Performance bound of dynamic forward link adaptation in cellular WCDMA networks using high-order modulation and multicode formats

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The problem of optimal dynamic rate and power adaptation for forward link data transmission using high-order modulation and multicode formats in cellular WCDMA networks is formulated as a constrained optimisation problem. Based upon this formulation, a combination of exhaustive search (over all the possible allocations of modulation order) and linear programming is used to obtain the upper bound on the achievable radio link level throughput.

System model: We consider the problem of dynamic rate and power adaptation for forward link data transmission under constrained frame error rate (FER) in a cellular multicode WCDMA network using high-order modulation (e.g. M-ary QAM for $M=4,\ 16,\ 64, \ldots,$ where 4-ary QAM is equivalent to QPSK) and multicode formats. Dynamic rate adaptation refers to the dynamic allocation of the modulation order ($\alpha_i^{(j)} \in \{1, 2, \ldots, K\}$ for ith mobile in cell j, $i = 1, 2, \ldots, g_j$ and $j = 0,\ 1, \ldots,\ J$) along with the number of code channels ($\phi_i^{(j)} \in \{1, 2, \ldots, M\}$ for ith mobile in cell j) corresponding to transmissions to the mobiles in the downlink. Power adaptation refers to the dynamic proration of a fixed amount of transmission power among the different users in a cluster of $(J+1)$ cells. A fixed spreading gain (N) is assumed for each code channel and the same modulation order is assumed to be used in each of the code channels allocated for downlink transmission to a mobile. The criterion for optimal rate and power adaptation is the maximisation of the average number of radio link level frames transmitted per frame-time (or sum-rate throughput) under common FER constraint (for all users) and constrained power budget in the base station (BS) transmitters.

Assuming that the signal power received at the ith mobile in cell j from $(\mathrm{BS})_j$ is attenuated by the δth power of the distance $r_{i,j}$ and shadowing follows a log-normal distribution, the long-term fading in terms of path-loss and shadowing can be expressed as

$$L_j(j, i) = r_{i,j}^{-\delta} \cdot 10^{\xi_{i,j}/10} \tag{1}$$

where $\xi_{i,j}$ is a zero-mean Gaussian random variable with variance σ^2. Ignoring the background noise and assuming asynchronous transmission among all base stations $(\mathrm{BS})_j$ ($j = 0,\ 1, \ldots,\ J$) [1] and that the downlink channel is interference-limited so that the composite of in-cell and other-cell interferences becomes dominant, for a standard RAKE receiver the downlink SIR can be formulated as [2]

$$(\mathrm{SIR})_{o,i}^{(j)} = \frac{P_{b,i}^{(j)} N}{\varepsilon \bar{P}_{B,j}[(1-v) + \sum_{j' \neq j} \beta_{j'/j} \cdot \eta_{j'/j}(i)]} \tag{2}$$

where v denotes the in-cell orthogonality factor ($v = 1$ corresponds to perfectly orthogonal in-cell mobiles), ε is the chip pulse-shape factor, $P_{b,i}^{(j)}$ represents the power allocation to the ith mobile in cell j at basic rate (i.e. for $\alpha_i^{(j)} = \phi_i^{(j)} = 1$ which corresponds to QPSK modulation), $\bar{P}_{B,j}$ is the average total power allocation to $(\mathrm{BS})_j$, $\beta_{j'/j} = \bar{P}_{B,j'}/\bar{P}_{B,j}$, and $\eta_{j'/j}(i)$ is the inter-cell interference factor defined as

$$\eta_{j'/j}(i) \triangleq \frac{\zeta_i^{(j')} L_{j'}(j, i)}{\zeta_i^{(j)} L_j(j, i)} \tag{3}$$

In (3), $\zeta_i^{(j)} \triangleq \sum_{l=1}^{V} a_{i,j,l}^2$ models the short-term fading where $a_{i,j,l}$ is the lth path gain (V = number of multipaths). For transmission using high-order modulation and multicode formats $\bar{P}_{B,j}$ can be expressed as

$$\bar{P}_{B,j} = \sum_{i=1}^{g_j} \frac{1}{3}(4^{\alpha_i^{(j)}} - 1)\phi_i^{(j)} P_{b,i}^{(j)} + P_c \tag{4}$$

where P_c is power allocation to the pilot signal. In (4), $\alpha = 1,\ 2,\ 3$ correspond to QPSK, 16-QAM and 64-QAM, respectively. The rate allocation corresponding to ith mobile in cell j is determined by two parameters $\alpha_i^{(j)}$ and $\phi_i^{(j)}$, and transmission rate $m_i^{(j)}$ is given by $m_i^{(j)} \triangleq \alpha_i^{(j)} \times \phi_i^{(j)}$. Note that, for ith mobile in cell j achieved SIR (i.e. $(\mathrm{SIR})_{o,i}^{(j)}$) depends on rate (hence modulation order and number of code channels) and power allocations corresponding to mobiles in different cells.

Using the downlink SIR in (2), the symbol error rate (SER) for the ith mobile in cell j, denoted by $p_{e,i}^{(j)}$, can be evaluated as in (5) [3], where $Q(x) = 1/\sqrt{(2\pi)}\int_x^\infty e^{-y^2/2}\,dy$:

$$p_{e,i}^{(j)} = 4(1 - 2^{-\alpha_i^{(j)}})Q\left(\sqrt{(\mathrm{SIR})_{o,i}^{(j)}}\right) - 4(1 - 2^{-\alpha_i^{(j)}})^2 Q^2\left(\sqrt{(\mathrm{SIR})_{o,i}^{(j)}}\right) \tag{5}$$

For a specific modulation order $\alpha_i^{(j)}$, the bit error rate (BER) $p_{b,i}^{(j)}$ can be approximated by

$$p_{b,i}^{(j)} \cong (1/2\alpha_i^{(j)})\, p_{e,i}^{(j)} \tag{6}$$

when Gray encoding is used.

For radio link level error control we consider a selective-repeat (SR) type of hybrid ARQ error control scheme using an individual frame decoding for which the FER can be expressed as

$$P_{E,i}^{(j)} = 1 - \sum_{e=0}^{t} \binom{L}{e} (p_{b,i}^{(j)})^e (1 - p_{b,i}^{(j)})^{L-e} \tag{7}$$

where t-bit error correction capability is assumed for each radio frame of length L bits and $\binom{k}{l} \triangleq k!/(k-l)!/l!$. For given t and target FER the required SIR (when the modulation order is α) can be obtained by combining (5) and (6) with (7).

Optimal rate and power allocation:

A Problem formulation: The optimal multi-cell (i.e. $(J+1)$ cell) rate (in terms of $\{\alpha_i^{(j)}\}$ and $\{\phi_i^{(j)}\}$) and power allocation problem can be formulated as a constrained optimisation problem as follows:

$$\text{maximise} \quad f = \sum_{j=0}^{J} \sum_{i=1}^{g_j} \phi_i^{(j)} \alpha_i^{(j)} \tag{8}$$

$$\text{subject to} \quad H_1 \ (\text{Domain for } \phi)\colon \sum_{i=1}^{g_j} \phi_i^{(j)} \leq M, \quad \phi_i^{(j)} \geq 0 \tag{9}$$

$$H_2 \ (\text{Domain for } \alpha)\colon 0 \leq \alpha_i^{(j)} \leq (1/2p_{b,i}^{(j)}), \quad \alpha_i^{(j)} \leq K \tag{10}$$

$$H_3 \ (\text{Power budget})\colon \sum_{j=0}^{J} \bar{P}_{B,j} \leq (J+1)P_B \tag{11}$$

$$H_4 \ (\text{FER constraint})\colon (\mathrm{SIR})_{o,i}^{(j)} \geq (\mathrm{SIR})_{o,\alpha_i^{(j)}} \tag{12}$$

Here, P_B is the average power budget available at the base station of a cell, and M is the total number of code channels available per cell with $M \leq N$. Note that instead of an absolute power constraint per cell an average power constraint is assumed in (11). With a limited power budget across $(J+1)$ cells, better throughput performance can be obtained due to the statistical multiplexing of the power resource when a mean power constraint is assumed. The first constraint in (10) ensures non-negative values for $Q(\cdot)$ in (5).

The optimal allocation of modulation order $\{\alpha_i^{(j)}\}$, number of codes $\{\phi_i^{(j)}\}$ (hence the transmission rate $m_i^{(j)}$) and total power ($\{\bar{P}_{B,j}\}$) can be found by cycling through all possible combinations of modulation order and multicode formats to determine the rate allocations which maximise the sum-rate throughput while satisfying all the constraints. This would incur exponential time-complexity and, hence, the problem of optimal rate selection (which is similar to the satisfiability problem ([4], p. 673)) is NP-complete. An upper bound on the achievable throughput, which is reasonably close to the optimal performance, would be useful to assess the performances of sub-optimal rate allocation procedures.

B Upper bound on sum-rate throughput: Denoting $P_{b,i}^{(j)}(4^{\alpha_i^{(j)}} - 1)$ as $c_{1,i}^{(j)}$ and $3(1 - P_c/P_B)$ as ψ, the constraints in (11) and (12) can be expressed as follows:

$$\sum_{j=0}^{J} \sum_{i=1}^{j} c_{1,i}^{(j)} \phi_i^{(j)} \leq \psi(J+1)P_B \tag{13}$$

$$\sum_{l=1}^{g_j} c_{2,l}^{(j)} \phi_l^{(j)} + \sum_{j' \neq j} \sum_{l=1}^{g_{j'}} c_{3,l}^{(j'/j)}(i) \phi_l^{(j')} \leq c_{4,i}^{(j)} \tag{14}$$

where $c_{2,l}^{(j)} = c_{1,l}^{(j)} \cdot (1 - v)$, $c_{3,l}^{(j'/j)}(i) = c_{1,l}^{(j')} \cdot \eta_{j'/j}(i)$, and $c_{4,i}^{(j)} = (\psi \cdot P_{b,i}^{(j)} \cdot N)/(\varepsilon (\mathrm{SIR})_{o,\alpha_i^{(j)}})$.

Since for a particular allocation of the modulation order α the objective function and all the constraints are now linear in $\phi_i^{(j)}$, rate allocations can be determined using linear programming. However, to find the optimal allocation an exhaustive search over the modulation order is still required. That is, by searching over all possible combinations of α, the set of $\alpha_i^{(j)}$ and $\phi_i^{(j)}$ which gives rise to the maximum sum-rate throughput can be determined. Since the search space is now reduced only to the α subspace, the complexity of computation reduces greatly compared to the pure exhaustive search.

However, since using linear programming may result in non-integer values for $\{\phi_i^{(j)}\}$, this approach essentially gives the upper bound on the sum-rate throughput. Note that $P_{b,i}^{(j)}$ is a key parameter in this formulation. As we will show later, the upper bound on sum-rate throughput can be obtained by choosing any value of $P_{b,i}^{(j)}$ within an operating region for all mobiles.

Simulation model, results and discussions: Three cells (i.e. $J=2$) are considered in a hexagonal cell-layout. For a given number of modulation orders and code channels, similar transmission rate may be achieved for different combinations of modulation order and number of code channels. In such cases transmission rates corresponding to the minimum modulation orders are selected. For example, with $K=3$ and $M=3$, $(\alpha, \phi) \in \{(1, 1), (1, 2), (1, 3), (2, 2), (2, 3), (3, 3)\}$.

The inter-cell interference factor $\eta_{j'/j}(i)$ is calculated using (3) such that $L_j(j, i) > L_{j'}(j, i)$ for all j' (i.e. effect of soft-handoff is ignored). Over a radio frame transmission time, the values of $L_{j'}(j, i)$ and $L_j(j, i)$ are assumed to be constant, and the value of $\eta_{j'/j}(i)$ is calculated by using the average of n $(=16)$ independent values of $\zeta_i^{(j')}/\zeta_i^{(j)}$.

Variation in the upper bound on sum-rate throughput with the target FER follows the same trend as that achievable by optimal rate allocation (Fig. 1). Also, they are observed to be reasonably close.

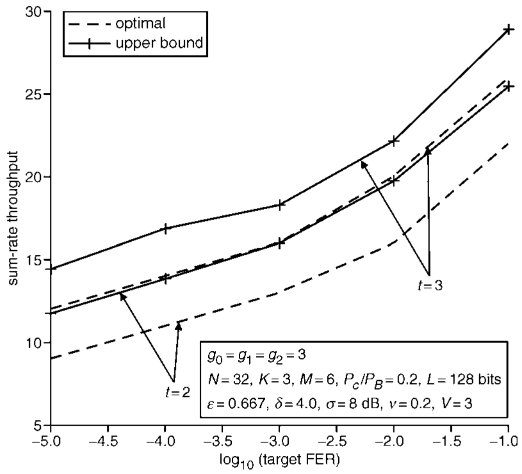

Fig. 1 *Comparison between optimal sum-rate throughput and upper bound on sum-rate throughput*

Evaluation of the upper bound on sum-rate throughput reveals that the throughput does not monotonically decrease with increasing $P_{b,i}^{(j)}$ (Fig. 2). As $P_{b,i}^{(j)}$ varies, the throughput remains constant until $P_{b,i}^{(j)}$ reaches a threshold, and then it falls off rapidly with increasing $P_{b,i}^{(j)}$. When $P_{b,i}^{(j)}$ is sufficiently small, throughput is constrained primarily by the limited number of modulation order and number of code channels. However, as $P_{b,i}^{(j)}$ increases, throughput is constrained by the inability to allocate high rate due to limited power budget. Note that, in a practical system, a minimum value of $P_{b,i}^{(j)}$ would be required to combat the effect of noise. The threshold value for $P_{b,i}^{(j)}$ can be considered as the optimal value of $P_{b,i}^{(j)}$ in the sense that it is the maximum basic power level to combat noise without incurring any reduction in throughput due to the interference.

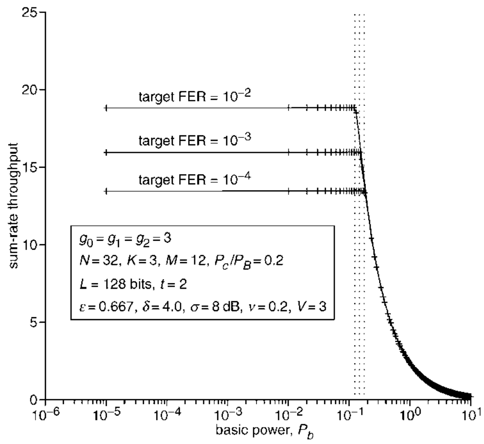

Fig. 2 *Variations in upper bound on sum-rate throughput with basic power*

The region to the left of the threshold $P_{b,i}^{(j)}$ (Fig. 2) can be considered as the operating region from which the different values of $P_{b,i}^{(j)}$ can be chosen for dynamic rate and power adaptation such that the maximum possible throughput can be achieved. As long as the value of $P_{b,i}^{(j)}$ (which may be chosen to be same or different for each user) falls within the operating region the upper bound on the sum-rate throughput does not change.

As the target FER increases/decreases, the upper bound on sum-rate throughput increases/decreases (Fig. 2). Therefore, for a given power budget, as the FER requirements become more stringent/relaxed, the threshold value of $P_{b,i}^{(j)}$ increases/decreases.

Acknowledgments: This work was supported in part by a scholarship from TR*Labs*, Winnipeg, Canada, and in part by the Natural Sciences and Engineering Research Council (NSERC) of Canada.

27 November 2003
Electronics Letters online no: 20040038
doi: 10.1049/el:20040038

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