Soft Morphological Structuring Element Decomposition

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Abstract **:** In this paper a decomposition technique suitable for any grey-scale soft morphological structuring element is presented. A hardware structure implementing this technique is also presented.

Key words : soft mathematical morphology, structuring elements.

1. Introduction

Mathematical morphology is a methodology for image analysis based on set theory and topology [1]. Its two basic operations are erosion and dilation, from which all the other operations are composed. A relatively new approach to mathematical morphology is soft mathematical morphology. This combines mathematical morphology with weighted order statistics. It has been shown that soft morphological operations are less sensitive to additive noise and to small variations in object shape [2].

Applications such as textural segmentation and granulometry require the application of successively larger size structuring elements [1]. However, in morphological processors the size of the structuring element is usually restricted to 3x3 pixels [3-4]. The handling of larger size structuring elements on existing machines is not a straightforward process. Structuring element's decomposition is required in such cases. This practical problem has concerned many researchers; binary morphology [5-6] and grey-scale morphology [7-8]. One decomposition strategy is to represent the structuring element as successive dilations of smaller structuring elements. Algorithms for optimal structuring element decomposition based on the latter strategy have been presented [5-7]. Also, several methods for decomposing structuring elements into combined structures of segmented small components have been presented [8].

In this paper, a decomposition technique for any grey-scale soft morphological structuring element is presented for the first time. The difference between a standard morphological structuring element and a soft morphological structuring element is that the domain of the soft morphological structuring element is split into two subsets, i.e. the core and the soft boundary. According to the proposed decomposition technique the domain of a large size structuring element (both the core and the soft boundary) is divided into non-overlapping sub-domains. The small structuring elements which obtain values from these sub-domains are used to form the soft morphological operations locally. The final result is composed from the latter local results. An architecture implementing this technique is also presented.

2. Notations and Definitions

Let $\{k\Diamond f(x)\}$ denote the k times repetition of $f(x)$, i.e. $\{k\Diamond f(x)\}=\{f(x), f(x), \ldots f(x)\}$ (k times). Also let $\min^{(k)}$ denote the kth order statistic of a multiset with N elements (N>k), i.e.

the kth smallest element of the multiset and let $\max^{(k)}$ denote the (N-k+1)th order statistic of the multiset, i.e. the kth largest element of the multiset. A multiset is a collection of objects, where the repetition of objects is allowed. Soft morphological dilation and erosion of a grey-scale image f by a soft grey-scale structuring element [a, $ß$, k], have been defined as follows [9] :

$$f \oplus [a, ß, k](x) = \max^{(k)}_{\substack{(x-y)\in \mathrm{G_A} \\ (x-z)\in \mathrm{G_B}}} \left(\left\{ k \Diamond \left(f(y) + a(x-y) \right) \right\} \cup \left\{ f(z) + ß(x-z) \right\} \right) \text{ and } \tag{1}$$

$$f \,\mathrm{T} [a, ß, k](x) = \min^{(k)}_{\substack{(x+y)\in \mathrm{G_A} \\ (x+z)\in \mathrm{G_B}}} \left(\left\{ k \Diamond \left(f(y) - a(x+y) \right) \right\} \cup \left\{ f(z) - ß(x+z) \right\} \right) \tag{2}$$

respectively,

where k is the order index

$x, y, z \in \mathrm{E}^{\mathrm{N}}$ are the spatial coordinates,

$f: \mathrm{F} \rightarrow \mathrm{E}$ is the grey-scale image,

$a: \mathrm{G_A} \rightarrow \mathrm{E}$ is the core of the structuring element,

$ß: \mathrm{G_B} \rightarrow \mathrm{E}$ is the soft boundary of the structuring element,

$\mathrm{F}, \mathrm{G_A}, \mathrm{G_B} \subseteq \mathrm{E}^{\mathrm{N}}$ are the domains of the grey-scale image, the core and the soft boundary of the structuring element, respectively, and $\mathrm{G_B}=\mathrm{G}\backslash\mathrm{G_A}$, where $\mathrm{G} \subseteq \mathrm{E}^{\mathrm{N}}$ is the domain of the grey-scale structuring element and "\" stands for set difference.

3. Soft Morphological Structuring Element Decomposition Technique

The proposed decomposition technique is described in this section. The domain G of the structuring element is divided into smaller non-overlapping subdomains G_1, G_2, ... G_n. Also, $\mathrm{G}_1 \cup \mathrm{G}_2 \cup ... \mathrm{G}_n = \mathrm{G}$. The soft morphological structuring elements obtain values from these domains and they are denoted by $[?_1, \mu_1, k]$, $[?_2, \mu_2, k]$, ... $[?_n, \mu_n, k]$, respectively. These have common origin, which is the origin of the original structuring element. Additionally, the points of G which belong to its core are also points of the cores of G_1, G_2, ... G_n and the points of G which belong to the soft boundary are also points of the soft boundaries of G_1, G_2, ... G_n. This process is graphically illustrated in Figure 1. In this figure the core of the structuring element is denoted by the shaded area.

Then, soft dilation and erosion are computed as follows :

$$f \oplus [a,ß,k](x) = \max_{i=1}^{n}{}^{(k)} [\max_{\substack{(x\text{-}y)\in G_A \\ (x\text{-}z)\in G_B \\ j=1}}^{k}{}^{(j)} \left(\left\{ k\Diamond\left(f(y) + ?_i(x-y) \right) \right\} \cup \left\{ f(z) + \mu_i(x-z) \right\} \right)] \tag{3}$$

$$f\,\Theta[a,ß,k](x) = \min_{i=1}^{n}{}^{(k)} [\min_{\substack{(x\text{-}y)\in G_A \\ (x\text{-}z)\in G_B \\ j=1}}^{k}{}^{(j)} \left(\left\{ k\Diamond\left(f(y) - ?_i(x+y) \right) \right\} \cup \left\{ f(z) - \mu_i(x+z) \right\} \right)] \tag{4}$$

respectively.

Proof :

$$\forall (x-y) \in G_A : a(x-y) = \bigcup_{i=1}^{n} ?_i(x\text{-}y)$$

$$\Rightarrow f(y) + a(x-y) = \bigcup_{i=1}^{n} \left[f(y) + ?_i(x\text{-}y) \right]$$

$$\Rightarrow k\Diamond\left(f(y) + a(x-y) \right) = k\Diamond\left(\bigcup_{i=1}^{n} \left[f(y) + ?_i(x\text{-}y) \right] \right) =$$
$$k\Diamond\left(f(y) + ?_1(x-y), f(y) + ?_2(x-y), \ldots f(y) + ?_n(x-y) \right) \tag{5}$$

Also, $\forall (x-z) \in G_B : ß(x-z) = \bigcup_{i=1}^{n} \mu_i(x\text{-}z)$

$$\Rightarrow f(z) + ß(x-z) = \bigcup_{i=1}^{n} \left[f(z) + \mu_i(x\text{-}z) \right] =$$
$$f(z) + \mu_1(x-z), f(z) + \mu_2(x-z), \ldots f(z) + \mu_n(x-z) \tag{6}$$

Therefore

$$f \oplus [a,ß,k](x) = \max_{\substack{(x-y)\in G_A \\ (x-z)\in G_B}}{}^{(k)} \begin{pmatrix} \left\{ k\Diamond\left(f(y) + ?_1(x-y), f(y) + ?_2(x-y), \ldots f(y) + ?_n(x-y) \right) \right\} \cup \\ \left\{ f(z) + \mu_1(x-z), f(z) + \mu_2(x-z), \ldots f(z) + \mu_n(x-z) \right\} \end{pmatrix}$$

$$= \max_{\substack{(x\text{-}y)\in G_A \\ (x-z)\in G_B}}{}^{(k)} \begin{pmatrix} \left\{ k\Diamond\left(f(y) + ?_1(x-y) \right) \right\} \cup \left\{ f(z) + \mu_1(x-z) \right\}, \\ \left\{ k\Diamond\left(f(y) + ?_2(x-y) \right) \right\} \cup \left\{ f(z) + \mu_2(x-z) \right\}, \\ \ldots \\ \left\{ k\Diamond\left(f(y) + ?_n(x-y) \right) \right\} \cup \left\{ f(z) + \mu_n(x-z) \right\} \end{pmatrix}$$

$$= \max_{\substack{(x\text{-}y)\in G_A \\ (x\text{-}z)\in G_B \\ i=1}}^{n}{}^{(k)} \left[\left\{ k\Diamond\left(f(y) + ?_i(x-y) \right) \right\} \cup \left\{ f(z) + \mu_i(x-z) \right\} \right]$$

The above equation can be expressed, in terms of order statistics of the multiset, as follows :

$$f \oplus [a,ß,k](x) = \max_{i=1}^{n}{}^{(k)} [\max_{\substack{(x\text{-}y)\in G_A \\ (x\text{-}z)\in G_B}}{}^{(N)} \left(\left\{ k \Diamond \left(f(y) + ?_i(x-y) \right) \right\} \cup \left\{ f(z) + \mu_i(x-z) \right\} \right),$$

$$\max_{\substack{(x\text{-}y)\in G_A \\ (x\text{-}z)\in G_B}}{}^{(N-1)} \left(\left\{ k \Diamond \left(f(y) + ?_i(x-y) \right) \right\} \cup \left\{ f(z) + \mu_i(x-z) \right\} \right),$$

$$\ldots$$

$$\max_{\substack{(x\text{-}y)\in G_A \\ (x\text{-}z)\in G_B}} \left(\left\{ k \Diamond \left(f(y) + ?_i(x-y) \right) \right\} \cup \left\{ f(z) + \mu_i(x-z) \right\} \right)]$$

where N is the number of the elements of the multiset.

However, if an element is not greater than the local (N-k)th order statistic, then it cannot be greater than the global (N-k)th order statistic. Therefore, the terms $\max^{(N)} \ldots \max^{(k+1)}$ can be omitted :

$$f \oplus [a,ß,k](x) = \max_{i=1}^{n}{}^{(k)} [\max_{\substack{(x\text{-}y)\in G_A \\ (x\text{-}z)\in G_B}}{}^{(k)} \left(\left\{ k \Diamond \left(f(y) + ?_i(x-y) \right) \right\} \cup \left\{ f(z) + \mu_i(x-z) \right\} \right),$$

$$\max_{\substack{(x\text{-}y)\in G_A \\ (x\text{-}z)\in G_B}}{}^{(k-1)} \left(\left\{ k \Diamond \left(f(y) + ?_i(x-y) \right) \right\} \cup \left\{ f(z) + \mu_i(x-z) \right\} \right),$$

$$\ldots$$

$$\max_{\substack{(x\text{-}y)\in G_A \\ (x\text{-}z)\in G_B}} \left(\left\{ k \Diamond \left(f(y) + ?_i(x-y) \right) \right\} \cup \left\{ f(z) + \mu_i(x-z) \right\} \right)]$$

$$= \max_{i=1}^{n}{}^{(k)} [\max_{\substack{(x\text{-}y)\in G_A \\ (x\text{-}z)\in G_B \\ j=1}}^{k}{}^{(j)} \left(\left\{ k \Diamond \left(f(y) + ?_i(x-y) \right) \right\} \cup \left\{ f(z) + \mu_i(x-z) \right\} \right)]$$

Similarly eqn (4) can be proven.

Example : Let us consider the following image f and soft structuring element [a, $ß$] :

11	15	10	0	0	12

f

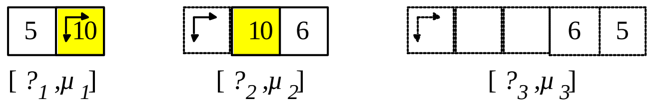
5	10	10	6	6	5

[a,$ß$]

Soft dilation at point (0,0) for k=2, according to eqn. (1) is :

$$f \oplus [a,ß,\ 2](0,0) = \max^{(2)} \left(\{ 2 \Diamond (10,10) \} \cup \{16,21,16,17\} \right) = \max^{(2)} (10,10,10,10,16,21,16,17) = 17$$

According to the proposed technique the structuring is divided into three structuring elements :



The following multisets are obtained from the above structuring elements :{2◊(10),17} , {2◊(10),16} and {16,15}, for the first the second and the third stucturing elements, respectively. From these multisets the max and the max$^{(2)}$ elements are retained : ({17,10} and {16,10} and {21,16}). The max$^{(2)}$ of the union of these multisets, i.e. 17, is the result of soft dilation at point (0,0). It should be noticed that although 17 is the max of the first multiset, it is the max$^{(2)}$ of the global multiset.

4. Architecture for Decomposition of Soft Morphological Structuring Elements

An architecture for the implementation of decomposition of soft morphological structuring elements is depicted in Figure 2. The structuring element is loaded to the *structuring element management* module. This divides the structuring element into n smaller structuring elements and provides the appropriate one to the next stage. The pixels of the image are imported to the *image window management* module. This provides an image window which interacts with the appropriate structuring element, provided by the *structuring element management* module. Both the previous modules consist of registers and multiplexers (MUXs), controlled by a counter mod$n$, as shown in Figure 3. The second stage, i.e. the *arithmetic unit* (Figure 4) consists of adders/subtractors (dilation/erosion) and of an array of MUXs that are controlled by the order index k. The MUXs provide the multiple copies of the addition/subtraction results to the next stage, i.e. an array of *order statistic modules* (OSMs). An OSM is capable of computing any order statistic of a multiset [4, 10]. Of course, instead of using an array of k OSMs, only one OSM may be used, at the expense of speed. The max$^{(l)}$/min$^{(l)}$ results (l=1, ... k) of every multiset are collected by means of an *array of registers*. These registers provide the $n \times k$ max$^{(l)}$/min$^{(l)}$ of the n multisets concurrently to the last stage OSM which computes the final result according to eqns. (3) or (4).

5. Conclusions

A decomposition technique for any grey-scale soft morphological structuring element has been presented in this paper. According to this technique the domain of the structuring element is divided into non-overlapping sub-domains. These sub-domains are used to compute the soft morphological operations locally. From these local results the global result of the soft morphological operation is composed. An architecture suitable for real time applications implementing this technique has been also presented.

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FIGURE CAPTIONS

Figure 1 : Example of a 4x4 soft structuring element decomposition.

Figure 2 : Architecture for the implementation of the decomposition technique.

Figure 3 : Data window management

Figure 4 : Arithmetic unit

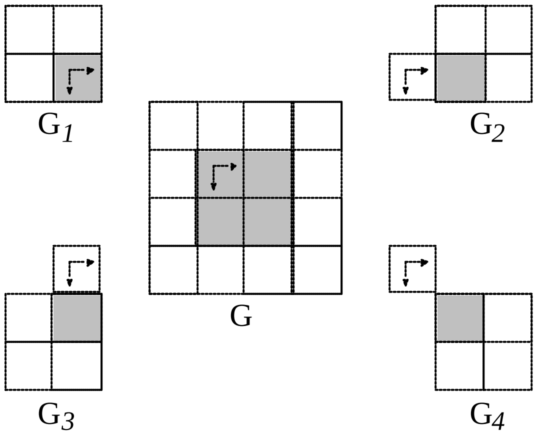

Figure 1

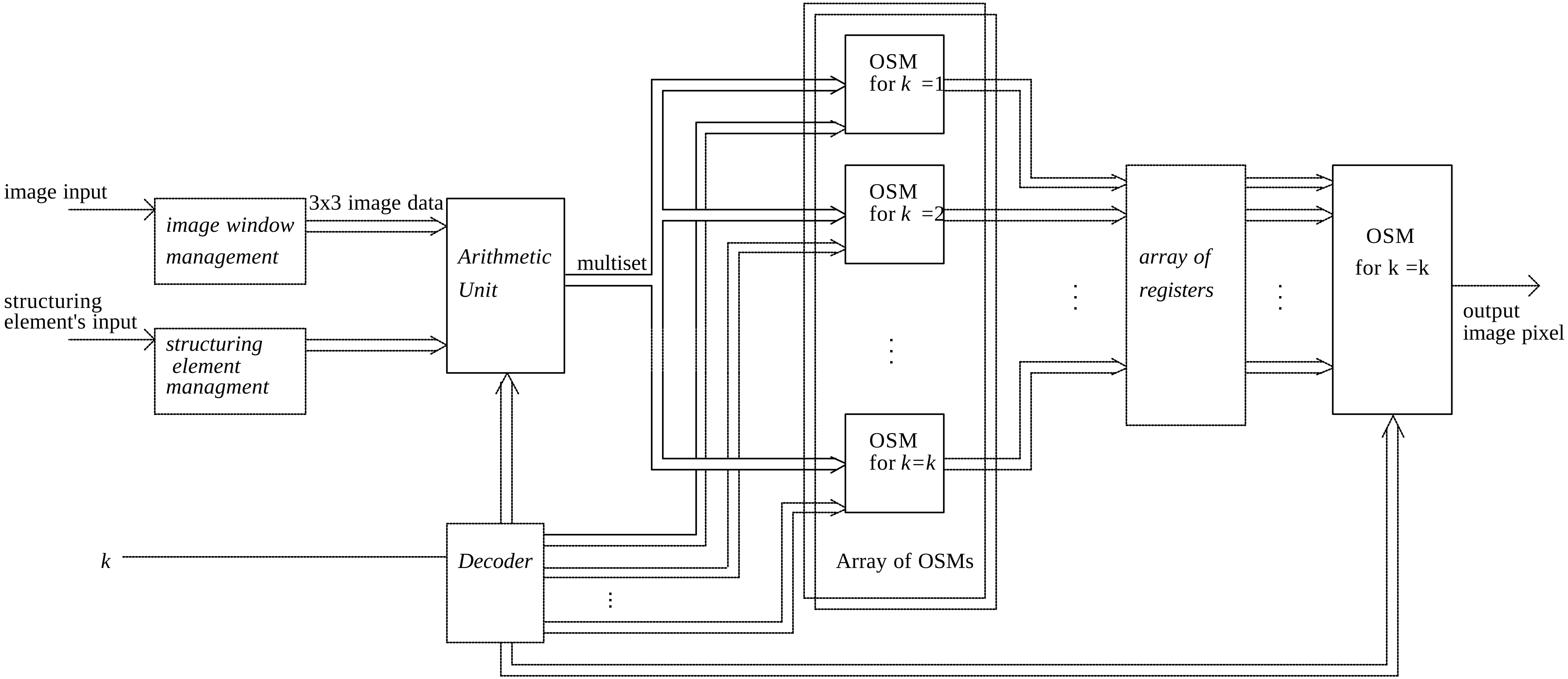
image input
image window management
3x3 image data
structuring element's input
structuring element managment
Arithmetic Unit
multiset
OSM for k =1
OSM for k =2
OSM for k=k
Array of OSMs
array of registers
OSM for k =k
output image pixel
k
Decoder

Figure 2

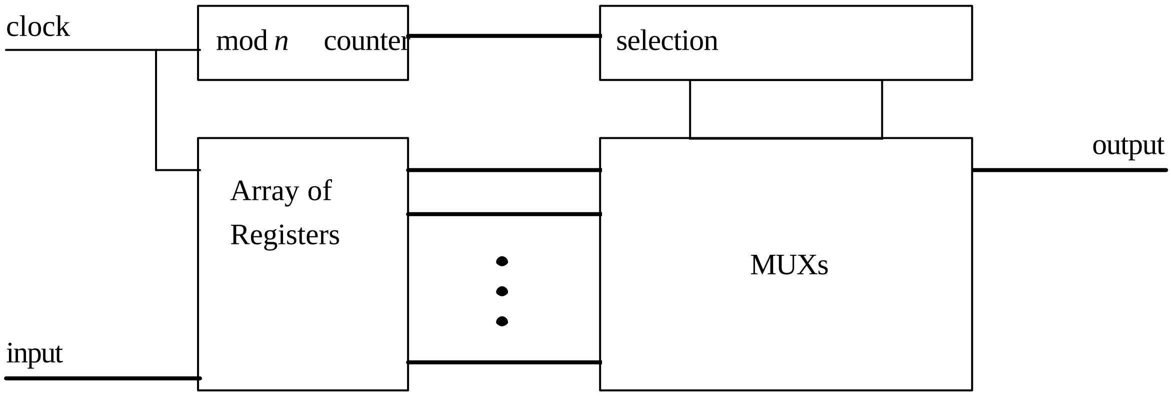
clock
mod n counter
selection
output
Array of
Registers
MUXs
input

Figure 3

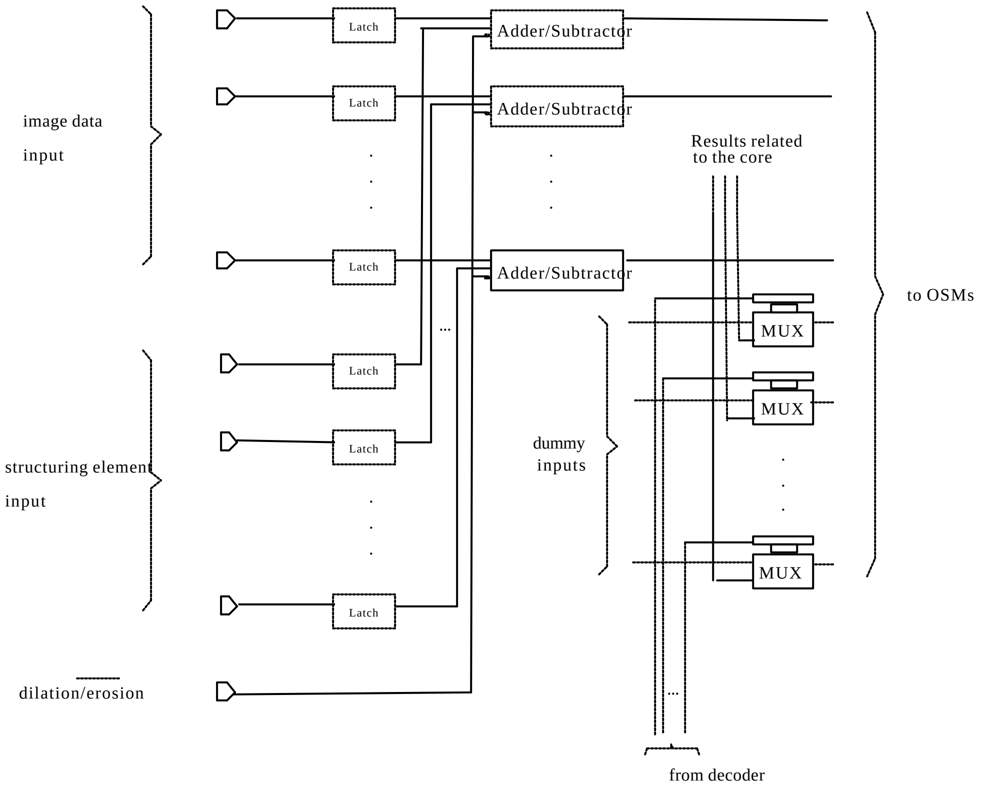
image data
input
Latch
Latch
Latch
Latch
Latch
Latch
Adder/Subtractor
Adder/Subtractor
Adder/Subtractor
Results related
to the core
to OSMs
structuring element
input
dummy
inputs
MUX
MUX
MUX
dilation/erosion
from decoder

Figure 4