

PERFORMANCE ANALYSIS AND DESIGN OF STBC'S FOR FADING ISI CHANNELS

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Abstract — In this paper, space–time block–coded transmission over fading intersymbol interference (ISI) channels is investigated. A lower bound on the pairwise error probability for optimum detection is given. Also an approximation for the bit error rate is derived and compared with simulation results for maximum–likelihood sequence estimation (MLSE) for the GSM/EDGE (Enhanced Data Rates for GSM Evolution) system. Furthermore, a novel design rule for space–time block codes (STBC's) with arbitrary rate for fading ISI channels is provided. A corresponding full–rate code is designed and shown to yield higher performance than Alamouti's code. It is demonstrated that for fading channels with L independent impulse response coefficients, STBC's designed for the flat fading channel can achieve at most a diversity order of $(N_T + L - 1)N_R$ if N_T transmit antennas and N_R receive antennas are used. On the other hand, the maximum diversity order employing the proposed code design rule is LN_TN_R .

1. Introduction

Recently, it has been shown that space–time codes (STC's) can significantly improve the achievable performance for transmission over *flat* fading channels [1, 2]. Unfortunately, wireless communication channels are usually *frequency–selective*. Since existing STC's are designed for channels without intersymbol interference (ISI), it is not obvious how they perform in frequency–selective channels which suffer from ISI.

The performance of space–time codes in unequalized multipath fading has been investigated in [3]. Furthermore, an analysis for multipath fading and maximum–likelihood sequence estimation (MLSE) is given in [4]. The results in [3] and [4] show that multipath fading does not decrease the diversity order of space–time codes designed for flat fading, i.e., a diversity order of $N_T \cdot N_R$ can be achieved, if N_T transmit antennas and N_R receive antennas are used. Recently, design criteria for space–time trellis codes (STTC's) for fading ISI channels have been derived in [5]. Here, we concentrate on space–time block codes (STBC's) since, in contrast to STTC's, STBC's can be easily combined with suboptimum low–complexity equalization, e.g. [6, 7]. The technique used here for derivation of the STBC design rule is completely different from that used in [5]. Since it is well known that the *matched filter bound* (MFB) is a simple but very useful tool to analyze the performance limits for transmission over fading ISI channels, e.g. [8], we adopt a similar approach. Namely, we consider the transmission of

a single STBC word and evaluate the corresponding pairwise error probability (PEP) and bit error rate (BER) for maximum–likelihood detection. However, for practical reasons we do not employ the optimum matrix matched filter (MF) as receiver input filter but identical fixed square–root Nyquist (SRN) filters for each receive branch, respectively.

2. Preliminaries

2.1. Notation

Bold upper case (\mathbf{X}) and lower case (\mathbf{x}) letters denote matrices and vectors, respectively. $\det(\cdot)$, $\text{rank}(\cdot)$, $(\cdot)^T$, $(\cdot)^H$, and $(\cdot)^*$ refer to the determinant of a matrix, the rank of a matrix, transposition, Hermitian transposition, and complex conjugation, respectively. $\text{diag}\{x_1, x_2, \dots, x_M\}$ is a diagonal matrix with main diagonal elements x_1, x_2, \dots, x_M , whereas \mathbf{I}_M , $\mathbf{0}_{M \times N}$, and $\mathbf{0}_{M \times 1} = \mathbf{0}_M$ are the $M \times M$ identity matrix, the $M \times N$ all–zero matrix, and the all–zero column vector of length M , respectively. $\text{Pr}\{\cdot\}$, $\mathcal{E}\{\cdot\}$, $j \triangleq \sqrt{-1}$, $\Re\{\cdot\}$, $\|\cdot\|$, and \otimes denote the probability of the event in brackets, expectation, the imaginary unit, the real part of a complex number, the L_2 –norm of a vector, and the Kronecker product, respectively. Throughout this paper, all signals are represented by their complex–baseband equivalents.

2.2. Transmission Model

We consider a transmission scheme using N_T transmit antennas and N_R receive antennas. In this paper, we focus on the transmission of a single $N \times N_T$ STBC matrix \mathbf{C} , whose element $c_{k\nu}$ is transmitted at time k , $1 \leq k \leq N$, from antenna ν , $1 \leq \nu \leq N_T$. Thereby, \mathbf{C} is taken from a set \mathcal{C} of 2^{N_b} different code matrices. N_b information bits are mapped to the elements of \mathcal{C} and if we want to refer to a particular element of the set, we use a subscript α , $1 \leq \alpha \leq 2^{N_b}$ ($\mathbf{C}_\alpha \in \mathcal{C}$). In order to make the transmitted energy independent from the number of transmit antennas N_T , the normalization $\sum_{k=1}^N \sum_{\nu=1}^{N_T} \mathcal{E}\{|c_{k\nu}|^2\} = N \cdot E_c$ is adopted, where E_c is the average transmitted energy per symbol interval. After T –spaced sampling (T denotes the symbol duration) the received signal at antenna μ can be written as [9]

$$r_\mu[k] = \sum_{\nu=1}^{N_T} \sum_{l=0}^{L-1} h_{\nu\mu}[l]c_{k-l\nu} + n_\mu[k], \quad (1)$$

with the discrete–time overall channel impulse response (CIR) $h_{\nu\mu}[\cdot]$ from transmit antenna ν to receiver antenna μ and the noise process $n_\mu[\cdot]$. $h_{\nu\mu}[\cdot]$ is truncated to a suitable length

L and includes the combined effects of transmit filter, multi-path fading, receiver input filter, and sampling. As usual, we assume that the path weights of the wireless channel are complex Gaussian random variables (RV's). Therefore, the coefficients of the discrete-time overall CIR are (generally correlated) Gaussian RV's. Furthermore, a block fading channel model is adopted, i.e., $h_{\nu\mu}[k]$ is assumed to be time-invariant within one block (and thus, for our analysis) but varies from block to block, e.g. due to frequency hopping.

As it is customary, we assume that the transmit filters and receiver input filters for all branches are identical, respectively. It can be expected that properly chosen fixed receiver input filters cause only a negligible loss in performance [10].

$n_\nu[k]$ is assumed to be spatially and temporally white. Transmit and receiver input filters are normalized to guarantee a noise variance $\sigma_n^2 \triangleq \mathcal{E}\{|n_\nu[k]|^2\} = N_0/T$ and $E_h = \sum_{\nu=1}^{N_T} \sum_{\mu=1}^{N_R} \sum_{l=0}^{L-1} \mathcal{E}\{|h_{\nu\mu}[l]|^2\} = N_R N_T$, where N_0 denotes the single-sided power spectral density of the underlying continuous-time noise process.

3. Performance Analysis

Similar to the classical MFB (e.g. [8]), we consider in the following the transmission of a single (matrix) symbol. The resulting PEP constitutes a lower bound for continuous transmission (employing the same transmit and receiver input filters).

3.1. Optimum Detector

Before the PEP can be analyzed, the optimum detection rule for the transmitted matrix \mathbf{C} has to be derived. From Eq. (1) it can be observed that only the samples $r_\mu[k]$, $1 \leq k \leq N + L - 1$, depend on elements of \mathbf{C} . Hence, without loss of optimality, we may restrict our attention to the vectors $\mathbf{r}_\mu \triangleq [r_\mu[1] \ r_\mu[2] \ \dots \ r_\mu[N + L - 1]]^T$, $1 \leq \mu \leq N_R$. Regarding Eq. (1), \mathbf{r}_μ can be expressed as [9]

$$\mathbf{r}_\mu = \mathbf{C}'_m \mathbf{h}_\mu + \mathbf{n}_\mu \quad (2)$$

with the definitions $\mathbf{h}_\mu \triangleq [h_{1\mu}^T \ \dots \ h_{N_T\mu}^T]^T$, $\mathbf{h}_{\nu\mu} \triangleq [h_{\nu\mu}[0] \ \dots \ h_{\nu\mu}[L-1]]^T$, $\mathbf{C}'_m \triangleq [\mathbf{C}'_1 \ \dots \ \mathbf{C}'_{N_T}]$, $\mathbf{C}'_\nu \triangleq [c_{1\nu} \ \dots \ c_{L\nu}]$, $\mathbf{c}_{l\nu} \triangleq [\mathbf{0}_{l-1}^T \ c_{1\nu} \ \dots \ c_{N\nu} \ \mathbf{0}_{L-l}^T]^T$, and $\mathbf{n}_\mu \triangleq [n_\mu[1] \ \dots \ n_\mu[N + L - 1]]^T$. \mathbf{C}'_m is the $(N+L-1) \times LN_T$ modified STBC matrix. Now, we stack the signal vectors \mathbf{r}_μ of all receive antennas μ into a single vector $\mathbf{r} \triangleq [\mathbf{r}_1^T \ \mathbf{r}_2^T \ \dots \ \mathbf{r}_{N_R}^T]^T$ and obtain

$$\mathbf{r} = \mathbf{C}_m \mathbf{h} + \mathbf{n}, \quad (3)$$

where the definitions $\mathbf{h} \triangleq [h_1^T \ \dots \ h_{N_R}^T]^T$, $\mathbf{n} \triangleq [n_1^T \ \dots \ n_{N_R}^T]^T$, and $\mathbf{C}_m \triangleq \mathbf{I}_{N_R} \otimes \mathbf{C}'_m$ have been used. Since we assume that the channel (i.e., \mathbf{h}) is perfectly known at the receiver and the elements of \mathbf{n} are independent zero-mean complex Gaussian random variables with variance σ_n^2 , respectively, the (optimum) maximum-likelihood detector determines the estimate $\hat{\mathbf{C}}$ for the transmitted matrix \mathbf{C} (which

is uniquely associated with \mathbf{C}_m) according to the decision rule

$$\hat{\mathbf{C}} = \underset{\mathbf{c}}{\operatorname{argmin}} \{ \|\mathbf{r} - \mathbf{C}_m \mathbf{h}\|^2 \}. \quad (4)$$

3.2. Pairwise Error Probability

The PEP $P_e(\alpha, \beta)$ is the probability that $\hat{\mathbf{C}} = \mathbf{C}_\beta$ is detected if $\mathbf{C} = \mathbf{C}_\alpha$ ($\mathbf{C}_\alpha, \mathbf{C}_\beta \in \mathcal{C}$, $\alpha \neq \beta$) is transmitted. From Eq. (4) it is easy to see that PEP is given by

$$P_e(\alpha, \beta) = \Pr\{2\Re\{\mathbf{n}^H \mathbf{C}_m^{\beta|\alpha} \mathbf{h}\} > (\mathbf{C}_m^{\beta|\alpha} \mathbf{h})^H \mathbf{C}_m^{\beta|\alpha} \mathbf{h}\}, \quad (5)$$

with $\mathbf{C}_m^{\beta|\alpha} \triangleq \mathbf{C}_m^\beta - \mathbf{C}_m^\alpha$, where \mathbf{C}_m^α and \mathbf{C}_m^β are obtained from \mathbf{C}_α and \mathbf{C}_β , respectively, in the same way as \mathbf{C}_m from \mathbf{C} .

Since we assume a Ricean channel model, \mathbf{h} may be written as $\mathbf{h} = \bar{\mathbf{h}} + \mathbf{h}_s$, where $\bar{\mathbf{h}} \triangleq \mathcal{E}\{\mathbf{h}\}$ and \mathbf{h}_s denote the direct and the specular (Rayleigh) component of \mathbf{h} , respectively. The covariance matrix of \mathbf{h} is given by $\Phi_{h_s h_s} \triangleq \mathcal{E}\{\mathbf{h}_s \mathbf{h}_s^H\}$.

By introducing the vector $\mathbf{l} \triangleq [(\mathbf{C}_m^{\beta|\alpha} \mathbf{h})^T \ \mathbf{n}^T]^T$, Eq. (5) can be simplified to

$$P_e(\alpha, \beta) = \Pr\{\Delta(\alpha, \beta) < 0\}, \quad (6)$$

with $\Delta(\alpha, \beta) \triangleq \mathbf{l}^H \mathbf{F} \mathbf{l}$ and

$$\mathbf{F} \triangleq \begin{pmatrix} \mathbf{I}_{(N+L-1)N_R} & -\mathbf{I}_{(N+L-1)N_R} \\ -\mathbf{I}_{(N+L-1)N_R} & \mathbf{0}_{(N+L-1)N_R \times (N+L-1)N_R} \end{pmatrix}.$$

Since $\Delta(\alpha, \beta)$ is a quadratic form of Gaussian random variables, the Laplace transform of its probability density function can be easily calculated to [11]

$$\Phi_{\Delta(\alpha, \beta)}(s) = \frac{\exp\left(-s \bar{\mathbf{l}}^H (\mathbf{F}^{-1} + s \Phi_{ll})^{-1} \bar{\mathbf{l}}\right)}{\det(\mathbf{I}_{2(N+L-1)N_R} + s \Phi_{ll} \mathbf{F})}, \quad (7)$$

where the mean and the covariance matrix of \mathbf{l} are given by

$$\bar{\mathbf{l}} \triangleq [(\mathbf{C}_m^{\beta|\alpha} \bar{\mathbf{h}})^T \ \mathbf{0}_{(N+L-1)N_R}^T]^T \text{ and } \Phi_{ll} \triangleq \begin{pmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \mathbf{A}_{12}^T & \mathbf{A}_{22} \end{pmatrix}$$

$[\mathbf{A}_{11} \triangleq \mathbf{C}_m^{\beta|\alpha} \Phi_{h_s h_s} (\mathbf{C}_m^{\beta|\alpha})^H, \mathbf{A}_{22} \triangleq \sigma_n^2 \cdot \mathbf{I}_{(N+L-1)N_R}, \mathbf{A}_{12} \triangleq \mathbf{0}_{(N+L-1)N_R \times (N+L-1)N_R}]$, respectively. The PEP can be calculated directly from $\Phi_{\Delta(\alpha, \beta)}(s)$ [12]

$$P_e(\alpha, \beta) = \frac{1}{2\pi j} \int_{\gamma-j\infty}^{\gamma+j\infty} \Phi_{\Delta(\alpha, \beta)}(s) \frac{ds}{s} \quad (8)$$

for $0 < \gamma < \Re\{s_1\}$, where s_1 denotes that pole of $\Phi_{\Delta(\alpha, \beta)}(s)$ which has minimum positive real part. The integral in Eq. (8) may be evaluated using efficient numerical methods (cf. [12]).

3.3. Approximation for BER

The exact calculation of BER is quite involved if not impossible. Therefore, we may resort to the union bound which is given by

$$P_b = \frac{1}{2^{N_b}} \sum_{\alpha=1}^{2^{N_b}} \sum_{\substack{\beta=1 \\ \beta \neq \alpha}}^{2^{N_b}} \frac{n(\alpha, \beta)}{N_b} \cdot P_e(\alpha, \beta), \quad (9)$$

where $n(\alpha, \beta)$ denotes the number of bit errors if C_α is transmitted and C_β is detected. Since the union bound is an *upper* bound, unlike $P_e(\alpha, \beta)$, P_b is not a lower bound. However, for moderate to high signal-to-noise ratios (SNR's) the union bound becomes tight and P_b is a very good approximation for the achievable BER for space-time block coded transmission over fading ISI channels. A disadvantage of Eq. (9) is that the PEP's for all possible pairs of α and β have to be calculated. In order to decrease the computational load, we propose to approximate P_b by

$$P_b \approx \frac{N_{\max}}{N_b 2^{N_b}} \cdot P_{\max}, \quad (10)$$

with

$$P_{\max} \triangleq \max_{\substack{(\alpha, \beta) \\ \alpha \neq \beta}} \{P_e(\alpha, \beta)\}. \quad (11)$$

N_{\max} denotes the number of pairs (α, β) for which $P_e(\alpha, \beta) = P_{\max}$ is true. Note that, for simplicity, in Eq. (10) it is assumed that each (matrix) symbol error causes only one bit error. For an efficient method to determine those pairs of (α, β) with $P_e(\alpha, \beta) = P_{\max}$ without calculating $P_e(\alpha, \beta)$ itself we refer to [9]. Once the relevant pairs (and their number N_{\max}) are available, P_b is obtained from Eqs. (10), (11). Despite the simplicity of the proposed approximation, comparisons with simulation results for MLSE show its accuracy and relevance.

4. Diversity Order and Code Design

For simplicity, in this section, we restrict ourselves to the case of pure Rayleigh fading, where the largest performance gains with STBC's can be expected.

4.1. Bound on Diversity Order

Before we proceed, we note that matrix $C_m^{\beta|\alpha} \Phi_{h_s h_s} (C_m^{\beta|\alpha})^H$ is a positive semidefinite Hermitian matrix and thus, there is a factorization (eigendecomposition) [13]

$$C_m^{\beta|\alpha} \Phi_{h_s h_s} (C_m^{\beta|\alpha})^H = \mathbf{M} \mathbf{\Lambda} \mathbf{M}^H = \mathbf{G} \mathbf{G}^H, \quad (12)$$

where \mathbf{M} is a unitary matrix and $\mathbf{\Lambda} = \text{diag}\{\lambda_1, \dots, \lambda_{(N+L-1)N_R}\}$, $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_{(N+L-1)N_R} \geq 0$, is the diagonal matrix of the eigenvalues. \mathbf{G} is given by $\mathbf{G} \triangleq \mathbf{M} \mathbf{\Lambda}^{1/2}$, with $\mathbf{\Lambda}^{1/2} \triangleq \text{diag}\{\sqrt{\lambda_1}, \dots, \sqrt{\lambda_{(N+L-1)N_R}}\}$. Now $C_m^{\beta|\alpha} \mathbf{h} = C_m^{\beta|\alpha} \mathbf{h}_s$ may be expressed as

$$C_m^{\beta|\alpha} \mathbf{h} = \mathbf{G} \mathbf{h}'_s, \quad (13)$$

where \mathbf{h}'_s is a zero-mean complex Gaussian vector with $\Phi_{h'_s h'_s} \triangleq \mathcal{E}\{\mathbf{h}'_s (\mathbf{h}'_s)^H\} = \mathbf{I}_{LN_T N_R}$. Taking the above considerations into account, Eq. (5) can be rewritten as [9]

$$P_e(\alpha, \beta) = \Pr\{2\Re\{\mathbf{n}'^H \mathbf{h}''_s\} > \mathbf{h}''_s{}^H \mathbf{h}''_s\}, \quad (14)$$

with $\mathbf{n}' \triangleq \mathbf{M}^H \mathbf{n}$ and $\mathbf{h}''_s \triangleq \mathbf{\Lambda}^{1/2} \mathbf{h}'_s$. Note that the elements of \mathbf{n}' are independent zero-mean Gaussian random variables

with variance σ_n^2 , respectively. Hence, using standard Chernoff bounding techniques [1], the PEP may be upper bounded as

$$P_e(\alpha, \beta) \leq \frac{1}{\prod_{n=1}^q \lambda_n \cdot \left(\frac{1}{4\sigma_n^2}\right)^q}, \quad (15)$$

where q denotes the number of non-zero eigenvalues of $C_m^{\beta|\alpha} \Phi_{h_s h_s} (C_m^{\beta|\alpha})^H$ and is referred to as the *diversity order* (or *diversity advantage* [1]) of the STBC for a given fading ISI channel. $q = \text{rank}(C_m^{\beta|\alpha} \Phi_{h_s h_s} (C_m^{\beta|\alpha})^H)$ can be bounded by [13]

$$q \leq \min\{\text{rank}(C_m^{\beta|\alpha}), \text{rank}(\Phi_{h_s h_s})\}. \quad (16)$$

Since $\text{rank}(C_m^{\beta|\alpha}) \leq (N + L - 1)N_R$ and $\text{rank}(\Phi_{h_s h_s}) \leq LN_T N_R$ is valid, Eq. (16) can be rewritten as

$$q \leq N_R \cdot \min\{N + L - 1, LN_T\}. \quad (17)$$

Obviously, for $L = 1$ we obtain the well-known result that the maximum diversity order $q = N_T N_R$ can be achieved with $N = N_T$ (cf. e.g. [2]). However, for $L > 1$ Eq. (17) indicates that choosing $N > N_T$ may be beneficial. On the other hand, since Eq. (17) implies $q \leq LN_T N_R$, N may be restricted to $N \leq (N_T - 1)L + 1$. Hence, in the following, we assume $N_T \leq N \leq (N_T - 1)L + 1$. If $N = N_T$ is adopted $q \leq (N_T + L - 1)N_R$ holds, i.e., increasing the number of transmit antennas has the same effect as a longer channel impulse response.

4.2. Design of STBC's

For simplicity, we restrict ourselves to the case where all coefficients $h_{\nu\mu}[l]$ of the discrete-time overall CIR's are mutually independent and have equal variances $1/L$. The general case, where the coefficients are correlated and/or have different variances is considered in [9].

For convenience, we define the $(N + L - 1) \times LN_T$ matrix

$$\mathbf{S}_{\beta|\alpha} \triangleq [\mathbf{S}_1^{\beta|\alpha} \quad \mathbf{S}_2^{\beta|\alpha} \quad \dots \quad \mathbf{S}_L^{\beta|\alpha}] \quad (18)$$

with

$$\mathbf{S}_l^{\beta|\alpha} \triangleq \begin{pmatrix} \mathbf{0}_{(l-1) \times N_T} \\ \mathbf{C}_\beta - \mathbf{C}_\alpha \\ \mathbf{0}_{(L-l) \times N_T} \end{pmatrix}. \quad (19)$$

From Eq. (15) the following STBC design rules can be derived ($N_T \leq N \leq (N_T - 1)L + 1$ is assumed):

- **Rank Criterion:** Select the code set \mathcal{C} in such a way that matrix $\mathbf{S}_{\beta|\alpha}$ has rank $N + L - 1$ for $1 \leq \alpha, \beta \leq 2^{N_b}$, $\alpha \neq \beta$.
- **Eigenvalue Criterion:** Choose that code set among all code sets that satisfy the rank criterion, which maximizes the minimum value of the product of the eigenvalues of matrix $\mathbf{S}_{\beta|\alpha} \mathbf{S}_{\beta|\alpha}^H$.

Due to space limitations we refer to [9] for the derivation of the proposed design rule. Note that for $L = 1$ and uncorrelated fading coefficients our design rule is identical to the conventional design rule for frequency–nonselective channels given in [1, 2].

For mutually uncorrelated CIR coefficients, STBC’s designed according to the given rule achieve a diversity order of $q = (N + L - 1)N_R$. Note that for $N = (N_T - 1)L + 1$, $q = LN_T N_R$ results and the diversity order grows linearly with L , N_T , and N_R , whereas for (full–rate) STBC’s designed for flat fading channels $N = N_T$ holds and q grows linearly only with $N_T + L$ and N_R ($q = (N_T + L - 1)N_R$).

4.3. Example: New Code for $N_T = 2$

In this section, again we assume that all coefficients of all discrete–time overall CIR’s are mutually uncorrelated. We give a full–rate STBC for $N_T = 2$ which yields maximum diversity order for $L = 2$ ¹. For this, we assume that N binary phase–shift keying (BPSK) symbols $b_k \in \{1, -1\}$, $1 \leq k \leq N$, are mapped to matrix \mathbf{C} . For $L = 2$, $q = LN_T = 4$ can be achieved if $N = 3$ is chosen and \mathbf{C} is properly designed ($N_R = 1$ is assumed). An example for a mapping which yields $q = 4$ is $c_{11} = 0.6 b_1 + 0.4 b_2 + 0.35 b_3$, $c_{12} = 0.6 b_1 - 0.5 b_2 + 0.4 b_3$, $c_{21} = 0.25 b_2 - 0.2 b_3$, $c_{22} = 0.25 b_2 + 0.2 b_3$, $c_{31} = -0.35 b_1 + 0.5 b_2 + 0.6 b_3$, and $c_{32} = 0.4 b_1 + 0.4 b_2 - 0.6 b_3$. This code has been found by a random search. For this code $\mathbf{S}_{\beta|\alpha} \mathbf{S}_{\beta|\alpha}^H$ had the largest minimum eigenvalue product of all found codes with $q = 4$. Of course, this code is not orthogonal [14], however, orthogonality is of no importance for ISI channels. Note that Alamouti’s orthogonal code

$$\mathbf{C} = \frac{1}{\sqrt{2}} \begin{pmatrix} b_1 & b_2 \\ -b_2 & b_1 \end{pmatrix} \quad (20)$$

yields a diversity order of $q = N_T + L - 1 = 3$.

In Fig. 1, the approximation for the BER according to Eq. (10) vs. $10 \log_{10}(E_b/(N_0 N_R))$ (E_b is the mean received energy per information bit) is shown for the above STBC’s ($N_R = 1$) and BPSK with $N_R = 2$ for different L . For $L = 1$, i.e., flat Rayleigh fading, $q = N_T = 2$ is valid for both STBC’s and $q = N_R = 2$ holds for BPSK (BPSK with receive diversity and Alamouti’s code yield identical BER’s). However, for $L = 2$ Alamouti’s code achieves $q = N_T + L - 1 = 3$, whereas $q = N + L - 1 = LN_T = 4$ and $q = LN_R = 4$ is realized by the new design and BPSK, respectively, i.e., the slope of the corresponding BER curves is larger. Similarly, for $L = 3$, $q = N_T + L - 1 = 4$ and $q = N + L - 1 = 5$ is realized by Alamouti’s code and the new design, respectively. Note that a properly designed STBC with $N = 4$ could achieve $q = 6$ (cf. Eq. (17)), i.e., the same diversity order as BPSK with $N_R = 2$.

5. Numerical Results for GSM/EDGE

In this section, we investigate the performance of STBC’s for (filtered) BPSK and 8PSK modulation. Filtered BPSK can be

¹Note that $L = 2$ with independent CIR taps is specified as test case for IS–136.

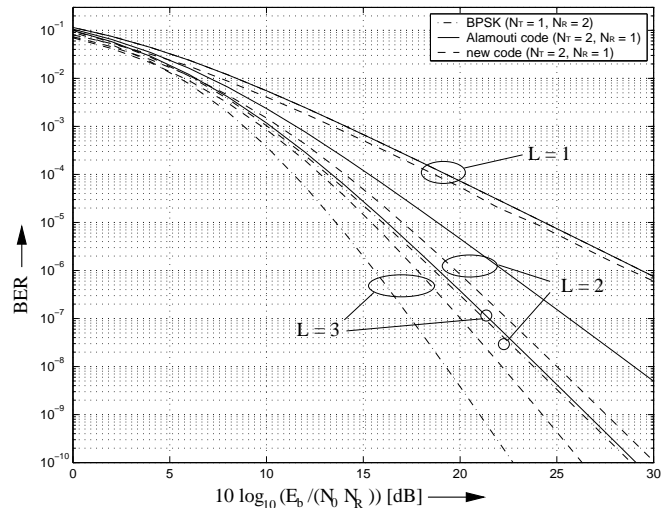


Figure 1: Approximation for BER vs. $10 \log_{10}(E_b/(N_0 N_R))$ for the new code, Alamouti’s code, and BPSK with receive diversity.

used to approximate Gaussian minimum–shift keying (GMSK) modulation which is employed for GSM. 8PSK is adopted for EDGE. For the numerical results two different power delay profiles with mutually independent paths are employed: rural area (RA) and typical urban area (TU). In contrast to the TU profile, the RA profile also contains a direct path. For each transmit and receive branch a linearized GMSK pulse and a square–root raised cosine filter with roll–off factor 0.3 are used, respectively. Furthermore, it is assumed that the CIR’s between different antenna pairs are mutually independent. The novel STBC given in Section 4.3 did not yield a significant performance improvement over STBC’s designed for flat fading for the investigated power delay profiles. On the other hand, the search for STBC’s optimized for the GSM/EDGE system is beyond the scope of this paper and a topic for future research. Therefore, for simplicity, STBC’s designed for flat fading are used exclusively in the following. For the numerical results BER is approximated by Eq. (10) and compared with simulation results for MLSE [7] wherever possible.

Fig. 2 shows BER vs. $10 \log_{10}(E_b/(N_0 N_R))$ for BPSK transmission over a TU channel. BPSK with receive diversity, Alamouti’s code, and the orthogonal STBC for $N_T = 4$ given in [14] are considered. It can be observed that for the TU channel a considerable gap between BPSK with receive diversity and the corresponding transmit diversity schemes remains, i.e., it can be expected that a significant gain can be achieved by STBC’s optimized for frequency–selective fading channels. Remarkably, the BER approximation proposed in Section 3.3 is well confirmed by simulation results for MLSE.

8PSK transmission over an RA channel is considered in Fig. 3. Obviously, for the RA profile Alamouti’s code is near optimum. This is not surprising since the RA channel is practically a flat Ricean fading channel. Also for 8PSK the BER results predicted by the proposed approximation coincide very well

with simulation results for MLSE.
For further numerical results we refer to [9].

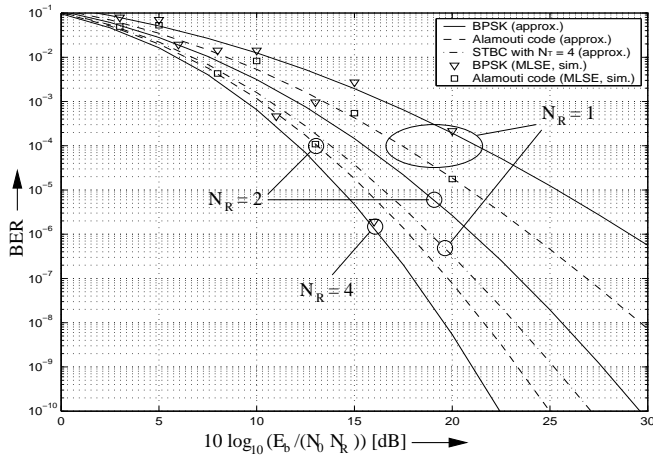


Figure 2: Approximation for BER vs. $10 \log_{10}(E_b / (N_0 N_R))$ for BPSK transmission over TU channel. For comparison also simulation results for MLSE are shown.

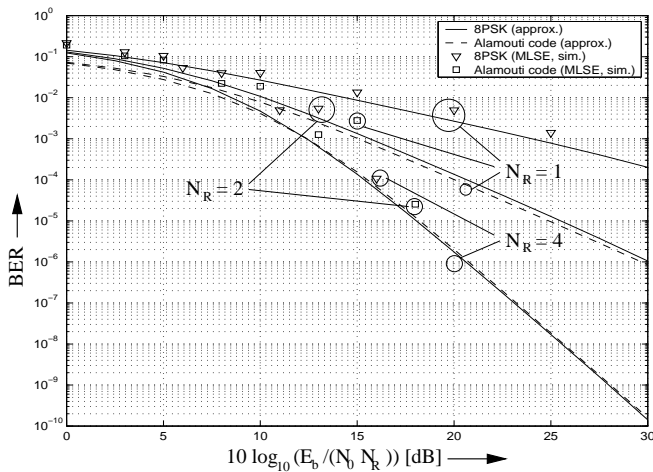


Figure 3: Approximation for BER vs. $10 \log_{10}(E_b / (N_0 N_R))$ for 8PSK transmission over RA channel. For comparison also simulation results for MLSE are shown.

6. Conclusions

In this paper, the performance of space-time block-coded transmission over fading ISI channels has been investigated and a novel design rule for STBC's has been given. A lower bound on the pairwise error probability and an approximation for the BER have been derived. It has been shown that codes designed according to the novel rule can yield a higher diversity order and superior performance than STBC's optimized for flat fading channels. The maximum achievable diversity order for fading channels with L independent impulse response taps, N_T

transmit and N_R receive antennas is $LN_T N_R$. Only one specific code has been given, however, the results of this paper motivate and enable the search for STBC's matched to fading ISI channels.

The proposed approximation for BER is easy to compute and comparisons with simulation results for MLSE have shown its relevance. Finally, it has been shown that space-time block-coded transmission can be highly beneficial for the GSM/EDGE system. Since in most cases MLSE is too complex, our findings should also stimulate the search for low-complexity suboptimum equalizers for STBC's.

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