# Time and Range Averaging of Lidar Echoes Using APD Based Receivers

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## ABSTRACT

Lidar receivers perform time and/or space averaging to decrease the variance of the optical power estimates. In this paper we study an Avalanche PhotoDiode (APD) based receiver. The number samples to reach a given minimum variance depends on the receiver transfer function. Herein, we review the linear receiver and derive the number of samples for the logarithmic pre-amplifier. Comparing the two receivers, we show that the signal variance for the logarithmic case is degraded by a factor that vanishes as the receiver aperture increases. These results can be readily applied to the problem of estimating log-power returns in the context of Differential Absorption lidar (DIAL) systems. As an application example, we study two different log-power estimators and compare their performance.

**Keywords:** lidar, noise, statistical characterization, averaging, nonlinear pre-amplifier.

## 1. INTRODUCTION

The goal of lidar inversion is to retrieve the extinction and the backscatter coefficients from the return optical power.[2] This procedure is hampered by noise of different sources such as speckle noise, signal induced and dark current shot noise, and electronic thermal noise.[6] Time and/or space averaging is an ever present procedure to decrease the variance of power estimates to an acceptable level. The idea underlying time and/or space averaging is that the random media being sampled is stationary and homogeneous. This assumptions are meaningful only for a limited time interval and space volume. Therefore, it is very important to select the minimum number of space and/or time samples leading to the maximum acceptable variance.

The statistics of linear APD based receivers have already been studied (see e.g.,[6 10]). However, to our knowledge, the logarithmic pre-amplifier case has not yet been addressed.

We begin by briefly reviewing the linear scenario, and then we consider the logarithmic receiver. The logarithmic nonlinearity modifies the APD output Poisson statistics in a non-trivial way. However, depending on the size of the receiver aperture compared to the correlation length of the backscattered signal, we take one of the following approaches:

(a) Small aperture: for powers where, typically, the logarithmic amplifier operates, the APD output shot noise is small compared to the speckle noise. Therefore, we can neglect the shot noise term and compute the mean and variance of the pre-amplifier output by using the gamma statistics of the input signal[4];

(b) Large aperture: as the speckle count $m$ increases, the signal variance due to speckle decreases and may become of the order of the signal induced shot noise. On the other hand, for high speckle count ($m > 10$) the gamma distribution can be approximated with a Gaussian one. Furthermore, if the intensity of the Poisson process is high, i.e., the signal strength is more than 50 photons over the APD receiver integration time (inverse of the thermal noise equivalent bandwidth), the process at the pre-amplifier input can also be assumed Gaussian.[5]

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By taking advantage of these approximations we derive, in a manageable way, the second order statistical characterization of the output signal. Based on these statistics, we propose a mean optical power estimator and compute its bias and variance. Furthermore, we present results for the so-called number of independent samples, for a given signal-to-noise ratio (SNR) of the input power estimator.

These results can be applied to the problem of estimating the logarithm of the backscattered optical power signal in DIAL systems. The properties of log-power estimators in the presence of speckle and electronic thermal noises were addressed by Rye,[3] in the context of direct detection systems. However, the effect of shot noise was neglected and the moments of the estimators were computed in a non-closed form. The statistical characterization herein derived of the logarithmic receiver takes into account shot noise and leads to closed forms for the mean and variance of the proposed log-power estimators.

The layout of this paper is as follows: In Section 2 we review the estimation problem and the second order statistical charaterization of the APD output; Section 3 describes the linear receiver in the presence of speckle and shot noises. Next, in Section 4 we address the logarithmic pre-amplifier, breaking the problem in two different limits, depending on the size of the receiver aperture; and in Section 5 we examine the behaviour of two log-power estimators as a small application example in the context of DIAL systems.

## 2. BACKGROUND

Fig.1 schematizes a lidar based remote sensing scenario. The transmitter illuminates a random media with a train of light pulses. The receiver reads the backscattered light and converts it into an electrical signal. The power estimator should then infer characteristics of the random media. Let $x(t)$ be the optical signal scattered by the random media

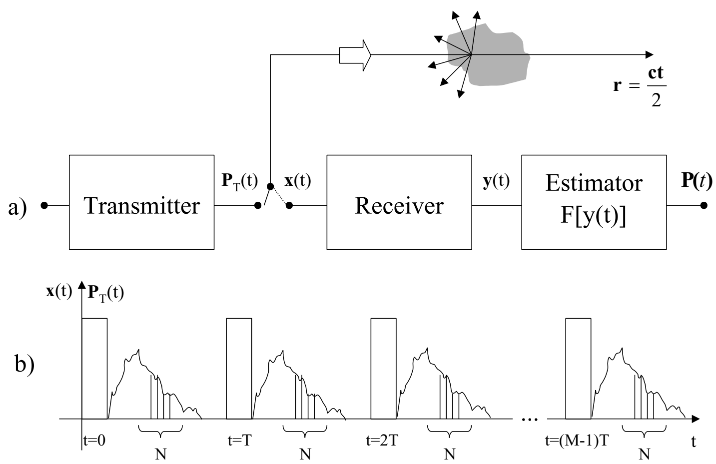


**Figure 1.** Lidar remote sensing scenario.

due to a train of light pulses. As a light pulse propagates through the line of sight, a number $N$ of samples are taken within a small range interval (space sampling). This sampling is then repeated for a number $M$ of consecutive pulses (time sampling). The numbers $N$ and $M$ are limited by the above mentioned homogenity and stationarity assumptions, respectively. According to this procedure, we define the set of sampling instants as

$$\mathcal{T} \equiv \{t \in \mathbb{R} : t = t_0 + i\Delta t + kT, \\ i = 0, \ldots, N; k = 0, \ldots, M\}. \tag{1}$$

where $t_0 \equiv 2r_0/c$, $\Delta t$ is the time interval between consecutive range samples, and $T$ is the time interval between consecutive light pulses. We shall denote the total number of samples by $N_{\mathcal{T}}$.

Fig.2 schematizes a receiver consisting of an APD, a pre-amplifier, an A/D converter, and an estimator of the input power. We denote $L[y(t)]$ as the pre-amplifier gain function. Two cases are studied. Firstly, we briefly review the linear pre-amplifier based receiver and then we analyze the logarithmic case.

The statistics of the input power signal can be understood if we regard the receiving aperture as consisting of $m$ independent spatial correlation cells. Within each cell the energy density is approximately constant and statistically independent of the energy density of the other cells. Since the incident energy density on each cell has a negative exponential distribution, it follows that the total power signal is approximately gamma distributed.[4] We represent the total power signal and its expected value as $P(t)$ and $P_s(t)$, respectively. The expected power signal is the entity to be estimated.

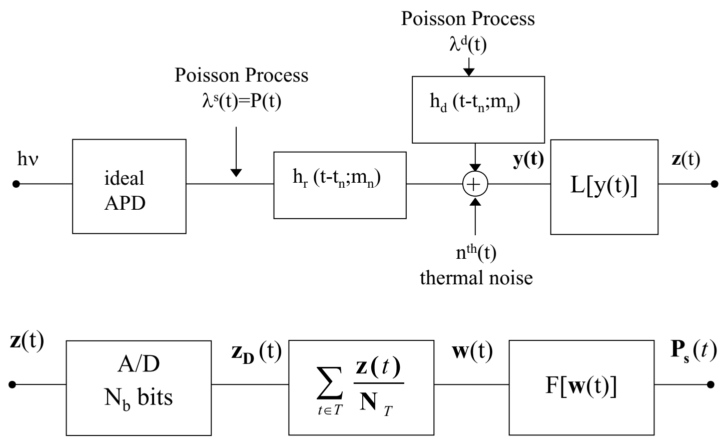


**Figure 2.** Receiver and power estimator.

Assuming that the quantization interval is *small* and that the number of samples is greater than 10, then $w(t)$ is with good approximation Gaussian distributed, i.e., $p_w(w) = \mathcal{N}(\eta_w, \sigma_w^2)$,[1],[7] where

$$\eta_W \equiv E\{w(t)\} = \frac{1}{N_{\mathcal{T}}} \sum_{t \in \mathcal{T}} E\{z(t)\} \tag{2}$$

$$\sigma_W^2 \equiv \mathrm{var}\{w(t)\} = E\left\{\frac{1}{N_{\mathcal{T}}} \sum_{t \in \mathcal{T}} [z(t) - E\{z(t)\}]\right\}^2 . \tag{3}$$

The equivalent number of independent samples is defined as $N_{eq} \equiv \mathrm{var}\{z(t)\}/\mathrm{var}\{w(t)\}$. This number is equal to the number of samples $N_{\mathcal{T}}$, provided that the samples, taken at instants $t \in \mathcal{T}$, are independent. We shall assume that the sampling rate and the pulse repetition rate are such that these samples are in fact independent.

The statistical characterization of the mean optical power estimator $\widehat{P_s}$ depends on the function $F[w(t)]$, which in turn is related to the receiver output statistics. For a given value of $P$, the APD output signal is a filtered marked point Poisson process,[8] as illustrated in Fig.2, where $h_r(t - t_n; m_n)$ and $h_d(t - t_n; m_n)$ represent the APD impulse response to the signal and dark current, respectively. The conditional mean and variance of $y$ are

$$E\{y \mid P\} = \mathcal{R}_v(P + P_d) \tag{4}$$

$$\mathrm{var}\{y \mid P\} = F\mathcal{R}_v^2 \frac{Bq}{\mathcal{R}_i}(P + P_d) + 4kBTR, \tag{5}$$

**Table 1.** Simulation parameters.

| Parameter | Value |
|---|---|
| APD quantum efficiency | 50% |
| APD multiplication gain $M$ | 100 |
| APD excess noise factor $F$ | 4 |
| APD dark-current NEP $P_d$ | 60pW |
| Pre-amplifier equivalent bandwidth $B$ | 40 MHz |

where $\mathcal{R}_v = RM\mathcal{R}_i$ is the APD voltage responsivity, $\mathcal{R}_i = \left(\frac{q\eta\lambda}{hc}\right)$ is the APD current responsivity without multiplication for the laser wavelength $\lambda$, $M$ is the APD avalanche gain, $F = M^x$ stands for the excess noise factor, $R$ is the receiver's equivalent resistance, $P_d$ is the dark current noise equivalent power, $T$ is the detector equivalent temperature, and $B$ is the receiver signal bandwidth. The symbols $q$, $h$, $k$ and $c$ represent the electron charge, Planck's constant, Boltzmann's constant and speed of light, respectively. In order to have a more compact notation, the explicit dependence on $t$ was dropped. Note that (5) does not contain the speckle noise contribution since it is conditioned to $P$.

## 3. LINEAR PRE-AMPLIFIER

For this type of pre-amplifier, the output voltage signal is $z = ay$ and, consequently, $E\{z\} = aE\{y\}$. Taking into account that $P$ is gamma distributed, i.e.,

$$p_P(P) = \left(\frac{m}{P_s}\right)^m \frac{(P)^{m-1}}{\Gamma(m)} \exp\left(-\frac{mP}{P_s}\right), \tag{6}$$

where $\Gamma(x)$ is the gamma function, and departing from the conditional mean and variance of $y$, (4) and (5), we have

$$\eta_y \equiv E\{E\{y \mid P\}\} = \mathcal{R}_v(P_s + P_d), \tag{7}$$

$$\sigma_y^2 \equiv E\{\text{var}\{y \mid P\}\} + \text{var}\{E\{y \mid P\}\} = \mathcal{R}_v^2\left[\frac{P_s^2}{m} + F\frac{Bq}{\mathcal{R}_i}(P_s + P_d)\right]. \tag{8}$$

From (2) and (7) we find the dependence of the mean $\eta_w$ on $P_s$ which suggests the power estimator

$$\widehat{P_s} = F(w) = \frac{w}{a\mathcal{R}_v} - P_d. \tag{9}$$

The mean and variance of $\widehat{P_s}$ are

$$E\{\widehat{P_s}\} = P_s \tag{10}$$

$$\text{var}\{\widehat{P_s}\} = \frac{1}{N_{\mathcal{T}}}\left[\frac{P_s^2}{m} + F\frac{Bq}{\mathcal{R}_i}(P_s + P_d)\right], \tag{11}$$

where the electronic thermal noise was neglected since we assume the avalanche gain is high.

Define SNR for the estimator as $(SNR)_{\widehat{P_s}} \equiv \frac{E^2\{\widehat{P_s}\}}{\text{var}\{\widehat{P_s}\}}$. From (10) and (11) the number of samples that satisfies a given SNR is

$$N_{\mathcal{T}} = (SNR)_{\widehat{P_s}}\left[\frac{1}{m} + F\frac{Bq}{\mathcal{R}_i}\left(\frac{1}{P_s} + \frac{P_d}{P_s^2}\right)\right]. \tag{12}$$

Fig.3 shows the behaviour of the number of samples $N_{\mathcal{T}}$ for $(SNR)_{\widehat{P_s}} = 10$ and for $(SNR)_{\widehat{P_s}} = 100$, using the system parameters listed in Table 1 and a speckle count $m = 1$. Note that as the power increases all the terms of (12) vanish except for the speckle noise one.

## 4. LOGARITHMIC PRE-AMPLIFIER

The output of the logarithmic pre-amplifier is $z = z_0 \log(ay + b)$. It is shown in[1] that if $\eta_y \gtrsim 10b/a$, then $b$ can be dropped for any practical purposes. We assume that the receiver operates under this condition.

As we have already mentioned, we consider two approximations: (a) small receiver aperture; (b) large receiver aperture.

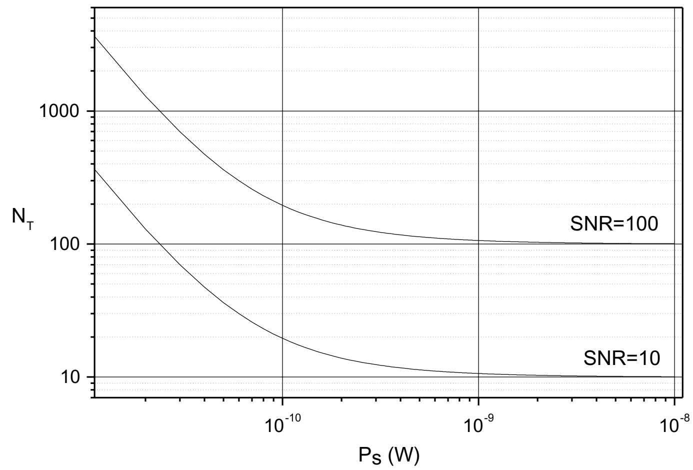


**Figure 3.** Number of independent samples for the linear pre-amplifier receiver.

## 4.1. Small Aperture

Assume that the input signal is sufficiently high (e.g., above 0.5 nW for m=1 and the system parameters listed in Table 1, which is in agreement with the typical log operation conditions) such that speckle noise is much higher than shot noise. We can, therefore, neglect the second term on the right hand side of (8). In this case, the pre-amplifier output can be approximated by $z = z_0 \log[aMR\mathcal{R}_i P]$. Since $P$ is gamma distributed given by (6), it follows that

$$\eta_z \equiv E\{z\} = z_0 \left[\log\left(a\mathcal{R}_v P_s/m\right) + (\log e)\psi(m)\right], \tag{13}$$

$$\sigma_z^2 \equiv \text{var}\{z\} = z_0^2 (\log e)^2 \psi^{(1)}(m) \tag{14}$$

where $\psi(x) = \Gamma'(x)/\Gamma(x)$ is the so-called digamma function and $\psi^{(n)}(x)$ denotes its $n^{\text{th}}$ derivative.

Expression (13) induces the functional relationship

$$\widehat{P_s} = F(w) = \frac{me^{-\psi(m)}}{a\mathcal{R}_v} 10^{w/z_0}. \tag{15}$$

Since $w$ is Gaussian distributed, $\widehat{P_s}$ is lognormal with mean

$$E\{\widehat{P_s}\} = \frac{me^{-\psi(m)}}{a\mathcal{R}_v} 10^{\eta_w/z_0} 10^{\ln 10 \sigma_w^2/2z_0^2} = P_s e^{\psi^{(1)}(m)/2N_{\mathcal{T}}}, \tag{16}$$

and variance

$$\text{var}\{\widehat{P_s}\} = \frac{m^2 e^{-2\psi(m)}}{(a\mathcal{R}_v)^2} 10^{2\eta_w/z_0} 10^{\ln 10\sigma_w^2/z_0^2} \left[10^{\ln 10\sigma_w^2/z_0^2} - 1\right] = P_s^2 e^{\psi^{(1)}(m)/N_{\mathcal{T}}} \left[e^{\psi^{(1)}(m)/N_{\mathcal{T}}} - 1\right]. \tag{17}$$

For a number of samples ($N_{\mathcal{T}} > 10$), $\sigma_w^2$ is small so that we can make a first order approximation on the exponential factor. Then, (16) and (17) become with good approximation

$$E\{\widehat{P_s}\} \approx P_s + \frac{\psi^{(1)}(m) P_s}{2N_{\mathcal{T}}} \approx P_s \tag{18}$$

$$\text{var}\{\widehat{P_s}\} \approx \frac{\psi^{(1)}(m)}{N_{\mathcal{T}}} \left[1 + \frac{\psi^{(1)}(m)}{N_{\mathcal{T}}}\right] P_s^2 \approx \frac{\psi^{(1)}(m) P_s^2}{N_{\mathcal{T}}}. \tag{19}$$

Note that this estimator has a small bias that is always lower that $0.1P_s$ since $\psi^{(1)}(m) < 2$ and $N_{\mathcal{T}} > 10$. This leads to the minimum number of samples for a given SNR

$$N_{\mathcal{T}} = (SNR)_{\widehat{P_s}} \psi^{(1)}(m). \tag{20}$$

Contrarily to the linear aperture, equation (20) does not depend on $P_s$. This is due to the assumption of low shot noise. This result can be compared with the linear case in the limit of high input power. For instance, $m = 1$ yields an asymptotic number of samples given by $(SNR)_{\widehat{P_s}}$ for the linear pre-amplifier and $\frac{\pi^2}{6}(SNR)_{\widehat{P_s}}$ for the logarithmic one. It is clear that the logarithmic nonlinearity increases the signal variance at the receiver output. This is a well known fact in the context of reflectivity radars.[9] As the speckle count $m$ increases, the variance due to speckle decreases and so does the number of samples in both types of receiver. In this case we should look more carefully at the Poisson statistics at the APD output since speckle noise may decrease to a level comparable to the signal-induced shot noise. For example, for $m = 100$ and the system parameters presented in Table 1 this happens for power levels below 50 nW.

### 4.2. Large Aperture

In this approximation we will take into account the signal-induced and dark-current shot noises but, still assuming that the input signal is high, we can approximate the conditional probability density function (p.d.f.) of $y$, for a given $P$ with the Gaussian density

$$p_{y|P}(y \mid P) = \frac{1}{\sqrt{2\pi}\sigma_{y|P}} \exp\left[-\frac{(y-\eta_{y|P})^2}{2\sigma_{y|P}^2}\right], \tag{21}$$

where, $\eta_{y|P}$ and $\sigma_{y|P}^2$ are given by (see, (4) and (5))

$$\eta_{y|P} = \mathcal{R}_v(P + P_d), \tag{22}$$

and

$$\sigma_{y|P}^2 = F\mathcal{R}_v^2 \frac{qB}{\mathcal{R}_i}(P + P_d). \tag{23}$$

The effect of speckle can be simplified in the limit of high speckle count by replacing the gamma distribution of $\eta_{y|P}$ with a Gaussian distribution with mean (7) and variance given by the first term of (8)

$$p_{\eta_{y|P}}(\eta_{y|P}) = \frac{1}{\sqrt{2\pi}\sigma_{\eta_{y|P}}} \exp\left[-\frac{(y-\eta_y)^2}{2\sigma_{\eta_{y|P}}^2}\right], \tag{24}$$

where,

$$\eta_y = \mathcal{R}_v(P_s + P_d), \tag{25}$$

and

$$\sigma_{\eta_{y|P}}^2 = \mathcal{R}_v^2 \frac{P_s^2}{m}. \tag{26}$$

The overall p.d.f. is the convolution of both distributions, yielding a Gaussian p.d.f. $p_y(y)$ with mean (7) and variance (8). In this way, the expected value of $z$ is

$$\eta_z = \int_{-\infty}^{+\infty} z_0 \log(ay) p_y(y) dy. \tag{27}$$

A good approximation of $E\{z\}$ can be obtained if $\sigma_y \ll \eta_y$ by expanding $z(y)$ in a Taylor series around $\eta_y$ up to the second order term. Making the change of variable $u = (y - \eta_y)/\sqrt{2}\sigma_y$, we find that

$$\eta_z = \frac{z_0 \log e}{\sqrt{\pi}} \int_0^{+\infty} \left[\log(a\eta_y) + \frac{\sqrt{2}\sigma_y u}{\eta_y} - \frac{2\sigma_y^2 u^2}{\eta_y^2}\right] \exp(-u^2) du. \tag{28}$$

The above integration yields

$$\eta_z = z_0 \left[\log(a\eta_y) - \frac{1}{2}\left(\frac{\sigma_y}{\eta_y}\right)^2\right]. \tag{29}$$

The variance can be computed in a similar way,

$$\sigma_z^2 = z_0^2(\log e)^2\left[\left(\frac{\sigma_y}{\eta_y}\right)^2 - \frac{1}{4}\left(\frac{\sigma_y}{\eta_y}\right)^4\right]. \tag{30}$$

The second term of the right hand side of (29) can be discarded provided that $\sigma_y^2 \ll \eta_y^2 \log(a\eta_y)$. Substitution of (7) and (8) on the above equations gives

$$\eta_w = \eta_z \approx z_0 \log\left[a\mathcal{R}_v(P_s + P_d))\right], \tag{31}$$

$$\sigma_w^2 = \frac{1}{N_{\mathcal{T}}}\sigma_z^2 \approx \frac{z_0^2(\log e)^2}{N_{\mathcal{T}}}\frac{\frac{P_s^2}{m} + F\frac{qB}{\mathcal{R}_i}(P_s + P_d)}{(P_s + P_d)^2}. \tag{32}$$

This result suggests the following power estimator

$$\widehat{P_s} = \frac{10^{w/z_0}}{a\mathcal{R}_v} - P_d. \tag{33}$$

Again, using the fact that $w$ is Gaussian, it follows that $\widehat{P_s}$ is lognormal, yielding

$$E\{\widehat{P_s}\} = \frac{1}{a\mathcal{R}_v}10^{\eta_w/z_0}10^{\ln 10\sigma_w^2/2z_0^2} - P_d \approx P_s, \tag{34}$$

$$\mathrm{var}\{\widehat{P_s}\} = \frac{1}{N_{\mathcal{T}}}\left(E\{\widehat{P_s}\} + P_d\right)^2\left[10^{\ln 10\sigma_w^2/z_0^2} - 1\right] \approx \frac{1}{N_{\mathcal{T}}}\left[\frac{P_s^2}{m} + F\frac{qB}{\mathcal{R}_i}(P_s + P_d)\right], \tag{35}$$

where we made the assumption that $\sigma_w \ll \eta_w$ and a first order approximation of the factor $10^{\ln 10\sigma_z^2/z_0^2}$ in (35). This leads to the expression for the number of samples already obtained in (12) for the linear pre-amplifier.

## 5. ESTIMATION OF LOG-POWER RETURNS IN DIAL SYSTEMS

In DIAL systems it is necessary to estimate the logarithm of the ratio of two optical power signals, in order to retrieve the concentration of a given component in the atmosphere. This reduces to the problem of estimating the log-power signal of each measurement channel and then subtracting the result. The properties of such log-power estimators can be derived in the same manner as in the logarithmic pre-amplifier case since the underlying statistics are identical.

In this example, the receiver follows the scheme of upper half of Fig.2. We assume that the pre-amplifier is linear and propose two estimators schematized in Fig.4: (a) In the first case we search for an estimator of the mean optical power before we estimate the log-power; (b) In the second case, we average over the logarithm of the APD output signal, in the same manner we did for the log-amplifier, and then obtain a log-power estimator.

### 5.1. Averaging the APD Output

Consider the estimator presented in the upper half of Fig.4. In the small aperture approach, the mean optical power estimator $\widehat{P_s}$ proposed in (9) has a Gaussian distribution with mean (10) and variance given by the first term of the right hand side of (11). We suggest the obvious estimator

$$\widehat{L} = \log\frac{\widehat{P_s}}{P_0}, \tag{36}$$

were $P_0$ is a normalizing constant power. Its mean and variance can be readily computed, recalling the procedure explained in Section 4.2 for the large aperture receiver, as

$$E\{\widehat{L}\} \approx \log\frac{P_s}{P_0}, \tag{37}$$

$$\mathrm{var}\{\widehat{L}\} \approx \frac{\sigma^2_{\widehat{P_s}}}{P_s^2} = \frac{1}{N_{\mathcal{T}}m}, \tag{38}$$

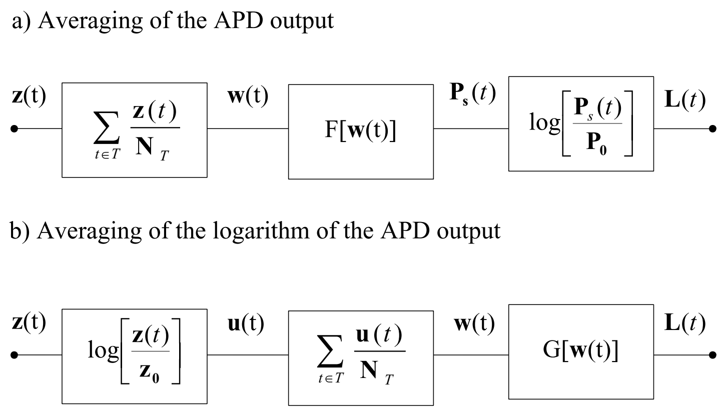


**Figure 4.** Log-power estimators.

provided that $\sigma^2_{\widehat{P_s}} \ll \eta^2_{\widehat{P_s}} \log(\widehat{P_s}/P_0)$. The large aperture receiver log-power estimator differs from this only by including the shot noise term on its variance

$$\mathrm{var}\{\widehat{L}\} \approx \frac{1}{N_{\mathcal{T}}}\left[\frac{1}{m} + F\frac{Bq}{\mathcal{R}_i}\left(\frac{1}{P_s} + \frac{P_d}{P_s^2}\right)\right]. \tag{39}$$

Next we propose another log-power estimator under a different averaging procedure.

## 5.2. Averaging the Logarithm of the APD Output

In this case we first construct the signal $u = \log(z/z_0)$ from the APD output $z$ (see the lower half of Fig.4). Under the small aperture assumption, the mean and variance of the sample average $w$ of this signal, using the results of Section 4.1, become

$$\eta_w = \log\frac{P_s}{mP_0} + \psi(m), \tag{40}$$

$$\sigma_w^2 = \frac{1}{N_{\mathcal{T}}}\psi^{(1)}(m), \tag{41}$$

since $\eta_{z|P}$ is gamma distributed. Expression (40) induces the estimator

$$\widehat{L} = w + \log m - \psi(m) \tag{42}$$

with mean and variance given by

$$E\{\widehat{L}\} = \log\frac{P_s}{P_0}, \tag{43}$$

$$\mathrm{var}\{\widehat{L}\} = \frac{1}{N_{\mathcal{T}}}\psi^{(1)}(m). \tag{44}$$

In the large aperture approach, the Gaussian statistics of the signal $z$ yields the same estimator properties as obtained in the previous section.

## 6. CONCLUDING REMARKS

We studied the influence of the common sources of noise in APD based receivers with a logarithmic pre-amplifier. The results were compared to those obtained in the linear case. In addition, taking advantage of this results, we compared two log-power estimators in the context of DIAL systems.

We considered the following situations:

(a) Speckle noise higher than shot noise
This happens in *small* aperture receivers having *low* speckle count values ($m \leq 10$). Compared with the linear case, the proposed mean power estimator exhibits a variance increased by an $m$ dependent factor ($\pi^2/6$ for $m = 1$).

(b) Speckle noise comparable to shot noise
This happens in *large* aperture receivers having *high* speckle count values ($m > 10$). Compared with the linear case, the proposed mean power estimator performs similarly.

With respect to the proposed log-power estimators, we conclude that:

(a) Small aperture
There is an advantage in averaging the APD output in first place since it leads to a smaller variance.

(b) Large aperture
The variance is the same regardless of the averaging and the log operation order.

## 7. ACKNOWLEDGMENTS

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## REFERENCES

1. J. M. Bioucas Dias and J. M. Nunes Leitao. *On the importance of noise figure in Reflectivity Radars.* International Weather Radar Networking. Kluver Academic Publishers, Ljubljana, 1991. COST73.
2. J.D.Klett. Stable analitical solution for processing lidar returns. *Appl. Opt.*, 20(2):211, 1981.
3. Barry J.Rye. Power ratio estimation in incoherent backscatter lidar: direct detection with gaussian noise. *Appl. Opt.*, 28(17):3639–3646, 1989.
4. J.W.Goodman. Some effects of target-induced scintillation on optical radar performance. *Proc. IEEE*, 53:1688–1700, 1965.
5. M.E.Tiuri. Radio astronomy receivers. *IEEE Trans. Antenas Propag*, (AP-12):930–938, 1964.
6. R.J.McIntyre. Multiplication noise in uniform avalanche diodes. *IEEE Trans. Electron Devices*, (13):164–168, 1966.
7. Dale Sirmans and R.J. Doviak. Meteorological radar signal. Technical report, National Oceanic and Atmospheric Administration, September 1973. NOAA Technical Memorandum ERL NSSL 64.
8. D. Snyder. *Random Point Processes.* Wiley-Interscience Publ., 1975.
9. G. Walker, P. Ray, D. Zrnic, and R. Doviak. Time, Angle and Range Sampling With Weather Radar. In *Preprints 16th Radar Meteorology Conference*, pages 156–162, Boston, 1975. American Meteorology Society.
10. W.B.Jones. *Introduction to Optical Fiber Communication Systems.* Rinehart & Winston, 1988.