Trellis Coding for Diagonally Layered Space-Time Systems

Adina Matache, Richard D. Wesel, and Jun Shi
Electrical Engineering Department,
University of California, Los Angeles, CA 90095-1594

***Abstract*— Foschini's diagonally layered space-time transmission system known as D-BLAST is an advanced architecture designed for a Rayleigh fading environment using multiple element antenna arrays at both the transmit and receive sites to achieve very high spectral efficiencies. In this paper we examine the performance of trellis codes that are designed to have a distance structure that is matched to the periodic signal-to-noise ratio variation of the channel created by D-BLAST, under the assumption that the channel is static during one burst but may change from burst to burst. We show that trellis coding comes within 2 dB of the best theoretical outage curve possible with D-BLAST.**

***Index Terms*— Multiple transmit antennas, fading channels, trellis codes, space-time processing.**

I. Introduction

With the growing bit-rate demand in wireless communications, more sophisticated technologies are needed to make better use of the limited radio spectrum available. The basic information theory results reported by Foschini and Gans [1] have promised extremely high spectral efficiencies possible through multiple element antenna arrays at both the transmitter and receiver. The diagonally layered space-time transmission system described in [2] (better known as D-BLAST) is a receiver architecture designed for systems with multi-element antenna arrays at both the transmit and receive sites. In [3] Ariyavisitakul showed that D-BLAST using minimum-mean squared error (MMSE) space-time processing achieves the Shannon capacity for a multiple-input, multiple-output Rayleigh fading system. The optimality of the layered space-time scheme makes D-BLAST attractive for high data-rate applications.

In this paper we examine the performance of robust trellis codes in conjunction with D-BLAST, under the assumption of quasistatic Rayleigh fading. We show that trellis coding comes within 2 dB of the best theoretical outage curve possible with D-BLAST. Other recent work includes the threaded space-time codes of El Gamal and Hammons [4] and the wrapped space-time codes of Caire and Colavolpe [5].

The paper is organized as follows. Section II describes in detail the layered space-time architecture of D-BLAST. Section III introduces a trellis code design strategy for transmission in systems with periodic signal-to-noise ratio variation and explains how to perform Viterbi traceback in a coded D-BLAST system. Section IV provides simulation results for trellis coded layered space-time systems under quasistatic Rayleigh fading channels.

II. Layered Space-Time Architecture

With an equal number n of transmit and receive antennas, the transmission process of the layered architecture of D-BLAST can be summarized as follows. The primitive data stream is demultiplexed into n data streams of equal rate, and each substream is modulated using the same constellation. The n substreams are transmitted simultaneously using n antenna elements. The receiver also uses n antennas to decouple and detect the transmitted signals.

The channel under consideration is the ideal Rayleigh channel model, meaning that the entries of the $n \times n$ channel matrix H are independent identically distributed zero-mean, unit-variance, complex Gaussian variables. The transmitter may use knowledge of the channel statistics (such as average SNR) to select a transmission rate, but the transmitter does not have access to the particular realization of the channel matrix. It is assumed that the receiver learns (tracks) the channel during a training phase. Communication takes place in a burst mode in which the channel characteristics are essentially unchanged (static) during a burst, but change randomly from burst to burst.

In a complex baseband representation, the basic vector equation describing the channel operating on the signal is

$$\underline{r} = H\underline{s} + \underline{\nu} = \sum_{i=1}^{n} s_i \underline{h}_i + \underline{\nu}, \tag{1}$$

where $H = [\underline{h}_1, \underline{h}_2, \ldots, \underline{h}_n]$ is the $n \times n$ channel matrix, $\underline{s} = [s_1, s_2, \ldots, s_n]^T$ is the complex transmitted signal vector and $\underline{\nu}$ is a $n \times 1$ vector of independent zero-mean complex Gaussian noise samples with variance $N_0/2$ per dimension. We assume that the power emitted from each transmit antenna is proportional to $1/n$ so that the total radiated power is constant and independent of the number of transmit antennas. The average signal-to-noise ratio at each receive antenna is also independent of n.

The detection process amounts to estimating the n components of the vector $\underline{s}$ from the received vector $\underline{r}$. From [2], the key aspects of spatial processing of a received vector in detection of transmitted signal components are:

1) *interference suppression*: interference from yet to be detected symbols is suppressed by linear operations.
2) *interference canceling*: interference from already detected symbols is subtracted out.

The interference canceling step subtracts out from $\underline{r}$ the detected signal components in a manner similar to a decisionfeedback equalizer (DFE), while the interference suppression step removes the interference stemming from the as yet undecided components using linear operations optimized under a zero-forcing (ZF) or minimum-mean squared error (MMSE) criterion.

The interference suppression steps (ZF and MMSE) are described in detail next. Assuming that the receiver has correctly detected the first $i-1$ signal components of $\underline{s}$, we can cancel the interference from these components by subtracting them out from the received vector $\underline{r}$. Then, the resulting $n \times 1$ vector $\underline{u}_i$ is

$$\underline{u}_i = s_i \underline{h}_i + s_{i+1} \underline{h}_{i+1} + \cdots + s_n \underline{h}_n + \underline{\nu}. \tag{2}$$

A. ZF Interference Suppression

Assuming the first $i-1$ signal components have been canceled out, the interference stemming from the simultaneous transmission of $s_{i+1}, \ldots, s_n$ is nulled out by projecting $\underline{u}_i$ onto the nullspace of the vectors $\underline{h}_{i+1}, ..., \underline{h}_n$. Let $H_{[i+1,n]}$ denote the vector space spanned by the column vectors $\underline{h}_{i+1}, ..., \underline{h}_n$, and $\tilde{\underline{u}}_i$ the result of the projection:

$$\tilde{\underline{u}}_i = \underline{u}_i - H_{[i+1,n]} \left(H^*_{[i+1,n]} H_{[i+1,n]} \right)^{-1} H^*_{[i+1,n]} \underline{u}_i, \tag{3}$$

where $(*)$ represents complex conjugate transpose. The above expression can be further simplified to obtain

$$\tilde{\underline{u}}_i = s_i \tilde{\underline{h}}_i + \tilde{\underline{\nu}}, \tag{4}$$

where $\tilde{\underline{h}}_i$ and $\tilde{\underline{\nu}}$ are the projection of $\underline{h}_i$ and $\underline{\nu}$ respectively onto the nullspace of $H_{[i+1,n]}$. Using standard maximum ratio combining, the decision statistic for s_i is given by the inner product $\langle \tilde{\underline{u}}_i, \tilde{\underline{h}}_i \rangle$ to obtain

$$\hat{s}_i = \frac{1}{||\tilde{\underline{h}}_i||^2} \langle \tilde{\underline{u}}_i, \tilde{\underline{h}}_i \rangle = s_i + \frac{1}{||\tilde{\underline{h}}_i||^2} \langle \tilde{\underline{\nu}}, \tilde{\underline{h}}_i \rangle. \tag{5}$$

The mean squared error between s_i and $\hat{s}_i$ is given by

$$\text{MSE}_i = \frac{1}{||\tilde{\underline{h}}_i||^4} \tilde{\underline{h}}_i^* \tilde{\Gamma} \tilde{\underline{h}}_i = \frac{N_0}{||\tilde{\underline{h}}_i||^2}, \tag{6}$$

where $\tilde{\Gamma} = E\left[\tilde{\underline{\nu}}\, \tilde{\underline{\nu}}^*\right]$ is the covariance matrix of the projected noise vector $\tilde{\nu}$. Then, the resulting signal-to-noise ratio for this decision statistic is

$$\text{SNR}_i = \frac{\varepsilon_s}{N_0} ||\tilde{\underline{h}}_i||^2, \tag{7}$$

where ε_s is the average constellation energy and $||\tilde{\underline{h}}_i||^2$ is a chi-square random variable with $2i$ degrees of freedom. Since the entries of the channel matrix H are zero-mean, unit-variance complex Gaussians, the mean of the chi-square variate is i. Note that interference suppression using zero-forcing creates in effect a periodic time-varying channel of period n whose gains are chi-square random variables with $2, 4, \ldots, 2n$ degrees of freedom.

B. MMSE Interference Suppression

Again, assuming the first $i-1$ signal components have been canceled out, the interference from components $s_{i+1}, \ldots, s_n$ is suppressed by minimizing the mean squared error between s_i and $\hat{s}_i$. The decision statistic for s_i is given by the inner product $\hat{s}_i = \langle \underline{u}_i, \underline{c}_i \rangle$, where $\underline{c}_i$ is a n-dimensional column vector chosen to minimize the mean squared error. It is easily shown [6] that the MMSE solution for $\underline{c}_i$ satisfies

$$\underline{c}_i = \varepsilon_s \left(1 + \varepsilon_s \underline{h}_i^* A^{-1} \underline{h}_i\right)^{-1} A^{-1} \underline{h}_i, \tag{8}$$

where $A = \varepsilon_s \sum_{j=i+1}^{n} \underline{h}_j \underline{h}_j^* + \Gamma$ and $\Gamma = N_0 I_n$ is the covariance matrix of the noise vector $\underline{\nu}$. The minimum mean squared error is given by

$$\text{MMSE}_i = \varepsilon_s \left(1 - \underline{c}_i^* \underline{h}_i\right) = \varepsilon_s \left(1 + \varepsilon_s \underline{h}_i^* A^{-1} \underline{h}_i\right)^{-1}, \tag{9}$$

and the corresponding signal-to-noise ratio is

$$\text{SNR}_i = \varepsilon_s \underline{h}_i^* A^{-1} \underline{h}_i. \tag{10}$$

Observe that under the MMSE criterion we again have a periodic time-varying channel of period n but with different statistics for the periodic gain factors.

III. Trellis Coding for D-BLAST

A. Trellis Codes for Periodic SNR

The design of trellis codes for channels that are characterized by additive white Gaussian noise with a distinct periodic variation in signal-to-noise ratio is investigated in [7], [8], [9], [10], and [11]. The special case of periodic erasure channels is treated in [7], where a family of trellis codes for periods 2 - 5 are designed using two different objective functions for selecting among the Pareto optimal codes. These codes are specifically designed to provide robust performance over all periodic erasure patterns (for which the number of unerased coded bits per period is at least equal to the number of information bits per period). However, because the erasure channels are the extreme points in the set of periodic SNR channels, these codes are also a good match to the periodic fading channel created by D-BLAST. A brief review of the design approach for these trellis codes is presented next.

Let the n-element vector $\tilde{\underline{a}} = [\tilde{a}_1, \ldots, \tilde{a}_n]$ contain the periodic scale factors with $\tilde{\underline{a}}^2 = [|\tilde{a}_1|^2, \ldots, |\tilde{a}_n|^2]$. Also let the normalized symbol-wise squared Euclidean distance between two constellation points be $d_i^2(s \rightarrow \hat{s}) = |\hat{s}_i - s_i|^2/\varepsilon_s$, where s_i and $\hat{s}_i$ are the correct and incorrect constellation points associated with the i^{th} symbols of a trellis error event $s \rightarrow \hat{s}$. The periodic erasure of symbols scales distances with the same index modulo n by the same binary scale factor $\tilde{a}_i$. Define the periodic squared distance $\tilde{d}_i^2(s \rightarrow \hat{s})$ for $i \in \{1, 2, \ldots, n\}$ as the sum of the square of the distances scaled by the same factor $\tilde{a}_i$ for a given error event: $\tilde{d}_i^2(s \rightarrow \hat{s}) = \sum_{m=0}^{\infty} d_{i+mn}^2$. The n values of $\tilde{\underline{d}}^2 = [\tilde{d}_1^2, \ldots, \tilde{d}_n^2]$ form the periodic distance vector.

The minimum over the periodic distance vectors of valid error events is referred to as the squared residual Euclidean distance (RED),

$$\text{RED}(\underline{\tilde{a}}) = \min_{\underline{\tilde{d}}^2} \langle \underline{\tilde{a}}^2, \underline{\tilde{d}}^2 \rangle. \quad (11)$$

Trellis code design for all possible erasure patterns is a multi-criterion problem since we have to minimize bit-error-rate (BER) (equivalently maximize RED) at all erasure patterns simultaneously.

One approach described in [7] is to seek to maintain the same required excess mutual information (MI) for all erasure patterns, which is motivated by the desire of having consistent performance relative to the required excess MI over all erasure patterns. This produces the design criterion based on maximizing J_{MI},

$$J_{\text{MI}} = \sum_j (n - q_j/n) \log_2(\text{RED}_j^2), \quad (12)$$

where q_j is the number of elements of the j^{th} erasure pattern equal to zero. Note that in the formulation of J_{MI} we give less importance to the $\log_2(\text{RED}_j^2)$ terms for more severe erasure patterns.

To illustrate why the codes in [7] are a good match to the periodic SNR variation of the D-BLAST channel, consider code $\#1$ designed using J_{MI} as the objective function. This code is a rate-$1/3$, 64-state, 8-PSK trellis code for period $n = 2$, with octal generators $(G_1, G_2, G_3) = (173, 62, 115)$. For a BER of 10^{-5}, this code requires an excess mutual information of 0.64 on the channel where every other symbol is an erasure ($\underline{\tilde{a}} = [0, 1]$), and an excess mutual information of 0.87 on the channel with no erasures ($\underline{\tilde{a}} = [1, 1]$). For a periodic channel, the excess mutual information is defined as the capacity margin between the channel MI where the desired BER is achieved given by

$$\text{MI} = \frac{1}{n} \sum_{i=1}^{n} \log_2 \left(1 + \frac{|\tilde{a}_i|^2 \varepsilon_s}{N_0} \right), \quad (13)$$

and the transmitted rate of the code.

Figure 1 shows an extension of the results presented in [7], by considering the excess mutual information required for BER $= 10^{-5}$ when the periodic scale factors have the form $\underline{\tilde{a}} = [\tilde{a}_1, 1]$, with $\tilde{a}_1$ as a parameter varying from 0 to 1. This plot is generated using a transfer function bound to obtain the SNR necessary to achieve BER $= 10^{-5}$ for values of $|\tilde{a}_1|^2$ between 0 and 1. The channel mutual information is calculated for each sample of $|\tilde{a}_1|^2$ and the corresponding SNR. It is interesting to note that as $|\tilde{a}_1|^2$ increases from 0 to 1 the excess mutual information is monotonically increasing from 0.64 to 0.87, which correspond to the excess MI on the two erasure channels.

Based on the monotonicity of this curve we can reason that the performance of code $\#1$ relative to mutual information on any periodic fading channel falls in between the performance on the AWGN channel and the erasure channel. Therefore it is reasonable to conclude that a trellis code designed by considering only the two end-points and making the MI gap between them as small as possible would yield a robust performance on any other periodic fading channel, regardless of the channel fading statistics.

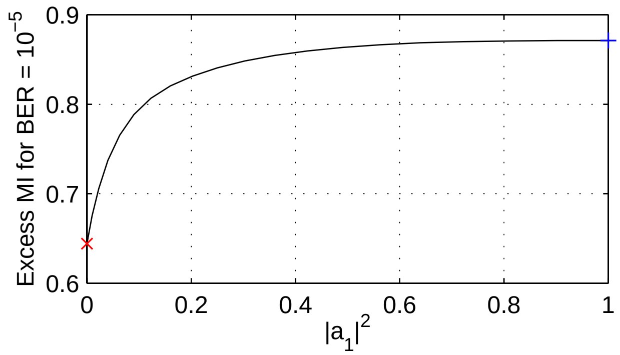


Fig. 1. Excess MI required by code $\#1$ to achieve BER $= 10^{-5}$.

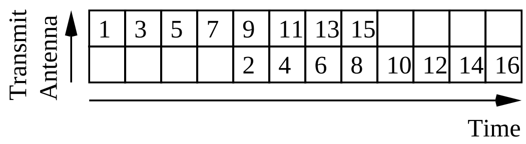
B. Continuous Viterbi Traceback in D-BLAST

In the following we describe a trellis coding technique for the layered space-time architecture of D-BLAST. For illustration consider the example in Fig. 2(a) of a 2×2 standard D-BLAST system. The symbol placement is a diagonal layering for Viterbi decoding with traceback depth $L_D = 7$ and block size of 16 symbols. Each square represents a symbol transmitted from a single antenna at a single symbol time. The numbers in the squares represent the processing order of the Viterbi decoder. The empty squares represent overhead symbols where nothing is transmitted or a known pilot is transmitted.

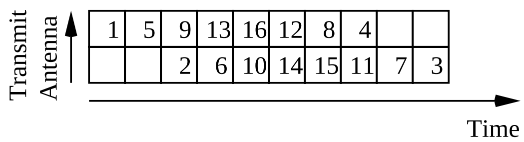
In Fig. 2(a), each upper space-time symbol is received by canceling the space-time symbol below it, and each lower space-time symbol is received by suppressing the symbol above it. Note that four overhead symbols in the lower left permit the first seven symbols to be received without any decoding. This is enough to permit the first Viterbi traceback. Thereafter, the bottom symbols are decoded "just-in-time" to provide the cancellation required for the next Viterbi traceback. This processing method leaves an additional four overhead symbols at the end of the block.

With this coding method, the basic idea is to choose a traceback depth mindful of the requirements in [12] and a diagonal layering which together permit the decoding of a symbol "just in time" to be subtracted out from the received signals. In general, in order to decode a symbol "just in time" the step-size of the diagonal layering must be $(L_D + 1)/n$. This determines the overhead penalty associated with D-BLAST to be equal to $(L_D + 1)(n - 1)$.

Figure 2(b) shows a variation on D-BLAST architecture with half the previously discussed overhead. In this archi-



(a) Standard D-BLAST for $L_D = 7$.



(b) Reduced-overhead D-BLAST for $L_D = 7$.

Fig. 2. Symbol placement for standard and reduced-overhead D-BLAST for a Viterbi traceback depth of $L_D = 7$ and a block length of 16 data symbols.

tecture, each odd-numbered space-time symbol is received by canceling the even-numbered space-time symbol above or below it, and each even-numbered space-time symbol is received by suppressing the odd-numbered space-time symbol above or below it. As before, the first seven symbols may be received without any decoding, and even-numbered space-time symbols are decoded "just-in-time" to provide the cancellation required for the next Viterbi traceback. Note that by working from both ends toward the middle, the overhead is cut in half.

While reducing the overhead by a factor of two, this technique also increases the period of the resulting time-varying channel by a factor of two. However, the new periodic channel has some additional structure within each period. For example, the SNR of a top symbol under cancellation is strongly correlated to the SNR of a top symbol under suppression. As a result, even though the period doubles the number of possible periodic erasure channels remains the same. Thus, while the specific codes will be different for this reduced-overhead architecture, the actual code design will be similar in complexity.

IV. Results and Conclusions

In this section we present performance results for the coded layered space-time architecture described above. The performance measure is the frame-error-rate (FER) under the assumption of an ideal Rayleigh channel and a burst-mode communication scenario. The results are obtained through Monte Carlo simulation. The FER is averaged over enough channel realizations so that 1000 erroneous frames are accumulated in each simulation.

In order to compare the performance results to channel capacity, we also provide the theoretical outage probability curves. Outage is dealt with probabilistically because the channel matrix H is random, and thus capacity is a random variable. Under ZF interference suppression, Foschini [2] has shown

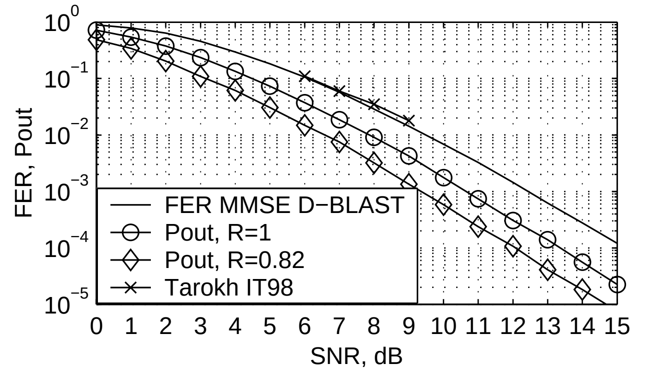


Fig. 3. FER and Pout for 2×2 standard MMSE D-BLAST, code $\#1$, block length 200 bits.

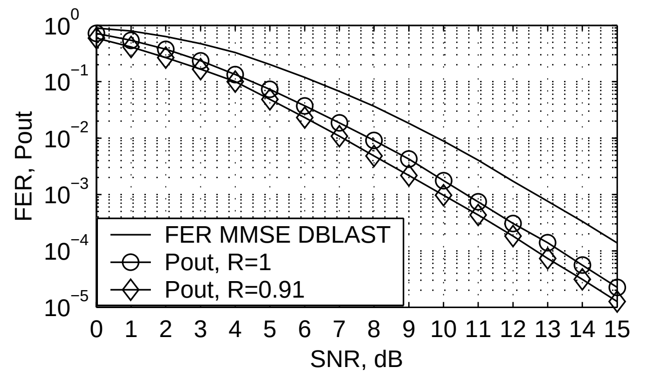


Fig. 4. FER and Pout for 2×2 reduced-overhead MMSE D-BLAST, code $\#1$, block length 200 bits.

that D-BLAST achieves the following bound on the Shannon capacity for a Rayleigh fading system with n transmit and n receive antennas

$$C_F = \sum_{i=1}^{n} \log_2 \left(1 + \frac{\rho}{n} \chi^2_{2i}\right), \tag{14}$$

where ρ is the average SNR per receive antenna and χ^2_{2i} is a chi-square random variable with $2i$ degrees of freedom. Furthermore, Ariyavisitakul [3] showed that under the MMSE criterion, Foschini's lower bound is actually the true Shannon capacity bound for a system with n transmit and n receive antennas

$$C = \log_2 \det \left(I_n + \frac{\varepsilon_s}{N_0} H H^*\right), \tag{15}$$

where $\det(\cdot)$ means determinant and I_n is the $n \times n$ identity matrix. For some instances of H, the channel capacity may be below the transmitted rate. In this case a channel outage is said to have occurred and the channel is considered to be in the OUT state.

Figure 3 shows the FER results versus the average SNR per receive antenna for code $\#1$ discussed above in conjunction with a 2×2 standard MMSE D-BLAST system. Each block contains 200 information bits (100 bits on each transmit antenna) and the traceback depth is $L_D = 41$. This figure compares performance of the robust trellis code on the periodic channel created by D-BLAST to that of a space-time trellis code designed and simulated by Tarokh [13] using the same complexity and similar block size. Even though our code was designed to be robust on all periodic erasure channels, and not specifically matched to the Rayleigh statistics, our performance in Rayleigh fading is essentially the same as Tarokh's. Similar performance is also achieved by the space-time turbo code in [14].

The previous paragraph ignores the overhead penalty required by D-BLAST to create the periodic channel. For demonstrating the promise of the robust code perspective, it is reasonable to neglect the overhead and look at how closely each code comes to the theoretical outage curve that applies to the code itself (the rate $R = 1$ outage curve in Fig. 3 for both Tarokh's code and the robust code on the periodic channel created by D-BLAST). However, to analyze the overall system performance, the overhead penalty must be considered. The overhead penalty associated with D-BLAST can be quantified by recomputing the theoretical outage probability at a rate of 0.82 b/s/Hz/antenna, also shown in Fig. 3. At 10% outage the performance of our robust code is within 1.5 dB from the best theoretical outage curve and within 3 dB from the theoretical outage curve which takes into account the overhead penalty required by D-BLAST.

Figure 4 shows the FER results for code $\#1$ in conjunction with a 2×2 reduced-overhead MMSE D-BLAST system. Although code $\#1$ was not designed for the period-4 SNR variation created by the reduced-overhead D-BLAST architecture, its loss in performance is very small compared to the standard D-BLAST system in Fig. 3. At the same time the overhead penalty is reduced in half, as shown in Fig. 4 with the outage probability curve at the new rate of 0.91 b/s/Hz/antenna. A search for robust trellis codes that are matched to the periodic time-varying channel created by the reduced-overhead D-BLAST system is currently in progress, however we expect to see an improvement in performance with the newly designed codes.

Simulation results for the period-4 trellis code C_2 designed in [9] in conjunction with a 4×4 standard MMSE D-BLAST system are shown in Fig. 5. This is a rate-$1/4$, 64-state, 4-PSK code, with a spectral efficiency of 0.5 b/s/Hz/antenna. The FER results are obtained for a block length of 400 bits and traceback depths $L_D = 23$ and $L_D = 43$. The longer traceback depth improves the FER performance by about 0.5 dB. However, the overhead is increased and the overall rate is lowered from $R = 0.424$ b/s/Hz/antenna to $R = 0.376$ b/s/Hz/antenna. At 10% outage this system performs within approximately 3 dB from the theoretical outage curve which takes into account the overhead penalty, and within 2 dB from the best theoretical outage curve possible with D-BLAST.

In conclusion, our trellis coding technique for the diagonally layered architecture of D-BLAST achieves a performance that is within 2 dB of theory and competitive with the best performance of previously reported space-time trellis codes and turbo codes.

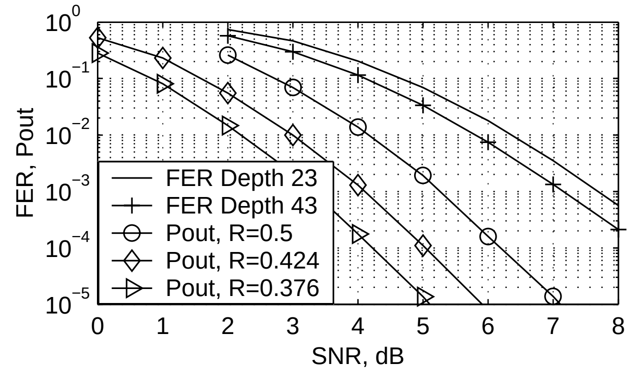


Fig. 5. FER and Pout for 4×4 standard MMSE D-BLAST, code C_2, block length 400 bits.

Acknowledgment

The authors would like to thank Gerard Foschini for his insightful comments and suggestions.

References

[1] G. J. Foschini and M. J. Gans, "On limits of wireless communication in a fading environment when using multiple antennas," *Wireless Pers. Commun.*, vol. 6, no. 3, pp. 311-335, Mar. 1998.
[2] G. J. Foschini, "Layered space-time architecture for wireless communication in a fading environment when using multi-element antennas," *Bell Labs Tech. J.*, vol. 1, no. 2, pp. 41-59, Autumn 1996.
[3] S. L. Ariyavisitakul, "Turbo space-time processing to improve wireless channel capacity," *IEEE Trans. Commun.*, vol. 48, no. 8, pp. 1347-1359, Aug. 2000.
[4] H. El Gamal and A. R. Hammons, "A new approach to layered space-time coding and signal processing," *IEEE Trans. Inform. Theory*, vol. 47, no. 6, pp.2321-2334, Sep. 2001.
[5] G. Caire and G. Colavolpe, "On space-time coding for quasi-static multiple-antenna channels," submitted to *IEEE Trans. Inform. Theory.*, available at http://www.eurecom.fr/~caire.
[6] U. Madhow and M. L. Honig, "MMSE interference suppression for direct-sequence spread-spectrum CDMA," *IEEE Trans. Commun.*, vol. 42, no. 12, pp. 3178-3188, Dec. 1994.
[7] R. D. Wesel, X. Liu, and W. Shi, "Trellis codes for periodic erasures," *IEEE Trans. Commun.*, vol. 48, no. 6, pp. 938-947, June 2000.
[8] R. D. Wesel and J. M. Cioffi, "Trellis code design for periodic interleavers," *IEEE Commun. Letters*, vol. 3, no. 4, pp. 103-105, April 1999.
[9] C. Fragouli, C. Komninakis, and R. D. Wesel, "Minimality for punctured convolutional codes," *Proc. IEEE ICC'01*, Helsinki, Finland, June 2001.
[10] R. Knopp and P. Humblet, "On coding for block fading channels," *IEEE Trans. Inform. Theory*, vol. 46, no. 1, pp. 189-205, January 2000.
[11] E. Malkamaki and H. Leib, "Evaluating the performance of convolutional codes over block fading channels," *IEEE Trans. Inform. Theory*, vol. 45, no. 5, pp. 1643-1646, July 1999.
[12] J. B. Anderson and K. Balachandran, "Decision depths of convolutional codes," *IEEE Trans. Inform. Theory*, vol. 35, pp. 455-459, Mar. 1989.
[13] V. Tarokh, N. Seshadri, and A. R. Calderbank, "Space-time codes for high data rate wireless communication: Performance criterion and code construction," *IEEE Trans. Inform. Theory*, vol. 44, no. 2, pp.744-765, Mar. 1998.
[14] Y. Liu, M. P. Fitz, and O. Y. Takeshita, "Full rate space-time turbo codes," *IEEE J. Select. Areas Commun.*,, vol. 19, no. 5, pp. 969-980, May 2001.