The Economics of Greenhouse Effect: Dynamic General Equilibrium Approach

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Abstract: This paper uses a primitive general equilibrium (GE) model to examine variation in carbon dioxide (CO_2) produced by human activity. This model consists of two goods; food and energy. Food is produced using labor and energy, while energy is produced using labor. It is assumed that food output decreases as CO_2 increases. The representative household has a utility function with food and energy as variables. The household maximizes utility given an income constraint. If production and utility functions are specified by Cobb-Douglas type, it is easy to compute a short-run GE, given CO_2 and population. The energy, produced and consumed in this short-run GE enhances CO_2, while CO_2 itself decreases in the long run by the sequestration of carbon in wood and the sea. The long-run variation of CO_2 is expressed by a differential equation. First, it is assumed that population is constant. Increases of CO_2 are a linear function of produced short-run energy, and decreases are constant. The long-run process has either stability in which CO_2 converges to zero, or instability in which CO_2 expands to infinity. In the case of instability, the policy of taxing energy is effective in the sense that the policy can prevent the divergence. Second, it is assumed that population increases at the fixed rate. Increases of CO_2 are an increasing function of produced short-run energy. Decreases are an increasing function of CO_2. A variety of simulations are conducted, some of which show the stability of CO_2. Finally, following T.R.Malthus it is assumed that population growth decreases as food output decreases, and vice versa. Simulation shows that the model has the final day in which population decreases to zero, while CO_2 does not necessarily diverge to infinity. The tax on energy is shown to be effective, in the sense that the final day can be somewhat postponed.

Keywords: *greenhouse effect, general equilibrium, Pigou, Malthus, simulation.*

1. INTRODUCTION

According to Brown [1987], through the combustion of fossil fuels through the human activity, 185 billion tons of carbon in total have been released into the atmosphere since the 1860's, when the first industrial revolution was accomplished. In the year 1860, 93 million tons of carbon are estimated to have been released, while it has risen to 5 billion tons per year around 1985. (According to IPCC, Intergovernmental Panel on Climate Change, more than 6 billion tons of carbon was released per year around 2000.)

As a result, the density of the carbon dioxide (CO_2) in the atmosphere rose more than 30 per cent during this period. If the present trend continues, it will be double by the middle of the 21st century. Although CO_2 allows the sun's rays to warm the earth's surface, it absorbs infrared radiation with longer wavelength which is emitted from the earth, which is the greenhouse effect. It is estimated that through the greenhouse effect the mean air temperature of the earth will rise 1.5 to 4.5° C above the present level by the middle of the 21st century. (According to 3rd IPCC Report, the estimate of temperature rise is modified to 1.4 to 5.8° C.) In such a "global warming", although the precipitation rises 7 through 11 per cent on the average, there are areas where the humidity of the soil is reduced. This is the case since the rise of precipitation activates evaporation. Thus, in such areas as North America or Russia (C.I.S.) the droughts during summer may well ruin the crops of wheat or corn. (Note, however, that there are some reports estimating the increased output of rice in Japan and other Asian countries).

While carbon is released into the atmosphere as mentioned above, there are two mechanisms which can get rid of the carbon from the atmosphere. One is the photosynthetic function of the trees and the other is a function of the sea as the greatest repository of carbon. Thus, CO_2 as a stock variable is subject to countervailing factors: one is the enhancing factor exhibited by the combustion of fossil fuels through the human activity and the other is a reducing factor just mentioned. This paper develops a primitive economic model to explore the variation of CO_2 through human activity and its effects on food production and population. A Pigouvian tax on energy is also examined.

2. GENERAL EQUILIBRIUM MODEL

A primitive general equilibrium (G.E.) model was constructed, for the purpose of examining the greenhouse effect. Suppose that there are two firms. The first firm is a farm which produces wheat; Z_f. Whereas wheat is produced by labor : L_1, and energy: H_{f1}, the output depends on CO_2: Y, in the atmosphere. Thus, this farm has the production function:

$$Z_f=g^1(L_1,H_{f1},Y) \text{ where } g^1_1>0,\ g^1_2>0, \text{ and } g^1_3<0 \quad (1)$$

and g^i_j is the partial derivative of g^i with respect to the j^{th} variable. The second firm is the energy industry which produces energy: H_{f2}, using only labor : L_2. It has the production function

$$H_{f2}=g^2(L_2) \text{ where } g^2_1>0 \quad (2)$$

There is only one household, which consumes wheat: Z_h, and energy: H_h. Household behavior is stipulated by the optimal problem:

$$\max u(Z_h,H_h)$$
$$\text{s.t. } p Z_h+p_H H_h=w\underline{N}+\pi_1+\pi_2 \quad (3)$$

where $u(Z_h,H_h)$ is the utility function, p is the wheat price, p_H is the energy price, w is the wage rate, $\underline{N}$ is the initial leisure hours (population), and π_i is the profit from the i^{th} firm (i=1,2). For the sake of simplification, in this model, leisure consumption is excluded from the utility function.

Given CO_2: Y, the short-run General Equilibrium (G.E.) is obtained, which satisfies

$$H_{f1}^d+H_h^d= H_{f2}^s=H,\ Z_h^d=Z_f^s, \text{ and } L_1^d+L_2^d=\underline{N} \quad (4)$$

where superscript d(s) implies "demand" ("supply").

As energy is produced, CO_2 in the atmosphere increases by the amount of η(H), while CO_2 decreases by the amount of G(Y) thanks to the activity of the sea and trees. Thus, we have a dynamic system

$$dY(t)/dt= \eta(H(t))-G(Y(t)) \quad (5)$$

where t stands for time.

First of all, we construct an unstable case for (5). In order to do so, we specify (1) through (5) by the following Cobb-Douglas type.

$$g^1(L_1,H_{f1},Y)=L_1^{\alpha 1} H_{f1}^{\alpha 2} A(Y)^{\alpha 3},$$
$$\alpha_1+\alpha_2+\alpha_3\leqq 1,\ A'(Y)<0 \quad (6)$$
$$g^2(L_2)=L_2\text{: constant returns to scale,} \quad (7)$$
$$u(Z_h,H_h)= Z_h^{\gamma} H_h^{1-\gamma},\ 0<\gamma<1, \quad (8)$$
$$\eta(H)=\underline{\eta}H\text{: } \underline{\eta}>0\text{, constant,} \quad (9)$$
$$G(Y)=\underline{G}\text{: } \underline{G}>0\text{, constant,} \quad (10)$$

This specification guarantees the short-run G.E. under (1) through (4): {Z*, L_1*, L_2*, H*, H_f*, H_h*, (p_H/p)*}. We have

$$(p_H/p)^*=(w/p)^*=\{(1-\gamma(1-\alpha_1-\alpha_2))/(\gamma\underline{N})\}^{1-\alpha 1-\alpha 2}\ \alpha_1^{\alpha 1}\alpha_2^{\alpha 2}\{A(Y)\}^{\alpha 3} \quad (11)$$
$$L_1^*=\{\alpha_1\gamma/(1-\gamma(1-\alpha_1-\alpha_2))\}\underline{N} \quad (12)$$
$$H_{f1}^*=\{\alpha_2\gamma/(1-\gamma(1-\alpha_1-\alpha_2))\}\underline{N} \quad (13)$$
$$H_h^*=\{(1-\gamma)/(1-\gamma(1-\alpha_1-\alpha_2))\}\underline{N} \quad (14)$$

Indeed, we have

$$L_1^*+L_2^*=\underline{N},$$

because from (13) and (14) it follows that

$$H^*=H_h^*+H_{f1}^*=H_{f2}^*=L_2^*=\{(1-\gamma(1-\alpha_2))/(1-\gamma(1-\alpha_1-\alpha_2))\}\underline{N}. \quad (15)$$

For this short-run G.E. the dynamic system (5) is

$$dY(t)/dt=\underline{\eta}H^*-\underline{G}$$
$$=\underline{\eta}\{(1-\gamma(1-\alpha_2))/(1-\gamma(1-\alpha_1-\alpha_2))\}\underline{N}-\underline{G}.$$

Note that as shown in (15), H* is independent of Y, and

if $\underline{\eta}\{(1-\gamma(1-\alpha_2))/\ (1-\gamma(1-\alpha_1-\alpha_2))\}\underline{N}-\underline{G}>0$,
then $Y(t)\rightarrow+\infty$, (16)
if $\underline{\eta}\{(1-\gamma(1-\alpha_2))/(1-\gamma(1-\alpha_1-\alpha_2))\}\underline{N}-\underline{G}<0$,
then $Y(t)\rightarrow 0$.

Thus, if parameters of the model satisfies (16), we have an unstable case in which wheat continues to decline.

3. PIGOUVIAN TAX

Traditionally, a Pigouvian tax is introduced in order to realize the static optimum when the external diseconomy exists. In this section, it is applied to a dynamic situation. Suppose that (16) is satisfied. We examine whether it is possible to remedy the unstable case into the stable case through suitable taxation. For this purpose, we levy a tax on energy; i.e. consumers of energy must pay $(1+\tau)p_H$ for one unit of energy. It is assumed that the tax revenue, $\tau p_H(H_h+H_{f1})$, is distributed to households.
Under this modification, the first firm's behavior is stipulated by the optimal problem:

$$\max\ \pi_1=pZ-wL_1-(1+\tau)p_HH_{f1},$$

and the second firm's behavior is stipulated by

$$\max\ \pi_2=p_HH_{f2}-wL_2,$$

while the household's behavior is stipulated by

$$\max u(Z_h,H_h)$$
$$\text{s.t. } pZ_h+(1+\tau)p_HH_h=w\underline{N}+\pi_1+\pi_2+\tau p_H(H_h+H_{f1}).$$

In the same way as above, by the tedious computation the short - run G.E. is given as

$$(p_H/p)[\tau]^*=(w/p)[\tau]^*=\{(1-\gamma(1-\alpha_1-\alpha_2)+\gamma\tau\alpha_1)/(\gamma\underline{N})\}^{1-\alpha_1-\alpha_2}(1+\tau)^{\alpha_1-1}\alpha_1^{\alpha_1}\alpha_2^{\alpha_2}\{A(Y)\}^{\alpha_3}$$
$$L_1[\tau]^*=\{\alpha_1\gamma(1+\tau)/(1-\gamma(1-\alpha_1-\alpha_2)+\gamma\tau\alpha_1)\}\underline{N} \quad (17)$$
$$H_{f1}[\tau]^*=\{\alpha_2\gamma/(1-\gamma(1-\alpha_1-\alpha_2)+\gamma\tau\alpha_1)\}\underline{N} \quad (18)$$
$$H_h[\tau]^*=\{(1-\gamma)/(1-\gamma(1-\alpha_1-\alpha_2)+\gamma\tau\alpha_1)\}\underline{N} \quad (19)$$

where $0<\tau<+\infty$.
As $\tau\rightarrow\infty$, we have $L_1[\tau]^*\rightarrow\underline{N}$ and $L_2[\tau]^*=H[\tau]^*=H_{f1}[\tau]^*+H_h[\tau]^*\rightarrow 0$. Thus, by a suitable tax rate, τ, we can modify the dynamic system into $dY(t)/dt=\underline{\eta}H[\tau]^*-\underline{G}<0$.

4. POPULATION GROWTH

The assumption (10) may be unrealistic. If $G(Y)=\underline{G}Y$: $\underline{G}$ is constant, is assumed instead of (10), the dynamic system (5) becomes stable. The assumption of constant population (leisure time), however, is also unrealistic. In this section, these assumptions are modified. Simulations are conducted as follows.
First, it is assumed that

$$(dN(t)/dt)/N(t)=n_0>0,\ \text{constant}, \quad (20)$$
$$dY(t)/dt=\underline{\eta}H(t)-\underline{G}Y(t). \quad (21)$$

If $n_0=0.01$, $N(t)=N(0)e^{0.01t}$. Suppose that $\gamma=1/2$, $\alpha_1=\alpha_2=1/4$, $Y(0)=10$, $\underline{\eta}=1$, $\underline{G}=1/10$, and $N(0)=1$.

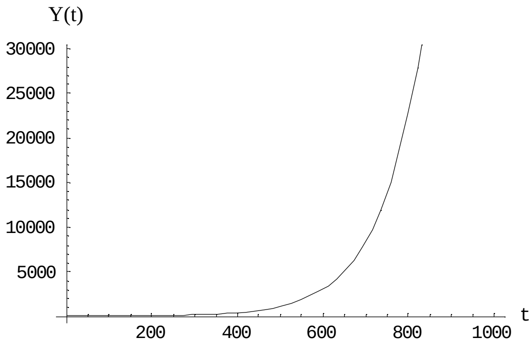

Figure 1: CO_2 when Constant Population Growth

Letting $\underline{N}=N(0)e^{0.01t}$ in (15), the dynamic path of Y(t) stipulated in (21) is shown in Figure 1. This pattern remains the same even if $\underline{G}$ becomes large. The dynamic path of Y(t) stipulated in (21) essentially depends on N(t). Thus, if N(t) is bounded above, such as ArcTan(t), Y(t) is also bounded, as shown in Figure 2.

Y(t)

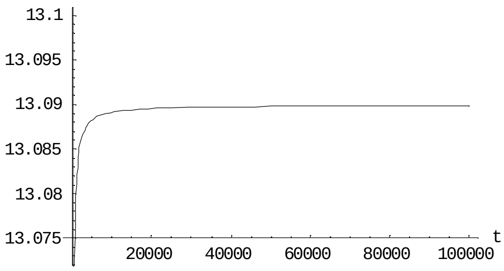

Figure 2: CO_2 when Population is Bounded

5. MALTHUS AND GLOBAL WARMING

T.R.Malthus argued that poverty is not a social phenomenon but a natural phenomenon, criticizing W. Godwin's argument, in which poverty is caused by social system: private ownership. According to Malthus, population growth is expressed as the geometry progression; $1, 2^2, 3^2, \ldots, n^2, \ldots$, while food production as the natural progression; 1, 2, 3, ... , n, ... , since the latter is under decreasing marginal productivity, which causes poverty. It was asserted that in the long run, a society's population growth is restricted by food production. In this section, this factor is examined. Thus, it is assumed that population expands at n_0% annually, while it is reduced if the wheat production decreases due to global warming.

5.1 Simulation 1

In this section, simulations are conducted, so that A(Y) in (6) is specified by

$$A(Y)=\underline{A}/Y: \underline{A}>0, \text{ constant} \qquad (22)$$

Furthermore, following the Malthus model the dynamic system is expressed as (21) and

$$(dN(t)/dt)/N(t)=n_0+\underline{B}\, dg^1(L_1(t),H_{f1}(t),Y(t))/dt, \qquad (23)$$

where $\underline{B}>0$ is constant, $L_1(t)$ and $H_{f1}(t)$ are given by setting $\underline{N}=N(t)$ in (12) and (13).
When $\gamma=1/2$, $\alpha_1=\alpha_2=\alpha_3=1/4$, $\underline{\eta}=1/100$, $\underline{G}=1/10$, $\underline{A}=1000$, and $\underline{B}=1/1000$, the dynamic system: (21) and (23), are expressed as

$$dN(t)/dt=-2\, 2^{3/4}\, 3^{1/2}\, 5^{1/4}\, N(t)^{1/2}Y(t)^{1/4}-2N(t)(-N(t)/160+3Y(t)/40)/(3Y(t))$$
$$dY(t)/dt=-(4/3)\,(-N(t)/160+3Y(t)/40)$$

Setting N(0)=10000, Y(0)=100, the dynamic path of Y(t) is shown in Figure 3. Note that Y(t) is bounded and decreasing after t=4. However, as shown in Figure 4, N(t) converges to zero. In other words, this society has the final day, in which population is zero.
Even if $\underline{B}$ is modified from 1/1000 to 1/10 and $\underline{A}$ is modified from 1000 to 10000, this society has the final day, as shown by the solid line in Figure 5, while the dynamic path of CO_2 in this society is shown as the thick solid line in Figure 6.

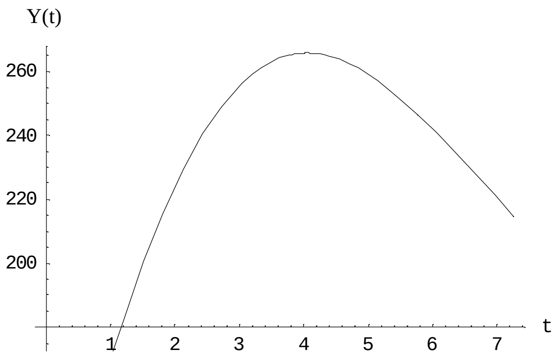

Figure 3: CO_2 in Simulation 1

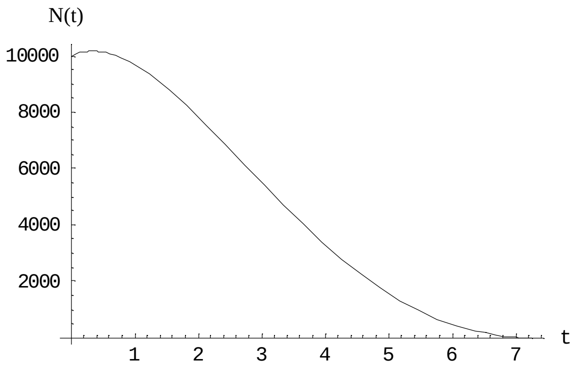

Figure 4: Population in Simulation 1

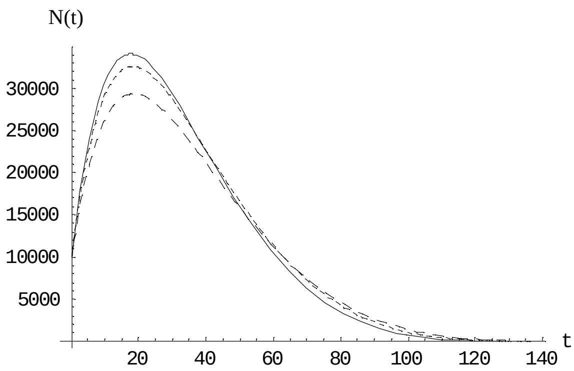

Figure 5: Pigouvian Tax and Population (1)

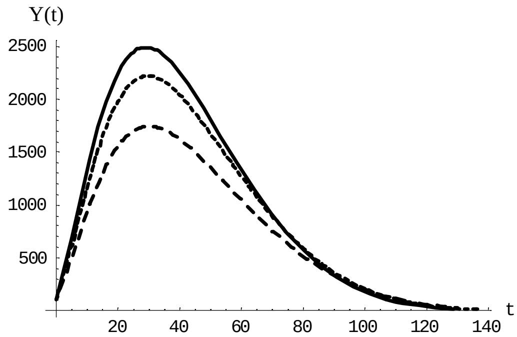

Figure 6: Pigouvian Tax and CO_2 (1)

Finally, the effect of tax on energy is examined. The dynamic system under Pigouvian taxing on energy can be formulated where $L_1(t)$ and $H_{f1}(t)$ in (23) are given by setting $\underline{N}$=N(t) in (17) and (18). As τ increases from 0 to 1/2 to 3/2, the dynamic path of N(t) is given by the solid line to the dense dashed line, to the sparse dashed line in Figure 5. Although the final days exist for all cases, the final day is postponed further as the tax rate increases. For example, when τ =0 the final day is t=137.346, when τ =1/2 it is t=138, finally when τ =3/2 it is t=145.287 in this simulation.

5.2 Simulation 2

What would happen if (22) is modified to the following

$$A(Y)=\underline{A}/\mathrm{ArcTan}[Y]: \underline{A}>0, \text{ constant.} \qquad (24)$$

Note that contrary to (22), $0 \leqq \mathrm{ArcTan}[Y] \leqq \pi/2$ holds in (24). In this specification: γ =1/2, $\alpha_1 = \alpha_2 = \alpha_3$=1/4, $\underline{\eta}$=1/100, $\underline{G}$=1/10, $\underline{A}$=10000, and $\underline{B}$=1/10, the dynamic system: (21) and (23), are expressed as

$$dN(t)/dt=-6^{1/2}N(t)^{1/2}(\mathrm{ArcTan}[Y(t)])^{1/4}/5-2N(t)(-N(t)/120+Y(t)/10)/(2\mathrm{ArcTan}[Y(t)](1+Y(t)^2))$$
$$dY(t)/dt=N(t)/120-Y(t)/10$$

As above, setting N(0)=10000, Y(0)=100, the dynamic path of Y(t) is shown as the thick solid line in Figure 7. Note that Y(t) is bounded and decreasing after t=40. However, as shown by the solid line in Figure 8, N(t) converges to zero. In other words, this society has the final day, in which population is zero.

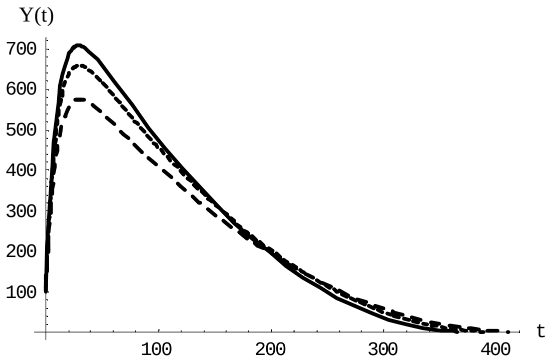

Figure 7: Pigouvian Tax and CO_2 (2)

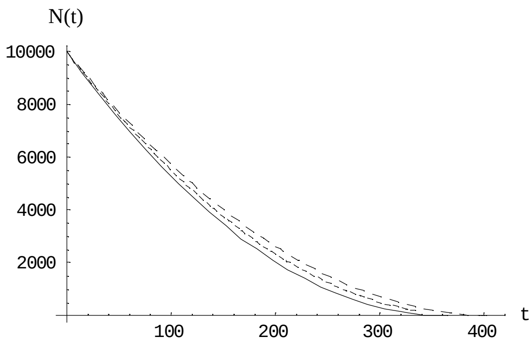

Figure 8: Pigouvian Tax and Population (2)

In the same way as above, the effect of tax on energy is examined. The dynamic system under Pigouvian taxing of energy can be formulated where $L_1(t)$ and $H_{f1}(t)$ in (23) are given by setting $\underline{N}$=N(t) in (17) and (18). As τ increases from 0 to 1/2 to 3/2, the dynamic paths of N(t) are given by the solid line, the dense dashed line, and the sparse dashed line, respectively, in Figure 8. Although the final days exist for these cases, the final day is postponed further as the tax rate increases. For example, the final day is t=365.373 when τ =0, it is t=388.491 when τ =1/2, finally it is t=410.951 when τ =3/2 in this simulation.

Remark 1: as for the Malthusian assumption on the relation between population, N, and food, Z, a remark is in order. As is seen presently in the developed countries, as the society becomes rich the population growth decreases due to presumably educational and other considerations. Note, however, this paper focuses its analysis on low-level Z.

Remark 2: Compare this result for an externality model (stock version) with Fukiharu [2002, Section 4], in which population (initial leisure hours) decreases when pollution worsens due to capital goods production. In the previous paper (externality model, flow version), there exists a possibility of stability; hence the conclusion of no final day. Note, however, the stability depends on the initial value of capital-labor ratio. In the present paper the initial value of CO_2-population ratio; Y/N, is fixed.

Remark 3: Capital accumulation is not considered in this paper, as in the traditional two-sector growth model. Incorporation of capital accumulation into the present model does not seem to prevent the final day, since in the traditional model the growth rate of capital accumulation converges to the growth rate of population. Thus, capital goods production cannot influence the population growth except for through the externality, as remarked above.

6. CONCLUSIONS

In 1995 the IPCC concluded that global warming is taking place due to greenhouse effect. The main aim of the 1997 Kyoto Protocol was to reach agreement for each country to reduce the global warming gases, such as CO_2. The aim of this paper is to examine the economic consequences of greenhouse effect, constructing a primitive general equilibrium model. In this model consumption of energy raises CO_2 in the atmosphere, which reduces food production. In Section 2, assuming that absorption of CO_2 by sea and trees is constant, it was shown that, there arises the possibility of divergence of CO_2 level. In Section 3, a Pigouvian tax on energy consumption, adopted by each Kyoto Protocol participant, was shown to be effective in preventing the divergence. Until this section, population is assumed to be constant. From Section 4, population is allowed to change. In Section 4, it was shown through simulations that the variation of CO_2 is determined by the population growth when the latter is not affected by the food production. In Section 5, following Malthus the population growth is assumed to be reduced if the food production declines due to greenhouse effect, while population grows at a fixed rate without food production effect. Simulations revealed that the society in this model has the final day in which population becomes zero. Pigouvian tax on energy consumption was shown to be effective in postponing the final day. In the previous paper it was shown that when the population is affected by the pollution due to capital good sector, there might not exist the final day, depending on the initial position of capital and population. There remains a study to examine if we have a range of initial positions of population and CO_2 stock for which the dynamic path does not face the final day. Such a study will be attempted in subsequent papers.

6. REFERENCES

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