# An Iterative Receiver Algorithm for Space-Time Encoded Signals*

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## Abstract

*An iterative receiver is proposed for a wireless communication system employing multiple transmit and receive antennas. The transmitted symbol sequences are space-time encoded. By exploiting the inherent structure in the space-time encoded data sequence, the receiver is able to significantly improve the initial estimate of the unknown channel leading to significant performance gains in terms of the bit-error-rate (BER) as a function of the signal-to-noise ratio (SNR). Computer simulations demonstrate the efficacy of the scheme in single-user and multi-user environments.*

## 1. Introduction

The use of multiple transmit antennas in conjunction with space-time coding techniques, has in recent years been recognized as a powerful method to combat the effects of signal fading in wireless communications [1]. For example, it has been demonstrated that for the transmit diversity scheme in [2], using 2 transmit and $m$ receive antennas, a diversity order of $2m$ is obtained. A generalization to more than two transmit antennas is given in [3]. The method in [2] assumes that perfect channel state information (CSI) is available at the receiver. Receiver algorithms for the realistic case when channel state information is not available, have been presented in [4, 5]. In [4], it was concluded that the lack of CSI incurred approximately a 3 dB penalty in performance relative to the ideal case. An expectation-maximization (EM) scheme was suggested in [5] and studied the effect of pilot-symbol spacing in fading environments. These references are concerned with single-user scenarios in the presence of additive white Gaussian noise (AWGN). Multiuser methods using space-time block codes for interference suppression have been presented in [6], but assume perfect CSI at the receiver.

The method presented here carries out detection and decoding of the transmitted symbols based on the initial channel estimates from a few training symbols. The quality of the CSI is then improved by employing the detected and re-encoded symbols, leading to significant improvement in BER relative to the known channel case. The proposed method is block based and assumes that the fading parameters are constant during each frame and vary from one frame to another (block fading). Simulation results indicate that our method performs similarly to the EM technique for the case of a single user, while relying on fewer assumptions and less side-information, specifically the transmit power. Furthermore, in a scenario with several users, results show that relying only on training symbols gives very poor performance. The proposed algorithm is able to significantly reduce the BER with only a small number of iterations. Computer simulation examples include both space-time block and trellis coded signals.

## 2. Signal Model and Algorithm

We consider a wireless communication scenario where $K$ users each transmit space-time encoded symbol sequences using $n$ antennas. The noisy superpo-

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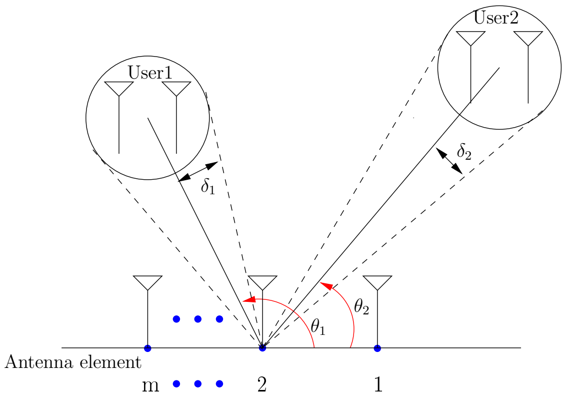


**Figure 1. Diagram of receiving array and two users.**

sition of all user signals impaired by frequency-flat Rayleigh fading are received by an array of $m$ antennas. The fading is assumed to be caused by scatterers in the neighborhood of the transmitting antennas. From the perspective of the receiving array, the fading is identical for each user transmit antenna. This models the fading seen from base stations located on high towers. Since the array elements cooperate on discriminating among the different users by providing spatial filtering, the receive array elements are correlated. Hence, only a diversity of order $n$ is obtained. In what follows, we assume $n = 2$ transmit antennas for ease of presentation. A diagram of the receiving array with two users in the field is presented in Figure 1, where the angles of arrival and angles of separation for user $k$ are denoted by $\theta_k$ and $\delta_k$, respectively.

With negligible delay-spread, the synchronously sampled output of the receiver matched-filter at time $t$ can thus be written as

$$\begin{aligned}\mathbf{x}_t &= \sum_{k=1}^{K}\sqrt{P_k}\left[\mathbf{a}(\theta_k^{(1)})\,\mathbf{a}(\theta_k^{(2)})\right]\begin{bmatrix}\gamma_k^{(1)} & 0\\ 0 & \gamma_k^{(2)}\end{bmatrix}\mathbf{c}_{k,t}+\mathbf{n}_t\\ &= \sum_{k=1}^{K}\mathbf{H}_k\mathbf{c}_{k,t}+\mathbf{n}_t \qquad (1)\end{aligned}$$

where $\gamma_k^{(i)}$ is the complex fading coefficient for user $k$ transmit antenna $i$, $P_k = P_{k,T}/n$ is the power transmitted from each transmit antenna, and $\mathbf{n}_t$ is spatially and temporally uncorrelated white Gaussian noise with variance $\sigma^2$ ($P_{k,T}$ is the total power transmitted by user $k$). Coefficients $\gamma_k^{(i)}$ are modeled as independent samples of a stationary complex Gaussian process with zero mean and unit variance. The average SNR for user $k$ is $\mathrm{SNR}_k = P_{k,T}/\sigma^2$. We denote the array response vector of user $k$ and transmit antenna $i$ as $\mathbf{a}(\theta_k^{(i)})$. If


**Figure 2. Diagram of frame structure.**

we let $\mathbf{X} = [\mathbf{x}_1, \cdots, \mathbf{x}_{N_T}]$, $\mathbf{N} = [\mathbf{n}_1, \cdots, \mathbf{n}_{N_T}]$, and $\mathbf{C}_k = [\mathbf{c}_{k,1}, \cdots, \mathbf{c}_{k,N_T}]$, and for a total frame length of $N_T$ symbols, we may write (1) as

$$\mathbf{X} = \mathbf{H}_1\mathbf{C}_1 + \sum_{k=2}^{K}\mathbf{H}_k\mathbf{C}_k + \mathbf{N}. \qquad (2)$$

In (2), we assume that user-1 is the desired signal and the quantity, $\sum_{k=2}^{K}\mathbf{H}_k\mathbf{C}_k + \mathbf{N}$, is the multi-user interference plus AWGN.

The receiver assumes no knowledge of array geometry or array calibration, and only works with the unstructured $m \times 2$ channel $\mathbf{H}_k$, which is implicitly defined in the model. Note that the assumption of $K$ synchronous users is only made for purposes of formulating the model, and that the algorithm to be presented only requires synchronization to the desired signal.

The user frame is partitioned into $N_p$ training symbols and $N$ information symbols for a total frame length of $N_T = N_p + N$. The desired user code matrix is partitioned as $\mathbf{C}_1 = [\mathbf{C}_p|\mathbf{C}_u]$, where $\mathbf{C}_p$ is a $2 \times N_p$ matrix containing the encoded training symbols, and $\mathbf{C}_u = [\mathbf{c}_{1,1}, \ldots, \mathbf{c}_{1,N}]$ is a $2 \times N$ information symbol matrix, see Figure 2. Likewise, the received data is partitioned as $\mathbf{X} = [\mathbf{X}_p|\mathbf{X}_u]$, where $\mathbf{X}_p$ is a $m \times N_p$ matrix, and $\mathbf{X}_u = [\mathbf{x}_1, \ldots, \mathbf{x}_N]$ is a $m \times N$ matrix.

## 2.1. Space-Time Block Coding Algorithm

Space-time block encoding is performed using the methods in [2], where the codewords are obtained from

$$[\mathbf{c}_{k,t}\ \mathbf{c}_{k,t+1}] = \begin{bmatrix} d_{k,t} & -d^*_{k,t+1}\\ d_{k,t+1} & d^*_{k,t}\end{bmatrix}, \qquad (3)$$

and $d_{k,t}$ is the $k$th user symbol at time $t$.

The algorithm performs the following operations:

1. Initial least-squares channel estimation of user-1 from training symbols:[1]
   $\hat{\mathbf{H}}_1 = \mathbf{X}_p\mathbf{C}_p^H(\mathbf{C}_p\mathbf{C}_p^H)^{-1}$.
2. Iterate between channel and symbol matrix estimation:
   (a) Weighted least-squares estimation of the signal matrix:

$\hat{\mathbf{C}}_u = \hat{\mathbf{H}}_1^H \hat{\mathcal{R}}_{xx}^{-1} \mathbf{X}_u$, where
$\hat{\mathcal{R}}_{xx} = \frac{1}{N_T} \sum_{t=1}^{N_T} \mathbf{x}_t \mathbf{x}_t^H$.

(b) Decode and detect transmitted symbols:[2]

$$\begin{aligned} \hat{d}_{1,t} &= \lfloor \hat{\mathbf{c}}_{1,t}(1) + \hat{\mathbf{c}}^*_{1,t+1}(2) \rfloor \\ \hat{d}_{1,t+1} &= \lfloor \hat{\mathbf{c}}_{1,t}(2) - \hat{\mathbf{c}}^*_{1,t+1}(1) \rfloor \end{aligned}$$

(c) Re-encode estimated bits:
$\{\hat{d}_{1,t}\}_{t=1}^{N} \longrightarrow \{\hat{\mathbf{c}}_{1,t}\}_{t=1}^{N}$ using (3).

(d) Re-estimate channel:
$\hat{\mathbf{H}}_1 = \mathbf{X}_u \hat{\mathbf{C}}_u^H (\hat{\mathbf{C}}_u \hat{\mathbf{C}}_u^H)^{-1}$.
Go to 2(a) and repeat.

Note that the estimate of the codewords in 2(a) is essentially a linear MMSE estimate using the iteratively refined estimate of the channel $\mathbf{H}_1$ and the sample data-covariance matrix, $\hat{\mathcal{R}}_{xx}^{-1}$.

A generalization of this algorithm to $n > 2$ transmit antennas can be obtained by using the appropriate space-time codes defined in [3].

### 2.2. Space-Time Trellis Coding Algorithm

If the transmitted symbols are encoded using a space-time trellis code [1], we must modify the decoding of step 2 in the proposed algorithm above.

Specifically, for the special case of $K = 1$ user, given an estimate of the desired user channel matrix $\mathbf{H}_1$, a maximum likelihood (ML) decoder should select the codewords according to:

$$\hat{\mathbf{C}}_u = \arg\min_{\mathbf{C}_u} \|\mathbf{X}_u - \hat{\mathbf{H}}_1 \mathbf{C}_u\|_F^2, \qquad (4)$$

where $\|\cdot\|_F$ is the Frobenius norm. This space-time ML decoder is implemented as a Viterbi algorithm, where the trellis path with the shortest accumulated metric is chosen

$$\hat{\mathbf{C}}_u = \arg\min_{\mathbf{C}_u} \sum_{t=1}^{N} \left[ \sum_{i=1}^{m} |x_{i,t} - \sum_{j=1}^{n} \hat{h}_{i,j}\, c_{j,t}|^2 \right]. \qquad (5)$$

In (5), $x_{i,t}$ is the received signal at antenna $i$ and time epoch $t$, $\hat{h}_{i,j}$ is the CSI estimate between transmit antenna $j$ and receive antenna $i$, and $c_{j,t}$ is the symbol transmitted from antenna $j$ at time epoch $t$.

[1]The operator $H$ denotes conjugate transposition.
[2]The symbol $\lfloor\cdot\rfloor$ signifies a slicing operation in the case of BPSK modulated symbols.

In a multi-user scenario, we modify (4) to account for data prewhitening, resulting in a weighted least-squares criterion,

$$\hat{\mathbf{C}}_u = \arg\min_{\mathbf{C}_u} \|\hat{\mathcal{R}}_{xx}^{-\frac{1}{2}} \left( \mathbf{X}_u - \hat{\mathbf{H}}_1 \mathbf{C}_u \right) \|_F^2. \qquad (6)$$

It has been shown in [7] that data prewhitening with the square-root inverse of the sample data covariance matrix $\hat{\mathcal{R}}_{xx}^{-\frac{1}{2}}$ as above, is asymptotically equivalent (i.e. for large frame lengths, $N_T$) to using the unknown square-root inverse of the interference-plus-noise covariance matrix. Thus, (6) should be interpreted as the asymptotic weighted least-squares criterion, and its minimization can be implemented using the Viterbi algorithm as in (5) for the $K = 1$ case.

## 3. Simulation Results

Performance is evaluated by Monte Carlo simulations using BPSK modulation for the space-time block codes, and QPSK for the space-time trellis codes. The probability of bit error is given as a function of the average SNR for both single-user and multi-user scenarios.

### 3.1. Single-user Scenario

Figure 3(a) shows the results obtained for a single user with two transmit antennas and one receive antenna. We block encoded the data using (3). Using 2 training symbols to estimate the channel, it is observed that the performance is 3 dB higher than the the bound derived from the perfect CSI (maximum likelihood genie bound), and that significant improvement is possible by iterating twice. We also plot in Figure 3(a) the results for the block-faded EM method presented in [5]. Despite the fact that our method and the EM scheme are derived using very different assumptions, they result in identical performance.

Similar behavior is observed for space-time trellis codes. In Figure 3(b) we compare our method with EM over block fading with one and two receive antennas using the 4-state, QPSK code in [1, Fig 4]. With a frame size of 120 symbols and 10 symbols dedicated to training, we find that only one iteration of both algorithms is necessary to approach the perfect CSI bound.

### 3.2. Multi-user Scenario

In a multi-user environment, the total interference-plus-noise seen by the receiver is no longer Gaussian. Thus, we choose to compare our method to the optimal linear MMSE receiver having perfect knowledge of all the users channels as well as the variance of the ambient background noise. The optimal LMMSE receiver [8, 12.27] estimates the codeword symbols according to $\hat{\mathbf{C}}_u = \mathbf{W}^H \mathbf{X}_u$, where

$$\begin{aligned} \mathbf{W} &= \mathcal{R}_{xx}^{-1} \mathcal{R}_{xc_1} \\ &= (\sum_{k=1}^{K} \mathbf{H}_k \mathbf{H}_k^H + \sigma^2 \mathbf{I}_m)^{-1} \mathbf{H}_1. \end{aligned} \quad (7)$$

In (7), $\mathbf{I}_m$ is the $m \times m$ identity matrix, $\mathcal{R}_{xx} = \mathrm{E}\left\{\mathbf{X}\mathbf{X}^H\right\}$, and $\mathcal{R}_{xc_1} = \mathrm{E}\left\{\mathbf{X}\mathbf{C}_1^H\right\}$. Optimal LMMSE codeword matrix estimation is followed by detection and decoding as in step 2(b).

We simulated a multi-user scenario with $K = 3$ users. Angles of arrival were normally distributed, $\theta_k^{(i)} \sim \mathcal{N}(\theta_{k,0}, 4°)$ with mean angles $[\theta_{1,0}, \theta_{2,0}, \theta_{3,0}] = [70°, 90°, 110°]$ and variance of 4° relative array endfire for the three users. Emulating perfect power control, all users had the same instantaneous power. The frame size is 180 symbols with 10 symbols allocated to training.

We observe in Figure 4, that using only the initial channel estimate obtained from the 10 training symbols results in unacceptably poor performance. We could improve the initial estimate by increasing the number of training symbols. However, this increases the frame overhead and reduces the system information rate. Instead, using our method with only a small number of iterations, we obtain a significant improvement in performance, as we see in Figure 4. The curve labeled "convergence" is obtained by iterating until the sequence of symbol estimates are identical in two consecutive iterations.

We note that the performance gain is especially large at high SNR. At high SNR, after the first iteration of the algorithm, the spatial filter (step 2a) attempts to steer a beam towards the desired user and place nulls in the direction of the interfering users. Consequently, the signal-to-interference-plus-noise ratio (SINR) improves considerably, thereby reducing the BER significantly. Further improvement is possible by performing additional iterations. However, at low SNR levels the system is AWGN dominated resulting in poor estimation of the channel matrix, $\mathbf{H}_1$. Thus, the performance enhancement is smaller at low SNR.

## 4. Conclusions

We have demonstrated an iterative receiver which exploits the structure of space-time codes for joint channel estimation and decoding. In a single user context, our method has performance similar to the EM method but does not require receiver knowledge of the user power. In a multi-user scenario we have shown large gains in performance with a few algorithm iterations.

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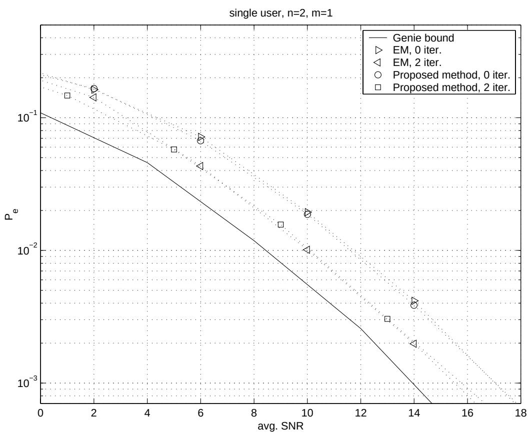
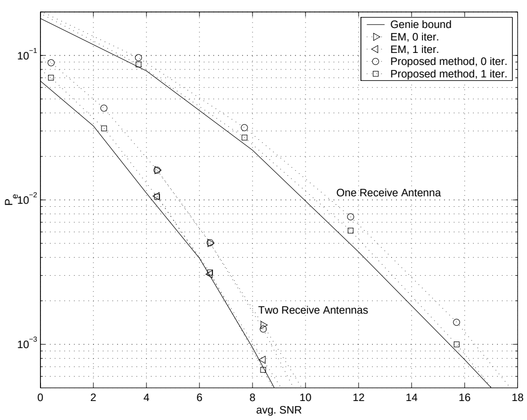



(a) Block code: $K = 1$, $n = 2$, $m = 1$, $N = 180$, $N_p = 2$ training symbols.




(b) Trellis code: $K = 1$, $n = 2$, $m = 1, 2$, $N = 120$, $N_p = 10$ training symbols.

**Figure 3. Simulation results for the proposed method in a single-user environment.**

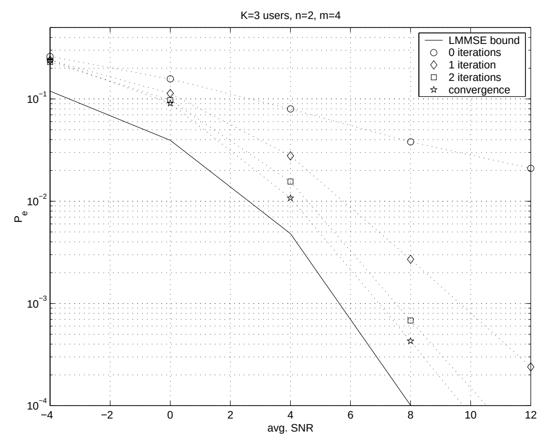



**Figure 4. Simulation results for the proposed method in a multi-user environment.**