

Relating Choice of Agent Rationality to Agent Model Uncertainty - an experimental study

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Abstract: The importance of model uncertainties arising from different assumptions about human behavior as opposed to parameter uncertainty is often neglected in integrated models for policy development. In this study, so-called agent model uncertainty is estimated in relation to the choice of agent rationality. A classification scheme is proposed which allows us to organize decision models according to their deviation from full rationality. Five decision models covering the whole range from full rationality to maximum deviation from rationality (random decisions) are classified. They are then used in an existing integrated model simulating crop fertilizer usage and related threshold policies for groundwater protection. Using this model and the different decision models, two hypotheses are tested: 1) that agent model uncertainty increases with increasing deviation from rationality and 2) that agent model uncertainty increases for all decision models similarly and uniformly in response to an increase in noise in the model. Results are analyzed with respect to changes in policy and with respect to the level of weather influence on crop yield. Results show that agent model uncertainty varies with deviation from the purely rational in a non-linear way. Hypothesis two also does not hold. The degree of sensitivity of results with respect to uncertain parameters that the agent needs to consider is very much dependent on the decision model. Therefore it is suggested to test agent-based models for robustness and validity with respect to agent model uncertainty by using different categories of decision models that sample the range of possible rationalities.

Keywords: bounded rationality, agent-based modeling, agent model uncertainty

1 INTRODUCTION

Agent-based models are increasingly used to develop integrated models for developing policy scenarios in resources management. To do so one has to make assumptions on human behavior. The possibility of making choices are numerous [Pahl-Wostl, 2002], but only a few investigations [e.g. Hare and Pahl-Wostl, 2001] exist about the implications of different choices. Uncertainty exists about choice of a decision model from the wide variety of possible models. Agent model uncertainty is defined here as the uncertainty propagated through an agent-based model because of the uncertainty about the decision model to be used [Hare and Pahl-Wostl, 2001]. Since choice of decision model normally reflects an alteration in model structure of some kind, agent model uncertainty can be seen as a sub-class of model form uncertainty as defined by Morgan and Henrion [1990].

Perfect rational behavior (*PRB*) as defined by the Von Neumann - Morgenstern utility [Kreps, 1988] is convenient and successful for many problems, but it does not accurately describe human behavior in many real-world situations [McFadden, 1999].

Bounded rationality has been proposed to be a better concept to describe human decision strategies. However, the term bounded rationality has at least two different meanings [Sent, 1997], which leads to several dimensions along which decision models may deviate from rationality. It is of interest how choice of agent rationality relates to agent model uncertainty. Hypotheses about this relationship are proposed in section 2. An agent-based model is introduced in section 3 and five exemplary decision models are described in section 4 together with a classification of rationality. The decision models are applied to the agent-based model as described in section 5. Agent model uncertainties are then calculated and used to test the proposed hypotheses (Section 6). Conclusions are presented in section 7.

2 HYPOTHESES

Working hypotheses are formulated to support the development of a more systematic approach about how to account for different assumptions of agent rationality in agent-based models. In order to formulate hypotheses relating choice of agent rationality to agent model uncertainty, a classification scheme for decision models is proposed as shown

1. information availability
 - full information about environment, parameter uncertainties as *objective* probability distribution (OPD)
 - full information about environment, parameter uncertainties as *subjective* probability distribution
 - full information about environment, parameters game theoretic treatment
 - percepts as cause and effect
 - percepts without further information
 - percepts from search process with stopping heuristic
2. sampling of alternatives before the decision
 - all alternatives
 - some/one alternatives
 - no beforehand assessment
3. measure for assessment before the decision
 - utility function
 - prospect function (cognitive limitations)
 - probability function
 - other measure
 - no measure
4. selection of an alternative
 - best possibility
 - probability distribution (prob. dist.)
 - selection heuristics
 - ignorance-based decision making
 - One-reason decision making
 - Elimination heuristics
 - Satisficing heuristics for sequential search
 - Random selection.

Figure 1: Decision model classification scheme

in figure 1. The distinction into four dimensions (1. through 4. in figure 1) has its origin in artificial intelligence [Russell and Norvig, 1995, p. 419]. However sampling and measure for the assessment of alternatives before the decision are discussed as one step by Russell and Norvig [1995]. Classes within each dimension are ordered with increasing deviation from rationality in figure 1. Rationality and deviation from rationality are used in a strictly economic sense (PRB) for the classification and hypotheses. Note, that the economic sense can differ from personal intuition about what may or may not be described as rational behavior. See Reusser [2004] for more details on the suggested classification. Based on the classification scheme hypotheses are derived to relate agent model uncertainty to choice of agent rationality. Note that agent model uncertainty requires a reference rationality to allow comparison.

Hypothesis 1 *Agent model uncertainty with reference to a classical rational decision model PRB increases with increasing deviation from rationality.*

Hypothesis 2 *Noise in the decision problem is expected to increase uncertainty for all decision models uniformly which is expected to result in higher agent model uncertainty if a rational decision model (PRB) without noise is used as a reference.*

3 AGENT-BASED MODEL

The agent-based model in this work is based on an existing model by Hare and Pahl-Wostl [2001] investigating agent model uncertainty for nitrogen fertilization on crop fields. The model provides the environment for the decision models (*farmer model*). Low fertilizer application levels lead to low yield and low income, while high levels lead to high yield, high income but groundwater contamination and fines.

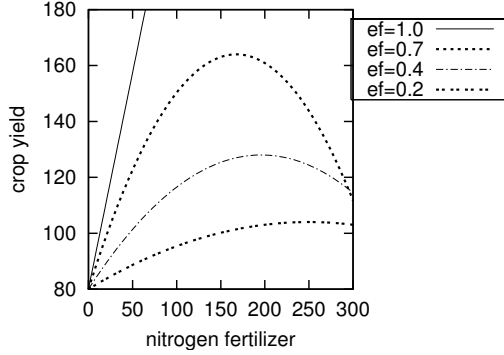
The *field* model determines the crop yield as a function of the fertilizer amount. The yield function (Figure 2) is parabolic and depends on the fertilizer nitrogen uptake effectiveness parameter values (ef). Saturation occurs for higher nitrogen fertilizer amounts. The yield function is also affected by the *climate* model, which reduces the uptake effectiveness randomly within a certain range (see Section 5) for each decision step. The varying uptake effectiveness due to the climate will be referred to as the model noise, about which the farmer decision models are ignorant. The income I is determined by the achieved yield Y together with parameters from the *market* model (price for crop, fertilizer and fertilizer application). The fertilizer that is not taken up by the crop enters the *groundwater* model. Groundwater pollution is not regarded as a problem by the win-oriented farmer *A regulator* model is in charge to decrease groundwater pollution in the presence of win-oriented farmers. If the applied fertilizer amount is higher than the *fertilizer threshold* level t_f , the *regulator* model may or may not *fine* the farmer with probability p_{fined} .

4 DECISION MODELS

Five decision models differing in the choice of agent rationality will be compared in order to test the two hypotheses. The five decision models chosen for this study are meant to be an exemplary set for testing the hypotheses. Many more decision models are possible such as for example a stubborn agent, evolutionary learning agents, and a rational agent with parameter learning abilities. This would be subject to further research. Since agent model uncertainty is a sub-class of model form uncertainty, comparing different decision models means that different models are compared. Structural differences will be

Table 1: Classification of farmer decision models

farmer model	information avail.	sampling	measure	selection
<i>RationalFarmer PRB</i>	full information, OPD	all alternatives	utility function	best alternative
<i>ProspectFarmer</i>	full information, OPD	all alternatives	prospect function	best alternative
<i>ExperientialFarmer</i>	percepts as cause or effect	some alternatives	probability function	prob. dist.
<i>HeuristicFarmer</i>	percepts as cause or effect	no assessment	no measure	selection heuristic
<i>RandomFarmer</i>	no information	no assessment	no measure	random selection


 Figure 2: Crop yield as a parabolic function of the nitrogen fertilizer amount for different uptake effectiveness ef .

come clear in the following sections. Table 1 shows the differences between the models according to the classification scheme (Figure 1).

Using the *RationalFarmer* as a reference point for full rationality, we assume that deviation from rationality increases from the *RationalFarmer* to the *ProspectFarmer*, to the *ExperientialFarmer*, to the *HeuristicFarmer*, and to the *RandomFarmer*. The goal of each decision model is to select the desired possible yield goals the farmer sees to attain. This decision then directly influences the rate of fertilizer applied. For purposes of comparison, each agent shares a common decision space, selecting from possible yield goals of 110, 120, 130, 140, 150, 160, 170, or 180 bundles of crop [Hare and Pahl-Wostl, 2001].

4.1 Rational Decision Model

In the *RationalFarmer* decision model (*PRB*), a Von Neumann-Morgenstern expected utility EU [Kreps, 1988] for each yield goal Y is calculated as

$$EU(Y) = R_f(Y) * p(f|Y) + R_{nf}(Y) * (1 - p(f|Y))$$

with $p(f|Y) = p_{fined}$ if $Y > t_f$ (fertilizer threshold) and $p(f|Y) = 0$ otherwise. The returns R_f and R_{nf} are defined as $R_f(Y) = I(Y) - fine$ and $R_{nf}(Y) = I(Y)$, respectively.

4.2 Decision Model Based on Prospect Theory

Prospect theory by Tversky and Kahneman [1992] reflects five important deviations from expected util-

ity theory, that consistently occur if humans are making decisions. Because of these deviations and in contrast to rational decisions, monetary returns transform to utility in a nonlinear way. Uncertain events such as the probability to be fined are weighted with a function $w()$, overestimating the probability for events with a low probability. In the *ProspectFarmer* decision model, the overall expected prospect $V(Y)$ for a specific yield goal is calculated as

$$V(Y) = -(fine)^\beta * w(p(f|Y)) + I^\alpha$$

with $\alpha \approx \beta \approx 0.88$, w as defined by Tversky and Kahneman [1992], and other symbols as defined before.

4.3 Experiential Decision Model

The *ExperientialFarmer* decision model is a more realistic decision model, which makes it possible to reproduce several stylized facts about human decision behavior as observed in psychological experiments [Arthur, 1993]. The basic idea is to make decisions based on accrued experience, which implies that learning occurs. Probabilities to choose a certain option change proportionally to the accumulated income from using this option. Therefore lock-in on suboptimal strategies may occur if differences between options are small or random processes are present in the decision environment, such as the noise due to the climate model.

4.4 Heuristic Based Decision Model

According to Bock, farmers often use the following heuristic to determine a yield goal: The average yield is calculated for a given field over a 4- or 5-year period and a safety margin $s = 5\%$ is added to that average [Bock and Hergert, 1991, therein Wiese et al., 1987]. Since this method lacks possible reactions to fines, the following extended version is used, where the *HeuristicFarmer* decision model does not add the safety margin if a fine was paid the year before:

$$Y_{Goal} = \frac{\sum_{y=0}^t Y_y}{t} * \left(1 + s - \sum_{y=0}^t b * s * r^y \right)$$

t is the time horizon used by the farmer to average the yield, Y_y is the yield achieved y years ago, and

r is a discount rate. b decides whether a past fine is considered or not: $b = 1$ if $\frac{fine_y}{I_y} > f$ and $b = 0$ otherwise. $fine_y$ is the fine payed y years ago, I_y is the income achieved y years ago, and f is a decision model parameter.

4.5 Random Decision Model

The *RandomFarmer* decision model uses a dice throw to choose the yield goal. It is used as reference for maximum deviation from rationality.

5 EXPERIMENTAL SETUP

The five decision models are applied to the agent-based model with four different fertilizer threshold levels $t_f \in (50, 75, 100, \text{unlimited})$. Additionally, the agent-based model is run on three noise levels (no noise, ef not reduced; medium noise, ef reduced between 0 and 0.2, and high noise, ef reduced between 0 and 0.4). This results in $5 * 4 * 3 = 60$ scenarios. Parameter uncertainty is accounted for with Monte Carlo simulations. For the resulting cumulative probability distribution, we want the 95 % confidence interval for all percentiles to be about plus or minus 3 estimated percentiles. According to Morgan and Henrion [1990, p. 202] this requires 1000 Monte Carlo runs. Each simulation lasts 125 decision steps except for the *ExperientialFarmer* with 1250 decision steps in accordance to Hare and Pahl-Wostl [2001]. The higher number of decision steps for the *ExperientialFarmer* was chosen in order to reduce the effect of the learning phase.

Groundwater pollution level is used to estimate agent model uncertainties since it is a key variable for judging the efficiency of policy measures in this model. Pollution levels are averaged over all decision steps. Cumulative probabilities [Morgan and Henrion, 1990] are obtained by sorting average pollutions x from all Monte Carlo runs and assigning each result a probability of $\frac{1}{n}$.

5.1 Calculation of Agent Model Uncertainty

The mean square difference between two cumulative probability distributions obtained from different agent models is proposed as a measure for agent model uncertainty AMU .

$$AMU = \frac{1}{m} \sum_{i=1}^m (p(x_i) - p_{ref}(x_i))^2$$

with p_{ref} and p the cumulative probabilities for pollution level x_i for the reference scenario and the scenario of interest, respectively. Pollution levels were selected from the wide variety of possible model output parameters for calculation of AMU , because in the context of policy analysis, the main interest is whether the policy used by the regulator to reduce

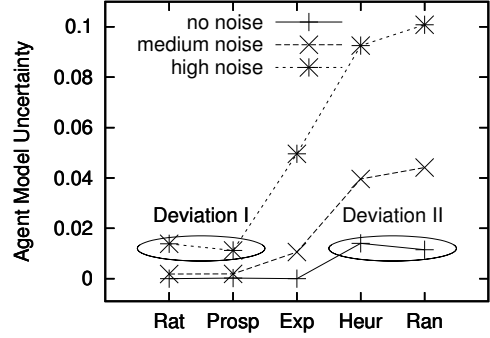


Figure 3: Agent model uncertainty for scenarios with a fertilizer threshold of $t_f = 50$ lbs.

Rat: *RationalFarmer*, Prosp: *ProspectFarmer*, Exp: *ExperientialFarmer*, Heur: *HeuristicFarmer*, Ran: *RandomFarmer*

groundwater pollution has the desired influence. The *RationalFarmer* scenarios with *no noise* are used as *reference on each fertilizer threshold level* (e.g. *RationalFarmer* scenario with $t_f = 50$ lbs and *no noise* is used for all scenarios with $t_f = 50$ lbs). x_i is set to $x_i \in (5, 10, 15, \dots, 445, 450 \text{ units})$ and therefore $m = 90$. $AMU = 0$ only for equal cumulative probability distributions. Therefore, for the reference runs (*Rational-50-no-noise*, *Rational-75-no-noise*, *Rational-100-no-noise*, and *Rational-Baseline-no-noise*) the agent model uncertainty is $AMU = 0$ by definition, since the same cumulative probability function is used as data and as reference.

6 RESULTS AND DISCUSSION

Agent model uncertainties AMU are discussed in the light of the need to test the two hypotheses (Section 2). However, for a more detailed discussion see Reusser [2004]. Figure 3 shows AMU on the y-axis for all scenarios with a fertilizer threshold of $t_f = 50$ lbs. The decision models are ordered along the x-axis with assumed increasing deviation from rationality. Different symbols indicate different noise levels. AMU for the other fertilizer threshold levels t_f are shown in figures 4 through 6. According to hypothesis 1 we expect increasing AMU for increasing deviation from rationality (left to right in the four figures). A first deviation from expectations (Deviation I) is observed in figures 3 and 4, with the AMU for the *Prospect-50-high-noise* scenario being lower than for the *Rational-50-high-noise* scenario. This is due to the fact that the *ProspectFarmer* decision model overestimates the probability to be fined. Because of this overestimation, the pollution level is slightly lower compared to the *RationalFarmer* decision model. Since the pollution for the reference scenario (*Rational-50-no-noise*) is lower as well (be-

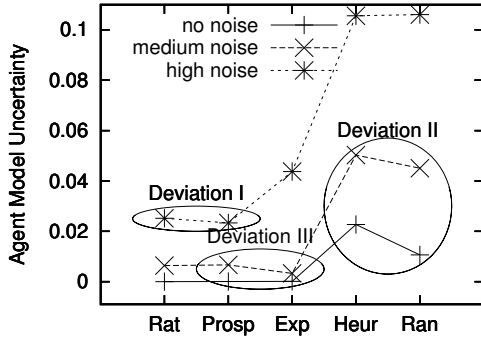


Figure 4: Agent model uncertainty for scenarios with a fertilizer threshold of $t_f = 75$ lbs.

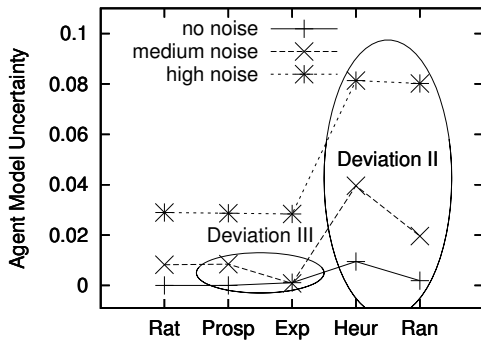


Figure 5: Agent model uncertainty for scenarios with a fertilizer threshold of $t_f = 100$ lbs.

cause of the lower noise level), a lower AMU is observed for the *ProspectFarmer* decision model at high noise levels.

The AMU for the the *HeuristicFarmer* decision model is higher than for the *RandomFarmer* decision model (Deviation II), which can be observed in all four figures. Note that the *RandomFarmer* decision model is the reference for maximum deviation from rationality. However, the *HeuristicFarmer* decision model results in a AMU that is even greater than this. The primary goal of the decision models is to optimize fertilizer usage for high income (it is not to avoid high pollution levels) and therefore such higher deviations are possible. It is noteworthy that the decision model which is supposed to imitate behavior of real farmers shows highest deviations (compared to the other decision models) from rational decisions.

A third deviation is observed in figures 4 through 6. AMU for the *ExperientialFarmer* is smaller than for the *ProspectFarmer* decision model. In fact, the *ExperientialFarmer* and *RationalFarmer* decision models are expected to result in equal decisions, „if alternatives are distinct, non-random and clearly different“[Arthur, 1993] (observed in figure 3). Equal results for the *Experiential-* and the

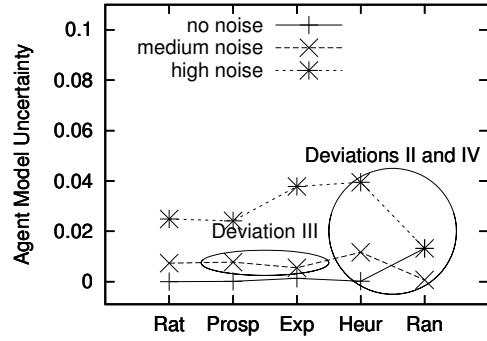


Figure 6: Agent model uncertainty for scenarios with no fertilizer threshold.

RationalFarmer decision model if noise is absent are not observed for higher fertilizer thresholds t_f because of the saturation in the parabolic yield function (Figure 2). Note, that the accrued experience based decision allows the *ExperientialFarmer* decision model to perform better than the *RationalFarmer* decision model on medium noise levels (lower AMU)

In figure 6 the increasing AMU for increasing deviation from rationality can not be observed as expected. In the case of the scenario with no fertilizer limitation, the maximum achievable yield is usually the most rational choice, since additional fertilizer costs are generally low compared to increase in income. All decision algorithms except for the *RandomFarmer* are able to choose a high yield goal if not restricted by a fertilizer threshold. Therefore, almost no difference is observed for the various decision models.

We expect higher AMU as the noise level increases (hypothesis 2). Higher agent model uncertainties are observed for higher noise levels in all four figures, as expected. The single exception is observed in figure 6, where the AMU for the medium noise level is lower than for the no noise level (Deviation IV). However, noise does not increase uniformly for all decision models in contrast to the expectations raised by hypothesis 2. Sensitivity of agent model uncertainty on noise (defined as the difference in AMU between the high-noise and no-noise scenario) is higher for the *Heuristic-* and *RandomFarmer* decision model compared to the other three decision models in figures 3 through 5. No explanation is available for the differing sensitivities toward noise.

7 CONCLUSIONS

In view of the simulation results, the two hypotheses fall. If our assumptions hold, agent model uncertainty does not increase linearly with increasing deviation away from full rationality. The agent model

uncertainty of different decision models changes non-uniformly in response to increases in model noise. Even if the assumed ordering of decision models according to increasing deviation from full rationality is wrong, there is no possible order under which hypothesis 1 would stand given the results of our experimental model simulation. Under the assumption that *AMU* is a useful measure for agent model uncertainty, this means that it is necessary to consider testing decision models with different choice of rationality in agent based models, since uncertainty about which decision model to use can be a significant source of uncertainty in the outcome of such models [Hare and Pahl-Wostl, 2001, see also]. Special care is also necessary in interpreting the basic uncertainty of different decision models if the level of noise is high in the model, since different decision models have different sensitivities toward noise.

More research is necessary to test whether the uncertainty about which decision model to use is still a significant source of uncertainty with a non-linear influence if different model output parameters (such as income) for calculation of *AMU* are used. We expect this to be the case based on the findings of Hare and Pahl-Wostl [2001], where results for profit also showed increases in uncertainty level as a result of changing away from rational farmer towards a more boundedly rational one. While learning is not included as a separate topic in the classification scheme in figure 1, simulation results from the *ExperientialFarmer* decision model suggest that learning processes are of high importance for agent rationality and should be further investigated. This includes farmer agents with a heterogeneous mix of decision models.

In order to investigate the full range of agent rationality, it is suggested to test agent-based models with a set of at least four decision models, one from each of the following four categories: 1) A fully rational model can be used as a reference for a decision model that is based on optimization in a perfectly known world. As an alternative, a somewhat more realistic decision model according to prospect theory could be used. Differences in results are small for the two models and both models are based on *full information availability*. 2) A *learning* decision model such as the *ExperientialFarmer* should be included as well in order to investigate importance of non-linearities for learning processes. 3) A „likeliest case” empirical model (e.g. the *Heuristic-Farmer*) imitating decision mechanisms of relevant stakeholders will allow to estimate the degree of deviation between results from the „likeliest case” model and results from the other decision models. Of course such a „likeliest case” model must be

based on investigations to try to understand how people actually decide. 4) A random decision model should finally be the reference for maximum deviation from rationality.

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