Absolute Phase Estimation with Discontinuities: A Stochastic Nonlinear Filtering Approach*

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Abstract

The paper proposes a new method for the absolute phase (not simply modulo-2π) reconstruction in *Interferometric Synthetic Aperture Radar* (InSAR) applications, when discontinuities are present. By adopting a Bayesian viewpoint, the proposed approach integrates the absolute phase observation probabilistic model jointly with its prior knowledge; the observation mechanism takes into account the joint probability of the InSAR image pair, given the absolute phase; the *a priori* probability of the absolute phase is modelled by a *Compound Gauss Markov random field* (CGMRF) tailored to piecewise smooth absolute phase images. The *maximum a posteriori probability* (MAP) criterion is adopted. An iterative scheme embodying *stochastic nonlinear filtering* and *iterative conditional modes* (ICM) steps is used to compute the MAP solution. A set of experimental results illustrate the appropriateness of method.

1 Introduction

Synthetic Aperture Radar (SAR) is a coherent system that produces high resolution images of the electric field backscatterd by the surface being illuminated [1]. SAR images are typically acquired by a single antenna. By using two antennas (actually two antennas in single-pass mode, or one antenna in repeat-pass mode) separated by a baseline, it becomes possible to *interfere* the two images in such a way that the common scene reflectivity is canceled and the geometric information contained in the scene topography is retained in the phase difference. It is the so-called *interferometric synthetic aperture radar* (InSAR).

In a SAR system, as in any coherent system, only noisy versions of the *principal phase values* (modulo-2π) are available, as computed from the argument of the received wave. However, in InSAR applications, the objective is the estimation of the absolute phase (phase unwrapping in the InSAR jargon), and not simply its modulo-2π.

Classical phase unwrapping methods are either of *path following type* or of least-squares type (e.g., [2], [3]. In the path following schemes, phase is unwrapped through selected image patches. In the presence of discontinuities or noise, different patches between two points may lead to different absolute phase differences. Heuristic rules are applied to resolve or mitigate the inconsistencies [2], [4]. Typically, unwrapping methods that do no rely on path following cast the problem in the least-squares formalism [5]. Recently, it has been shown that the least-squares solution to phase unwrapping is equivalent to the solution of the discretized Poisson equation with Newmann boundary conditions. This solution can be computed efficiently by using fast cosine or Fourier transforms [6]. Due to decorrelation (temporal and spatial), no-return or low return areas (e.g., due to layover phenomena), the modulo-2π phase estimates corresponding to those areas might be extremely biased/noisy. In an attempt to include this information in the unwrapping procedures, the weighted least-squares approach has been used [7], taking as weights the correlation coefficients. This idea can be extended to discontinuities by assuming that the correlation coefficient between two sites split by a discontinuity is zero.

In a quite different vein, papers [8], [9] have adopted the Bayesian viewpoint, in addressing absolute phase reconstruction in interferometric applications. The Bayesian approach supplies a model-based framework suited to describe the data observation mechanism and its prior knowledge. In this fashion, paper [8] proposes a nonlinear recursive filtering approach to the absolute phase reconstruction in interferometric type applications. Paper [9] considers the InSAR observation model, taking into account not only the image phase, but also the *backscattering coefficient* and the *correlation factor* images, which are jointly recovered from SAR image pairs. Contrarily to the weighted least-squares approach, the methodology followed in [9] uses the information conveyed by the correlation coefficients in a model-based fashion.

In a way or another, most phase unwrapping algorithms assume that the phase difference between two neighbor sites varies smoothly (less than π in a deterministic or stochastic sense, depending on the paradigm); based on this prior knowledge, it is possible, by *looking* at the neighboring site phases, to infer the 2π multiple component of the phase of a given site. However, in situations such as undersampling in terrains with high fringe rates, abrupt feature/objects, or the *layover* phenomena, the smoothness assumption can not be taken any more. Independently of its origin, phase discontinuities are the principal source of error in any unwrapping algorithm that does not take them into account.

Work [10] proposes a method in which the unwrapped phase is chosen to minimize a weighted sum of discontinuity magnitudes. The weights are set close to zero for noisy or discontinuity regions. In [11], a Markovian approach to phase unwrapping using a discontinuity model is followed. The phase unwrapping is cast into a labeling problem; the labels are the 2π multiples of the phase at each site. Neural

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networks, exploiting the structure of discontinuity phase pattern, are applied in [12].

1.1 Proposed Approach

The paper proposes a two step procedure to absolute phase estimation in presence of phase discontinuities: (a) determine the discontinuity field; (b) determine the absolute phase using the discontinuity field obtained in step (a). In this paper, we address only the step (b).

The approach is Bayesian and therefore model-based. SAR images are described as random fields whose statistical properties are built upon the physical mechanism of image generation. This random field is parametrized by the absolute phase. A noncausal first order CGMRF is taken as prior. The CGMRF is suited to piecewise smooth fields, and, therefore, to account for discontinuities between smooth regions.

The MAP estimation criteria is selected. The estimate is determined by means of a scheme embodying nonlinear recursive stochastic filtering and ICM steps.

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2 Observation Model

For a given InSAR geometry, the terrain elevation is obtained from the phase $\phi = \phi_2 - \phi_1$, where ϕ_1 and ϕ_2 are the propagation path phases read by the two antennas. Phase ϕ relates, in a noisy and nonlinear way, with the observed SAR images (interferometric pair).

Denote x_1 and x_2 the complex amplitudes (inphase and quadrature components packed into complex numbers) of the backscattered field read by each antenna at a given pixel. These amplitudes are given by

$$\begin{aligned} x_1 &= z_1 e^{-j\phi_1} + n_1 && (1)\\ x_2 &= z_2 e^{-j\phi_2} + n_2, && (2) \end{aligned}$$

where z_i, for $i = 1, 2$, is the complex amplitude originated by the scatterers illuminated by aperture i, and n_i is the respective electronic noise.

Assuming that the surface being illuminated is rough compared to the wavelength, that there are no strong specular reflectors, and that there are a *large* number of scatterers per resolution cell, then the complex amplitude z_i are circularly symmetric and Gaussian [13]. Noises n_i are independent of complex amplitudes z_i, and also circularly symmetric and Gaussian. Complex amplitudes z_1 and z_2 are different due to spatial and temporal decorrelations. The former is originated by non-overlapping portions or the aperture regions of each antenna; the latter is originated by scatterer displacements. We assume that[1] $E[|z_1|^2] = E[|z_2|^2] := \theta^2$ and that $E[z_1 z_2^*] := \alpha\theta^2$, where α stands for the correlation factor between z_1 and z_2, also termed *change parameter* or *degree of coherence* [13]. We assume that $\alpha \in [0, 1]$, which is valid whenever the scatterer displacements have an even distribution.

[1] For compactness, lowercase latters will denote random variables and their values as well. Also, the generic probability density function of the generic random vector $\mathbf{x}$, $p_{\mathbf{x}}(\mathbf{x})$, or of the generic random variable,x, $p_x(x)$, will be denoted by $p(\mathbf{x})$ and $p(x)$, respectively.

Defining $x := [x_1\ x_2]^T$, $\theta := E[|z_1|^2]$, $E[|n_1|^2] := \sigma_n^2$, and assuming that $E[|n_1|^2] = E[|n_2|^2]$, the probability density function of x is written as [13]

$$p(x|\phi,\theta,\alpha) = \frac{1}{\pi^2|\mathbf{Q}|} e^{-x^H \mathbf{Q}^{-1} x}, \qquad (3)$$

where $\mathbf{Q} := E[xx^H]$ is given by

$$\mathbf{Q} = \begin{bmatrix} \theta^2 + \sigma_n^2 & \alpha\theta^2 e^{-j\phi} \\ \alpha\theta^2 e^{j\phi} & \theta^2 + \sigma_n^2 \end{bmatrix}. \qquad (4)$$

Developing the quadratic form in (3), one is led to

$$p(x|\phi,\theta,\alpha) = c e^{\lambda \cos(\phi - \eta)}, \qquad (5)$$

where $c = c(x, \theta, \alpha)$

$$\begin{aligned} \eta &= \arg(x_1^* x_2) && (6)\\ \lambda &= \frac{2\alpha\theta^2 |x_1 x_2|}{|\mathbf{Q}|}. && (7) \end{aligned}$$

Often, parameters θ and α are unknown and must be jointly estimated with the absolute phase ϕ. This issue is addressed in [9], where the images formed by those parameters are modelled as independent noncausal first order *Gauss Markov random fields* (GMRF) [14]. Since our main target in this paper is the determination of ϕ in the presence of discontinuities, we assume that images $\boldsymbol{\theta}$ and $\boldsymbol{\alpha}$ are known. For this reason, parameters θ and α will be omitted from now on.

The formal structure of (5) would be exactly the same if the observation mechanism was

$$x = e^{j\phi} + n, \qquad (8)$$

with n being complex circular symmetric and Gaussian noise of variance σ, and

$$\begin{aligned} \eta &= \arg(x) && (9)\\ \lambda &= \frac{|x|}{\sigma_n^2}. && (10) \end{aligned}$$

Model (8) is adopted in [8], where the absolute phase reconstruction is addressed under a stochastic nonlinear filtering approach.

Let $x_{ij} := [x_{1ij}\ x_{2ij}]^T$ denote the random vector of complex amplitudes associated to the site (i,j) and $\mathbf{x} := \{x_{ij},\, i,j = 1,\ldots,N\}$ (we assume without lack of generality that images are squared), and $\boldsymbol{\phi} = \{\phi_{ij},\, i,j = 1,\ldots,N\}$. Assuming that the components of $\mathbf{x}$ are conditionally independent, then

$$p(\mathbf{x}|\boldsymbol{\phi}) = \prod_{ij=1}^{N} p(x_{ij}|\phi_{ij}). \qquad (11)$$

The conditional independence assumption is valid if the resolution cells associated to any pair of pixels are disjoint. Usually this is a good approximation, since the *point spread function* (PSF) of the SAR system is only slightly larger than the corresponding inter-pixel distance [15]. Anyway, the correlation introduced by the PSF can be modelled by assuming that the observed vector is $\mathbf{y} = \mathbf{B}\mathbf{x}$, where $\mathbf{B}$ is the associated blur matrix.

2.1 Prior Model

Image $\boldsymbol{\phi}$ is assumed to be *piecewise smooth*, with abrupt variations between neighboring regions. This variations may arise due to undersampling in terrains with high fringe rates, due to the presence of abrupt feature/objects, or due to the *layover* phenomena. Independently of its origin, discontinuities of the absolute phase ϕ are the principal source of error in any unwrapping algorithm that does not take them into account.

Gauss-Markov random fields [16] are both mathematically and computationally suitable for representing local interactions, and particularly continuity between neighboring pixels. However, the continuity constraint must be discarded for those pixels near the discontinuities. For this purpose we take the *first order* noncausal CGMRF

$$p(\boldsymbol{\phi}|\boldsymbol{l}) \propto \exp\left\{-\frac{\mu}{2}\sum_{ij}(\Delta_{ij}^{h})^2\bar{v}_{ij} + (\Delta_{ij}^{v})^2\bar{h}_{ij}\right\}, \qquad (12)$$

where $\mathbf{l} := \{v_{ij}, h_{ij}\}$ is the so-called *line field* process, $\bar{v}_{ij} := (1 - v_{ij})$, $\bar{h}_{ij} := (1 - h_{ij})$, $\Delta_{ij}^{h} := (\phi_{ij} - \phi_{i,j-1})$, $\Delta_{ij}^{v} := (\phi_{ij} - \phi_{i-1,j})$, and μ^{-1} means the variance of increments Δ_{ij}^{h} and Δ_{ij}^{v}. Notice that continuity constraint between sites (i,j) and $(i,j-1)$ is removed if variable v_{ij} is set to one; the same is true concerning horizontal lines.

2.2 Posterior Density

Consider that the line field process $\mathbf{l}$ is known. Invoking the Bayes rule, and noting that $p(\mathbf{x}|\boldsymbol{\phi},\mathbf{l}) = p(\mathbf{x}|\boldsymbol{\phi})$, we obtain the posterior probability density function of $\boldsymbol{\phi}$, given $(\mathbf{x},\mathbf{l})$, as

$$p(\boldsymbol{\phi}|\mathbf{x},\mathbf{l}) \propto p(\mathbf{x}|\boldsymbol{\phi})p(\boldsymbol{\phi}|\mathbf{l}), \qquad (13)$$

where the factors not depending on $\boldsymbol{\phi}$ where discarded. Introducing (5),(11), (12) in (13), one obtains

$$p(\boldsymbol{\phi}|\mathbf{x},\mathbf{l}) \propto e^{\sum_{ij} \lambda_{ij}\cos(\phi_{ij}-\eta_{ij}) - \frac{\mu}{2}\left((\Delta_{ij}^{h})^2\bar{v}_{ij} + (\Delta_{ij}^{v})^2\bar{h}_{ij}\right)} \qquad (14)$$

Since our approach is Bayesian,the posterior distribution (14) contains all information one needs to compute the absolute phase estimate $\widehat{\boldsymbol{\phi}}$.

3 Estimation Procedure

The MAP criterion is adopted for computing the phase $\boldsymbol{\phi}$. Accordingly

$$\widehat{\boldsymbol{\phi}}_{MAP} = \arg\max_{\boldsymbol{\phi}} p(\boldsymbol{\phi}|\mathbf{x},\mathbf{l}). \qquad (15)$$

Due to the periodic structure of $p(x|\phi)$, also termed the *observation factor*, computing the MAP solution leads to a huge non-convex optimization problem, with unberable computation burdeen. Instead of computing the exact estimate $\widehat{\boldsymbol{\phi}}_{MAP}$, we resort to a suboptimal scheme that delivers nearly optimum estimates, with a reasonable computational load.

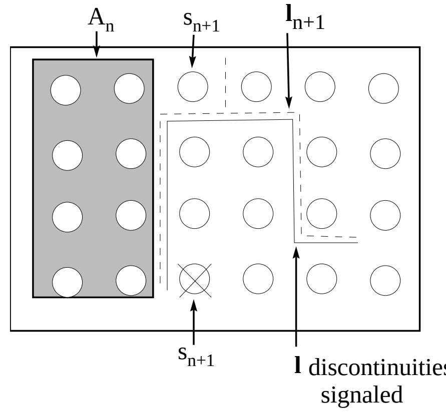

Figure 1: Illustration of sequence $s_n \in Z_N$. The crossed site cannot be the next sequence element.

Let V_s, with $s \in Z_N := \{(i,j)|\, i,j = 1,\ldots,N\}$, be the set of neighbors of site s in the Markovian sense: i.e.,

$$p(\phi_s|\phi_r, r \neq s,\, r \in Z_N) = p(\phi_s|\phi_r, r \in V_s). \qquad (16)$$

Since we have adopted a noncausal first order random Markov field, the neighbors of site (i,j) are all elements (boundary apart) of the set $\{(i+1,j),(i-1,j),(i,j+1),(i,j-1)\}$ that do not have a discontinuity signaled between them and site (i,j).

Define the site sequence $s_n \in Z_N$, with $n = 1,2,\ldots,N^2$, and the sets $A_n := \{s_1, s_2, \ldots, s_n\}$, where $s_i \in Z_N$, and $V_{A_n} := \bigcup_{i=1}^{n} V_{s_i}$, such that

$$\begin{array}{l} s_{n+1} \notin A_n \\ s_{n+1} \in V_{A_n}. \end{array} \qquad (17)$$

Fig. (1) schematizes a set A_n and a possible site s_{n+1}. The crossed site cannot be the next sequence element s_{n+1}, since it does not belong to V_{A_n} by virtue of discontinuity field configuration.

We assume that the discontinuity field does not divide the set of sites into independent sets. This means that there exits at least one path, made of neighbor sites, between any pair of sites. In this situation there exists at least one sequence verifying (17). In any real scenario, the number of sequences s_n verifying (17) is huge, some better than others. A possible way of choosing *good* sequences, from the estimate *goodness* viewpoint, is by selecting sites with higher parameter α (i.e., less noisy η_{ij} estimate) in first place. This topic is, however, out of the scope of this paper.

In addition to the above definitions, we introduce vector $\mathbf{l}_{n+1}$ denoting a discontinuity field configuration equal to $\mathbf{l}$ expect for those variables v_{ij} and h_{ij} signaling discontinuities between site s_{n+1} and the set of sites not in A_n: this variables are set to one, *disconnecting* site s_{n+1} from sites not in A_n. The dashed line in Fig. (1) schematizes the line field configuration $\mathbf{l}_{n+1}$ associated the the set A_n and to the next site s_{n+1}.

Algorithm

For $n := 1, 2, \ldots, N^2$

$$\widehat{\phi}_{s_n} = \arg\max_{\phi_{s_n}} p(\phi_{s_n}|\mathbf{x}, \mathbf{l}_n, \widehat{\phi}_r, r \in V_{s_n}) \qquad (18)$$

For $i := 1, \ldots, ; \quad n := (i \bmod N^2)$

$$\widehat{\phi}_{s_n} = \arg\max_{\phi_{s_n}} p(\phi_{s_n}|\mathbf{x}, \mathbf{l}, \widehat{\phi}_r, r \in V_{s_n}) \qquad (19)$$

The algorithm we propose is shown in the box above. It embodies two cycles: the first, implemented by (18), is an initialization step; the second, implemented by (19), is exactly the *iterated conditional modes* (ICM) proposed in [14]. ICM is known to be a suboptimal scheme that converges to local maxima. Therefore, the starting point is critical in obtaining *good* estimates. The first cycle is exactly designed aiming at the determination of a good starting point. It is also a ICM type procedure, but where the neighbors of a given pixel are constrained to be in the set of the already updated sites. This is achieved by using the line field configuration $\mathbf{l}_n$, instead of $\mathbf{l}$. The reason for this procedure is found in the observation mechanism: let $\phi = (\phi \bmod 2\pi) + 2\pi k$ be the phase, at a given site, decomposed into its principal value $(\phi \bmod 2\pi)$ and a 2π multiple; since $p(x|\phi)$ is 2π-periodic, the observation mechanism conveys only information about the principal value $(\phi \bmod 2\pi)$. Integer k must be inferred from neighbors absolute phase, under the assumption that the phase varies smoothly between neighbor sites. This implies a recursive scheme, at least for determining the term $2\pi k$ of phase. It is for this reason that in the first step of the algorithm, implemented by (18), the neighbors of a given pixel are constrained to be in the set of already updated sites.

3.1 Implementation

Assume that $s_n = (i, j)$; from the *a posteriori* distribution (14), it follows, after a simple but lengthy manipulation, that

$$p(\phi_{ij}|\mathbf{x}, \mathbf{l}, \phi_r, r \in V_{ij}) \propto e^{\lambda_{ij}\cos(\phi_{ij} - \eta_{ij})} e^{-\frac{\bar{l}_{ij}\mu}{2}(\phi_{ij} - \bar{\phi}_{ij})^2}, \qquad (20)$$

where

$$\bar{l}_{ij} := \bar{h}_{ij} + \bar{h}_{i+1,j} + \bar{v}_{ij} + \bar{h}_{i,j+1} \qquad (21)$$

$$\bar{\phi}_{ij} := \frac{\phi_{i-1,j}\bar{h}_{ij} + \phi_{i+1,j}\bar{h}_{i+1,j} + \phi_{i,j-1}\bar{v}_{ij} + \phi_{i,j+1}\bar{h}_{i,j+1}}{\bar{l}_{ij}}. \qquad (22)$$

Maximizing (20) with respect to ϕ_{ij} is not a trivial task, due to presence of term $\cos(\phi_{ij} - \eta_{ij})$. Aiming at this, we adopt the strategy followed in [8]. Essentially, it consists in representing the observation factor $p(x_{ij}|\phi_{ij})$ as a train of Gaussian terms centered on $\eta_{ij}^{(k)} := \eta_{ij} + 2\pi k$, for $k \in \mathcal{Z}$, and with a common variance γ_{ij}. This variance minimizes the *Kulback distance* (defined in a 2π-interval) between $p(x_{ij}|\phi_{ij})$ and the train of Gaussian terms. The relation between this optimal variance γ_n and the observation dependent parameter λ_n is provided by a look-up table

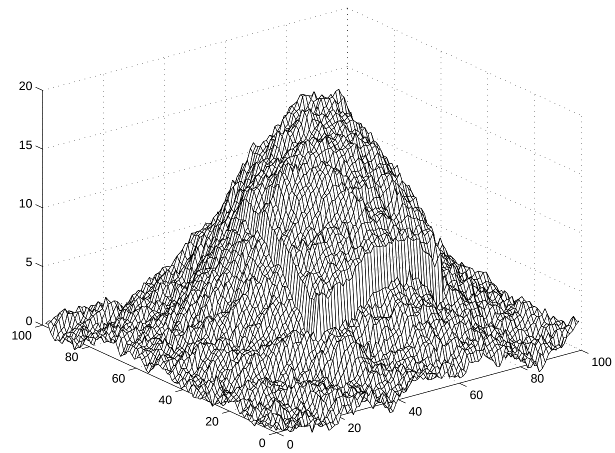

Figure 2: Original phase image.

of solutions of the above mentioned minimization. For details see [17].

The above approximation of $p(x_{ij}|\phi_{ij})$ would produce an infinite number of Gaussian terms in (20). We adopt the following simplification: in multiplication (20) take only the term of the Gaussian sum representation closest to $\bar{\phi}_{ij}$. The resulting density $p(\phi_{ij}|\mathbf{x}, \mathbf{l}, \widehat{\phi}_r, r \in V_{ij})$ turns out to be also Gaussian, with maximum

$$\widehat{\phi}_{ij} = \frac{\frac{\eta^\star_{ij}}{\gamma_{ij}} + \widehat{\bar{\phi}}_{ij}\bar{l}_{ij}\mu}{\frac{1}{\gamma_{ij}} + \bar{l}_{ij}\mu} = \frac{\eta^\star_{ij} + \widehat{\bar{\phi}}_{ij}\bar{l}_{ij}\mu\gamma_{ij}}{1 + \bar{l}_{ij}\mu\gamma_{ij}}, \qquad (23)$$

where $\eta^\star_{ij} := \eta_{ij} + 2\pi k^*$, and $k^* = \arg\min_k |\eta_{ij}^{(k)} - \widehat{\bar{\phi}}_{ij}|$. We note that both maximizations (18) and (19) are given by (23): in the first case $\bar{l}_{ij}$ depends only on $\mathbf{l}_n$, while in the second case $\mathbf{l}_{ij}$ depends only on $\mathbf{l}$.

The interpretation of (23) is clear: estimate $\widehat{\phi}_{ij}$ is given by the weighted mean of $\eta^\star_{ij}$, and $\widehat{\bar{\phi}}_{ij}$; the weights are respectively $1/\gamma_{ij}$ and $\bar{l}_{ij}\mu$.

4 Experimental Results

Fig. 2 displays a simulated phase to be estimated; it is composed of three additive terms: a random component, a Gaussian shaped component, and a ramp modeling a discontinuity. The random component is generated by the autoregressive process

$$\phi_{ij} = \frac{1}{2}\phi_{i-1,j} + \frac{1}{2}\phi_{i,j-1} + u_{ij}, \qquad (24)$$

where index (i, j) are swept in a lexicographic order, and u_{ij} is a sequence of independent zero-mean Gaussian random variables with standard deviation $\sigma_u = 0.3$; the Gaussian shaped component is given by

$$\phi_{ij} = 3\pi\, e^{-\frac{(i-m)^2}{2\sigma^2} - \frac{(j-m)^2}{2\sigma^2}}, \qquad (25)$$

with $m = 50$ and $\sigma = 30$; the third component is a 50 pixels wide ramp having an increasing rate of 0.1 rad/pixel.

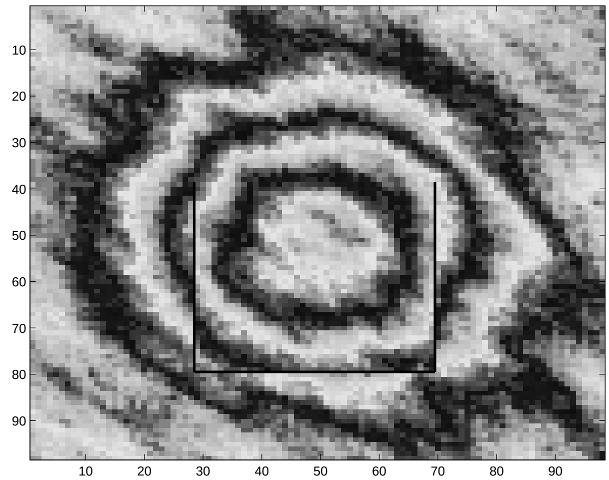

Figure 3: Interferogram corresponding to a additive noise model using phase shown in Fig. 2.

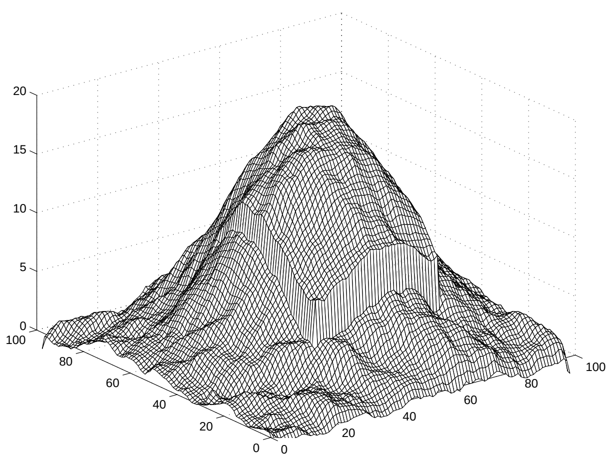

Figure 4: Reconstructed absolute phase.

Since the observation models (1)-(2) and (8) lead to equivalent observation factors $p(x|\phi)$, we use the latter for simplicity.

Fig. 3 shows the interferogram $\{\eta_{ij}\}$, where

$$\eta_{ij} = \arg(x_{ij}), \tag{26}$$

and $x_{ij} = e^{j\phi_{ij}} + n_{ij}$ with ϕ_{ij} represented in Fig. 2, and n_{ij} being a sequence of indenpendent zero-mean Gaussian random variables with standard deviation $\sigma_n = 0.3$. The discontinuity field configuration is superimposed.

Fig. 4 displays the reconstructed phase image using the proposed methodology. The site sequence s_n is the lexicographical one. Although this is a valid sequence for the discontinuity configuration considered, that is not the case for most discontinuity configurations. The algorithm was parametrized with the noise standard deviation $\sigma_n = 0.3$, and $\mu = 1/(2\sigma_u^2)$. Ten iteration of ICM step were applied.

The reconstruction is correct is the sense that discontinuities were preserved and no *cycle slips* have occurred. Fig. 5, part (b), shows a slice (line 70) of the the original phase (dashed line) and of the reconstructed phase (doted line).

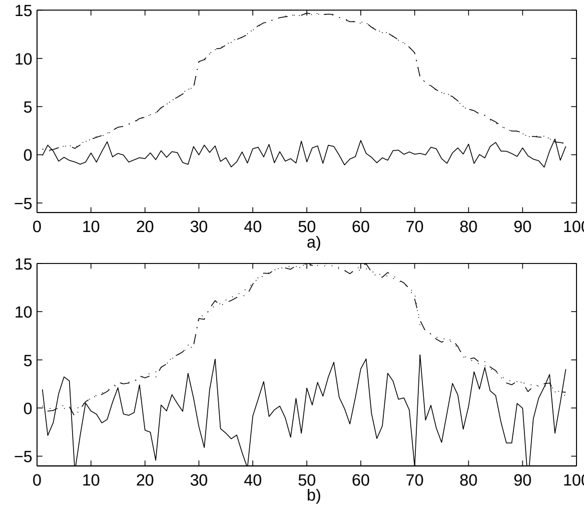

Figure 5: Original phase (dashed line), reconstructed phase (dotted line), and reconstruction error, 10 times magnified (solid line). Parameters used: part (a), $\sigma_u = \sigma_n = 0.1$; part (b), $\sigma_u = \sigma_n = 0.3$.

The solid line represents the reconstruction error 10 times magnified. Fig. 5, part (a), likewise, but for $\sigma_u = \sigma_n = 0.1$. The sample mean error is nearly zero and the sample error standard deviation is 0.07 and 0.3, for $\sigma_n = \sigma_u = 0.1$, and $\sigma_n = \sigma_u = 0.3$, respectively.

5 Concluding Remarks

The paper addressed the problem of InSAR absolute phase reconstruction, in presence of discontinuities. The approach was Bayesian; the adopted observation model reflects the physics of the InSAR problem, whereas the prior, a *compound Gauss Markov random field*, is suited to piecewise smooth phase images. In this way, the discontinuities ever present in InSAR applications are taken into account.

The proposed algorithm combines recursive and batch aspects: while the recursive facet, inspired on stochastic nonlinear filtering, provides a cycle sleep free though rough absolute phase estimate, the batch facet, implemented by ICM, improves the the bias and variance.

The line field configuration (i.e., discontinuities) were herein assumed known. Its estimation is an important issue to be addressed in future work.

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