

## **New binary morphological operations for effective low-cost boundary detection**

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In this paper, new operational definitions of binary morphological, both conditional and non-conditional, operations are proposed. The new operations are applied to detect boundary points from binary images. Comparisons of boundary detection algorithms using proposed, standard morphological, and gradient-based operations, showing the effectiveness of the proposed operations, are given. Comparative hardware implementations of standard and proposed morphological operations are also given. Main distinguishing aspects of the new operations are: high efficiency and low hardware implementation (i.e., low number of buffers and D-Flip-Flops).

*Keywords:* Image segmentation; boundary detection; morphological operations; dilation; erosion; conditional operation; low-complexity operations.

### **1. Introduction**

Mathematical morphology is a well-founded non-linear theory of image processing<sup>1,2,3</sup>. Its geometry-oriented nature provides an efficient framework for analysing object shape characteristics such as size and connectivity, which are not easily accessed by linear approaches. Morphological operations take into consideration the geometrical shape of the image objects to be analysed. The initial form of mathematical morphology is applied to binary images and usually referred to as standard mathematical morphology in the literature in order to be discriminated by its later extensions such as the gray-scale and the soft mathematical morphology. Mathematical morphology is theoretically founded on set theory. It contributes a wide range of operators to image processing, based on a few simple mathematical concepts. The operators are particularly useful for the analysis of binary images, boundary detection, noise removal, image enhancement, and image segmentation. The advantages of morphological approaches over linear approaches are 1) direct geometric interpretation, 2) simplicity, and 3) efficiency in hardware implementation.

An image can be represented by a set of pixels. A morphological operation uses two sets of pixels, i.e., two images: the original data image to be analysed and a structuring element (also called kernel) which is a set of pixels constituting a specific

shape such as a line, a disk, or a square. A structuring element is characterised by a well-defined shape (such as line, segment, or ball), size, and origin. Its shape can be regarded as a parameter to a morphological operation.

Basic operation of a morphology-based approach is the translation of a structuring element over the image and the erosion and/or dilation of the image content based on the shape of the structuring element. A morphological operation analyse and manipulate the structure of an image by marking the locations where the structuring element fits. In mathematical morphology, neighbourhoods are, therefore, defined by the structuring element, i.e., the shape of the structuring element determines the shape of the neighbourhood in the image.

The hardware complexity of implementing morphological operations depends on the size of the structuring elements. The complexity increases even exponentially in some cases. Known hardware implementations of morphological operations are capable of processing structuring elements only up to  $3 \times 3$  pixels<sup>4</sup>. If higher-order structuring elements are needed, they are decomposed into smaller elements. One decomposition strategy is, for example, to present the structuring element as successive dilation of smaller structuring elements. This is known as the “chain rule for dilation”<sup>3</sup>. Note that not all structuring elements can be decomposed.

In this paper, new operational definitions of binary morphological erosion and dilation are proposed. The goal is to provide boundary detectors based on shape preserving, low implementation costs, and fast morphological operations. Comparative implementation issues of the proposed and standard morphological operations are studied showing that with the new operations less memory and implementation costs (e.g., numbers of D-Flip-Flops) can be achieved.

This paper is organised into five additional sections. Section 2 summarises standard morphological operations, Section 3 motivates the introduction of the proposed operations, Section 4 introduces the new operations where in Section 4.4 conditional operations are proposed, Section 5 presents and discusses experimental results, and Section 6 concludes this paper.

## 2. Standard binary morphological operations

### 2.1. *Dilation and erosion*

The basic morphological operations are dilation and erosion (cf. Fig. 1). They are expressed by a kernel operating on an input binary image,  $B$ , where white pixels denote uniform regions and black pixels denote region boundaries. Erosion and dilation work conceptually by translating a structuring element,  $K$ , over the image points and examining the intersection between the translated kernel coordinates and the image coordinates. When specific conditions are met the image content is manipulated using the following rules (for set-theoretical definitions see<sup>3,5</sup>):

- **Standard dilation:** Move a kernel  $K$  line-wise over the binary image  $B$ . If the origin of  $K$  intersects a white pixel in  $B$ , then set all pixels covered by  $K$  in  $B$  to white if the respective pixel in  $K$  is set white.

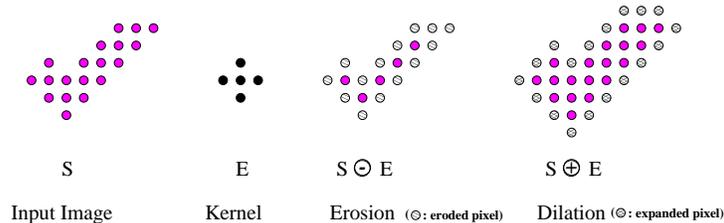


Fig. 1. Dilation and erosion (note that they are applied here to the black pixels).

- **Standard erosion:** Move a kernel  $K$  line-wise over the binary image  $B$ . If the origin of  $K$  intersects a white pixel in  $B$  and if *all* pixels of  $K$  intersect white pixels in  $B$  (i.e.,  $K$  fits), then keep the pixel of  $B$  that intersect the origin of  $K$  white. Otherwise set that pixel to black.

The dilation is an expansion operator that enlarges binary objects. The erosion is a thinning operator that shrinks objects. By applying erosion to an image, narrow regions can be eliminated while wider ones are thinned. In order to restore regions after erosion, dilation can be applied using a mask of the same size.

## 2.2. Dilation and erosion-based operations

Erosion and dilation can be combined to solve specific filtering tasks. Widely used combinations are opening, closing, and boundary detection. Opening (erosion followed by dilation) filters details and simplifies images by rounding corners from inside the object where the kernel used fits. Closing (dilation followed by erosion) protects coarse structures, closes small gaps, and rounds concave corners. Morphological operations are very effective for detection of boundaries in a binary image  $B$ <sup>5,3</sup>. The following boundary detectors are widely used:

$$\begin{aligned}
 E &= B - \mathcal{E}[B, K_{(m \times m)}], \\
 E &= \mathcal{D}[B, K_{(m \times m)}] - B, \quad \text{or} \\
 E &= \mathcal{D}[B, K_{(m \times m)}] - \mathcal{E}[B, K_{(m \times m)}].
 \end{aligned} \tag{1}$$

$E$  is the boundary image.  $\mathcal{E}$  ( $\mathcal{D}$ ) is the erosion (dilation) operator (erosion is often represented by  $\ominus$  and dilation by  $\oplus$ ).  $K_{m \times m}$  is the erosion (dilation)  $m \times m$  kernel used.  $-$  denotes the set-theoretical subtraction.

## 3. Motivation for new operations

### 3.1. Motivation for new erosion and dilation

Standard morphological erosion and dilation are defined around an origin of a structuring element. The position of this origin is crucial for the detection of boundary points in the image. For each step of an erosion or dilation, one pixel is set (at a time) in  $B$ . To achieve precise boundaries with single-pixel width,  $3 \times 3$  kernels (defined around the origin) are used (kernel examples are in Fig. 2): when a  $3 \times 3$  cross

kernel is used, an incomplete corner detection is obtained (as shown in Fig. 3); a  $3 \times 3$  *square* kernel gives complete boundary points but requires more computation (which grows rapidly with increased input data, Fig. 6(a)); and the use of a  $2 \times 2$  square kernel will produce incomplete boundaries (see Fig. 4).



Fig. 2. A  $3 \times 3$  *square*, a  $3 \times 3$  *cross*, and a  $2 \times 2$  square kernel.

To avoid these drawbacks, new operational rules for boundary detection by erosion or dilation are proposed in the following sections. A fixed-size ( $2 \times 2$  square) kernel is used and the rules set all four pixels of this kernel at a time in  $B$ . For boundary detection based on the new rules, accurate complete boundaries are achieved and the computational cost is significantly reduced.

### 3.2. *Motivation for conditional operations*

When extracting binary images from gray-level ones, the binary images are often enhanced by applying morphological operations<sup>2</sup>. Applying standard morphological operations for enhancement, however, can connect some object areas or erode some important information. This paper contributes, in Sec. 4.4, definitions to conditional morphological operations to solve this problem.

## 4. Proposed morphological operations

### 4.1. *Proposed erosion*

#### 4.1.1. *Definition*

Move the fixed-size  $2 \times 2$  square kernel line-wise over the binary image  $B$ . If at least one of the four pixels inside the kernel is black, then set all the four pixels in the output image  $E$  to black. If all four pixels inside the  $2 \times 2$  kernel are white, then set all (at a time) four pixels in  $E$  to white if they were not eroded previously.

#### 4.1.2. *Set-theoretical formulation*

An advantage of the proposed erosion is that it can be formally defined based on set-theoretical intersection, union, and translation in analogy to the formal definitions of the standard erosion<sup>5</sup>. The standard erosion satisfies the following property<sup>5</sup>: the erosion of an image by the union of kernels is equivalent to erosion by each kernel independently and then intersecting the result (see Eq. 2). So given image  $A$  and kernels  $B$  and  $C$  in  $\mathbb{R}^2$ ,

$$\mathcal{E}_s[A, B \cup C] = \mathcal{E}_s[A, B] \cap \mathcal{E}_s[A, C] \quad (2)$$

where  $\mathcal{E}_s$  denotes the standard erosion. The proposed erosion can be expressed as

$$\begin{aligned} \mathcal{E}_p[A, K_{2 \times 2}] &= \mathcal{E}_s[A, S_{3 \times 3}] = \\ \mathcal{E}_s[A, K_{2 \times 2}^{ul} \cup K_{2 \times 2}^{ur} \cup K_{2 \times 2}^{ll} \cup K_{2 \times 2}^{lr}] &= \\ \mathcal{E}_s[A, K_{2 \times 2}^{ul}] \cap \mathcal{E}_s[A, K_{2 \times 2}^{ur}] \cap \mathcal{E}_s[A, K_{2 \times 2}^{ll}] \cap \mathcal{E}_s[A, K_{2 \times 2}^{lr}] \end{aligned} \quad (3)$$

where  $\mathcal{E}_p$  denotes the proposed erosion,  $S_{3 \times 3}$  is a  $3 \times 3$  square kernel, and  $K_{2 \times 2}^{ul}$  is a  $2 \times 2$  kernel with origin at the upper left (equivalently upper right, lower left, lower right) corner (cf. Fig. 2). Thus the proposed erosion gives the same results as the standard erosion when using a  $3 \times 3$  square kernel. However, the proposed erosion is significantly faster. Using a  $3 \times 3$  cross kernel with the standard erosion accelerates processing but gives incomplete results, especially at corners (see Fig. 3).

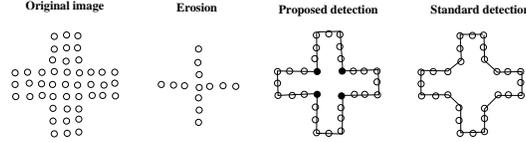


Fig. 3. Proposed versus standard erosion (standard erosion uses a  $3 \times 3$  cross kernel).

## 4.2. Proposed dilation

### 4.2.1. Definition

Move the  $2 \times 2$  kernel line-wise over the binary image  $B$ . If at least one of the four binary-image pixels inside the kernel is white, then set all (at a time) the four pixels in the output image  $E$  to white.

### 4.2.2. Set-theoretical formulation

The standard dilation satisfies the following property<sup>5</sup>: one dilation by the union of kernels corresponds to dilation by each kernel and then the union of the resulting images (see Eq. 4). This means that given sed in the following sections. A fixed-size ( $2 \times 2$  square) kernel is used and the rules set all four pixels of this kernel at a time in  $B$ . For boundary detection based on the new rules, accurate complete boundaries are achieved and the computational cost is significantly reduced.

image  $A$  and kernels  $B$  and  $C$  in  $\mathbb{R}^2$ ,

$$\mathcal{D}_s[A, B \cup C] = \mathcal{D}_s[A, B] \cup \mathcal{D}_s[A, C] \quad (4)$$

where  $\mathcal{D}_s$  denotes the standard dilation. The proposed dilation is then given by:

$$\begin{aligned} \mathcal{D}_p[A, K_{2 \times 2}] &= \mathcal{D}_s[A, S_{3 \times 3}] = \\ \mathcal{D}_s[A, K_{2 \times 2}^{ul} \cup K_{2 \times 2}^{ur} \cup K_{2 \times 2}^{ll} \cup K_{2 \times 2}^{lr}] &= \\ \mathcal{D}_s[A, K_{2 \times 2}^{ul}] \cup \mathcal{D}_s[A, K_{2 \times 2}^{ur}] \cup \mathcal{D}_s[A, K_{2 \times 2}^{ll}] \cup \mathcal{D}_s[A, K_{2 \times 2}^{lr}] \end{aligned} \quad (5)$$

where  $\mathcal{D}_p$  denotes the new dilation.

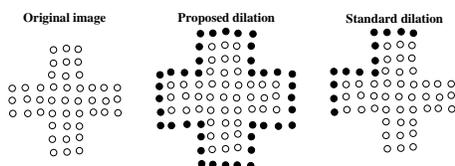


Fig. 4. Proposed versus standard (using a  $2 \times 2$  kernel with origin at the left upper pixel) dilation.

### 4.3. Proposed boundary detection

In this section, the need to explicitly use erosion or dilation (as defined by Eq. 1) for morphological boundary detection is questioned. Here, we propose to detect boundaries by implicitly using erosion or dilation. Such an implicit detection aims at reducing the complexity of morphological boundary detection.

#### 4.3.1. Definition

Move the  $2 \times 2$  kernel over the binary image  $B$ . If at least one of the four pixels of  $2 \times 2$  kernel is black, then set the four pixels of the same positions in the output boundary image  $E$  to white if their equivalent pixels in  $B$  are white. Otherwise set the pixels to black.

If the  $2 \times 2$  kernel fits in a white area of the image, all four pixels of that area are implicitly eroded, but boundary points (where the kernel does not fit) are kept. Fig. 6(b) gives a complexity comparison of the new binary boundary detection, boundary detection with the proposed erosion and boundary detection using standard erosion (with a  $3 \times 3$  square kernel). As shown, the cost of boundary detection using the proposed implicit boundary detection is significantly reduced.

### 4.4. Proposed conditional morphological operations

Image segmentation methods use, in general, a post-processing step to simplify segmented objects. The most popular post-processing filters are the median and morphological filters such as opening and closing. This is because of their efficiency. The difficulty with these, however, is that they may connect object regions that

do not belong together or disconnect regions that are part of the same object. To support morphological filters, this paper suggests conditional dilation and erosion for the purpose of object segmentation. They are topology preserving filters in the sense that they are applied if specific conditions are met as defined below.

#### 4.4.1. Conditional erosion

Using conditional erosion, a white pixel is eroded only if it has at least three black neighbours. This ensures that object regions are not connected. It is performed mainly at object boundaries. The basic idea is that if the majority of the  $2 \times 2$  quadrant kernel points are black then this is most likely a border point and can be eroded. This is useful when holes inside the object had to be kept.

#### 4.4.2. Conditional dilation

With conditional dilation, a black pixel is set to white if the majority of the  $2 \times 2$  kernel pixels are white. If this condition is met then it is more likely that this pixel is inside an object and not a border pixel. Conditional dilation sets pixels mainly inside the object and stops at object boundaries to avoid connection of neighbouring objects. This condition ensures that objects are not connected in the horizontal and vertical directions. In some rare cases, however, object regions may be connected *diagonally* as shown in Fig. 5. In this figure some  $\circ$  pixels become connected and so the two object regions.

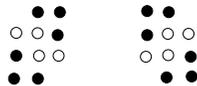


Fig. 5. Cases where object regions are diagonally connected using proposed conditional dilation.

## 5. Comparison and discussion

### 5.1. Non-conditional operations

Simulations using real images have shown the proposed boundary detectors are significantly faster, significantly reduced complexity (cf. Fig. 6), and give more accurate (cf. Figs. 7-9) result than the standard morphological boundary detectors. This is confirmed using different natural image data.

Fig. 6(a) shows that the computational cost using the standard erosion with a  $3 \times 3$  square kernel grows rapidly with the amount of input data, while the cost of the proposed erosion stays almost constant. Computations can be further reduced by applying the new morphological boundary detection with implicit erosion (Fig. 6(b)).

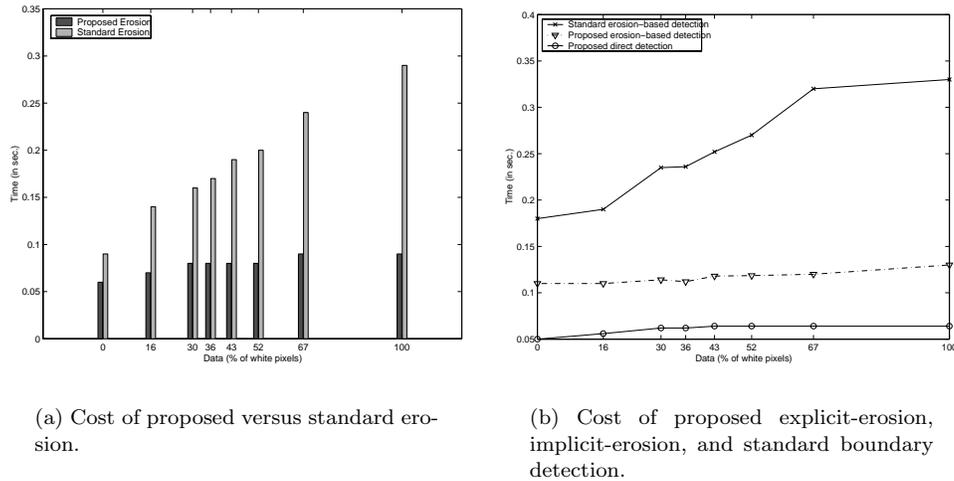


Fig. 6. Computational efficiency comparison.

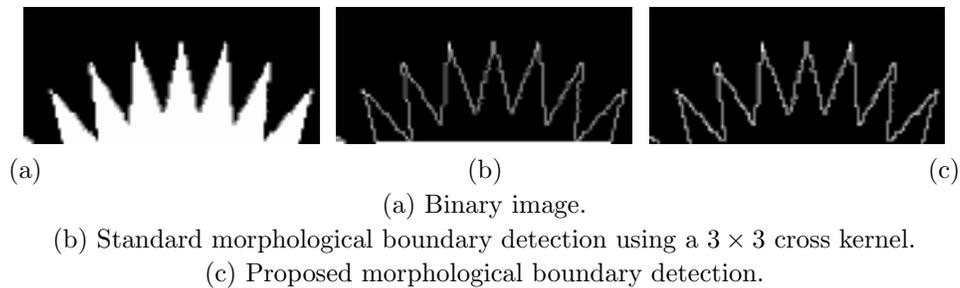
Fig. 7. Comparison: proposed boundary detection is preserve better details that the standard (with a  $3 \times 3$  cross) morphological boundary detection. In addition, the proposed detection is significantly faster.

Fig. 7, Fig. 8, and Fig. 9 show comparisons of proposed and standard boundary detection. As can be seen, proposed detection preserves the shape of the objects better than standard erosion-based detection as given in Eq. 1.

The proposed boundary detectors have been compared to gradient-based methods such as the Canny method<sup>6</sup>. Canny boundary detector is powerful method that is widely used in various imaging systems. The difficulty of using this method is that its parameters need to be tuned for different applications and images. Compared to the Canny-boundary detector, the proposed methods show higher detection accuracy resulting in better shapes (as shown in Fig. 8(d)). A better shape accuracy using the Canny method can be achieved when its parameters are tuned accordingly. This is, however, not appropriate for automated video and image processing. This

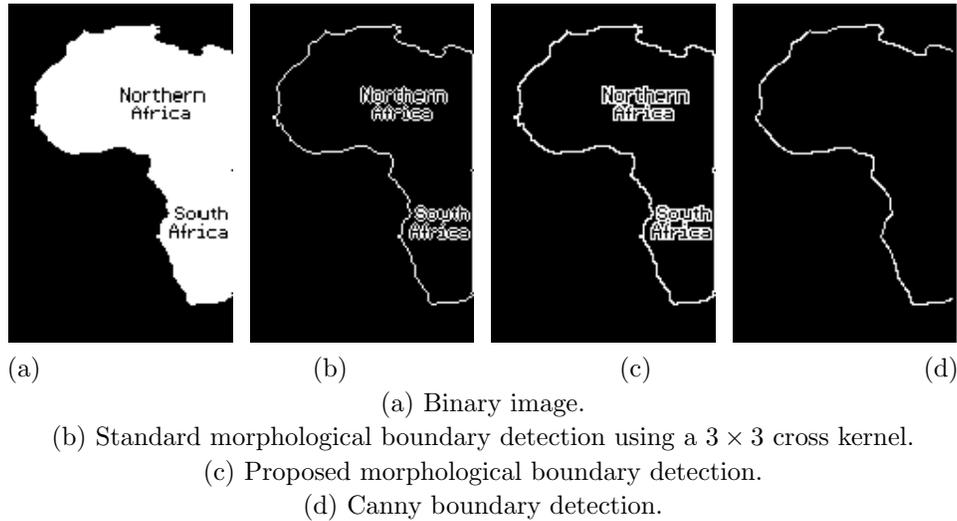


Fig. 8. Boundary detection comparison. Note the shape distortion when using Canny detector. The proposed detection gives more accurate results than the standard morphological detector using a  $3 \times 3$  cross kernel.

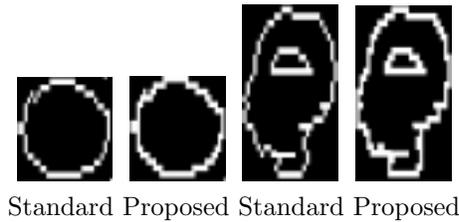
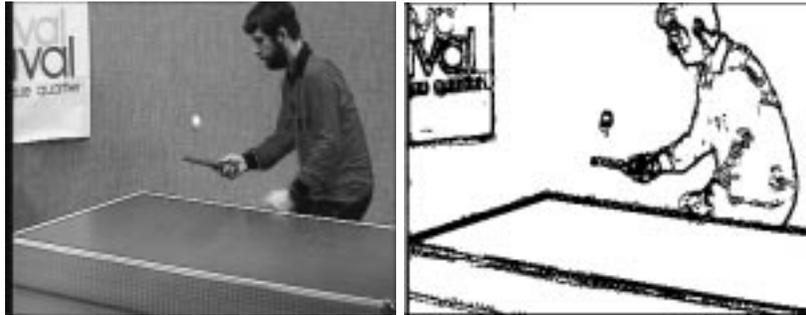


Fig. 9. Comparison: proposed morphological boundary detection preserve the details of the object better than standard erosion-based detection (see Eq. 1).

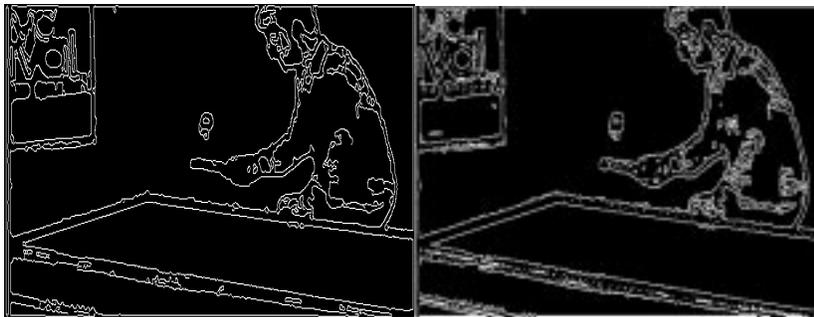
is mainly because the Canny detector uses a smoothing filter. In addition, the proposed boundary detectors have lower complexity and produce gap-free boundaries so that no boundary points linking is necessary.

Fig. 10(c) shows that boundary detection using Canny detector gives boundaries with gaps thus postprocessing and boundary point linking is necessary. Such postprocessing would increase the computational cost and does not guaranty that the correct points will be linked together. Using the proposed detector higher detection accuracy (e.g., gap free boundaries as in Fig. 10(d)) and significantly lower computations are achieved.



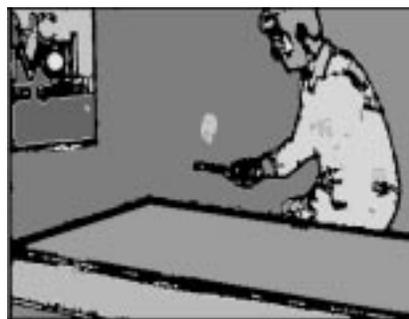
(a) Original image.

(b) Binary image.



(c) Canny boundary detection: boundaries with gaps, point linking is necessary.

(d) Morphological boundary detection: one-pixel wide and gap-free boundaries.



(e) Detected object based on the proposed morph. boundary detection.

Fig. 10. Boundary detection comparison. Note in Fig. 10(e) the detected single closed contours for the outline of the object regions (e.g., person, table).

## 5.2. Conditional morphological operations

A binary image resulting from a binarization of a gray-level image may contain artifacts, particularly at object boundaries<sup>7,8,9</sup>. Most image segmentation techniques that use binarization have a post-processing step, usually performed by non-linear filters, such as median or morphological opening and closing. Non-linear filters are effective and efficient and, therefore, widely used<sup>2</sup>.

This Section examines the usefulness of applying a post-processing step using standard and proposed conditional morphological operations to the binary image. To this end, erosion, dilation, closing, opening, and a  $3 \times 3$  median operation were applied to binary images and results were compared. The temporal stability of these filters throughout an image sequence has been also evaluated. Based on our experiments, we draw the following conclusion:

- Erosion removes some important details and dilation may connect objects.
- Standard opening with a  $3 \times 3$  cross kernel smoothes the image but some significant object details may be removed and objects may get disconnected.
- Standard closing performs better smoothing but may connect objects.
- Conditional closing (see Sec. 4.4) is significantly faster than standard closing and is more conservative in smoothing results. It may, however, connect objects diagonally as illustrated in Fig. 5.

To compensate for the above mentioned disadvantages, two post-processing solutions were further examined:

- Conditional erosion followed by conditional closing.
- Erosion, a  $3 \times 3$  median filter, and a conditional dilation.

Conditional erosion before closing does not connect objects but filters many details and can change the shape of objects. Erosion, median, and conditional dilation perform better by preserving edges and corners.

Our conclusion for morphological operations as a postprocessing step is as follows: applying smoothing filters can introduce artifacts, remove significant object parts, or disconnect object parts. This complicates subsequent object-based video and image processing such as object tracking, object-based motion estimation, and object-based image retrieval. These effects are more severe when objects are small or when their parts are thin compared to the used morphological or median masks. Use of the above operations is recommended when objects and their connected parts are large. Such information is, however, rarely a priori known. Therefore, an explicit post-processing step should be applied carefully. Often it is safer to avoid a post-processing step and to enhance object segmentation at higher levels of processing where more information is available<sup>10</sup>.

## 6. Conclusion

In this paper, new morphological operations are proposed showing significantly reduced computations and higher or equal performance compared to standard mor-

phological operations. Boundary detection is performed based on implicit morphological erosion with a significantly reduced number of computations. The advantage of morphological detection is that it produces gap-free and single-pixel-wide boundaries without need for post-processing. Both objective and subjective evaluation and comparisons show the reliability of the proposed operations also in noisy images while being of reduced complexity.

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