# FILTERING INTERFEROMETRIC PHASE IMAGES BY ANISOTROPIC DIFFUSION

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**ABSTRACT**

We present an anisotropic diffusion equation designed to restore interferometric images. It has two main purposes. the first is to preserve the structures and discontinuities formed by the fringes. The second is to incorporate the noise modeling which is specific to this kind of images. Besides we show that our model formalizes previous related work in interferometry filtering.

## 1. INTRODUCTION

Interferometry with Synthetic Aperture Radars (SAR) has become of main interest in order to obtain topographic measurements. The SAR radar images the same scene from two slightly different look angles. The interferometric product gives the interferogram and the coherence map. The interferogram represents the phase difference between the two acquisitions and only contains the information to estimate terrain height. This phase image has fringes representing the phase within the range of $-\pi$ to $\pi$. Any phase values greater than $\pi$ are wrapped back around to $-\pi$. To reconstruct the geometry for each point in the image, the phase needs to be unwrapped. Due to phase discontinuities this problem is ambiguous and makes phase unwrapping a complex and crucial processing. Another difficulty comes from the high level of speckle noise which will introduce some errors in the reconstruction. To reduce phase noise, multilook [1] processing can be used, but this implies a loss of spatial resolution. For that reason, noise filtering is applied before phase unwrapping.

Several filters have been applied to this kind of images [2, 3], but they are not adapted to local noise level variations. To preserve phase discontinuities, most of them unwrap the phase in the small filtering window before smoothing, and wrapping it again. Amongst the class of filtering approaches, we focus on the J.S. Lee et al. [4] filtering method which preserves phase gradient and reduces phase noise according to the coherence. It contains two main features. The first is that the specificity of the interferometric noise is taken into account. The variance of the phase noise depends on the coherence, that is the best estimator of the quality of the phase data extracted from the interferometric radar data. The second is the use of a directional window dependent of the noise level along interferometric fringes, in order to smooth within a fringe and prevent smoothing across fringes.

This idea of filtering while preserving some structures has been widely expressed in the framework of partial differential equations (PDE's). The so called anisotropic diffusion equations allow to remove noise while keeping the edges ( see [5] for a review). So it appears natural to introduce such a methodology for interferometric image filtering.

We propose in this paper an anisotropic diffusion model which incorporates the specificities of noise statistics. In Section 2, we recall the *probability density function (pdf)* of interferometric phase images, and the noise decomposition usually taken for these images. Section 3 presents the first step of our new noise filter, which is the restoration of the coherence map. In Section 4 we describe the anisotropic filter proposed by Weickert [6], and adapt it to interferograms. some results showing the advantages of this approach are discussed in Section 5.

## 2. CHARACTERISTICS OF PHASE NOISE

### 2.1. Construction of the interferometric product

The optimal strategy for topographic mapping requires simultaneous acquisition of images forming the interferometric pair. Let $(y_1, y_2) = (|y_1|e^{i\varphi_1}, |y_2|e^{i\varphi_2})$ be the two complex valued images of the same scene with slight different geometry, acquired by the sensor. Interferometry process combines $y_1$ and $y_2$ to extract informations of the surface topography. But SAR images are affected by speckle. One way to improve SAR images, at some expense of geometric resolution, is given by the multilook process. The multilook interferogram $z$ is implemented by averaging the correlation on neighboring pixels of a window of size $N_l$:

$$z = \frac{1}{N_l} \sum_{k=1}^{N_l} y_1(k)\overline{y_2}(k) = |z|e^{i\varphi} \tag{1}$$

where $\varphi = \varphi_1 - \varphi_2$ is the interferometric phase.

Moreover, the interferometric phase depends on the correlation between the images of an interferometric pair. The complex coefficient

$$\rho_c = \frac{E[y_1 y_2^*]}{\sqrt{E[|y_1|^2]E[|y_2|^2]}} = \rho_0 e^{i\theta}$$

is a statistical measure of the correlation between images. The coherence magnitude $\rho_0$ between the two images, given by the interferometric product, is a measure of the data quality. It is the best estimator of the quality of the phase data extracted from interferometric data. Comparison between the correlation map and the phase data shows that the decorrelated (noisy) regions of the phase data corresponds to the lowest-quality regions of the coherence image.

### 2.2. Phase noise model

Complex SAR images can be characterized as circular Gaussian random variables [7]. The expression of the 1-look *pdf* of the phase $\varphi$ is given by ($N_l = 1$ in (1)):

$$\left\{ \begin{array}{l} f_\varphi(\varphi) = \dfrac{(1-{\rho_0}^2)[(1-\beta(\varphi)^2)^{1/2}+\beta(\varphi)(\pi - arccos\ \beta(\varphi))]}{2\pi(1-\beta(\varphi)^2)^{3/2}} \\ \beta(\varphi) = \rho_0\ cos(\varphi - \theta),\ \varphi \in [-\pi+\theta, \pi+\theta[ \end{array} \right. \quad (2)$$

Let us note that the standard deviation $\sigma$ of the interferogram depends on the coherence $\rho_0$.

Since $\beta(\theta - \varphi) = \beta(\theta + \varphi)$, the *pdf* of the interferogram $\varphi$, defined in (2), is symmetrical with respect to $\theta$. It follows that $E[\varphi] = \theta$, and we can easily show that the standard deviation of $\varphi$ is not a function of the parameter $\theta$. This fact enables us to eliminate the multiplicative noise model. Every random variable $Z$, such that $E[Z] = \theta$, can be split in the form $Z = \theta + (Z - \theta)$. From this observation, if we note $\tilde{\varphi}$ the interferometric phase without noise and $b$ the noise with zero mean, an additive noise model is often proposed:

$$\varphi = \tilde{\varphi} + b \text{ where } \left\{ \begin{array}{l} \tilde{\varphi} \text{ and } b \text{ are independent} \\ \text{and } E[b] = 0 \end{array} \right. \quad (3)$$

## 3. VARIATIONAL FILTER FOR THE COHERENCE

The coherence magnitude between the images is classically used as a measure of the quality of the interferogram. Let us remind that our filter will take into account the variance of the interferometric noise that depends on the coherence. Because the coherence map looks "grainy" it is commonly smoothed by averaging in $3\times3$ windows. But such a method does not preserve edges.

We propose here to filter the given coherence $\rho_0$ through the classical Perona-Malik nonlinear diffusion equation in order to keep the edges as much as possible [5].

$$\left\{ \begin{array}{ll} \partial_t \rho = div(d(|\nabla\rho|)\nabla\rho), & \text{in } \Omega \times (0,\infty) \\ \langle \nabla\rho, n \rangle = 0, & \text{on } \partial\Omega \times (0,\infty) \\ \rho(x,0) = \rho_0(x), & \text{in } \Omega \end{array} \right. \quad (4)$$

where $n$ denotes the outer normal, $\langle .,. \rangle$ is the usual inner product and $d(s) = 1/(1+s^2)$. Figure 1 shows the result on the coherence map of the region of Utah.

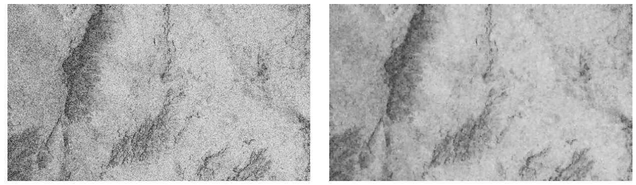

**Fig. 1**. Filtering of the coherence map with total variation on the region of Utah. Left: initial coherence $\rho_0$. Right: filtered coherence $\rho$.

## 4. NOISE ADAPTIVE ANISOTROPIC FILTER

Variational filters are often used for optical images where the noise is usually considered as gaussian. Observing the *pdf* of the interferometric phase noise, given in equation (2), it is then clear that standard variational methods will not be adapted. In this section, we propose a new noise adaptive filtering model, linked to anisotropic diffusion equations, that preserves phase jumps discontinuities (fringes boundary) and reduces noise according to the coherence. Moreover phase interferometric images present typical structures because of fringes. It is crucial to take into account the direction of fringes to smooth interferograms. To do that, we use an edge-descriptor different from the gradient. For optical images, Weickert [6] has introduced a structure tensor that describes the local image structure at each pixel. Then, instead of the scalar diffusivity $d$ in equation (4), Weickert propose to use in the nonlinear diffusion process, a diffusion tensor, adapted to the image for enhancing coherent structures. The preferred direction is determined according to the phase angle of the structure tensor. We will show how to compute this structure tensor taking into account noise information according to Lee et al. [4].

### 4.1. The structure tensor

The preferred smoothing direction is the one that minimizes gray value variations. The idea to take into account local variations is to consider the quadratic form:

$$f(d) = d^T \nabla\varphi . \nabla\varphi^T d$$

The direction $d$ which maximizes (resp. minimizes) $f(d)$, is the one of highest (resp. lowest) variations. It corresponds to the highest (resp. lowest) eigenvector of $\nabla\varphi.\nabla\varphi^T$, and is parallel (resp. perpendicular) to $\nabla\varphi$. To avoid noise amplification in computing $\nabla\varphi$, the interferogram is regularized:

$$\varphi_\sigma(x,t) = (K_\sigma * \overline{\varphi}(.,t))(x),\ \forall x \in \Omega,$$

where $K_\sigma$ is a Gaussian kernel, $\overline{\varphi}$ is defined on $\mathbb{R}^2$ such that $\overline{\varphi}_{|\Omega} = \varphi$ and is obtained by mirroring with respect to $\partial\Omega$.

In our case, we will choose $\sigma$ as the standard deviation of the interferogram, depending on the smoothing coherence (see section 2.2). Unfortunately, image information coming out from the neighborhood of point are not taken into account. The *structure tensor*, $J_\mu$, proposed by Weickert, is constructed by convolving componentwise $\nabla\varphi_\sigma \nabla\varphi_\sigma^T$ with a Gaussian kernel $K_\mu$.

$$J_\mu(\nabla\varphi_\sigma) = K_\mu * \left(\nabla\varphi_\sigma \nabla\varphi_\sigma^T\right).$$

Hence the matrix $J_\mu = (j_{lk})|_{l,k=1,2}$ is symmetric positive semidefinite. Thus $J_\mu$ possesses two orthonormal eigenvectors $w_1, w_2$:

$$w_1 = \begin{pmatrix} \dfrac{2j_{12}}{\sqrt{\left(j_{22}-j_{11}+\sqrt{(j_{11}-j_{22})^2+4j_{12}^2}\right)^2+4j_{12}^2}} \\ \\ \dfrac{j_{22}-j_{11}+\sqrt{(j_{11}-j_{22})^2+4j_{12}^2}}{\sqrt{\left(j_{22}-j_{11}+\sqrt{(j_{11}-j_{22})^2+4j_{12}^2}\right)^2+4j_{12}^2}} \end{pmatrix},$$

and $w_2$ is taken such as $\langle w_1, w_2\rangle = 0$, where $\langle ., .\rangle$ is the usual inner product. The corresponding eigenvalues $\lambda_1 \geq \lambda_2$ are given by:

$$\lambda_{1,2} = \frac{1}{2}\left(j_{11} + j_{22} \pm \sqrt{(j_{11} - j_{22})^2 + 4j_{12}^2}\right)$$

The vector $w_1$ indicates the orientation maximizing the gray value fluctuations while $w_2$ gives the preferred local direction of smoothing. The interpretation of the eigenvalues in terms of image structure is described in the following table.

| **Comparison of $\lambda_1$ and $\lambda_2$** | **Type of areas** |
|---|---|
| $\lambda_1 = \lambda_2 = 0$ | constant areas |
| $\lambda_1 = \lambda_2$ | isotropic structures |
| $\lambda_1 \gg \lambda_2 = 0$ | straight edges |
| $\lambda_1 \geq \lambda_2 \gg 0$ | corners |

Moreover, it is important to point out that the two parameters $\sigma$ and $\mu$ play two different roles. $\mu$ is an integration scale parameter that reflect the characteristic size of the texture. The eigenvalues describe average contrast in the eigendirections within a neighborhood of size $O(\mu)$. The noise parameter $\sigma$ makes the descriptor insensible to details of scale smaller than $O(\sigma)$.

### 4.2. Noise level estimation

The noise level is not constant in the image (low coherence areas correspond to high noise level areas in the interferogram), then the filter must be adaptive. Since the *pdf* of the phase is given by (2), the standard deviation $\sigma$ can be evaluated. But the dependence of $\sigma$ with respect to the coherence, plotted in Figure 2, has to be taken into account in the proposed model. From a numerical point of view, the filtered coherence map $\rho$ is first quantified, in order to approximate the standard deviation $\sigma$ with piecewise constant function.

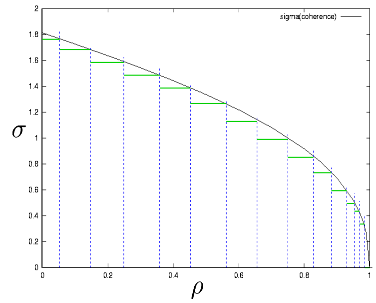


**Fig. 2**. Plot of the variance $\sigma$ of the interferometric phase with respect to the smoothed coherence $\rho$.

### 4.3. Diffusion equation

Let $\varphi_0(x)$ be the noisy interferogram represented by a bounded function $\varphi_0 : \Omega \longrightarrow \mathbb{R}$. It is then proposed to solve:

$$\begin{cases} \partial_t\varphi = div(D(J_\mu(\nabla\varphi_\sigma))\nabla\varphi) & \text{in } \Omega \times (0, \infty) \\ \langle D(J_\mu(\nabla\varphi_\sigma))\nabla\varphi, n\rangle = 0 & \text{on } \partial\Omega \times (0, \infty) \\ \varphi(x, 0) = \varphi_0(x) & \text{in } \Omega \end{cases} \quad (5)$$

where $n$ denotes the outer normal. The diffusion tensor $D \in \mathbb{R}^{2\times 2}$ is chosen as a function of the local image structure. We adapt the diffusion tensor $D(J_\mu(\nabla\varphi_\sigma))$ to the structure tensor $J_\mu$, as follows:

$$D(J_\mu(\nabla\varphi_\sigma)) = (w_1|w_2)\begin{pmatrix} \tilde\lambda_1 & 0 \\ 0 & \tilde\lambda_2 \end{pmatrix}\begin{pmatrix} w_1^T \\ w_2^T \end{pmatrix}$$

The eigenvectors of the diffusion tensor must correspond to the local structure of the image, so we choose them equal to the eigenvectors of the structure tensor. Its eigenvalues $\tilde\lambda_1$, $\tilde\lambda_2$ must be adapted in the context of interferometric phase images. Hence because we do not want any diffusion in the perpendicular direction to fringes, we impose $\tilde\lambda_1 = 0$. For the corresponding eigenvalue in the preferred smoothing direction $w_2$, we take a function of $(\lambda_1 - \lambda_2)^2$, that characterizes the saliency of the structures. In a window of size $\mu$, $w_2$ is the direction where the mean contrast is the lowest.

$$\tilde\lambda_2 = \begin{cases} \alpha \text{ if } \det(J_\mu(\nabla\varphi_\sigma)) = 0 \\ \alpha + (1-\alpha)e^{-\frac{1}{(\lambda_1-\lambda_2)^2}} \text{ otherwise} \end{cases}$$

The parameter $\alpha$ is introduced for theorical reasons (see [6] for the demonstrations).

J-S. Lee et al. were the first who took into account the orientation of the gradients. To incorporate fringes structures in their model, they consider sixteen directional windows, for which variance is calculated. The one corresponding to the minimal variance is selected. Our filter contributes to an automatic research of the principal smoothing direction. In fact, at each pixel, the use of the structure tensor allows us to avoid the calculus of the minimal variance of Lee. In addition, the noise adaptivity appears in two steps of the construction of our filter: via the calculus of $\sigma$ with respect to the quantified filtered coherence, and via the structure tensor.

## 5. NUMERICAL RESULTS

The performance of the filter is demonstrated with SAR data from ERS satellites of Utah region in Fig. 3, using the filtered coherence from Fig. 1. We focus on a region of interest containing both fringes and a strongly noisy part. To estimate the quality of the representation, we display the residue which is classically used in SAR [1].

We recall that the residues are based on the most common assumption, that is the desired unwrapped phase has local phase derivatives that are less than $\pi$ in magnitude everywhere. It enables us to define the corrected phase difference. The sum of the corrected phase difference along a closed contour, defined by four adjacents pixels, is necessarily $0$, $2\pi$ or $-2\pi$. Points for which the sum is not $0$ are called residues. We refer to [1] for more details.

The aim of filtering is to have residues concentrated on fringes locations and removed in noisy areas. We first present the results obtained with the Perona-Malik model applied directly to the interferogram. This naive approach which does not take into account noise statistics does not smooth properly fringes and some residus are still present in noisy areas. By using the coherence information, our approach is effective in removing isolated residues. Compared to Perona-Malik, we can observe that more residues are found on fringes lines which is due to the scale parameter $\mu$. Discontinuities are also well smoothed.

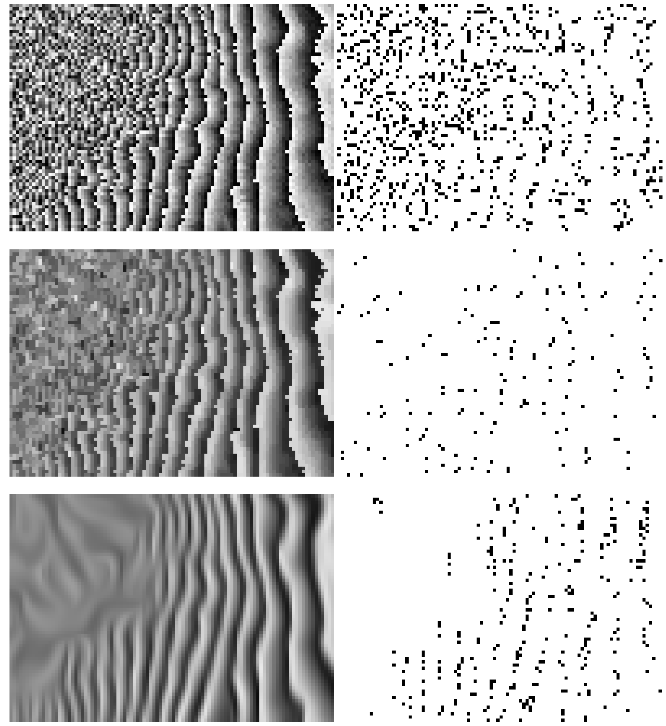

**Fig. 3**. Results of the noise adaptive anisotropic filter, on the region of Utah. $\mu = 2.0$, $\delta t = 0.1$, $\alpha = 0.001$. Left: Interferograms. Right: Residue maps. From top to bottom: the data, the result of Perona-Malik model, the result of our filtering.

## 6. CONCLUSION

The filtering model we describe in this paper takes into account the noise information given through the coherence, as in Lee et al. [4], and also preserves fringe discontinuities from smoothing by using anisotropic diffusion equations given by Weickert [6]. We have also shown that this framework allows to interpret and extend previous Lee et al. contribution. Future work will use these filtered interferograms to retrieve the absolute phase.

## 7. REFERENCES


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