix with size  $(N_b \times N)$  that registers the activation pattern for each bin. Thence, the elements in  $i$ -th row of

the matrix are assigned as follows,

$$F_{(ij)}^{\hat{\Delta}} = \begin{cases} 1 & p_{(i)} \leq |b_{y(j+\hat{\Delta})}| < p_{(i+1)} \\ 0 & \text{otherwise} \end{cases} \quad (18)$$

So that  $C(\hat{\Delta}) = \mathbf{c}(\hat{\Delta})^H \mathbf{c}(\hat{\Delta})$  can be calculated thus obtaining the TDS for the range  $R$  defined.

#### IV. SIMULATION RESULTS

Although the synchronization algorithm described in previous section performs independently of the non-linear characteristic of the HPA, for the baseband signal distortion expressed in (4) we select the widely accepted Saleh model [10] for memoryless Traveling Wave Tube Amplifiers (TWTA), as it introduces more significant AM/PM distortion than the Solid State Power Amplifiers (SSPA) models available. Equations (19) and (20) define this memoryless base-band model of HPA as two modulus-dependent transfer functions given by,

$$A[u_x] = \frac{\alpha_a u_x}{1 + \beta_a u_x^2} \quad (19)$$

$$\Phi[u_x] = \frac{\alpha_\Phi u_x^2}{1 + \beta_\Phi u_x^2} \quad (20)$$

where we set the small-signal gain term to  $\alpha_a = 2$ , while  $\beta_a = \beta_\Phi = 1$  and  $\alpha_\Phi = \pi/3$ , so that the input saturation voltage  $A_s = 1/\sqrt{\beta_a}$ , and the maximum output amplitude  $A_{max} = \max\{A[u_x]\} = \alpha_a A_s/2$ , are both normalized to 1. Hence, the phase distortion at the saturation point is  $\Phi_{sat} = \Phi[1/\sqrt{\beta_a}] = \pi/6 [rad] = 30^\circ$ . These HPA settings were used to distort a 64-QAM OFDM base band signal, generated with 48 useful sub-carriers according to a basic format defined in HIPERLAN II PHY standard. This testing signal has been normalized to match the input dynamic range of the HPA model. Two different oversampling rates have been tested and the TDS results are shown in figures 3 and 4, where the sampling frequency has been reduced in a 6/8 ratio in order to confirm that temporal correlation of data determines proportionally the temporal bandwidth of the main lobe in the TDS. Subsequently, these results of TDS lead us to obtain the S-curves by means of normalized differentiation of  $C(\hat{\Delta})$ . Given a range  $R = 80$  and a real delay  $\Delta = 40$ , results were obtained using different vector lengths:  $L = \{128, 256, 512, 1024, 2048\}$ , and several number of bins  $N_b = \{8, 16, 32, 64\}$ . Results with  $L = 128$  were omitted in figures as they confirm that  $N = L - R = 48$  doesn't fulfill the minimum length requirement  $L \geq 2R$  to compute the TDS using (17). When using  $L < 2R$ , deficient main-to-secondary lobe discrimination ranges are obtained in the spectrum.

#### V. CONCLUSIONS

An efficient low-complexity algorithm has been presented for the raw estimation of the time delay in a digital base-

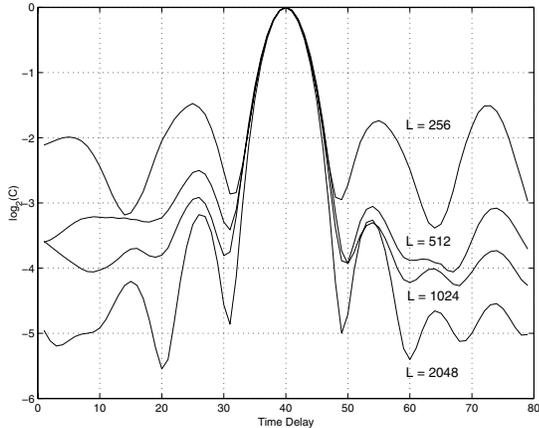


Fig. 3. Time delay spectrum for several lengths of data sets. Spectra are normalized and scaled as logarithmic in base 2 to observe the discrimination margin of the main lobe which is proportional to the length  $N$  of the windowed subsets.

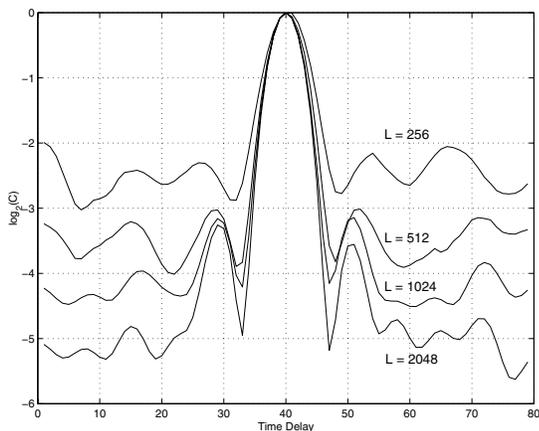


Fig. 4. Narrowing effect on the main lobe of TDS for a 8/6 higher oversampling rate than used in figure 3.

band OFDM model with non-linear distortion. Digital Pre-Distortion (PD) is a suitable solution for adaptive linearization in wideband OFDM systems, because of its relative implementation simplicity and its adaptability to be included into an existing non-linear transmitter, as a low power consumption separate stand-alone unit [11] [12]. However its implementation requires an estimation of the HPA's inherent time delay as well as the group delay due to the filters in U/C and D/C blocks. For a more realistic approach it is necessary to consider that  $\Delta/T_s$  is a non-integer value, within a continuous range  $\{0, R\}$  of possible delay values. This range  $R$  is normally defined as an integer multiple of  $T_s$ , several times greater than the design-specified nominal delay of the equipment. Digital implementation of time-shift of a discrete sampled signal, in fractions of the sampling time  $T_s$ , can be obtained with an interpolation stage. This enables to perform fine synchronization after fast-coarse time delay estimation. This can be done by applying a symmetry criterion over the TDS coefficients around its absolute minimum. The bin activation probability in (18) is determined in this case by the probability

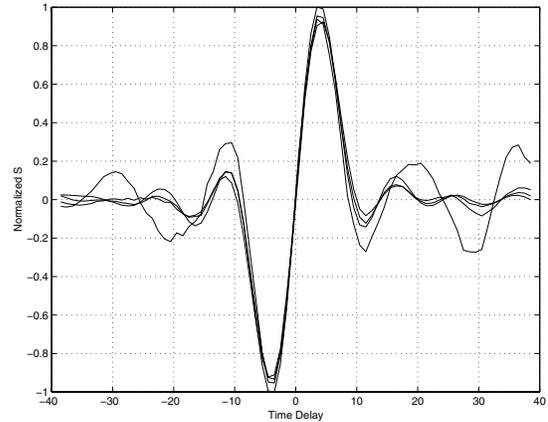


Fig. 5. Family of S-curves corresponding to the TDS shown in figure 3. The lack of discrimination margin when using  $L = 256$  samples cause slow-decaying tails in the S curve.

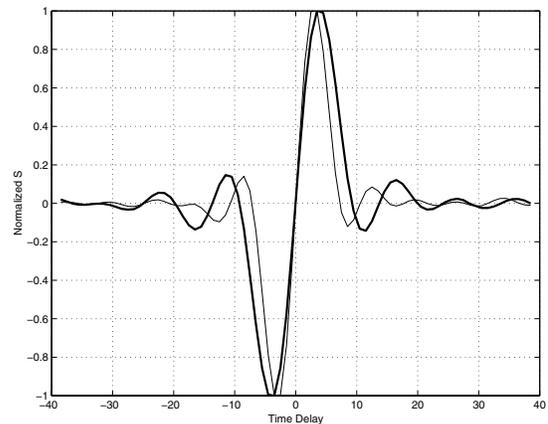


Fig. 6. Comparison of S-curves obtained for two different oversampling rates. Darker corresponds to a 6/8 reduction in sampling frequency. A signal length of  $N = 2048$  samples is considered in both cases.

distribution (PDF) of an OFDM signal which is very similar to the complex Gaussian distribution for a sufficient number of subcarriers (Central Limit Theorem). Hence, it is assumed that its modulus and phase are independent random variables drawn from a Rayleigh and uniform distribution, respectively. This statistic influence could be included for an optimal definition of the bins, adapting the limits in the vector  $\mathbf{p}$  according to the bin activation probability.

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