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A Study on Robustness of Iterative Learning Controller with Input Saturation against Time-Delay

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Abstract

In this paper, it is first pointed out that, when a typical iterative learning control(ILC) algorithm is applied to a class of dynamic systems with time-delay, erratic estimation of delay time may cause the control input to diverge. In order to resolve such a limitation of the conventional ILC algorithms due to uncertainty of the delay time, a new ILC algorithm with input saturation is proposed to prevent the divergence of the control input. Then, robustness of the proposed algorithm is studied against uncertainty of delay time. To show the effectiveness of the proposed algorithm, two numerical examples are given.

1 Introduction

The iterative learning control (ILC) method has been found to be a good alternative to overcome limitations of conventional control method against inaccuracy in modeling and/or uncertainty of some system parameters when the required task is repetitive[1,2,3,4]. However, the existing ILC algorithms have a limitation that erratic estimation of delay time may cause the control input to diverge when the ILC algorithms are applied to a class of linear dynamic systems with time-delay, as mentioned in [5,6].

To be more specific, consider the linear time-invariant system with time delay described by

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t-\tau') \\ y(t) &= Cx(t) \end{aligned} \tag{1}$$

where $x \in \mathcal{R}^n, u \in \mathcal{R}^r$ and $y \in \mathcal{R}^m$ denote the state, the input and the output, respectively, and τ' is the actual delay time. A, B and C are matrices with appropriate dimensions and it is assumed that CB is a full rank matrix. Let $y_d(\cdot)$ be the desired output trajectory and $x_d(\cdot)$ be the corresponding state trajectory, and assume that they are continuously differentiable on $[0, T]$.

The ILC algorithm in consideration of the delay time is generally expressed in the following form:

$$u_{k+1}(t) = u_k(t) + f\left(e_k(\cdot)\right)(t+\tau_e). \tag{2}$$

Here, τ_e is an estimated delay time and f is a functional of error function, $e_k(t), 0 \le t \le T$, between the desired output and the actual output at the kth iteration. In Eqn. (2), the control input $u_{k+1}(t)$ is updated by input $u_k(t)$ and the value of the functional $f\left(e_k(\cdot)\right)$ at time $t+\tau_e$. If the estimated delay time τ_e is different from the actual delay time τ', however, the control input $u_{k+1}(t)$ is to be updated from incorrect response error, and in this case, there is no guarantee that the algorithm is convergent. In fact, it is not difficult to observe an example in which the control input is divergent due to mis-estimation of the delay time for a system of the Eqn. (1) for which the ILC algorithm (2) is applied (See Example 1).

In this paper, we show that the input saturation function can be positively utilized to guarantee robustness against uncertainty of delay time.

In the sequel, for n-dimensional Euclidean space $\mathcal{R}^n$, $\|x\|$ denotes the Euclidean norm of a vector x. For the vector $x = (x^1, x^2, \cdots, x^n)^T$, the i-th component of x is denoted as x^i. For a matrix A, $\|A\|$ denotes its induced matrix norm. $\|x\|_\infty$ denotes the sup-norm defined by

$$\|x\|_\infty = \sup_{1 \le i \le n} |x^i|.$$

For an $n \times r$ matrix A with components a^{ij}, $\|A\|_\infty$ is defined by

$$\|A\|_\infty = \sup_{1\le i\le n} \sum_{j=1}^{r} |a^{ij}|.$$

For a function $h : [0,T] \to \mathcal{R}^n$ and a real number $\lambda > 0$, $\|h(\cdot)\|_\lambda$ denotes the λ-norm defined by

$$\|h(\cdot)\|_\lambda = \sup_{t\in[0,T]} e^{-\lambda t}\|h(t)\|_\infty.$$

As in the notation $u_k(t)$, the subscript k denotes the iteration number.

2 Robustness of ILC with Input Saturation for LTI Systems with Time-Delay

In this section, the effect of the estimation error of delay time is investigated. Suppose the delay time $\tau' > 0$ is estimated in terms of a lower bound τ_1 and an upper bound τ_2 so that

$$\tau_1 < \tau' < \tau_2.$$

Consider the ILC algorithm described by (3):

$$\begin{aligned} v_{k+1}(t) &= \begin{cases} u_k(t) + \Gamma\left[e_k(t+\tau_2) - e_k(t+\tau_1)\right] \\ \qquad , t\in[0, T-\tau_2] \\ v_k(T-\tau_2), t \in [T-\tau_2, T] \end{cases} \\ u_k(t) &= sat\left(v_k(t)\right). \end{aligned} \quad (3)$$

Here, the input saturation function $sat : \mathcal{R}^r \to \mathcal{R}^r$ is defined as $sat(v) = [sat(v^1), \cdots, sat(v^r)]^T$, where

$$sat(v^i) = \begin{cases} v^i & , |v^i| \le \delta \\ \frac{v^i}{|v^i|}\delta & , |v^i| > \delta. \end{cases}$$

The following assumption is made for the system (1) and the update law (3).

A1 The desired output trajectory $y_d(\cdot)$ is achievable by the desired input within the input saturation bound, i.e., $u_d(t) = sat\left(u_d(t)\right), t \in [0,T]$.

The following result explains the effect of the estimation error.

Theorem 1 *Let the system (1) satisfy the assumption* ***A1****. Suppose that the update law (3) is applied to the system (1) and the initial state at each iteration is the same as the desired initial state, i.e., $x_k(0) = x_d(0)$, for $k = 0, 1, 2, \cdots$. If*

$$\|I - (\tau_2-\tau_1)\,\Gamma CB\|_\infty \le \rho < 1 \quad (4)$$

then, as $k\to\infty$, the output error between $y_d(t)$ and $y_k(t)$ is bounded and this bound directly depends on the input satuation bound, δ, and the estimation bound of delay time, $\tau_2 - \tau_1$.

Proof:

Let

$$\begin{aligned} \Delta u_k(t) &= u_d(t) - u_k(t) \\ \Delta x_k(t) &= x_d(t) - x_k(t). \end{aligned}$$

It follow from Eqns. (1) and (3) that

$$\begin{aligned} \Delta v_{k+1}(t) &= \Delta u_k(t) - \Gamma\left[e_k(t+\tau_2) - e_k(t+\tau_1)\right] \\ &= \Delta u_k(t) - \Gamma \int_{t+\tau_1}^{t+\tau_2} \dot{e}_k(\sigma)d\sigma \\ &= \Delta u_k(t) \\ &\quad -\Gamma\int_{t+\tau_1}^{t+\tau_2}\left[CA\Delta x_k(\sigma) + CB\Delta u_k(\sigma-\tau')\right]d\sigma \\ &= \left[I - (\tau_2-\tau_1)\,\Gamma CB\right]\Delta u_k(t) \\ &\quad -\Gamma CA\int_{t+\tau_1}^{t+\tau_2}\Delta x_k(\sigma)d\sigma \\ &\quad -\Gamma CB\int_{t+\tau_1}^{t+\tau_2}\left[\Delta u_k(\sigma-\tau') - \Delta u_k(t)\right]d\sigma \end{aligned} \quad (5)$$

Taking the norm $\|\cdot\|_\infty$ on both sides of Eqn. (5), we have

$$\begin{aligned} \|\Delta v_{k+1}(t)\|_\infty &\le \rho\|\Delta u_k(t)\|_\infty \\ &\quad +\|\Gamma CA\|_\infty\int_{t+\tau_1}^{t+\tau_2}\|\Delta x_k(\sigma)\|_\infty d\sigma \\ &\quad +\|\Gamma CB\|_\infty\int_{t+\tau_1}^{t+\tau_2}\|\Delta u_k(\sigma-\tau')\|_\infty d\sigma \\ &\quad +\int_{t+\tau_1}^{t+\tau_2}\|\Delta u_k(t)\|_\infty d\sigma \\ &\le \rho\|\Delta v_k(t)\|_\infty \\ &\quad +\|\Gamma CA\|_\infty\int_{t+\tau_1}^{t+\tau_2}\|\Delta x_k(\sigma)\|_\infty d\sigma \\ &\quad +4\delta\|\Gamma CB\|_\infty(\tau_2-\tau_1) \end{aligned} \quad (6)$$

From Eqn. (1), we can obtain

$$\Delta x_k(t) = \int_0^t \left[A\Delta x_k(\sigma) + B\Delta u_k(\sigma-\tau')\right]d\sigma. \quad (7)$$

Taking the norm $\|\cdot\|_\infty$ on both sides of Eqn. (7) and applying the Grownwall-Bellman inequality, we find that

$$\|\Delta x_k(t)\|_\infty \le \int_0^t e^{a(t-\sigma)}\|B\|_\infty\|\Delta u_k(\sigma-\tau')\|_\infty d\sigma \quad (8)$$

where

$$a = \|A\|_\infty.$$

Multiplying $e^{-\lambda t}$ on both sides of Eqn. (8), we find that

$$\|\Delta v_{k+1}(\cdot)\|_\lambda \leq \rho \|\Delta v_k(\cdot)\|_\lambda + k_1 \delta(\tau_2 - \tau_1) \quad (9)$$

where

$$k_1 = 4\|\Gamma CB\|_\infty + 2\|\Gamma C\|_\infty \|B\|_\infty \left(e^{aT} - 1\right).$$

Since $0 \leq \rho < 1$ by assumption, we can further find that

$$\lim_{k\to\infty} \|\Delta v_k(\cdot)\|_\lambda \leq \frac{k_1}{1-\rho}\delta(\tau_2 - \tau_1). \quad (10)$$

From Eqns. (8) and (10), we can finally conclude that

$$\lim_{k\to\infty} \|e_k(\cdot)\|_\lambda \leq \frac{1 - e^{-(\lambda - a)T}}{\lambda - a} \frac{k_2}{1-\rho}\delta(\tau_2 - \tau_1)$$

where

$$k_2 = k_1 \|C\|_\infty \|B\|_\infty.$$

This completes the proof.

Theorem 1 implies that, if the input is saturated by some constant δ, then the control input does not diverge and furthermore the output error is bounded by the input saturation bound, δ, and the estimation bound of delay time, $\tau_2 - \tau_1$. From this observation, if we can estimate delay time more exactly, we can achieve better performance of ILC with high accuracy.

In the convergence condition (4), one may easily note that selection of Γ is possible even when the inequality relation contains the uncertainties in the system matrices, B and C, and uncertainty in the value τ'. One can estimate the bound of $(\tau_2 - \tau_1)CB$ if some ranges of each components of B and C are available, noting that $\tau_2 - \tau_1 > 0$. Selection of the input saturation bound δ which satisfies the assumption **A1**, is also possible. If the ranges of each components of B and C are available, one can estimate the range of the desired input trajectory $u_d(\cdot)$ from the desired output trajectory $y_d(\cdot)$, and then one can select the input saturation bound δ such that δ is larger than the maximum value of the estimated range of $u_d(\cdot)$.

In conventional feedback systems, input saturation mechanism may cause the worse performance, since it hampers the better control input value at each time instance if the desirable input is larger than the saturation bound. However, in the proposed ILC algorithm, the best control input does not prevented by the input saturation, since the saturation bound is larger than the maximum value of the desired control input. The saturation function restricts the control input against the divergence only during the forepart of the iterations.

3 Numerical Examples

To illustrate effectiveness of the proposed algorithm, consider the following linear time-invariant dynamic system.

$$\begin{aligned} \dot{x} &= \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t - 0.225) \\ y(t) &= \begin{bmatrix} 0 & 1 \end{bmatrix} x(t). \end{aligned} \quad (11)$$

The desired output trajectory is given as

$$y_d(t) = \begin{cases} 0, & 0 \leq t < 0.3 \\ 4(t-0.3) - 4(t-0.3)^2, & 0.3 \leq t \leq 1. \end{cases}$$

Example 1: To confirm the undesirable phenomenon when a typical ILC algorithm is applied, suppose the D-type ILC algorithm is applied to the system Eqn. 11 with some naive modification as in the following Eqn. 12 using estimated delay time 0.23.

$$u_{k+1}(t) = u_k(t) + \Gamma \dot{e_k}(t + 0.23) \quad (12)$$

Then we find that as the iteration number k increases the control input becomes divergent as shown in Fig. 1.

Example 2: Now, let us apply the proposed algorithm. For this, we consider two cases in which the delay time is estimated differently.

Suppose first the lower bound of delay time is 0.2 and the upper bound is 0.3, i.e., $0.2 < \tau' < 0.3$. Then, the proposed ILC algorithm can be represented as in the following Eqn. (13) using estimated delay time.

$$v_{k+1}(t) = u_k(t) + \frac{1}{1.3}\left[e_k(t+0.3) - e_k(t+0.2)\right] \quad (13)$$

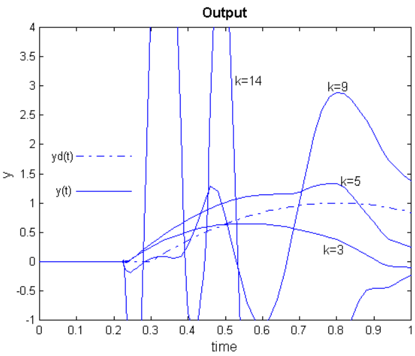
where

$$u_k^i(t) = \begin{cases} v_k^i(t) & , |v_k^i(t)| \leq 5 \\ 5 & , v_k^i(t) > 5 \\ -5 & , v_k^i(t) < -5 \end{cases}$$

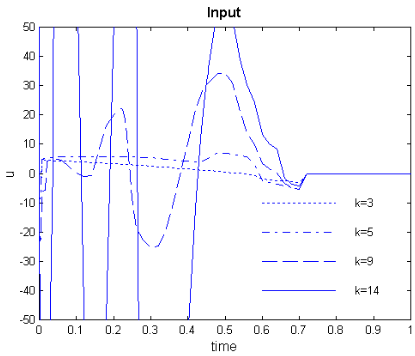
For another case, let the lower bound of the delay time be 0.22 and the upper bound be 0.23, i.e., $0.22 < \tau' \leq 0.23$. Then, the proposed ILC algorithm can be represented as

$$v_{k+1}(t) = u_k(t) + \frac{1}{1.3}\left[e_k(t+0.23) - e_k(t+0.22)\right] \quad (14)$$

Fig. 2 (a) and (b) show that the convergent output trajectories after 30th iteration and the trend of convergence at each case.



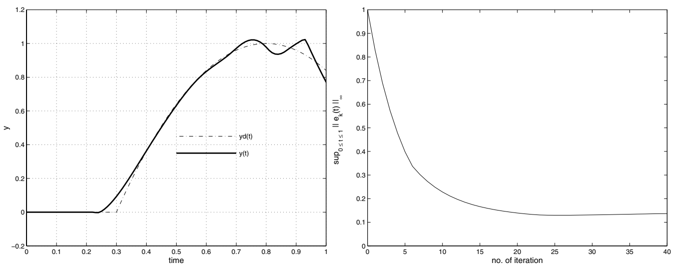
(a)



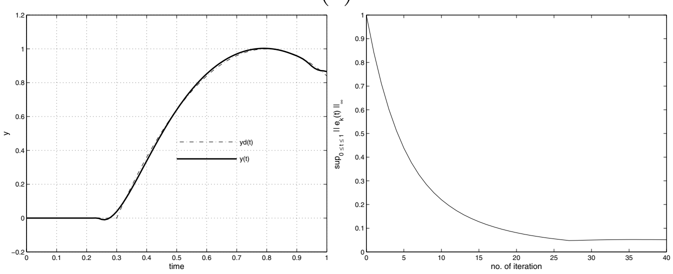
(b)

Figure 1: The diverged output trajectory and the control input

(a) plant output
(b) control input



(a)



(b)

Figure 2: The output trajectory and the trend of convergence for Example 5.2

(a) The case estimated by $0.2 < \tau < 0.3$
(b) The case estimated by $0.22 < \tau < 0.23$

4 Concluding Remarks

In this paper, a new ILC algorithm with input saturation was proposed and the robustness against uncertain delay time was investigated for linear time-invariant systems.

When the proposed ILC algorithm is applied, the output error can be bounded and this bound depends on the estimation bound of delay time and the input saturation bound, while the control input may diverge if the conventional ILC algorithms are applied.

Only the effect of the input saturation was studied in this paper, and we showed the robustness against time-delay. Use of time-varying saturation function can be a challenging problem to achieve better performance.

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