Denoising Quadrature Doppler Signals from Bi-directional Flow Using the Wavelet Frame

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***Abstract* --** A novel approach was proposed to denoise quadrature Doppler signals from bi-directional blood flow using the wavelet frame and a soft-thresholding algorithm. A direction separation step was firstly carried out to avoid the phase distortion of quadrature Doppler signals, which is induced from the non-linear soft-thresholding processing. Then real parts of separated complex signals from the uni-directional flow were denoised independently. Finally quadrature Doppler signals from the bi-directional flow were reconstructed from the denoised separated signals. The approach has been applied to the simulated Doppler signals from a femoral artery. It is concluded from the experimental results that this method is practical for denoising quadrature Doppler signals.

I. INTRODUCTION

Recently, soft-thresholding-based methods with the orthogonal wavelet transform [1],[2] and the wavelet frame [3] were respectively implemented for denoising real Doppler ultrasound signals, which contain no direction information of the blood flow. However, there may exist bi-directional blood flow in human arteries, such as femoral arteries. Quadrature demodulation

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methods are often used to generate complex directional signals [4]. The flow direction information is encoded in the phase relationship between in-phase and quadrature-phase channels. $I(t)$ and $Q(t)$ are used to represent signals from the quadrature channels, from which a complex Doppler signal can be defined as $z(t) = I(t) + jQ(t)$. When signals $I(t)$ and $Q(t)$ are denoised using the wavelet shrinkage method, the phase relationship may be distorted because of the non-linear soft-thresholding processing, which may lead to the wrong direction separation.

In this paper, a noise reduction method based on the wavelet frame is proposed to denoise quadrature Doppler signals from bi-directional blood flow. As we know, the Doppler signal from bi-directional flow can be presented as a linear combination of complex Doppler signals from the uni-directional flow. The complex signal from uni-directional flow can be also described as a combination of the real Doppler signal and the signal from its Hilbert transform. Thus the existing denoising algorithm based on the wavelet frame can be applied to the real parts of the complex signals from uni-directional flow. In this new method, the direction separation is firstly carried out to obtain complex Doppler signals from the forward and the reverse flow, respectively. Then real parts of the complex Doppler signals from uni-directional flow are denoised using the wavelet frame. Finally quadrature Doppler signals are reconstructed from the denoised separated signals. Experiments on simulated quadrature Doppler signals from a femoral artery have validated this approach.

II. METHODS

The basic schema of quadrature Doppler signal denoising is shown in Fig. 1. The method includes three procedures: the direction separation, the signal denoising and the signal reconstruction.

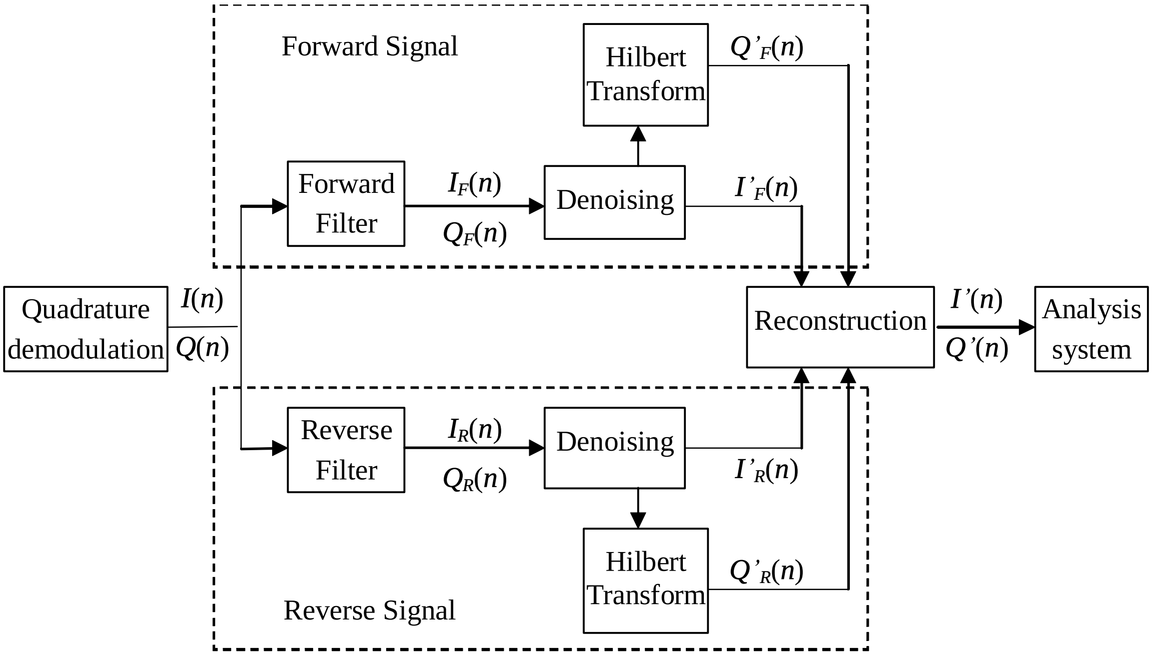

Fig. 1. The framework of quadrature Doppler signal denoising.

A. *The direction separation*

To separate the discrete Doppler signal $z(n)\left(=I(n)+jQ(n)\right)$ from bi-directional blood flow, two complex FIR filters are designed. Generally, the power spectrum of the quadrature Doppler signal from bi-directional flow as shown in Fig. 2(a) is supported for both positive and negative frequencies. After the signal $z(n)$ passes through the filters with frequency response as shown in Fig. 2(b) and (c), the complex Doppler signals $z_F(n)$ $\left(=I_F(n)+jQ_F(n)\right)$ and $z_R(n)$ $\left(=I_R(n)-jQ_R(n)\right)$ from uni-directional blood flow are respectively obtained. The power spectrum of $z_F(n)$ and $z_R(n)$ are shown in Fig. 2(d) and (e), respectively.

B. *Denoising Doppler signals using the wavelet shrinkage method*

It is well known that the imaginary part of the complex Doppler signal from uni-directional flow can be derived from the Hilbert transform of its real part [4]. Therefore, the real parts of the separated signals are extracted and denoised using the wavelet shrinkage method. The denoising

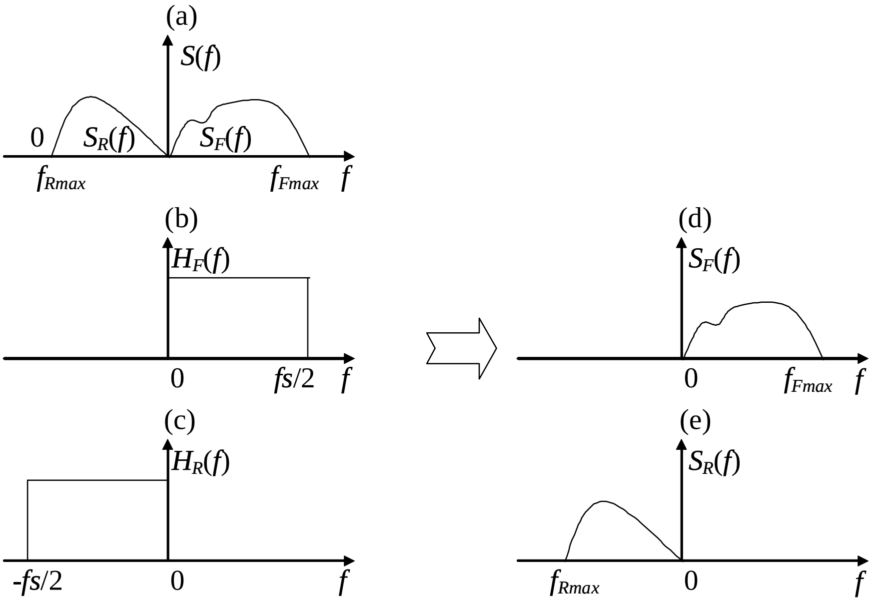

Fig. 2. The schematic diagram of the direction separation in frequency domain. (a) The quadrature Doppler signal, (b) the forward filter, (c) the reverse filter, (d) the forward signal and (e) the forward signal.

procedure can be implemented in three steps [3].

Step 1: wavelet frame decomposition

Here, $x(n)$ is used to represent the real parts ($I_F(n)$ and $I_R(n)$) of Doppler signals from uni-directional flow. The decomposition of the noisy signal $x(n)$ in the Hilbert space l_2 can be expressed as

$$\begin{aligned} s_i(l) &= \langle h_i(k-l), x(k) \rangle_{l_2} \\ d_i(l) &= \langle g_i(k-l), x(k) \rangle_{l_2} \end{aligned}$$

where h and g are the low-pass filter and the high-pass filter from the two-scale relation in time domain [5], i and l are the scale factor and the translation factor, $s_i(l)$ and $d_i(l)$ are the approximation coefficients and the detail coefficients at resolution i.

Step 2: soft-thresholding processing

A Gaussian white noise with a noise level s is presumed in Doppler signals. A soft-thresholding non-linearity is applied to all wavelet detail coefficients with a threshold defined as $t = r_1 \cdot s \cdot \sqrt{2\log(L)}$ [1], where r_1 is a constant, L is the length of the signal. Thus the thresholded wavelet coefficients $h_t(d_i(l))$ can be obtained by

$$h_t(d_i(l)) = \mathrm{sgn}(d_i(l))(|d_i(l)| - t)_+ ,$$

where the '+' term is a nonlinear process for keeping the positive values. It's easy to find the threshold t is proportional to the noise level s, which can be estimated from the standard deviation of the detail coefficients at the first resolution level [2],[3].

Step 3: wavelet frame reconstruction

The noise-free signal $x'(n)$ can be estimated from the reconstruction based on the original approximation coefficients at the coarsest resolution I and the modified detail coefficients at all resolutions:

$$x'(k) = \sum_{l \in Z} s_I(l) h_I(k-l) + \sum_{i=1}^{I} \sum_{l \in Z} d_i(l) g_i(k-l)$$

C. *The quadrature Doppler signal reconstruction*

The Hilbert transforms were carried out for the denoised signals $I'_F(n)$ and $I'_R(n)$ to obtain $Q'_F(n)$ and $Q'_R(n)$. The denoised complex Doppler signals $z'_F(n) = I'_F(n) + jQ'_F(n)$ and $z'_R(n) = I'_R(n) - jQ'_R(n)$ from the uni-directional flow can be easily formed. Finally, the denoised quadrature Doppler signal $z'(n)$ is reconstructed by making a summation of $z'_F(n)$ and $z'_R(n)$.

III. EXPERIMENTAL RESULTS AND DISCUSSION

To validate this novel denoising approach, experiments on simulated quadrature Doppler signals from a femoral artery were carried out. The SNR improvements, waveforms and spectral

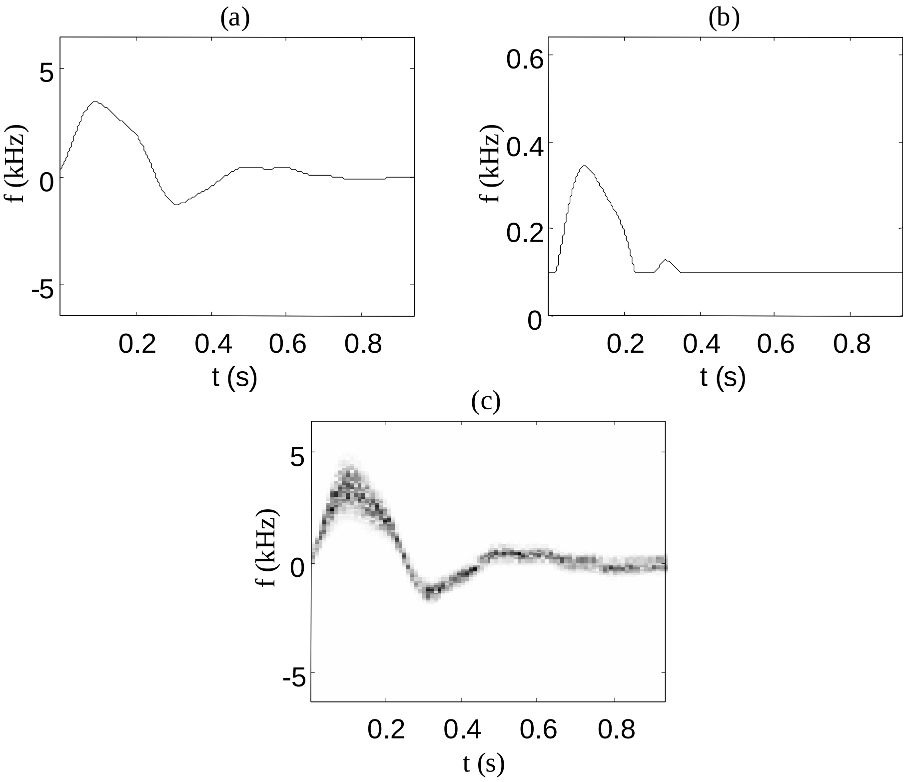

Fig. 3. (a) The mean frequency waveform and (b) the spectral width waveform used in the simulation, (c) the spectrogram of the simulated Doppler signal.

enhancement of the denoised signals were given in the results. As a comparison, the denoising method without the direction separation was also implemented. Both the signal simulation and denoising algorithms were implemented on an AMD Athlon XP personal computer using Matlab 5.3 software.

Doppler signals were simulated using the time-varying filtering method proposed by Wang and Fish [6]. The mean frequency and the spectral width waveforms used in the simulation were shown in Fig. 3(a) and (b), respectively. The power variation was set proportional to the spectral width. The sample rate was 12.8 kHz, and the length of the Doppler signal was 937 ms. The spectrogram of the simulated Doppler signal calculated using the short-time Fourier transform

Table I. SNR improvements after denoising

SNR improvement (dB)	The original SNR (dB)						
	0	2.5	5	7.5	10	12.5	15
Noise1	7.73	7.15	6.61	5.99	5.31	4.44	3.33
Noise2	7.23	6.74	6.31	5.71	5.06	4.22	3.15

with a 10-ms Hanning window, was shown in Fig. 3(c). It’s obvious that the spectra are supported for both positive frequencies and negative frequencies, which indicates the quadrature Doppler signal can represent the bi-directional flow within a femoral artery.

The noises in Doppler systems are often from the quadrature demodulation and the amplifier circuits within the independent quadrature channels, and the band-width of the circuit systems should be much higher than the sample rate. Thus, the Gaussian white noise was used to approximate the noise structure in the real Doppler systems. The Gaussian white noises were added into the original simulated Doppler signal to get the noisy signals with various SNRs. Two complex noise structures were considered in the experiments: (1) $n(t) = s\,e(t)\exp(jf)$, where $e(t)$ is a Gaussian white noise with $N(0,1)$, f is a constant; (2) $n(t) = s_1 e_1(t) + j s_2 e_2(t)$, where $e_1(t)$ and $e_2(t)$ are uncorrelated random variables. Both of the structures were tested in this paper.

Since Doppler signals are band-limited, there are few Doppler components near the fold frequency if the sample rate is set properly. There also exists the wall-filter in most Doppler ultrasound systems. Thus, the pass-band of the complex FIR filters can be set accordingly. The window method was used to calculate impulse response of the filters. A linear-phase FIR filter with a pass-band [100, 6300] Hz was designed by using the least-squares error minimization method. The length of digital filter is 129. The impulse response of the filter $hr(n)$ and its Hilbert transform result $hi(n)$ were calculated. The impulse responses of the complex filters used for separating Doppler signals from the forward and the reverse flow were defined as $hr(n)+jhi(n)$ and $hr(n)-jhi(n)$, respectively.

In the wavelet frame, as a compromise between the denoising performance and the computational complexity, the wavelet basis was a 4th-order function from the near symmetry wavelet family, and the coarsest resolution I was 6, which was chosen according to the cut-off frequency of the complex filter [3]. Quadrature Doppler signals were separated, denoised and reconstructed as described in Section II.

The SNR improvements of the denoised signals were calculated based on 100 independent realizations, and the results were shown in Table I. It’s found that considerable SNR improvements are obtained for all signals with various SNRs after applying this wavelet denoising method. Because the shrinkage not only wipes off most noisy components but also leaves out some Doppler signal components with low amplitudes, the SNR improvements can’t infinitely increase, which become more and more obvious while increasing the SNR of the noisy signal. It’s also shown that the SNR improvements are similar for both noise structures.

As an example, one segment of quadrature Doppler signals was extracted. The real parts and the amplitude spectra of the original signal, the noisy signal with 5 dB SNR and the denoised signals were shown on Fig. 4. In Fig. 4(c) and (d), it’s observed a mass of noises are presented both in the time domain and in the frequency domain. Fig. 4(e) and (f) showed the signal denoised without the direction separation while Fig. 4(g) and (h) showed the signal denoised with the direction separation. The noise reduction can be found both in the waveforms and the spectra. However, it is easy to found Fig. 4(f) contains more distortions than Fig. 4(h) although their real part waveforms are very similar, which indicates the phase distortion is induced by independently denoising the real part and the imaginary part of the quadrature signal.

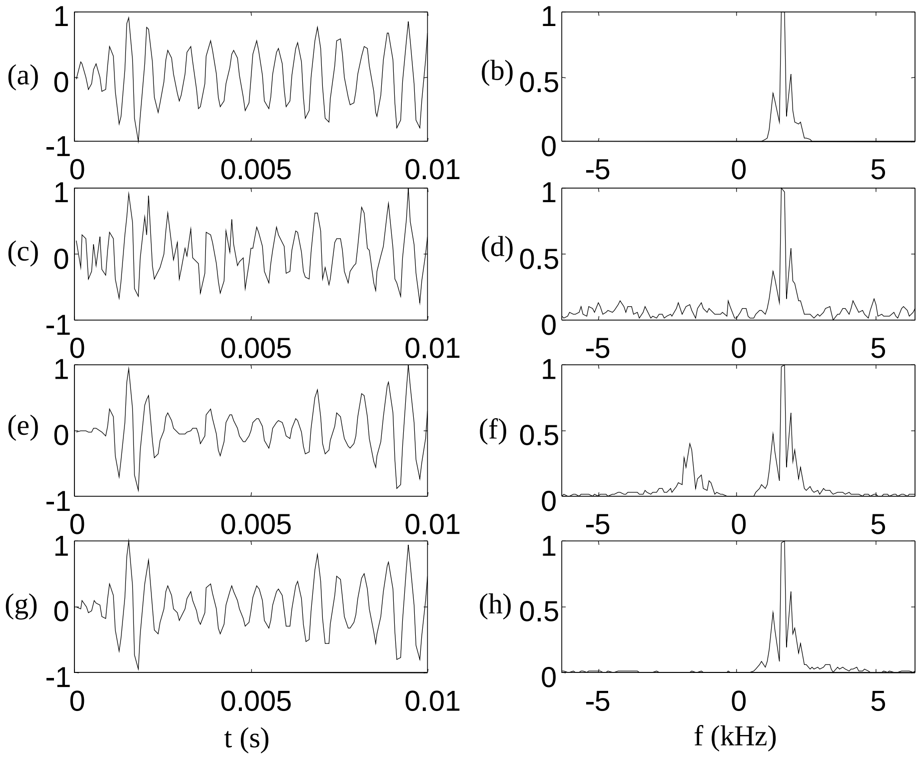

Fig. 4. The left are the real parts of the quadrature Doppler signal while the right are the corresponding spectra. (a),(b): the original signal; (c),(d): the noisy signal; (e),(f): the signal denoised without direction spearation; (g),(h): the signal denoised with direction spearation. (The amplitudes of waveforms and spectra are normalized.)

Furthermore, the overall performance of the spectrogram enhancement of a signal with 5 dB SNR was also given in Fig. 5. The maximum frequency waveforms were estimated from the spectrograms using the standard percentile method [7]. The fewer noise components and the smoother maximum frequency waveforms in Fig. 5(d) than those in Fig. 5(c), confirm the better performance achieved by this novel denoising approach.

IV. CONCLUSION

A soft-thresholding-based method with the wavelet frame is proposed to denoise quadrature Doppler signals from bi-directional flow. Two complex FIR filters are used to get Doppler signals from uni-directional blood flow. Then the wavelet shrinkage is applied to the separated signals. Finally the denoised quadrature Doppler signal is reconstructed from the separated signals. From the experiments on simulated Doppler signals from a femoral artery, it's shown the novel denoising method has good performance and efficiently avoids the phase distortion in the denoising method without the direction separation. It is concluded that the wavelet shrinkage method with the direction separation is a practical algorithm for the quadrature Doppler signal denoising.

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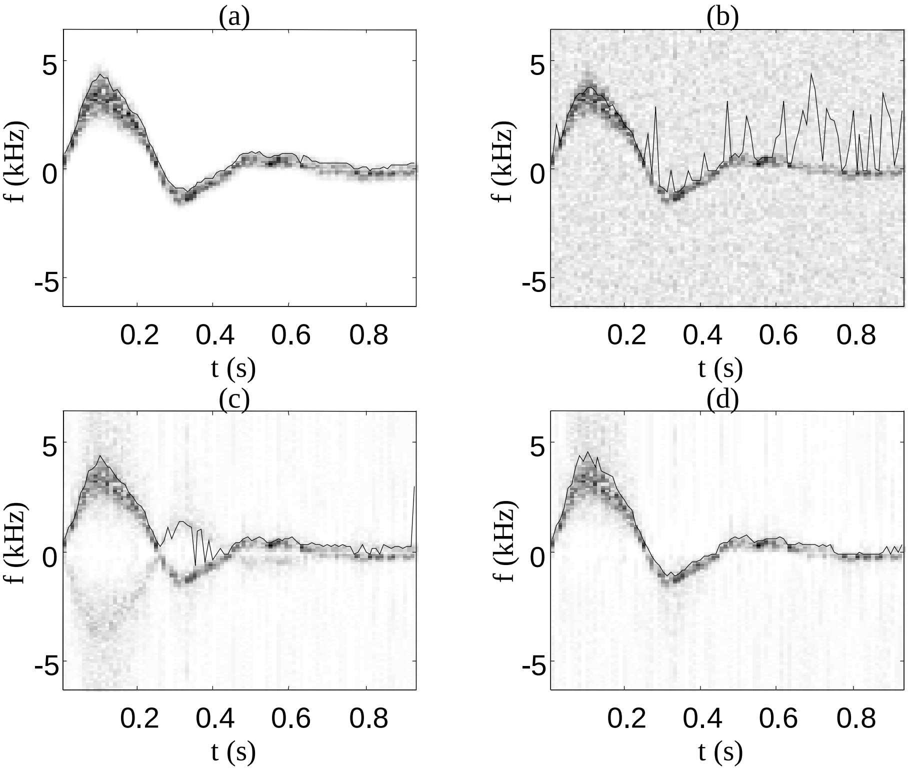

Fig. 5. Overall performance of the spectrogram enhancement. The solid curve superimposed on the spectrogram is the maximum frequency waveform. (a) the original signal, (b) the noisy signal with SNR of 5 dB, (c) the signal denoised without direction separation, (d) the signal denoised with direction separation.

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