Design of an Iterative Learning Controller for a Class of Linear Dynamic Systems with Time-Delay

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Abstract

In this paper, it is first observed that, when a typical iterative learning control(ILC) algorithm is applied to a class of linear dynamic systems with time-delay, erratic estimation of delay time may cause the control input to diverge. In order to resolve such a difficulty due to uncertainty of the delay time, a new ILC algorithm is proposed, in which the holding mechanism is adopted to hold the control input at a constant value for the duration of the delay time uncertainty. As a consequence, the output of the system tracks a given desired trajectory at the discrete points which are spaced by the size of the uncertainty of delay time. A numerical example is given to show the effectiveness of the proposed algorithm.

Keywords: Iterative Learning Control, Time-Delay

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1 Introduction

In recent years, the demand for high precision control methodology is increasing to improve the performance of the automation systems such as petro-chemical processes, industrial robot manipulators, NC machine-based manufacturing systems, Magneto-Optical Disk Drives(MODD), and so on. Among tasks utilizing these systems/machines, many tasks, such as batch job of certain chemical processes, spray painting, and arc-welding, are repetitive and require a controller which can track the given whole trajectory completely in a specified time interval.

Due to inaccuracy in modeling and/or uncertainty of some system parameters, a conventional feedback control system adopting PID control, state feedback control, or optimal path tracking control[1] is found to be unsatisfactory in its performance, and also, well-known advanced techniques such as adaptive control may not work[2]. Especially, unsatisfactory tracking performance is much more prominent for the nonminimum phase processes or time-delay systems. As a method to overcome the limitation of the conventional controllers, the iterative learning control(ILC) method was proposed by Arimoto et al. [3], and has been further developed by many researchers[4-9] since then. The ILC algorithm is generally expressed in the following form:

$$u_{k+1}(t) = u_k(t) + f\left(e_k(\cdot)\right)(t), 0 \leq t \leq T \tag{1}$$

where $u_k(t)$ is the control input and f is a functional of error function, $e_k(t)$, $0 \leq t \leq T$, between the actual output and the desired output at the k-th iteration[10]. The functional f can be designed in various ways according to the objective of the control[3,10-12].

Most of the results up to now on the ILC are for the dynamic systems with no time-delay. On the other hand, in many batch chemical processes as shown in Fig. 1 [16], the time-delay effect cannot be ignored. For example, consider the system in Fig. 1 in which the flow rates of cooling water and steam are the control inputs, and the temparature of the reactor is the output. In this system, there exists time-delay between the control input and the plant output. Suppose the temperature should track a given desired trajectory. As a means for complete tracking of the desired temperature output trajectory, the ILC method can be adopted. If the general form given in Eqn. 1 is to be applied to such a system with time-delay, the form of the controller may be modified as in the following Eqn. 2, taken into consideration of the delay time.

$$u_{k+1}(t) = u_k(t) + f\left(e_k(\cdot)\right)(t + \tau_e) \tag{2}$$

Here, τ_e is an estimated delay time. In Eqn. 2, the control input $u_{k+1}(t)$ is updated by input $u_k(t)$ and value of the functional $f\left(e_k(\cdot)\right)$ at time $t + \tau_e$. If the estimated delay time τ_e is different from the actual dealy time, however, the intput $u_{k+1}(t)$ is to be updated from incorrect response error, and in this case, there is no guarantee that the algorithm is convergent. That is, a typical ILC algorithm with some naive modification can be applied to a system with time-delay only in case the delay time is known exactly. Otherwise, the control input may be divergent due to uncertainty of the delay time. Hideg[13,14] investigated the possibility of divergence of an ILC for a plant with time-delay in the frequency domain via some computer simulations. As an example of quick remedy, he presented an ILC algorithm with windowing technique applied. Chen et al. [15] also investigated a higher order ILC algorithm for a class of nonlinear systems with state delays. In this paper, a new ILC algorithm for a class of linear dynamic systems with time-delay is proposed using the holding mechanism, and the convergence of the proposed algorithm is shown.

This paper is organized as follows. In Section 2, the problem under consideration is formally described. In Section 3, a new ILC algorithm is proposed, and further the condition for convergence is presented. In Section 4, a numerical example is presented to show the effectiveness of the proposed algorithm, and concluding remarks follow in Section 5.

In the sequel, for the n-dimensional Euclidean space R^n, $\|x\|$ denotes the Euclidean norm of a vector $x = (x_1, \cdots x_n)^T$. For a matrix A, $\|A\|$ denotes its induced matrix norm. $\|x\|_\infty$ denotes the ∞-norm defined by

$$\|x\|_\infty = \max_{1 \le i \le n} |x_i|$$

and for $n \times r$ matrix A with elements a_{ij},

$$\|A\|_\infty = \max_{1 \le i \le n} \sum_{j=1}^{r} |a_{ij}|.$$

As in the notation $u_k(t)$, the suffix k is employed to denote the iteration number.

2 Problem Formulation

Consider the linear time invariant system with time-delay described by

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t - \tau') \\ y(t) &= Cx(t) \end{aligned} \tag{3}$$

where $x \in R^n, u \in R^r$ and $y \in R^q$ denote the state, the input and the output, respectively, and τ' is the actual delay time. A, B and C are matrices with appropriate dimensions. Let $y_d(t)$ be the desired output trajectory and $x_d(t)$ be the corresponding state trajectory. Assume that they are continuously differentiable on $[0, T]$.

Suppose the delay time τ' is estimated in terms of some lower and upper bounds, τ_1 and τ_2 so that $\tau_1 \leq \tau' \leq \tau_2$ as shown in Fig. 2. The size of uncertainty in delay time can be defined as $h = \tau_2 - \tau_1$. Now, divide the time interval $[\tau_2, T]$ forward from time τ_2 by h and the time interval $[0, \tau_1]$ backward from time τ_1 also by h. The time interval $[0, \tau_1]$ cannot be an integer multiple of h, so some initial remainder ξ exists, which is smaller than h as shown in Fig. 2. A similar statement applies for $[\tau_2, T]$. Let m be the index that represent the sequence of divided intervals, and let $t = mh + \xi$ to denote each discrete point as shown in Fig. 2. Let M be the maximum among the numbers m that satisfy $mh + \xi \leq T$. Then we may write $m \in \{0, 1, \cdots, M\}$. Here, if T' is defined by $T' = Mh + \xi$, then the divided time interval in Fig. 2 is the same as the time interval which $[0, T]$ is divided by h from T' backward. Let d be the number that satisfies $dh + \xi = \tau_2$; then the actual delay time τ' can be represented by Eqn. 4.

$$\tau' = (d-1)h + \tau + \xi \tag{4}$$

where $\tau, 0 \leq \tau < h$, is unknown value in τ'.

Now, we reformulate the problem of the ILC for a class of linear dynamic systems with uncertain time-delay as a discrete-time tracking problem.

Problem 1 *Suppose a desired trajectory $y_d(t)$, $t \in [0, T]$, is given and the initial state at each iteration is the same as the desired initial state, i.e., $x_k(0) = x_d(0)$ for $k = 0, 1, 2, \cdots$. The problem is to find a control input $u(t), 0 \leq t \leq T$, such that the output of the given linear dynamic system Eqn. 3 matches the desired trajectory $y_d(t)$ exactly at the discrete points $y_d(mh + \xi), m \in \{d, d+1, \cdots, M\}$.*

3 Iterative Learning Control Law for Linear Systems with Time-Delay

In this section, it is shown that the output tracking performance is obtained by holding the control input at constant value over the time interval h.

The following ILC algorithm is proposed as a solution for the Problem 1 described in the previous section.

$$\begin{aligned} u_{k+1}(t) &= u_k(mh) + \Gamma e_k(mh + dh + \xi), \\ &\quad \forall t \in [mh, mh+h), m \in \{0, 1, \cdots, M-d\} \end{aligned} \tag{5}$$

where $e_k(mh + dh + \xi) = y_d(mh + dh + \xi) - y_k(mh + dh + \xi)$.

Before showing the convergence of the ILC algorithm Eqn. 5, we need the following Lemma 1, whose result is utilized in the proof of main result on convergence.

Lemma 1 *Let $a_k(mh)$ be a nonnegative function for every integer k and nonnegative integer $m \geq 0$, and let α and β be nonnegative constants. Suppose $a_k(mh) = 0, \forall m < 0$ and $0 \leq \rho < 1$. Then the inequality*

$$0 \leq a_{k+1}(mh) \leq \rho a_k(mh) + \beta \sum_{j=0}^{m-1} \alpha^{m-j} a_k(jh), \forall k, m \geq 0$$

implies

$$\lim_{k \to \infty} a_k(mh) = 0, \forall m \geq 0. \tag{6}$$

Proof:

For the proof, we are going to employ the method of mathematical induction.

For each $m \geq 0$, let P_m be the statement that

$$\lim_{k \to \infty} a_k(mh) = 0.$$

From assumption, we obtain that $0 \leq a_{k+1}(0) \leq \rho a_k(0)$ and $0 \leq \rho < 1$. So, the statement P_0 is true. That is

$$\lim_{k\to\infty} a_k(0) = 0.$$

Now suppose the statement P_n is true for every integer n with $0 \leq n < m$. Then it is easily seen that

$$\lim_{k\to\infty} \beta \sum_{j=0}^{m-1} \alpha^{m-j} a_k(jh) = 0$$

This implies that for any $\epsilon > 0$, there exists a positive K such that

$$\beta \sum_{j=0}^{m-1} \alpha^{m-j} a_k(jh) < \epsilon$$

for all $k \geq K$. This gives us

$$a_{k+1}(mh) < \rho a_k(mh) + \epsilon, \forall k \geq K.$$

We can choose ϵ' as follows:

$$a_k(mh) - \rho^k a_0(mh) + \frac{\rho^k}{1-\rho}\epsilon < \frac{\epsilon}{1-\rho} = \epsilon'.$$

This means that for any $\epsilon' > 0$, there exists a positive K' such that

$$a_k(mh) - \rho^k a_0(mh) + \frac{\rho^k}{1-\rho}\epsilon < \epsilon'$$

for all $k \geq K'$. So, we can write

$$\lim_{k\to\infty} \left(a_k(mh) - \rho^k a_0(mh) + \frac{\rho^k}{1-\rho}\epsilon \right) = 0$$

$$\lim_{k\to\infty} a_k(mh) - \lim_{k\to\infty} \rho^k a_0(mh) + \lim_{k\to\infty} \frac{\rho^k}{1-\rho}\epsilon = 0.$$

Since $0 \leq \rho < 1$, we can conclude that

$$\lim_{k\to\infty} a_k(mh) = 0$$

which establishes the truth of the statement P_m.

By mathematical induction, Eqn. 6 is true. This completes the proof.

Now, the convergence of the ILC algorithm Eqn. 5 will be shown.

Theorem 1 *Suppose that the update law Eqn. 5 is applied to the system Eqn. 3 and the initial state at each iteration is the same as the desired initial state, i.e.,* $x_k(0) = x_d(0)$*, for* $k = 0, 1, 2, \cdots$*. If*

$$\|I - \Gamma C \int_0^{h-\tau} e^{A\sigma} d\sigma B\|_\infty \leq \rho < 1 \tag{7}$$

then,

$$\lim_{k \to \infty} y_k(mh + \xi) = y_d(mh + \xi), \forall m \in \{d, d+1, \cdots, M\}.$$

Proof:

From Eqn. 3, the state value at $t = mh + dh + \xi$ is represented by the state value at $t = mh + dh - h + \xi$ and input $u(t)$ as follows:

$$\begin{aligned} x_k(mh + dh + \xi) &= e^{Ah} x_k(mh + dh - h + \xi) \\ &\quad + \int_{mh+(d-1)h+\xi}^{mh+dh+\xi} e^{A(mh+dh+\xi-\sigma)} B u_k(\sigma - \tau') d\sigma \\ &= e^{Ah} x_k(mh + dh - h + \xi) \\ &\quad + \int_{mh-\tau}^{mh+h-\tau} e^{A(mh+h-\tau-\sigma)} B u_k(\sigma) d\sigma \end{aligned}$$

The input $u(t)$ is constant over the time interval h. The integral above can now be separated into two parts: one where $u_k(t) = u_k(mh - h), mh - \tau \leq t < mh$; and the other one where $u_k(t) = u_k(mh), mh \leq t < mh + h - \tau$. This gives

$$\begin{aligned} x_k(mh + dh + \xi) &= e^{Ah} x_k(mh + dh - h + \xi) \\ &\quad + \int_0^{\tau} e^{A(h-\tau)} e^{A\sigma'} d\sigma' B u_k(mh - h) \\ &\quad + \int_0^{h-\tau} e^{A\sigma'} d\sigma' B u_k(mh). \end{aligned} \tag{8}$$

For simplicity of presentation, introduce the following notations:

$$\begin{aligned}
\Phi(t) &= e^{At} \\
\Theta(t) &= \int_0^t e^{A\sigma} B d\sigma \\
z_k(mh+dh+\xi) &= \begin{bmatrix} x_k(mh+dh+\xi) \\ u_k(mh) \end{bmatrix} \\
F &= \begin{bmatrix} \Phi(h) & \Phi(h-\tau)\Theta(\tau) \\ 0 & 0 \end{bmatrix} \\
G &= \begin{bmatrix} \Theta(h-\tau) \\ I \end{bmatrix} \\
H &= \begin{bmatrix} C & 0 \end{bmatrix}.
\end{aligned}$$

And then, rearrange Eqn. 8 in matrix form to obtain:

$$\begin{aligned}
z_k(mh+dh+\xi) &= F z_k(mh+dh-h+\xi) + G u_k(mh) \\
y_k(mh+dh+\xi) &= H z_k(mh+dh+\xi).
\end{aligned} \tag{9}$$

From Eqn. 9, $y_k(mh+dh+\xi)$ is represented as follows.

$$y_k(mh+dh+\xi) = HF^{m+1} \begin{bmatrix} e^{A(dh-h+\xi)} x_k(0) \\ 0 \end{bmatrix} + H \sum_{j=0}^{m} F^{m-j} G u_k(jh).$$

Now, let $u_d(mh)$ be a control input such that

$$y_d(mh+dh+\xi) = HF^{m+1} \begin{bmatrix} e^{A(dh-h+\xi)} x_d(0) \\ 0 \end{bmatrix} + H \sum_{j=0}^{m} F^{m-j} G u_d(jh). \tag{10}$$

and define

$$\Delta u_k(mh) = u_d(mh) - u_k(mh).$$

Then it follows from Eqns. 5 and 10 that

$$\begin{aligned}\Delta u_{k+1}(mh) &= \left(I - \Gamma C \int_0^{h-\tau} e^{A\sigma} d\sigma B\right) \Delta u_k(mh) \\ &\quad -\Gamma H \sum_{j=0}^{m-1} F^{m-j} G \Delta u_k(jh). \qquad (11)\end{aligned}$$

Taking the norm $\|\cdot\|_\infty$ on both sides of Eqn. 11, the following is obtained.

$$\begin{aligned}\|\Delta u_{k+1}(mh)\|_\infty &\leq \|I - \Gamma C \int_0^{h-\tau} e^{A\sigma} d\sigma B\|_\infty \|\Delta u_k(mh)\|_\infty \\ &\quad +\|\Gamma H\|_\infty \sum_{j=0}^{m-1} \left(\|F\|_\infty\right)^{m-j} \|G\|_\infty \|\Delta u_k(jh)\|_\infty.\end{aligned}$$

By Lemma 1,

$$\lim_{k\to\infty} \|\Delta u_k(mh)\|_\infty = 0.$$

From Eqns. 8 and 9, the following can be concluded:

$$\lim_{k\to\infty} y_k(mh+\xi) = y_d(mh+\xi), \forall m \in \{d, d+1, \cdots, M\}$$

This completes the proof.

Theorem 1 implies that, if the bound h of estimation error of the delay time is known and the sufficient condition of Theorem 1 is satisfied, then the output trajectory exactly track the discrete points of the desired output trajectory $y_d(mh+\xi)$, which are spaced by the size of the uncertainty h.

Note that h can be considered as a measure of uncertainty of delay time. Thus, the smaller the h is, the better the estimation results. In implementation of the iterative learning controller using computer or μ-processor, the desired trajectory is mostly discretized, and stored in memory. Therefore, if h is smaller than the sampling interval, the output can exactly track the discretized trajectory. That is, if the bound of estimation error of the delay time is known to be within some prespecified bound, then a satisfied performance is achived in real applications.

In the convergence condition Eqn. 7, one may easily note that selection of Γ is possible even when the inequality relation contains the uncertainties in the system matrices A, B, and C, and uncertainty in the value τ[7]. One can estimate the bound of integral $C \int_0^{h-\tau} e^{A\sigma} d\sigma B$ if some range of each elements of A, B, and C is available, noting $0 < h - \tau \leq h$ and $e^{A\sigma}$ is positive definite.

As shown in the ILC algorithm Eqn. 5 and Theorem 1, the convergence of the output is not guaranteed if the control input is not held at constant value over the time interval h.

If time interval $[0, T]$ is divided by h from $t = 0$ forward, i.e., a relation of $t = mh$ is satisfied at each discrete point, then Eqn. 5 is changed into the following equations.

Case 1: $\tau + \xi < h$

$$u_{k+1}(t) = \begin{cases} u_k(0) + \Gamma e_k(dh), t \in [0, h - \xi] \\ u_k(mh) + \Gamma e_k(mh + dh), \\ \qquad \forall t \in [mh, mh + h), m \in \{1, 2, \cdots, M - d\} \end{cases}$$

Case 2: $\tau + \xi \geq h$

$$u_{k+1}(t) = \begin{cases} u_k(0) + \Gamma e_k(dh + h), t \in [0, h - \xi] \\ u_k(mh) + \Gamma e_k(mh + dh + h), \\ \qquad \forall t \in [mh, mh + h), m \in \{1, 2, \cdots, M - d\} \end{cases}$$

That is, the control input is held over the time interval $[0, h - \xi]$ and over h after $t = h - \xi$. However, if τ is not known, it can not be determined which algorithm is applied.

It is remarked that the above result can be applied to linear systems with delays in the output or in the connection of subsystems. To be specific, let us consider the following system with output-delay.

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t-\tau') \end{aligned}$$

In this case, if we change the state variable by $w(t) = x(t-\tau')$, then

$$\begin{aligned} \dot{w}(t) &= Aw(t) + Bu(t-\tau') \\ y(t) &= Cw(t). \end{aligned}$$

Since this is a form similar to the one in Eqn. 3, the case of a system with output-delay can be handled by the result of Theorem 1.

In case that there is time-delay between two subsystems linked by a cascaded form as shown in Fig. 3 (a), the system dynamics can be represented by the following Eqn. 12.

$$\begin{aligned} \dot{x}_1(t) &= A_1x_1(t) + B_1u(t) \\ \dot{x}_2(t) &= A_2x_2(t) + B_2C_1x_1(t-\tau') \\ y(t) &= C_2x_2(t) \end{aligned} \tag{12}$$

If we change the state variable by $w_1(t) = x_1(t-\tau')$, then it is rearranged as shown in Fig. 3 (b).

$$\begin{aligned} \dot{w}_1(t) &= A_1w_1(t) + B_1u(t-\tau') \\ \dot{x}_2(t) &= A_2x_2(t) + B_2C_1w_1(t) \\ y(t) &= C_2x_2(t). \end{aligned}$$

This form also can be taken care of by the result of Theorem 1.

4 Numerical Example

To illustrate effectiveness of the proposed algorithm, consider the following linear time-invariant dynamic system.

$$\begin{aligned} \dot{x} &= \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t-0.225) \\ y(t) &= \begin{bmatrix} 0 & 1 \end{bmatrix} x(t). \end{aligned} \tag{13}$$

The desired output trajectory is given as

$$y_d(t) = 4(t-0.25) - 4(t-0.25)^2, 0 \le t \le 1.$$

To confirm the undesirable phenomenon when a typical ILC algorithm is applied, suppose the D-type ILC algorithm[3] is applied to the system Eqn. 13 with some naive modificaion as in the following Eqn. 14 using estimated delay time 0.22.

$$u_{k+1}(t) = u_k(t) + \Gamma \dot{e_k}(t+0.22) \tag{14}$$

Then we find that as the iteration number k increases the control input becomes divergent as shown in Fig. 4.

Now, let us apply the proposed algorithm. For this, we consider two cases in which the delay time is estimated differently.

Suppose first the lower bound of delay time is 0.22 and the upper bound is 0.27, i.e., $0.22 \le \tau' \le 0.27$. The size of uncertainty h is 0.05. Then the delay time τ' can be represented as

$$\begin{aligned} \tau' &= (d-1)h + \tau + \xi \\ &= (5-1) * 0.05 + \tau + 0.02,\ \ 0 \le \tau < 0.05. \end{aligned}$$

For another case, let the lower bound of the delay time be 0.22 and the upper bound be 0.32, i.e., $0.22 \le \tau' \le 0.32$. The size of uncertainty h is 0.1. Then the delay time τ' can be represented as

$$\tau' = (3-1)*0.1 + \tau + 0.02,\ 0 \le \tau < 0.1.$$

The best choices of Γ from Theorem 1 are $\left(C\int_0^{h-\tau} e^{A\sigma} d\sigma B\right)^{-1} = 1/0.0241$ and $1/0.067$. $C\int_0^{h-\tau} e^{A\sigma} d\sigma B$ have been guessed 0.03 and 0.08 in each case assuming 20% uncertainty. Γ can be 1/0.03 and 1/0.08 from the guessed values. Fig. 5 (a) and (b) shows the convergent output trajectories after 60th and 50th iterations, respectively. The output $y(t)$ is perfectly track discrete points of $y_d(t)$ at $t = mh+\xi, m \in \{d, d+1, \cdots, M\}$. Comparing (a) and (b), less h shows better performance.

5 Concluding Remarks

In this paper, the problem caused by estimation error of delay time was investigated when a typical ILC algorithm was applied, and a new ILC algorithm was proposed. If the new ILC algorithm is applied, the output of the plant can be convergent even if delay time estimation error exists, and tracks the discrete points of a given desired output trajectory.

Many previous works[15] on ILC for time-delay system deal with the case when the initial state errors are zero for every iteration. When an ILC algorithm is applied for the system with state-delay as described by Eqn. 15, and in case of initial value mismatch in the state, however, the time-delay may cause very complicated effect on the responses. The problem of designing ILC algorithm and their analysis is yet to be further investigated.

$$\begin{aligned} \dot{x}(t) &= Ax(t-\tau') + Bu(t) \\ y(t) &= Cx(t) \end{aligned} \tag{15}$$

On the other hand, since the performance of the ILC mainly depends on the error bounds, an analysis of inter-sample behavior is important. A rigorous analysis based on sampled data technique should be challenging problem.

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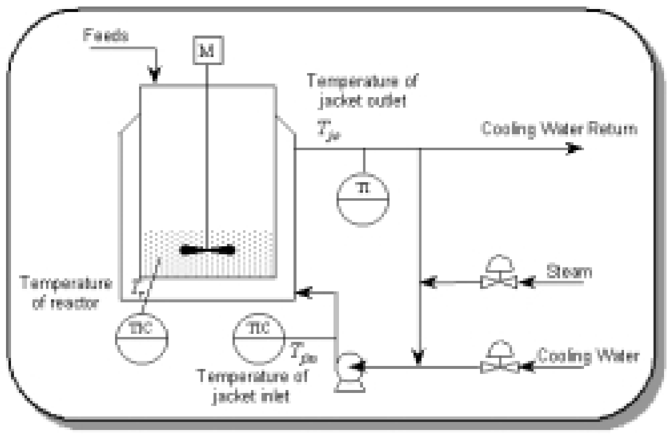


Figure 1: Process schematics of graft ABS (Acrylonitrile-Butadiene-Styrene) polymerization reactor

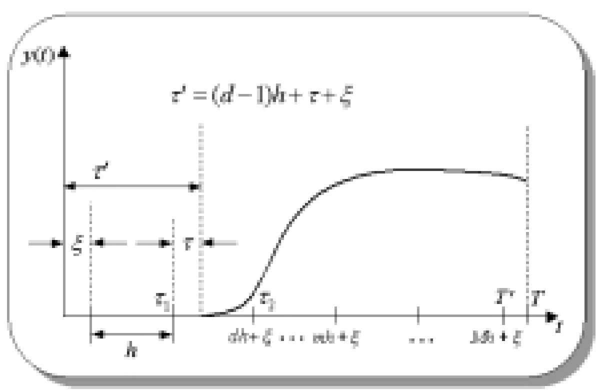


Figure 2: Representation of delay time

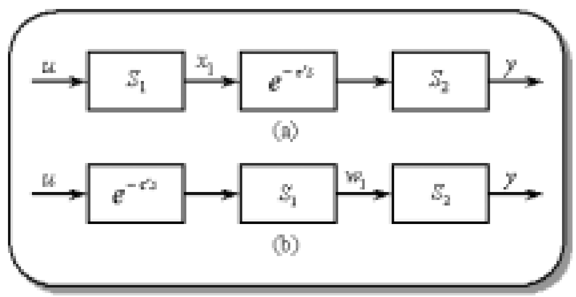
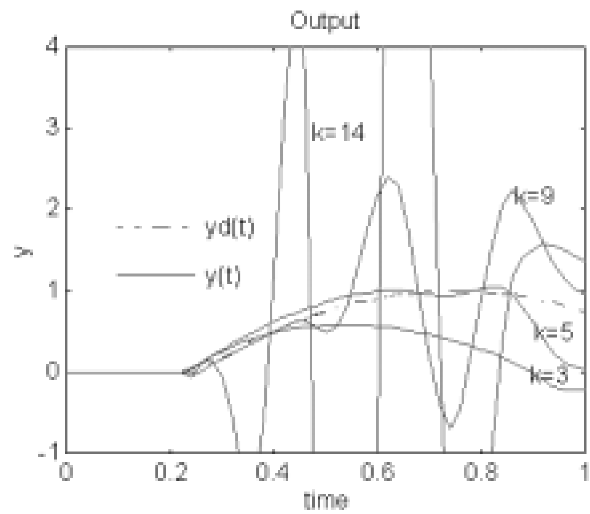


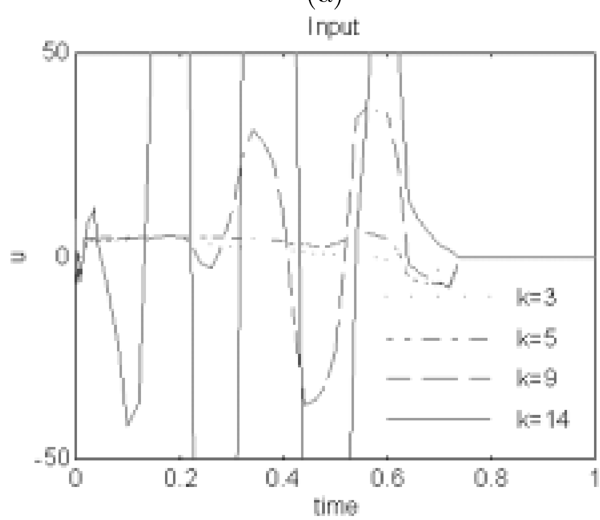
Figure 3: The time-delay system linked by cascade form

(a) The system with inner time-delay

(b) Rearrangement of (a)



(a)

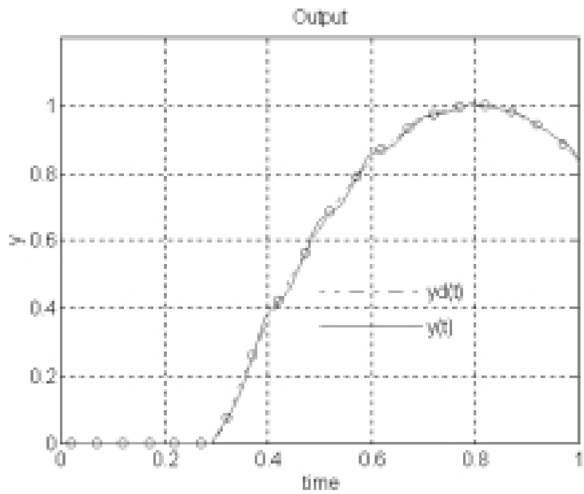


(b)

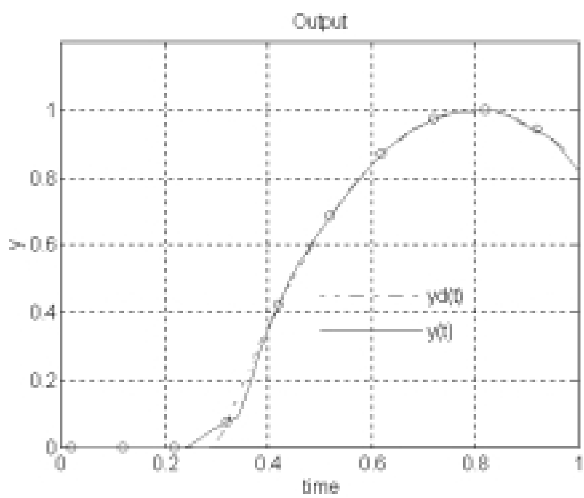
Figure 4: the diverged output $y(t)$ and input $u(t)$

(a) plant output

(b) control input



(a)



(b)

Figure 5: the desired output $y_d(t)$ and the plant output $y(t)$

(a) the case estimated by $0.22 < \tau' < 0.27$

(b) the case estimated by $0.22 < \tau' < 0.32$