Control of Cooperative Manipulators with Passive Joints

Renato Tinós and Marco Henrique Terra
Electrical Eng. Department, EESC, University of São Paulo C.P.359
São Carlos, SP, 13560-970, Brazil
{tinos} terra@sel.eesc.sc.usp.br

Abstract

The problem of the control of cooperative manipulators with passive joints and rigidly connected to a solid object is addressed in this paper. Passive joints can appear due to free-swinging joint failures or can be an intrinsic characteristic of the robots. A hybrid control of motion and squeeze force is proposed. For this purpose, a Jacobian matrix relating velocities in the actuated joints and load velocity is obtained based on the kinematic constraints of the cooperative system. Results of the control system applied in simulations and in real robots are presented.

1 Introduction

Passive joints can appear in robotic manipulators as an intinsic characteristc of the project (Bergerman, 1996) or as a result of free-swinging joint failures (Terra and Tinós, 2001). The intentional use of passive joints can be motivated by the need of minimization of weight, size, and energy in robotic applications. Free-swinging joint failures can occur for example because of a lost of electric power in electric motors or a rupture seal on hydraulic actuators (English and Maciejewski, 1998).

In some environments, human beings cannot be sent to make the necessary repairs in faulty robots because, for example, of the hazard and distance. In these cases, it is necessary to provide fault tolerance to the robot. Fault tolerance is also necessary when a fast response of the robot is required after a fault (this is the case of robotic mechanisms used to disarm explosives). Robotic systems with kinematic or actuation redundancy are interesting in applications where fault tolerance is needed because the number of degrees of freedom (dof) in these systems is greater than the dof required to execute the task. Actuation redundancy can be found in closed-link mechanisms like cooperative systems formed by two or more manipulators (Nakamura, 1991).

As in the human case, where the use of two arms presents an advantage over the use of only one arm in several cases, two or more robots can execute tasks that are difficult or even impossible for only one robot (Vukobratovic and Tuneski, 1998). Cooperative robots can be used in the manipulation of heavy, large or flexible loads, assembly of structures, and manipulation of objects that can slide from only one robot. Actuation redundancy makes the use of cooperative robots in unstructured or hazardous environments very appealing as robots are used to avoid the exposition of human beings to danger. However, the number of faults in these environments may increase due to external factors as extreme temperatures, obstacles, and radiation. In this case, also, a tolerance scheme is required.

The control of cooperative manipulators is a complex task due to the interaction among the arms caused by the forces and due to the kinematic constraints. The control should be coordinated and the squeeze force at the object should be minimized to avoid damage to the load. The squeeze force is formed by the components of the forces and torques in the object that do not contribute to the motion. The forces and torques that contribute to the motion are called motion force and are orthogonal to the squeeze force (Wen and Kreutz-Delgado, 1992).

In (Liu *et al.*, 1999), two arms with a number of actuated joints $n_a \geq k$, where k is the dof of the load, is controlled by a modified computed torque controller. A Jacobian matrix $\mathbf{Q}$ that relates the velocities of the actuated joints and the load velocity is obtained for $m = 2$, where m is the number of manipulators, and makes possible the control of the system. If $n_a > k$, where $n_a = n - n_p$ is the number of actuated joints, it is proposed a method to control $n_a - k$ components of the end-effector forces. Observe that the end-effector forces are controlled instead of the squeeze force. However, the end-effector forces are used to control the motion too, and the motion control affects the squeeze (section 2). Here, a new controller for the system with passive joints based on the decomposed control of the motion and squeeze (Wen and Kreutz-Delgado, 1992) is proposed. Futhermore, a new method to calculate the Jacobian $\mathbf{Q}$ is proposed for the general case $m > 1$.

This paper is organized as follows: the section 2 describes the kinematics and the dynamics of cooperative manipulators; the control system is presented in section 3; the section 4 presents the results of the control system in simulations and in a real system; and, finally, the conclusions are presented in section 5.

2 Cooperative Manipulators

Considering m arms rigidly connected to a solid object, the dynamics of the manipulator i is

$$\ddot{\mathbf{q}}_i = \mathbf{M}_i(\mathbf{q}_i)^{-1} * \\ [\tau_i + \mathbf{J}_i(\mathbf{q}_i)^T \mathbf{h}_i - \mathbf{g}_i(\mathbf{q}_i) - \mathbf{C}_i(\mathbf{q}_i)\dot{\mathbf{q}}_i] \quad (1)$$

where $\mathbf{q}_i$ is the vector of joint angles of arm i, $i = 1, \ldots, m$, τ_i is the vector of the torques at the joints of arm i, $\mathbf{M}_i$ is the inertia matrix, $\mathbf{C}_i$ is the matrix of the cenfrifugal and Coriolis terms, $\mathbf{g}_i$ is the gravitational vector, $\mathbf{J}_i$ is the Jacobian (from joint velocity to end-effector velocity) of arm i, and $\mathbf{h}_i$ is the force vector at the end-effector of arm i. The dynamics of all arms can be written as

$$\ddot{\mathbf{q}} = \mathbf{M}(\mathbf{q})^{-1}[\tau + \mathbf{J}(\mathbf{q})^T \mathbf{h} - \mathbf{g}(\mathbf{q}) - \mathbf{C}(\mathbf{q})\dot{\mathbf{q}}] \quad (2)$$

where $\mathbf{q} = [\mathbf{q}_1^T \mathbf{q}_2^T \ldots \mathbf{q}_m^T]^T$, $\tau = [\tau_1^T \tau_2^T \ldots \tau_m^T]^T$, $\mathbf{h} = [\mathbf{h}_1^T \mathbf{h}_2^T \ldots \mathbf{h}_m^T]^T$, $\mathbf{M}$ is formed by the individual inertia matrices of the arms, $\mathbf{C}$ is formed by the individual centrifugal and Coriolis matrices of the arms, $\mathbf{g}$ is formed by the individual gravitational vectors of the arms, and $\mathbf{J}$ is formed by the terms $\mathbf{J}_i$ for $i = 1, \ldots, m$.

The dynamics of the load is given by

$$\ddot{\mathbf{x}}_\mathbf{o} = \mathbf{M_o}^{-1}[-\mathbf{J_o}(\mathbf{x_o})^T \mathbf{h} - \mathbf{b_o}(\mathbf{x_o}, \dot{\mathbf{x}}_\mathbf{o})] \quad (3)$$

where $\mathbf{x_o}$ is vector of load position and orientation at the center of gravity (CG), $\mathbf{b_o}$ is the vector of centrifugal, Coriolis, and gravitational terms, $\mathbf{M_o}$ is the object's inertia matrix, and

$$\mathbf{J_o}(\mathbf{x_o}) = \begin{bmatrix} \mathbf{J_{o1}}(\mathbf{x_o})^T & \ldots & \mathbf{J_{om}}(\mathbf{x_o})^T \end{bmatrix}^T$$

where $\mathbf{J_{oi}}$ converts velocities at the CG into velocities at the end-effector of arm i.

As it is possible to compute the positions and orientations of the object using the positions of the joints of any arm of the cooperative system, the following kinematic constraint appears

$$\mathbf{x_o} = \varphi_1(\mathbf{q}_1) = \varphi_2(\mathbf{q}_2) = \ldots = \varphi_m(\mathbf{q}_m) \quad (4)$$

where $\varphi_i(\mathbf{q}_i)$ is the vector of the position and orientation of the object computed via joint positions of arm i, i.e., the direct kinematics of arm i. The following velocity constraint is also present

$$\dot{\mathbf{x}}_\mathbf{o} = \mathbf{D}_1(\mathbf{q}_1)\dot{\mathbf{q}}_1 = \ldots = \mathbf{D}_m(\mathbf{q}_m)\dot{\mathbf{q}}_m \quad (5)$$

where $\mathbf{D}_i$ is the Jacobian relating joint velocities of arm i and load velocities. The squeeze force are given by (Wen and Kreutz-Delgado, 1992)

$$\mathbf{h_{os}} = \mathbf{P_s}(\mathbf{x_o})\mathbf{h} \quad (6)$$

where the matrix $\mathbf{P_s}$ transforms the forces and torques at the end-effectors in the squeeze force.

Several solutions have been proposed to deal with the control problem in fault-free cooperative manipulators rigidly connected to a solid object, as the master/slave strategy (Luh and Zheng, 1987), the optimal division of the load control (Carignan and Akin, 1988), (Nahon and Angeles, 1992), the definition of new task objectives or variables (Koivo and Unseren, 1991), (Caccavale, 1997), and the hybrid control of motion and squeeze in the object (Uchiyama, 1998), (Wen and Kreutz-Delgado, 1992).

The hybrid control developed in (Wen and Kreutz-Delgado, 1992) is particularly interesting because the motion and the squeeze control in the load are decomposed, and because it does not utilize the detailed model of the system. In the cooperative system, joint torques in the form $\mathbf{D}^T \mathbf{h_{os}}$, where $\mathbf{D}$ is formed by the Jacobian matrices $\mathbf{D}_i$, do not affect the motion if there is not singular configuration in the arms. However, the motions of the arms affect the squeeze force due to the squeeze components of the d'Alembert (inertial) forces (Wen and Kreutz-Delgado, 1992). Thus, in (Wen and Kreutz-Delgado, 1992), a stable motion control with compensation of the gravitational torques is projected ignoring the squeeze terms. Then, the squeeze control is projected considering the inertial forces as perturbation.

3 Control of Cooperative Arms with Passive Joints

The control of the cooperative manipulators with passive joints is divided in motion control and squeeze force control. The Jacobian matrix $\mathbf{Q}$ that relates the velocities of the actuated joints and the load velocity is used by the motion control deduced in the following.

From eq. (4),

$$m\mathbf{x_o} = \varphi_1(\mathbf{q}_1) + \varphi_2(\mathbf{q}_2) + \ldots + \varphi_m(\mathbf{q}_m). \quad (7)$$

Assume that n_a joints are actuated and n_p joints are passive among the joints of all robots. The positions of the passive joints are grouped in the vector $\mathbf{q}_p$ and the positions of the actuated joints are grouped in the vector $\mathbf{q}_a$. Differentiating eq. (7) and partitioning in passive and actuated joints

$$m\dot{\mathbf{x}}_\mathbf{o} = \sum_{i=1}^{m} \frac{\partial \varphi_i(\mathbf{q}_i)}{\partial \mathbf{q}_a}\dot{\mathbf{q}}_a + \sum_{i=1}^{m} \frac{\partial \varphi_i(\mathbf{q}_i)}{\partial \mathbf{q}_p}\dot{\mathbf{q}}_p =$$

$$\mathbf{D}_a\dot{\mathbf{q}}_a + \mathbf{D}_p\dot{\mathbf{q}}_p \qquad (8)$$

where a is related to the actuated joints and p to the passive joints. From eq. (4), we can consider two cases. When m is even

$$\sum_{i=1}^{m}(-1)^{i+1}\varphi_i(\mathbf{q}_i) = \mathbf{0}. \qquad (9)$$

Differentiating eq. (9) and partitioning in passive and actuated joints,

$$\mathbf{R}_a\dot{\mathbf{q}}_a + \mathbf{R}_p\dot{\mathbf{q}}_p = \mathbf{0} \qquad (10)$$

that relates actuated and passive joint velocities when m is even. It is interesting to observe that such relation cannot be found in an individual manipulator with passive joints (Liu *et al.*, 1999). When m is odd

$$\sum_{i=1}^{m}(-1)^{i+1}\varphi_i(\mathbf{q}_i) = \mathbf{x}_o. \qquad (11)$$

Differentiating eq. (11) and partitioning in passive and actuated joints

$$\mathbf{R}_a\dot{\mathbf{q}}_a + \mathbf{R}_p\dot{\mathbf{q}}_p = \dot{\mathbf{x}}_o \qquad (12)$$

that relates actuated and passive joint velocities when m is odd. Using eq. (8), (10), and (12), we can write the relation between the load velocity and the actuated joints velocities

$$\dot{\mathbf{x}}_\mathbf{o} = \mathbf{Q}\dot{\mathbf{q}}_a \qquad (13)$$

where

$$\mathbf{Q} = \frac{1}{m}\left(\mathbf{D}_a - \mathbf{D}_p\mathbf{R}_p^{\#}\mathbf{R}_a\right) \qquad (14)$$

if m is even, and

$$\mathbf{Q} = \left(m\mathbf{I} - \mathbf{D}_p\mathbf{R}_p^{\#}\right)^{-1}\left(\mathbf{D}_a - \mathbf{D}_p\mathbf{R}_p^{\#}\mathbf{R}_a\right) \qquad (15)$$

if m is odd, where $\mathbf{I}$ is the identity matrix. Now the motion control problem can be adressed.

3.1 Motion Control

It is possible to partition eq. (2) as

$$\begin{bmatrix} \mathbf{M}_{aa} & \mathbf{M}_{ap} \\ \mathbf{M}_{pa} & \mathbf{M}_{pp} \end{bmatrix}\begin{bmatrix} \ddot{\mathbf{q}}_a \\ \ddot{\mathbf{q}}_p \end{bmatrix} + \begin{bmatrix} \mathbf{C}_a\dot{\mathbf{q}}_a \\ \mathbf{C}_p\dot{\mathbf{q}}_p \end{bmatrix} + \begin{bmatrix} \mathbf{g}_a \\ \mathbf{g}_p \end{bmatrix} = \begin{bmatrix} \tau_a \\ \mathbf{0} \end{bmatrix} + \begin{bmatrix} \mathbf{J}_a^T \\ \mathbf{J}_p^T \end{bmatrix}\mathbf{h} \qquad (16)$$

where a refers to the actuated joints and p to the passive joints. Consider now the Lyapunov function

$$V = \frac{1}{2}\dot{\mathbf{x}}_o^T\mathbf{M}_o\dot{\mathbf{x}}_o + \frac{1}{2}\dot{\bar{\mathbf{q}}}^T\bar{\mathbf{M}}\dot{\bar{\mathbf{q}}} + \frac{1}{2}\Delta\mathbf{x}_o^T\mathbf{K}_p\Delta\mathbf{x}_o \qquad (17)$$

where the two first terms are the kinetic energy of the system, $\Delta\mathbf{x}_o = (\mathbf{x}_{\mathbf{o}d} - \mathbf{x}_\mathbf{o})$ is the load position error, the diagonal matrix $\mathbf{K}_p$ is positive, $\dot{\bar{\mathbf{q}}} = \begin{bmatrix} \dot{\mathbf{q}}_a^T & \dot{\mathbf{q}}_p^T \end{bmatrix}^T$ and

$$\bar{\mathbf{M}} = \begin{bmatrix} \mathbf{M}_{aa} & \mathbf{M}_{ap} \\ \mathbf{M}_{pa} & \mathbf{M}_{pp} \end{bmatrix}.$$

Differentiating eq. (17)

$$\dot{V} = \dot{\mathbf{x}}_o^T\mathbf{M}_o\ddot{\mathbf{x}}_o + \dot{\bar{\mathbf{q}}}^T\bar{\mathbf{M}}\ddot{\bar{\mathbf{q}}} + \frac{1}{2}\dot{\bar{\mathbf{q}}}^T\dot{\bar{\mathbf{M}}}\dot{\bar{\mathbf{q}}} + \Delta\mathbf{x}_o^T\mathbf{K}_p\Delta\dot{\mathbf{x}}_o. \qquad (18)$$

Substituting eq. (3) and (16) in eq. (18), and considering that $(\dot{\mathbf{M}} - 2\mathbf{C})$ is antisymmetric (Spong and Vidyasagar, 1989), then

$$\dot{V} = -\dot{\mathbf{x}}_o^T\mathbf{b}_o - \dot{\mathbf{q}}_a^T\mathbf{g}_a - \dot{\mathbf{q}}_p^T\mathbf{g}_p + \dot{\mathbf{q}}_a^T\tau_a + \Delta\mathbf{x}_o^T\mathbf{K}_p\Delta\dot{\mathbf{x}}_o. \qquad (19)$$

Based on eq. (19), the following motion control law is proposed when $n_a \geq k$

$$\tau_a = \tau_m + \tau_g. \qquad (20)$$

The motion component of the control is given by

$$\tau_m = \mathbf{Q}^T(\mathbf{K}_v\Delta\dot{\mathbf{x}}_0 + \mathbf{K}_p\Delta\mathbf{x}_0) \qquad (21)$$

where $\mathbf{K}_v$ is a positive diagonal matrix. The conpensation for the gravity load, Centrifugal, and Coriolis forces is given by

$$\tau_g = \mathbf{g}_a - (\mathbf{R}_p^{\#}\mathbf{R}_a)^T\mathbf{g}_p + \mathbf{Q}^T\mathbf{b}_o \qquad (22)$$

if m is even, and

$$\tau_g = \mathbf{g}_a - \left(\mathbf{R}_p^{\#}(\mathbf{Q} - \mathbf{R}_a)\right)^T\mathbf{g}_p + \mathbf{Q}^T\mathbf{b}_o \qquad (23)$$

if m is odd.

Substituting eq. (20) in eq. (19) and considering a step set-point, then

$$\dot{V} = -\dot{\mathbf{x}}_o^T\mathbf{K}_v\dot{\mathbf{x}}_o \leq 0. \qquad (24)$$

Now, the problem of the squeeze control when the number of actuated joints (n_a) is greater than the dof of the load (k) is adressed.

3.2 Squeeze Force Control

As k dof are needed to control the k components of the motion, only $n_a - k$ components of the squeeze force can be simultaneously controlled. If torques proportional to the end-effector forces are applied, the stability is not garanteed because the inertial component affects the squeeze. As the problem is caused by the feedback of the end-effector forces, (Wen and Kreutz-Delgado, 1992) propose a pre-processing of them by a strictly proper linear filter, as an integrator, for the fault-free system. Thus, the following squeeze control law for the system with $(n_a > k)$ is used here

$$\tau_s = -\mathbf{E}_a(\mathbf{q}(t))^T \gamma_s(t) \tag{25}$$

where

$$\mathbf{E}_a = \begin{bmatrix} \mathbf{D}_{a1}(\mathbf{q}_1(t)) & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{D}_{a2}(\mathbf{q}_2(t)) & \dots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{D}_{am}(\mathbf{q}_m(t)) \end{bmatrix}$$

and $\mathbf{D}_{ai}$ relates the velocities of the arm i actuated joints and the load velocities. The vector γ_s gives the squeeze force that should be applied at the load by the squeeze force control when there are passive joints in the arms of the cooperative system. The integrator in the vector γ_s is defined as

$$\bar{\gamma}_s(t) = \mathbf{h_{os}}_d(t) + \mathbf{K}_i \int_{s=t_0}^{s=t} \left(\mathbf{h_{os}}_d(s) - \mathbf{h_{os}}(s)\right) ds \tag{26}$$

where $\mathbf{K}_i$ is a positive diagonal matrix and $\mathbf{h_{os}}_d$ is the vector of desired squeeze force. Consider now for simplicity that only one passive joint is present in the cooperative system. In the arm f with the passive joint j, we can write

$$\tau_{f_j}(t) = 0 = -\mathbf{D}_{f_j}(\mathbf{q}_f(t))^T \gamma_{s_f}(t) \tag{27}$$

where $\mathbf{D}_{f_j}$ is the j-th column of the matrix $\mathbf{D}_f$ and γ_{s_f} is the squeeze force that should be applied by the arm f. By the eq. (27)

$$0 = \mathbf{D}_{f_j}^T[1]\gamma_{s_f}[1] + \ldots + \mathbf{D}_{f_j}^T[k]\gamma_{s_f}[k] \tag{28}$$

where $\mathbf{a}[i]$ represents the i-th element of the vector $\mathbf{a}$ (the terms in parethesis were not written for simplicity). As there is one passive joint in the system, the l-th component of the squeeze force will not be controlled. Thus, the vector γ_{s_f} is defined as

$$\gamma_{s_f}[i] = \bar{\gamma}_{s_f}[i] \text{ if } i \neq l \tag{29}$$

where the vector $\bar{\gamma}_{s_f}$ is formed by the components of the vector $\bar{\gamma}_s$ given in eq. (26) related to arm f, and from eq. (28)

$$\gamma_{s_f}[l] = \frac{1}{\mathbf{D}_{f_j}^T[l]} \sum_{i=1, i\neq f}^{i=k} \mathbf{D}_{f_j}^T[i]\bar{\gamma}_{s_f}[i]. \tag{30}$$

In eq. (30), the l-th component of the squeeze force is calculated as a function of the components that are directly controlled. The components of the desired squeeze force that should be applied by the other arms are then calculated based on the components calculated for the arm f.

The l-th component of the squeeze force, which is not controlled, can be chosen based on the values of the components of vector $\mathbf{D}_{f_j}$. If the i-th component of $\mathbf{D}_{f_j}$ is equal to zero, the i-th component of the squeeze force must be controlled. A good way to choose the uncontrolled component of the squeeze force is looking for the component that gives the biggest value of $\mathbf{D}_{f_j}[l]$.

In this way, the control law for the system with passive joints is given by

$$\tau_a = \tau_m + \tau_g + \tau_s \tag{31}$$

where the terms in the right side are defined by eq. (21), (22), (23), and (25).

4 Results

First, the control system is applied in a simulation of two three-dof planar cooperative arms with passive joints manipulating an object in a x-y plane. The arms are equals and the gravity force is parallel to the y-axis (the x-axis passes throught the bases of the two arms). The parameters of the simulated system are presented in the Appendix. The sample period is 0.008s and measurement noise is added to the joint positions and velocities, and to the end-effector forces. Figures 1, 2, and 3 show a simulation of a trajectory of the system with one passive joint.

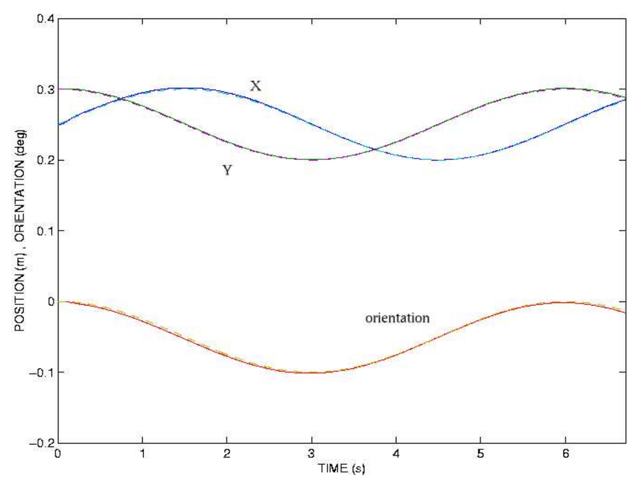

Figure 1: *Positions and orientation of the object in a trajectory of the simulated system with one passive joint (joint 1 at arm 1). The dashed lines show the desired trajectory.*

The next step is the application of the control system in a real cooperative system with two arms UARMII (figure 4). Each UARMII is a 3-joint, planar manipulator that floats on a thin air film on an "air table". The two arms are equals and the axis of each joint is parallel to the gravity force. The cooperative system is controlled by a PC running Matlab. This is possible because the

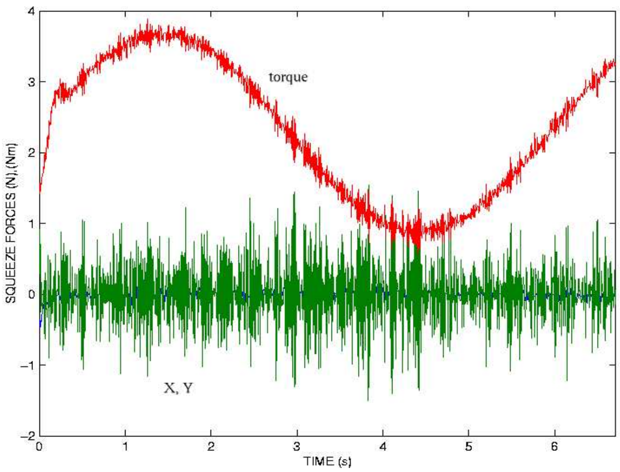

Figure 2: *Squeeze force in a trajectory of the simulated system with one passive joint (joint 1 at arm 1). The torque component of the squeeze force is not controlled.*

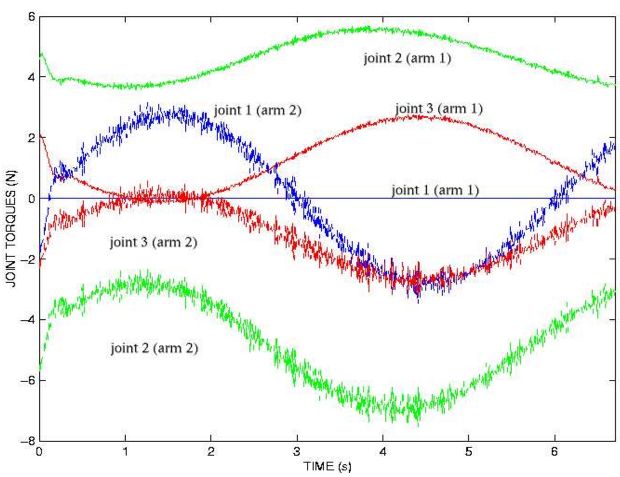

Figure 3: *Joint torques in the simulated system with one passive joint (joint 1 at arm 1).*

drivers for the UARMII servo board are written as Matlab mex-files. Each joint of the UARMII contains a brushless DC direct-drive motor, encoder, and pneumatic brake. The robot parameters are the same of the simulated system presented in the Appendix and a load of 0.025 kg is manipulated. The sample period is 0.06s. It is important to observe that this system is difficult to be correctly modelled because the flatness of the "air table" is irregular (the gravitational torques change with the position of the joints on the table). Other problem is that the joint velocities are obtained by differentiating the encoder readings, and force sensors are not used (the end-effector forces are estimated using the kinematic and dynamical models). Figures 5, 6, and 7 show the positions of the load, the squeeze force, and the joint torques in a trajectory with one passive joint. In this case, one component of the squeeze force (the torque) is not controlled.

Figure 4: *Real system.*

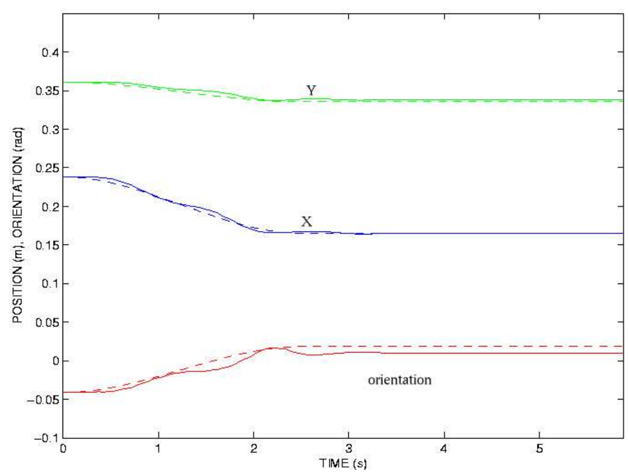

Figure 5: *Positions and orientation of the object in a trajectory of the real system with a passive joint (joint 1 at arm 2). The dashed lines show the desired trajectory.*

5 Conclusions

This work presents a control system for cooperative manipulators with passive joints. A hybrid control of motion and squeeze is utilized. For this purpose, a Jacobian matrix relating the velocities of the load and the velocities of the actuated joints is calculated.

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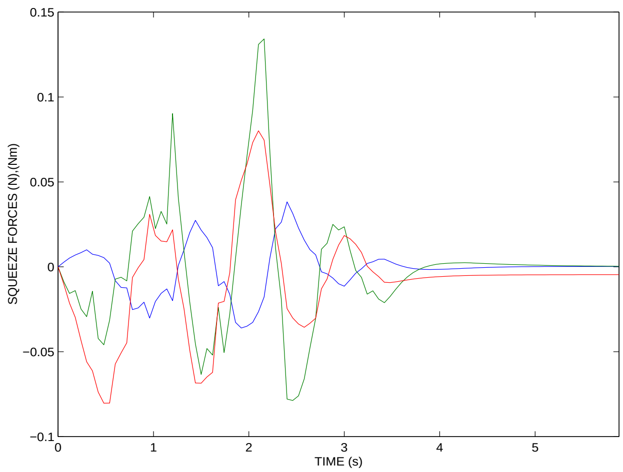

Figure 6: *Squeeze force in the real system with one passive joint (joint 1 at arm 2).*

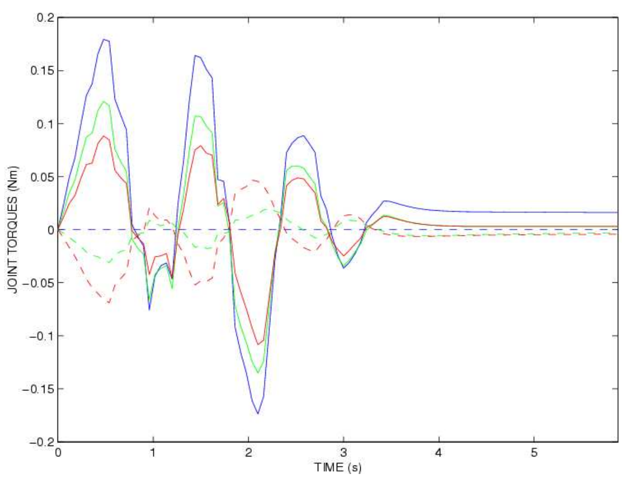

Figure 7: *Joint torques in the real system with one passive joint (joint 1 at arm 2). Solid lines: arm 1; Dashed lines: arm2.*

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Appendix

Table 1: *Parameters of the simulated system.*

Mass of links 1 and 2	0.85 kg
Mass of link 3	0.625 kg
Link length	0.203 m
Load length (between contact points)	0.1 m
Load mass	2.5 kg
Load moment of inertia	0.0022 kg m^2
Gravity	9.8 m/s^2