Modelling Interdependent Returns and Risk in the Environmental Industry

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Abstract: This paper stresses the importance of assessing the risk-return trade-off faced by environmental industries in financial markets. One of the most widely-used theoretical models in finance is the conditional CAPM, which describes the conditional risk-return tradeoff in financial markets, whereby both the conditional mean return and conditional beta risk are allowed to vary over time. This paper models the time-varying conditional mean and variances of returns, and the correlations between the individual asset return and the market return in the conditional CAPM in a multivariate GARCH framework. Specifically, the Vector ARMA-Asymmetric Multivariate GARCH model of Chan, Hoti and McAleer (2003), which incorporates the CCC-MGARCH of Bollerslev (1990) and the Vector ARMA-GARCH of Ling and McAleer (2002), is applied to the conditional CAPM, with the conditional CAPM specification replacing the ARMA specification in the conditional mean equation. The main motivation for using the Conditional CAPM-AMGARCH-in-mean model specification is not only because it has the ability to capture the stylized facts of financial asset returns such as persistence of volatility, volatility clusters and excess kurtosis, but it also considers the interdependencies of volatilities between the individual asset return and the market return, and the asymmetric effects of the unconditional shocks on the conditional variances of the individual asset return and the market return respectively. Further, the structural and statistical properties of this model have been established in Chan *et al*. (2003). Our dataset consists of monthly excess returns on the Australian Mining industry sectors including Gold Mining, Other Mining and Mining Finance from the period January 1980 to December 2002. The paper suggests that the conditional CAPM is inadequate in explaining the financial risk-return tradeoff for the environmental industry sectors; however, there appears to be some interdependent ARCH/GARCH effects between the Other Mining and Mining Finance excess returns and the market excess return, and no asymmetric ARCH/GARCH effects.

Keywords: Multivariate GARCH; Asymmetric effects; Conditional CAPM

1. INTRODUCTION

In the age of mining, oil and gas exploration, deforestation and water pollution, there is an increasing need for environmental industries to be "green" or "environmentally- friendly" in order to decelerate the depletion of earth's natural resources. The "greenness" of the environmental industries is, however, often the outcome of the risk-return trade-off that these industries face in financial markets, where industries are more pressured to exploit environmental resources when faced with higher risk levels and lower profit margins. Consequently, it is of importance to assess the risk-return trade-off faced by environmental industries in financial markets. One of the most widely-used theoretical models in finance is the conditional CAPM, which describes the conditional risk-return tradeoff in financial markets, whereby both the conditional mean return and conditional beta risk are allowed to vary over time (see Appendix). This paper models the time-varying conditional mean and variances of returns, and the correlations between the individual asset return and the market return in the conditional CAPM using various multivariate constant conditional correlation GARCH models.

2. MULTIVARIATE CONSTANT CONDITIONAL CORRELATION MODELS

In order to allow for interdependent effects, several different multivariate constant conditional correlation GARCH models have been proposed in the financial econometrics literature. They include the (i) the Constant Correlation Multivariate GARCH (CCC-MGARCH) of Bollerslev (1990), (ii) the Vector ARMA-GARCH model of Ling and McAleer (2003) and (iii) the Vector ARMA-Asymmetric Multivariate GARCH (or Vector ARMA-AMGARCH) model of Chan, Hoti and McAleer (2003). A more comprehensive survey of the multivariate GARCH models is provided in Chan, Hoti and McAleer (2003). It should be noted that the outline and content of this section follows closely to Chan *et al*. (2003).

CCC-MGARCH Model of Bollerslev (1990)

Bollerslev (1990) proposed a constant (or static) conditional correlation multivariate GARCH model, otherwise known as the CCC-MGARCH. The main feature of the CCC-MGARCH is that it assumes the conditional correlation to be constant over time. The m- dimensional CCC-MGARCH model is as follows:

$$Y_t = E(Y_t / Z_{t-1}) + \varepsilon_{0t} ,$$
$$\varepsilon_{0t} = D_{0t}\eta_{0t} ,$$
$$Var(\varepsilon_{0t} / Z_{t-1}) = D_{0t}\Gamma_0 D_{0t} ,$$

where Z_{t-1} is the information set available up to time $t-1$, $D_{0t} = diag(h_{0it}^{1/2})$ is a diagonal matrix of the conditional variances for the individual asset and the market return respectively with $i = 1,...,m$, for which m is the total number of assets on markets, and

$$\Gamma_0 = \begin{bmatrix} 1 & \rho_{012} & \cdots & \rho_{01m} \\ \rho_{021} & 1 & \rho_{023} & \cdots \\ & & \cdots & \\ \rho_{0m1} & \cdots & \rho_{0m,m-1} & 1 \end{bmatrix},$$

where $\rho_{0ij} = \rho_{0ji}$ for $i, j = 1,...,m$, and $\eta_{0t} = (\eta_{01t},...,\eta_{0mt})'$.

Under the assumption of constant correlations, the maximum likelihood estimate (MLE) of the correlation matrix is equal to the sample correlation matrix of the standardized residuals (i.e. $\Gamma_0 = E(\eta_{0t}\eta'_{0t})$), which is always positive definite. Furthermore, the correlation matrix can be concentrated out of the log-likelihood function, resulting in a reduction of parameters to be optimized.

Bollerslev (1990) assumed that:

$$h_{0it} = \omega_{0i} + \sum_{l=1}^{r} \alpha_{0it}\varepsilon_{0it-1}^2 + \sum_{l=1}^{s} \beta_{0il} h_{0it-l} , \qquad i = 1,..., m,$$

in which there are no interdependencies of volatilities across different assets and/ or markets, and no accommodation of asymmetric behaviour.

Vector ARMA-GARCH of Ling and McAleer (2003)

In order to allow for interdependencies of volatilities across different assets and/ or markets, Ling and McAleer (2003) proposed a static conditional correlation Vector ARMA-GARCH model, which states that:

$$\Phi_0(L)(Y_t - \mu_0) = \Phi_0(L)\varepsilon_{0t}$$
$$\varepsilon_{0t} = D_{0t}\eta_{0t}$$
$$H_{0t} = W_0 + \sum_{l=1}^{r} A_{0t}\vec{\varepsilon}_{0t-1} + \sum_{l=1}^{s} B_{0t} H_{0t-1}$$

where $D_{0t} = diag(h_{0t}^{1/2})$, A_{0t} and B_{0t} are $m \times m$ matrices with typical elements α_{0ij} and β_{0ij}, respectively for $i, j = 1,...,m$, $\Phi_0(L) = I_m - \Phi_{01}L - ... -$ are polynomials in L, $\vec{\varepsilon}_{0t} = (\varepsilon_{01}^2,...,\varepsilon_{0m}^2)$.

In their paper, Ling and McAleer established both the structural and statistical properties of the Vector ARMA-GARCH, including the necessary and sufficient conditions for stationarity and ergodicity, the sufficient conditions for the existence of moments and the asymptotic theory for the QMLE.

Vector ARMA-AGARCH Model of Chan, Hoti and McAleer (2003)

Chan, Hoti and McAleer (2003) extended the Vector ARMA-GARCH model to accommodate the asymmetric effects of the unconditional shocks on the conditional variances. The Vector ARMA-Asymmetric Multivariate GARCH (Vector ARMA – AMGARCH) is given by:

$$\Phi_0(L)(Y_t - \mu_0) = \Phi_0(L)\varepsilon_{0t} ,$$
$$\varepsilon_{0t} = D_{0t}\eta_{0t} ,$$
$$H_{0t} = W_0 + \sum_{l=1}^{r} A_{0i}\vec{\varepsilon}_{0t-l} + \sum_{l=1}^{r} C_{0l} I(\eta_{0t-l})\vec{\varepsilon}_{0t-l} + \sum_{l=1}^{s} B_{0t} H_{0t-l} ,$$

where C_{0t} is a $m \times m$ matrix with typical element γ_{0ij}, and $I(\eta_{0t-l})$ is an indicator variable such that:

$$I(\eta_{0t-l}) = \begin{cases} 1, & \varepsilon_{0it} \le 0 \\ 0, & \varepsilon_{0it} > 0. \end{cases}$$

As in Ling and McAleer (2003), Chan *et al.* (2003) also established the structural and statistical properties of their model, including the necessary and sufficient conditions for stationarity and ergodicity, the sufficient conditions for the existence of moments, and the sufficient conditions for consistency and asymmetric normality of the QMLE.

3. DATA

Our data sample comprises of monthly share price indices on three mining industry sectors - Gold Mining, Other Mining, Mining Finance – in Australia from the period January 1980 to December 2002. The return on the market portfolio is proxied by the return on an Australian market share price index, a value-weighted measure of share price indices for all industry sectors. Returns are calculated for each industry sector on a continuouscompounding basis, computed as the natural logarithm of the price differences. Excess returns are calculated by taking the difference between the industry sector returns and the risk-free rate, which is proxied by the continuously compounded (percentage change) monthly 30-day bank accepted bill rate. All data is obtained from Thompson Datastream Advance.

Table 1 provides descriptive statistics on the excess returns of the three mining industry sectors over the sample period. Gold Mining has the highest mean return and variance, while Other Mining has the lowest mean return and Mining Finance has the lowest variance. Only Mining Finance exhibits negative skewness (SK), while excess kurtosis (KU) is present in all industries. These findings suggest widespread departures from normality in returns. Consequently, all three mining industries failed the Jarque-Bera (JB) test of normality at the 1% significance level.

Table 1. Descriptive Statistics of Returns

	MEAN	***VAR***	***SK***	***KU***	***JB***
GOLDS	0.123	220.9	0.31	7.19	0.00
MINES	0.011	100.4	0.34	4.59	0.00
MIFIN	0.015	51.8	-0.04	3.76	0.00

Notes: Figures reported for the JB test are p-values

Diagnostic tests are conducted on the excess return market model, $r_{it} = \alpha_i + \beta_i r_{mt} + \varepsilon_{it}$, which is a widely used vehicle to transform the ex ante CAPM into ex post form. Table 2 reports the results of the OLS regression of the excess return market model for each of the three mining industry sectors distributed (iid) with zero mean and unit variance. The Newey-West (1987) method is employed to correct for the potential unspecified departures from homoskedasticity and no serial correlation.

Table 2. OLS Estimation of the Excess Return Model and the ARCH(LM) Test

	ALPHA	***BETA***	***ARCH(LM)***
GOLDS	-0.005	1.518	4.721
	(-0.006)	(10.386)	(0.030)
MINES	-0.085	1.183	11.713
	(-0.378)	(27.339)	(0.001)
MIFIN	-0.100	1.327	3.118
	(-0.229)	(15.737)	(0.077)

Notes: The entries in parentheses corresponding to the alpha and beta estimates are the Newey-West(1987) corrected t-ratios, while the entries in parentheses corresponding to the ARCH(LM) are p-values.

All intercept coefficients are insignificant as predicted by the excess return form of the CAPM, while the OLS beta estimates are all significant and of the expected (positive) sign. The test for conditional heteroskedasticity is the Lagrange multiplier test (LM) for autoregressive conditional heteroskedasticity (ARCH) by Engle (1982). The LM (ARCH) test statistic is asymptotically distributed as $\chi^2_{(p)}$, where we choose $p = 1$ to test for ARCH(1). The LM (ARCH) p-values indicate that there is considerable conditional heteroskedasticity in the return residuals, where the null hypothesis of homoskedasticity is rejected in all three mining industry sectors at the 10% significance level. This finding provides some justification for modelling conditional volatilities in the conditional CAPM using multivariate GARCH models.

4. EMPIRICAL RESULTS

This paper models the time-varying conditional means and variances of excess returns of the conditional CAPM using a variety of multivariate constant conditional correlation GARCH-in-mean models, including the CCC-MGARCH-in-mean, Conditional CAPM-GARCH-in-mean and the Conditional CAPM-AGARCH-in-mean. These models are derived by applying the Vector ARMA-Asymmetric Multivariate GARCH model of Chan, Hoti and McAleer (2003), which incorporates the CCC-MGARCH of Bollerslev (1990) and the Vector ARMA-GARCH of Ling and McAleer (2002) to the conditional CAPM, with the conditional CAPM specification replacing the ARMA specification in the conditional mean equation.

All the models are estimated using RATS Version 5.1 procedures. The Broyden, Fletcher, Goldfarb and Shanno (BFGS) algorithm described in Press, Flannery, Teukolsky and Vettering (1988)) is used to obtain the parameter estimates.

Table 3a and 3b present the CCC-MGARCH (1,1)-in-mean quasi-MLE estimates (QMLE) for the bivariate conditional CAPM for the Gold Mining and Other Mining industries. Asymptotic t-ratios for the QMLE are reported in the parentheses. It should be noted that results for Mining Finance are not reported here due to the absence of ARCH effects present.

Table 3a. CCC-MGARCH-(1,1)-In-Mean Estimates

CONDITIONAL	*CONDITIONAL MEAN*	
CAPM	α	λ
GOLDS	0.204	-0.012
	(0.232)	(-0.529)
RM	0.279	-
	(0.527)	-
MINES	-0.470	0.014
	(-0.265)	(0.250)
RM	-0.328	-
	(-0.225)	-

In general, the estimates for the conditional CAPM are unappealing. For Gold Mining, Other Mining and Mining Finance, the market price of risk estimates in the conditional mean specification for the three industry sectors, $\hat{\lambda}$, are statistically insignificant. Furthermore, the intercept terms in the conditional mean specification for these industry sectors are insignificant. These findings strongly refute the assumptions of the conditional CAPM in excess return form.

Table 3b. CCC-MGARCH-(1,1)-In-Mean Estimates

CONDITIONAL CAPM	*CONDITIONAL VARIANCE*		
	c	a	b
GOLDS	3.722	0.089	0.890
	(2.492)	(3.450)	(34.032)
RM	0.639	0.026	0.936
	(1.667)	(0.982)	(27.406)
MINES	28.327	0.059	0.380
	2.608	(1.302)	(1.663)
RM	9.259	0.092	0.547
	(1.958)	(2.494)	(2.934)

Empirical moment conditions of the CCC-MGARCH (1,1)-in-mean model are used to provide practical diagnostic checks of the regularity conditions. The second moment condition, namely $a+b<1$, is evaluated at its QMLE, while computation of the log-moment condition, namely $E(\log(a\eta_t^2+b))<0$, requires the QMLE values of the parameters together with the corresponding estimated standardized residuals. All three mining sectors and the market share price index satisfy the second moment condition, and the log-moment condition. The log-moment condition ensures that the QMLE are consistent and asymptotically normal in the presence of infinite second moments**.**

The Mining Finance industry has negative ARCH and GARCH estimates, while the market index counterpart to Mining Finance has a negative ARCH estimate. Apart from these cases, the ARCH and GARCH estimates of the remaining two sectors are characteristic of those estimated from typical financial time-series data.

For Gold Mining and its market index counterpart, the a (or ARCH) estimate is relatively small, while the b (or GARCH) estimate is generally large and close to one. As the long-run persistence (or $a+b$) is generally close to one, it suggests a near long memory process.

As the CCC-MGARCH assumes A_{0t} and B_{0t} are diagonal matrices, it does not contain any information regarding the covariance or interdependent effects between each industry sector and the market portfolio. In addition, the CCC-MGARCH does not capture the asymmetric behaviour of unconditional shocks on the conditional volatility. In order to examine the interdependent effects and to accommodate the asymmetric effects, the Conditional CAPM-GARCH and the Conditional CAPM-AGARCH are also estimated respectively.

Table 4a and 4b reports the quasi-MLE estimates of the Conditional CAPM-GARCH (1,1)-in-mean model. Estimates are reported for two of the mining industry sectors, as there was no convergence in the remaining sector (Other Mining) using the BFGS algorithm. Asymptotic t-ratios for the QMLE are reported in the parentheses.

From Table 4a, it is evident that the estimates for the conditional CAPM mean specification are unappealing, as with the CCC-MGARCH case. Both the market price of risk and the intercept terms are simultaneously insignificant for Gold Mining and Mining Finance. Such findings are in strong contradiction to the conditional CAPM theory in excess returns form.

The Conditional CAPM-GARCH suggests strong interdependent effects between the Mining Finance industry sector and the market index, as seen from Table 4b. The ARCH and GARCH covariance terms between the industry sector and the market index are statistically significant, as expected by the conditional CAPM. The conditional variances of Mining Finance are affected by previous short-run and long-run shocks from the market index, in addition to its own previous short-run and long-run shocks. The Gold Mining industry, however, has no significant interdependent effects with the market index.

Table 5a and 5b presents the quasi-MLE estimates and their asymptotic t-ratios of the Conditional CAPM-AGARCH (1,1)-in-mean model for the conditional CAPM. Estimates are reported for two of the mining industry sectors, as there was no convergence in the remaining sector (Mining Finance) using the BFGS algorithm.

From Table 5a, it is evident that the market price of risk and the intercept terms in the conditional CAPM mean specification are statistically insignificant for Gold Mining and Other Mining. Consistent with the results from the CCC-MGARCH-in-mean and the Conditional CAPM-GARCH-in-mean models, these findings suggest that there is no support for the conditional CAPM.

Table 5b shows that the interdependent effects between Other Mining and the market index are statistically significant. The conditional variances of Other Mining are affected by previous short-run shocks from the market index. The Gold Mining industry, however, has no significant interdependent effects with the market index. The asymmetric effects for both Gold Mining and Other Mining are insignificant.

Table 4a. Conditional CAPM-GARCH(1,1) – Conditional Mean Estimates

CONDITIONAL CAPM	α	λ
GOLDS	-0.024	-0.003
	(-0.025)	(-0.092)
RM	0.047	-
	(0.071)	-
MINES	N.C.	N.C.
	N.C.	N.C.
RM	N.C.	-
	N.C.	-
MIFIN	1.243	-0.038
	(1.178)	(-1.486)
RM	0.971	-
	(1.527)	-

Table 4b. Conditional CAPM-GARCH(1,1) – Conditional Variance Estimates

CONDITIONAL CAPM	C	A	B	$A_{im} = A_{mi}$	$B_{im} = B_{mi}$
GOLDS	3.510	0.082	0.895	0.001	0.015
	(2.452)	(3.315)	(36.564)	(0.149)	(1.827)
RM	2.876	-0.016	0.750	-	-
	(2.631)	(-0.391)	(8.511)	-	-
MINES	N.C.	N.C.	N.C.	N.C.	N.C.
	N.C.	N.C.	N.C.	N.C.	N.C.
RM	N.C.	N.C.	N.C.	N.C.	N.C.
	N.C.	N.C.	N.C.	N.C.	N.C.
MIFIN	92.030	0.012	0.097	0.007	-0.221
	(1730.358)	(27761.451)	(337.408)	(108.248)	(-3409.562)
RM	22.365	-0.026	0.960	-	-
	(3972.100)	(17330.670)	(4200.266)	-	-

Table 5a. Conditional CAPM-AGARCH(1,1) – Conditional Mean Estimates

CONDITIONAL CAPM	α	λ
GOLDS	0.016	-0.001
	(0.017)	(-0.034)
RM	0.022	-
	(0.032)	-
MINES	-2.091	0.071
	(-0.897)	(0.996)
RM	-1.664	-
	(-0.902)	-
MIFIN	N.C.	N.C.
	N.C.	N.C.
RM	N.C.	N.C.
	N.C.	N.C.

Table 5b. Conditional CAPM-AGARCH(1,1) – Conditional Variance Estimates

CONDITIONAL CAPM	C	A	B	$A_{im} = A_{mi}$	$B_{im} = B_{mi}$	D
GOLDS	3.715	0.096	0.890	0.001	0.015	-0.023
	(2.912)	(2.632)	(37.570)	(0.167)	(1.883)	(-0.659)
RM	2.859	-0.021	0.752	-	-	0.008
	(2.663)	(-0.408)	(8.241)	-	-	(0.140)
MINES	73.249	0.001	-0.076	-0.044	-0.694	-0.014
	(5.918)	(0.038)	(-0.391)	(-7.246)	(-4.258)	(-0.435)
RM	68.781	0.157	-0.375	-	-	-0.068
	(11.348)	(3.873)	(-1.461)	-	-	(1.412)
MIFIN	N.C.	N.C.	N.C.	N.C.	N.C.	N.C.
	N.C.	N.C.	N.C.	N.C.	N.C.	N.C.
RM	N.C.	N.C.	N.C.	N.C.	N.C.	N.C.
	N.C.	N.C.	N.C.	N.C.	N.C.	N.C.

5. CONCLUSIONS

This paper models the time-varying conditional mean and variances of excess returns, and the correlations between the individual asset excess return and the market excess return in the conditional CAPM using a variety of multivariate constant conditional correlation GARCH-in-mean models, including the CCC-MGARCH-in-mean, Conditional CAPM-GARCH-in-mean and the Conditional CAPM-AGARCH-in-mean. Our findings suggest that the conditional CAPM is inadequate in explaining the financial risk-return tradeoff for the Mining industry sectors; however, there appears to be some interdependent ARCH/GARCH effects between the Other Mining and Mining Finance excess returns and the market excess return, and no asymmetric ARCH/GARCH effects.

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8. APPENDIX

The conditional CAPM employed in this paper is that of Bollerslev, Engle and Wooldridge (1988), who used a conditional CAPM specification with a constant market price of risk:

(1) $E(R_{it}/Z_{t-1}) - r_{ft} = \lambda \operatorname{cov}(R_i, R_{mt}/Z_{t-1})$,

where $\lambda = \lfloor E(R_{mt}/Z_{t-1}) - r_{ft} \rfloor / \operatorname{var}(R_{mt}/Z_{t-1})$.

The assumption of a constant market price of risk implies that the conditional expected market returns $E(R_{mt}/Z_{t-1})$ is a linear function of the variance of the conditional market returns. Thus, by rearranging $\lambda = \lfloor E(R_{mt}/Z_{t-1}) - r_{ft} \rfloor / \operatorname{var}(R_{mt}/Z_{t-1})$, the model for time-varying conditional expected market returns can then be written as:

(2) $$E(R_{mt}/Z_{t-1}) - r_{ft} = \lambda \operatorname{var}(R_{mt}/Z_{t-1}) .$$

To convert the above *ex ante* conditional CAPM specification into *ex post* form, we use the fact that the return on any asset can be decomposed into an expected return and an unexpected return component. Thus, we can write $\varepsilon_{it} = R_{it} - E(R_{it}/Z_{t-1})$ for an individual asset i. Similarly, we can write $\varepsilon_{mt} = R_{mt} - E(R_{mt}/Z_{t-1})$ for a market portfolio of assets m. As a result,

(3) $$R_{it} = r_{ft} + \lambda \operatorname{cov}(R_{it}, R_{mt}/Z_{t-1}) + \varepsilon_{it}$$ and

(4) $$R_{mt} = r_{ft} + \lambda \operatorname{var}(R_{mt}/Z_{t-1}) + \varepsilon_{mt} .$$

The expressions $\varepsilon_{it} = R_{it} - E(R_{it}/Z_{t-1})$ and $v_{mt} = R_{mt} - E(R_{mt}/Z_{t-1})$ also imply that the conditional second moments are themselves equal to the forecast error variances and covariances. Thus, we can rewrite the conditional covariance between the return on asset i and the market return ($\operatorname{cov}(R_{it}, R_{mt}/Z_{t-1})$), along with the conditional variance of the market return ($\operatorname{var}(R_{mt}/Z_{t-1})$), respectively as:

(5) $$\operatorname{cov}(R_{it}, R_{mt}/Z_{t-1}) = E(\varepsilon_{it}\varepsilon_{mt}/Z_{t-1})$$ and

(6) $$\operatorname{var}(R_{mt}/Z_{t-1}) = E(\varepsilon_{mt}^2/Z_{t-1}) .$$

By incorporating Equations (5) and (6) into the *ex post* conditional CAPM specifications, we arrive at:

(7) $$R_{it} = r_{ft} + \lambda \operatorname{cov}(\varepsilon_{it}, \varepsilon_{mt}/Z_{t-1}) + \varepsilon_{it}$$ and

(8) $$R_{mt} = r_{ft} + \lambda \operatorname{var}(\varepsilon_{mt}^2/Z_{t-1}) + \varepsilon_{mt} ,$$

where we denote r_{ft} is the intercept term, α.