Nonparametric Estimation of Mean Velocity and Spectral Width in Weather Radar

José M. B. Dias and *José M. N. Leitão*
Instituto de Telecomunicações and
Departamento de Engenharia Electrotécnica e de Computadores.
Instituto Superior Técnico
1096 Lisboa Codex, PORTUGAL.
Email:edias@beta.ist.utl.pt Phone: +351 1 8418464 Fax: +351 1 8418472

Abstract--This paper proposes a new nonparametric approach to the estimation of the mean Doppler velocity (*first spectral moment*) and the spectral width (*square root of the second spectral centered moment*) of a zero-mean stationary complex Gaussian process immersed in independent additive white Gaussian noise. By assuming that the power spectral density of the underlying process is *bandlimited*, the exact *maximum likelihood* estimates of its spectral moments are derived. An estimate based on the sample covariances is also studied. Both methods are robust in the sense that they do not rely on any assumption concerning the power spectral density (besides being bandlimited). Under weak conditions, the estimates based on sample covariances are *best asymptotically normal*.

I. Introduction

The goal of spectral estimation is to infer the *power spectral density* (PSD) from a finite *observation* of the underlying process. This subject has been extensively studied, yielding a large set of techniques. However, in many applications the objective is the determination of PSD functionals, rather than the PSD itself. This is the case of *spectral moments* (SMs), particularly, the *mean power*, the *mean velocity*, and the *spectral width*, with applications to (naming only a few): weather radar [1], [2]; clear-air turbulence measurement [3]; ultrasound imaging in medicine [4]; and to synthetic aperture radar [5].

Concerning Doppler weather radars, the goal is the determination of the three first SMs. These moments are closely related with physical entities of interest within the *resolution volume* [2]; the mean power (zeroth moment) is related with the water content, the mean frequency (first moment) is related with the backscatterers mean radial velocity, and the spectral width (square root of the second centered moment) is a measure of the backscatterers velocity dispersion.

In a typical weather radar, the number of estimates per complete aerial revolution can be as large as $3 \times 360,000$. This has to be done in real or near real time, which means a few tens of seconds. On the other hand, in order to have, simultaneously, an acceptable azimuthal velocity (a few revolutions per minute) and to prevent the broadening of the lateral antenna pattern, the number of samples per estimate should not be *large*; typical values are 16, 32, 64, or 128 (depending on the azimuthal velocity and on the lateral aerial resolution). Consequently, any SM estimator in weather radar applications should meet the requirement of being *low* in complexity (in a computational sense) yielding estimates of *acceptable* quality based on *small* sample sizes.

A. Classical Estimators

Well known nonparametric techniques are the covariance or *pulse pair* (PP) estimate [6], [7], and the *periodogram based* (PB) estimate [1], which are representative of the covariance and the spectral approaches, respectively.

Concerning the PP statistical properties, we have the following: (1) the mean frequency is increasingly biased with the spectral skewness [7]; (2) the spectral width estimate is biased, regardless the spectral shape (its bias increases with the spectral width); (3) the relative variances (compared with the Cramer-Rao bound) of both estimates display parabolic like curves, with minima at intermediate spectral widths [8].

Concerning the PB statistical properties, we have the following: (1) the mean frequency estimator is biased due to the finite resolution associated to the FFT (this is a serious problem whenever the sample is *small*); (2) the spectral width estimator is biased due to the windowing effect associated to the DFT; (3) the relative variance of both estimates at low spectral widths display similar features to the PP ones. However, at high spectral widths the performance of the PB method is better than the PP one.

Despite the reported shortcomings, the PP and PB estimators exhibit a good tradeoff between computational complexity and performance. As a result, they are extensively used in weather radar applications.

By constraining the search space, parametric approaches deliver estimates with lower uncertainty, when compared with nonparametric procedures. Concerning this matter, the maximum likelihood (ML) criterion plays a prominent role, given its optimal properties, at least in asymptotic sense. In the field of parametric ML spectral moments estimation, various approaches have been proposed [1], [9], [10]. Besides ML, also *maximum entropy* [11] and *Bayesian* criteria [12] have been suggested.

Parametric methods have, generally speaking, higher complexity than nonparametric ones. The fact that none of the above parametric approaches has been used in real time or near real time weather radar applications strengthens this idea.

Gaussian spectral shape has been extensively used. Although having some experimental justification, the Gaussian shape assumption is not without weaknesses. This is put in evidence in [13], where a systematic and exhaustive measurement of spectral shape from precipitation echoes is reported. According to the authors' conclusions, ''... a Gaussian spectral shape agrees reasonably with a large fraction of the obtained spectra. However, in about a quarter of the cases the deviation from the Gaussian shape is considerable, e.g., one or both edges may be too steep or two slight, the peak may be off-center, there may be more than one peak." Thus, a more accurate spectral fitting should be looked for. This can be achieved, for example, with autoregressive moving average (ARMA) models of adequate dimension, of which the work reported in [14] is an example. However, the complexity inherent to the ARMA parameters estimation is unbearable in weather radar applications.

As a conclusion of the above considerations, the classical nonparametric PP and PB estimators are characterized by having low complexity and tolerable, sometimes poor, performance. On the other hand, parametric estimators rely strongly on spectral shape assumptions; in this sense they are not robust. Moreover, their complexity is incompatible with practical weather radar applications.

B. Rationale of the Proposed Approach

The method herein presented explores the fact that the PSD of weather echoes is bandlimited. In fact, the PSD associated to each resolution volume is a weighted replica of the hydrometeors velocity distribution [15]. Since in each resolution volume the hydrometeors have a maximum and a minimum velocity, the PSD $S_x(f)$ is bandlimited, i.e. $S_x(f) = 0$ for $f \notin [-f_m, f_m]$. Thus, the sampling theorem assures that the covariance function (CF) can be exactly recovered from its discrete samples, as long as the sampling rate $f_s = 1/T_s$ is equal or greater than the Nyquist limit ($f_s \geq 2f_m$). On the other hand, the SMs relate easily with the CF derivatives at $\tau = 0$, which are also given by a linear combinations of the CF samples. Accordingly, the name *bandlimited* (BL) will be used to designate the present approach.

In the PP approach the first and the second CF derivatives are replaced by discrete approximations based on the CF at lags $\tau = 0, 1, 2$ [7]. Therefore, the PP method can be viewed as an approximation of the BL method.

The simple line of attack we are proposing seems not to be fully explored in the literature. To our knowledge, only the work described in [4], in the field of ultrasound blood velocity measurements, explores the bandlimited concept. However, their approach and methodology diverge from the one herein developed.

II. Problem Formulation

Let the signal backsattered by the meteorological clutter, in a given resolution volume, be represented by the complex sample from a strictly stationary zero-mean complex Gaussian random process $\mathcal{X} = \{x(t),\, t \in \Re\}$, with covariance function $R_x(\tau) = E[x(t+\tau)x^*(t)]$ and PSD $S_x(f)$. These hypotheses find physical justification and have been experimentally confirmed [16].

The radar transmits pulses periodically at instants iT_s, with index i in the integer set $\mathbf{Z}$; the echo produced by the scatterers lying in a resolution volume having range r arrives at instants $t_i = 2r/c + iT_s$ (c is the speed of light). Consequently, the SMs associated with the mentioned resolution volume have to be inferred from samples $x(t_i)$, with $t_i \in \mathcal{T} = \{t : t = 2r/c + (i-1)T_s,\, i \in \mathbf{Z}\}$.

The equivalent electronic noise at the receiver input is modeled by an additive and independent zero-mean complex Gaussian white process $\mathcal{N} = \{n(t),\, t \in \Re\}$, with covariance verifying, for any $t_i, t_j \in \mathcal{T}$, $E[n(t_i)n(t_j)] = 0$ and $E[n(t_i)n^*(t_j)] = N_0\delta_{i-j}$ (δ_τ denotes the Kronecker symbol). The resulting signal plus noise process is denoted by $\mathcal{Y} = \{y(t) = x(t) + n(t),\, t \in \Re\}$. Since parameter N_0 can be estimated with arbitrary precision (this can be done by switching off the transmitted pulses), it will be assumed known.

The problem is, then, stated as follows: given the N-dimensional sample vector $\mathbf{Y} = [Y_1, \ldots, Y_N]^T$, with $Y_i = y(t_i)$, derive estimators of the spectral moments (assumed to exist) given by

$$\mu_k(S_x) \equiv \frac{\displaystyle\int_{-\infty}^{\infty} f^k S_x(f)\, df}{\displaystyle\int_{-\infty}^{\infty} S_x(f)\, df} \qquad k = 1, 2, \ldots,$$

and of the spectral width

$$\sigma(S_x) = [\mu_2(S_x) - \mu_1^2(S_x)]^{1/2}.$$

Estimators of $\mu_k(S_x)$ and $\sigma(S_x)$ will be denoted by $\widehat{\mu}_k(\mathbf{Y})$ and by $\widehat{\sigma}(\mathbf{Y})$, respectively.

III. The Bandlimited Approach

We assume that **a)** $S_x(f)$ is continuous almost everywhere; **b)** $S_x(f) = 0$, $f \notin [-f_m, f_m]$ and $f_m \leq f_s/2$; and **c)** $S_x \in L_2[-f_m, f_m]$.

Hypothesis **c)** implies (invoking the Schwarz's inequality) that $\int_{-\infty}^{\infty} |f|^k S_x(f)\, df < \infty$, which assures that μ_k exists for $k = 1, 2, \ldots$ and allows writing the SMs as functions of the CF derivatives at $\tau = 0$:

$$\mu_k(S_x) = \frac{(2\pi j)^{-k}}{R_x(0)} R_x^{(k)}(0), \qquad k = 1, 2, \ldots, \qquad (1)$$

where $R_x^{(k)}$ is the k-th derivative of R_x. Given the banlimited assumption **b)** jointly with **c)**, it can be shown (a variation of the the sampling theorem) that

$$R_x^{(k)}(\tau) = \sum_{i=-\infty}^{\infty} R_x(iT_s) h^{(k)}(\tau - iT_s), \qquad k = 1, 2, \ldots, \qquad (2)$$

$|f| \leq f_m$. Inserting (2) into (1), one is led to

$$\mu_k(S_x) = \frac{(2\pi j)^{-k}}{r_0} \sum_{i=-\infty}^{\infty} r_i h_i^k, \qquad k = 1, 2, \ldots, \tag{3}$$

with $r_i = R_x(iT_s)$ and $h_i^k = h^{(k)}(-iT_s)$ for $i \in \mathbf{Z}$.

Assume that for each $k = 1, 2, \ldots$, given a positive number ε_k, there exists an integer M_k such that

$$\frac{1}{(2\pi)^k} \sum_{|i|>M_k} |h_i^k| < \varepsilon_k.$$

Under these conditions, the error magnitude between μ_k and sum (3) truncated to $|i| \leq M_k$ is smaller than ε_k (notice that the nonnegative nature of $\{r_i,\, i \in \mathbf{Z}\}$ implies that $|r_i/r_0| \leq 1$). In what follows we consider that ε_k is negligible comparing with the estimation error and denote the k-th SM as the sum (3) truncated to M_k.

The invariance principle of ML estimation [17] states that if $g(\boldsymbol{\theta}) : \mathbf{\Theta} \rightarrow \mathbf{\Phi}$ is a function from $\mathbf{\Theta}$ onto a subset $\mathbf{\Phi}$ of $\Re^{n_1}$ with $n_1 \leq N$, then $g(\widehat{\boldsymbol{\theta}}^{ml})$ is the ML estimate of $g(\boldsymbol{\theta})$. By applying this principle to the truncated version of (3), we obtain ML estimates of the SMs given by

$$\widehat{\mu}_k^{ml} = \frac{(2\pi j)^{-k}}{\widehat{r}_0^{ml}} \sum_{i=-M_k}^{M_k} \widehat{r}_i^{ml} h_i^k, \qquad k = 1, 2, \ldots \tag{4}$$

The same set of concepts applies equally to the ML spectral width estimator. Hence,

$$\widehat{\sigma}^{ml} = [\widehat{\mu}_2^{ml} - (\widehat{\mu}_1^{ml})^2]^{1/2}. \tag{5}$$

Expressions (4) and (5) are exact ML estimates (under the banlimited and negligible ε_k assumptions). They are simple functions of ML sequences $\{\widehat{r}_i^{ml}, i = 0, 1, \ldots, M_k\}$.

A. *The Interpolation Filter*

The vanishing rate of coefficients h_i^k should be as large as possible in order to assure a negligible truncation error, with M_k being as low as possible. In case of $f_s = 2f_m$ (Nyquit rate), there is no alternative to the ideal low pass filter $H(f) = T_s\,\mathrm{rect}(f/f_s)$. However, if the sampling rate is greater than the Nyquist rate ($f_s > 2f_m$), then the vanishing rate of coefficients h_i^k can be significantly increased. This is achieved by choosing an interpolation filter $H(f)$ with a finite *rolloff* rate within the interval $[f_m, f_s - f_m]$ of length $(2f_m - f_s) \equiv \rho(f_s/2)$. In this work we have chosen $H(f)$ to be the *raised cosine* filter with a rolloff factor ρ, as it is used in digital transmission with the purpose of minimizing the intersymbol interference [18].

The impulse response of the raised cosine filter is

$$h(\tau) = T_s \frac{\sin(\pi f_s \tau)}{(\pi f_s \tau)} \frac{\cos(\pi f_s \rho \tau)}{1 - 4\rho^2 \tau^2}.$$

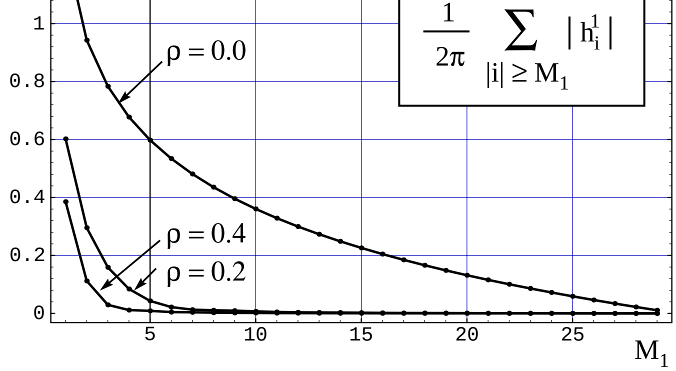

Figure 1: Upper bound interpolation error associated to interpolation coefficients h_i^1. Parameter ρ is the rolloff factor of interpolation the filter.

If $\rho > 0$, the vanishing rate of coefficients h_i^k is of order $O(1/\tau^3)$ as opposed to $O(1/\tau)$ for $\rho = 0$. Fig. 1 plots the upper bound $(\sum_{|i|\geq M_k} |h_i^1|)$ for different values of the rolloff factor ρ. For example, if $\rho = 0.2$, which demands a sampling rate 1.25 above the Nyquist rate, the value $M_1 = 7$ leads to $\varepsilon_1 < 0.01$. In the case of h_i^2, the same value $M_1 = 7$ leads to $\varepsilon_1 < 0.0025$.

B. *Computing the Estimate*

Under weak conditions, the estimates given by (4) and (5), are *best asymptotically normal* [19]. Despite the goodness of ML estimates, they have a degree of complexity incompatible with weather radar requirements, whatever the technique used to compute $\{\widehat{r}_k^{ml}, k = 0, 1, \ldots, M_k\}$. Therefore, considering the formal structure of ML estimators (4) and (5), it seems reasonable to analyze the properties of

$$\widehat{\mu}_k = \frac{(2\pi j)^{-k}}{\widehat{r}_0} \sum_{i=-M_k}^{M_k} \widehat{r}_i h_i^k, \qquad k = 1, 2, \ldots \tag{6}$$

$$\widehat{\sigma} = [\widehat{\mu}_2 - (\widehat{\mu}_1)^2]^{1/2}, \tag{7}$$

where $\{\widehat{r}_k, k = 0, 1, \ldots, M_k\}$ is a sequence of unbiased sample covariance estimates. Given the sample vector $\mathbf{Y}$, of size $N > M_k$, $\widehat{r}_i$ is given by

$$\widehat{r}_i \equiv \frac{1}{N - |i|} \sum_{n=0}^{N-|i|} Y_{n+i} Y_n^* - \delta_i N_0 \tag{8}$$

$$= \mathrm{tr}\{\mathbf{J}_i \mathbf{Y}\mathbf{Y}^H\} - \delta_i N_0 \qquad i = 1, \ldots, M_k, \tag{9}$$

with $\mathrm{tr}\{\cdot\}$ being the trace operator, $(\cdot)^H$ meaning transpose conjugate, $\mathbf{J}_i$ a null matrix except for the i-th diagonal where its entries are set to $(N - |i|)^{-1}$. For negative indices we have $\widehat{r}_{-i} \equiv \widehat{r}_i^*$. Notice that definition (9) is valid for $i = -M_k, \ldots, 0, \ldots, M_k$. Since the number M_k of sample covariances needed to determine the SMs is moderate (almost surely inferior to 10), the sequence $\{\widehat{r}_i,\, i = 0, \ldots, M_k\}$ can be by inverting the periodogram, which demands, approximately, $N \ln_2 N$ complex floating point multiplications.

As a last remark, we note that the even nature of $h(\tau)$ and the Hermitian property $\widehat{r}_{-i} = \widehat{r}_i^*$ allow to write

$$\widehat{\mu}_k = \left(\frac{j}{2\pi}\right)^k \frac{1}{\widehat{r}_0} \left\{ 2 \sum_{i=1}^{M_k} \mathrm{Im}[\widehat{r}_i]\, h_i^k \right\} \qquad k = 1, 3, \ldots$$

$$\widehat{\mu}_k = \left(\frac{j}{2\pi}\right)^k \frac{1}{\widehat{r}_0} \left\{ \widehat{r}_0 h_0^k + 2 \sum_{i=1}^{M_k} \mathrm{Re}[\widehat{r}_i]\, h_i^k \right\} \quad k = 2, 4, \ldots,$$

meaning that even SMs depend only on $\mathrm{Re}[\widehat{r}_i]$ and odd SMs depend only on $\mathrm{Im}[\widehat{r}_i]$.

IV. Statistical Properties and Numerical Examples

The statistical characterization of $\widehat{\mu}_k$, for $k = 1, 2, \ldots$ and of $\widehat{\sigma}$, given by (6) and (7), is carried out in [19]. The full extension of the results are out of the scope of this paper. We simply stress the following main points:

1. estimates $\widehat{\mu}_k$, for $k = 1, 2, \ldots$ are asymptotically unbiased. Namely, $E[\widehat{\mu}_k - \mu_k] = O(N)$

2. for CFs with linear phase (equivalently, having symmetric PSD with respect to μ_1), estimates $\widehat{\mu}_k$, for $k = 1, 2, \ldots$, are unbiased

3. estimate $\widehat{\sigma}$ is asymptotically unbiased. Namely, $E[\widehat{\mu}_k - \mu_k] = O(N)$

4. estimates $\widehat{\mu}_k$, for $k = 1, 2, \ldots$ and $\widehat{\sigma}$ are best asymptotically normal.

Given a generic spectral shape, the proposed set of estimators are asymptotically unbiased. For $N = 32$ (a typical sample size in weather radar) and for $\sigma \geq 0.01$, they exhibit negligible bias [19]. This is a great advantage over the PP and PB estimators, given that they are not uniformly unbiased.

Fig. 2 plots the standard deviations of $\widehat{\mu}_1$ (part a) and $\widehat{\sigma}$ (part b), assuming Gaussian shaped spectrum, sample size $N = 32$, and sampling rate $f_s = 1$. The Cramer-Rao bound is also plotted. We note that the estimate $\widehat{\mu}_1$ is efficient, at least for $\sigma \in [0.01, 0.15]$ and for SNR$\in [0.5, 100]$. The estimate $\widehat{\sigma}$ is also efficient for SNR$= 3\,dB$. For higher signal to noise ratios, and low spectral widths, the relative variance of $\widehat{\sigma}$ increases slightly.

The solid lines in Fig. 3 plot (in percentage) the asymptotic bias of the PP mean frequency (part a) and spectral width (part b) estimates, normalized by f_s. The coordinate γ parametrizes the PSD according to

$$S_x(f, \gamma) = \frac{8}{4 + \gamma} \left[\mathrm{rect}(2f) + \gamma\ \mathrm{rect}\big(8(f - 0.3125)\big) \right]. \tag{10}$$

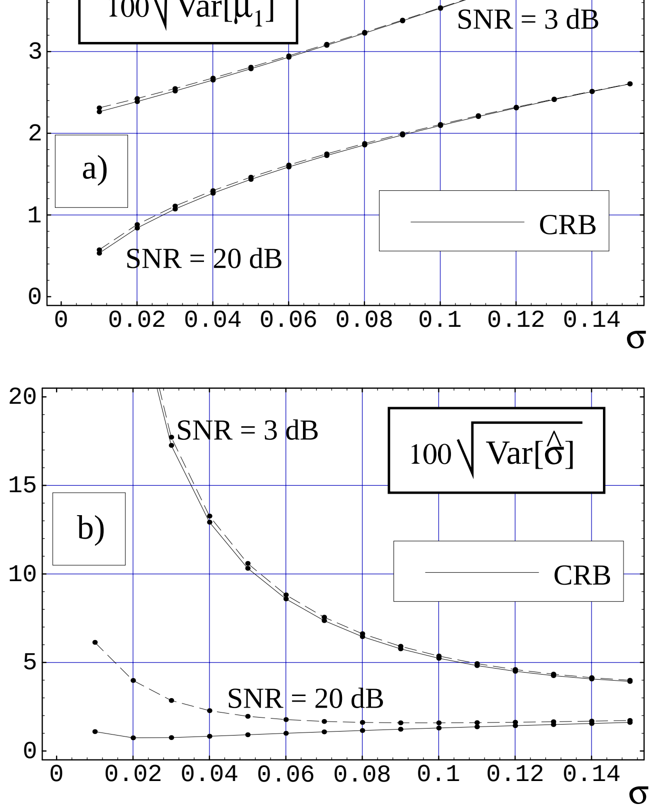

Figure 2: Standard deviations of mean frequency and spectral width estimators, for a Gaussian shaped spectrum.

By varying γ, a wide set of situations is modelled: for $\gamma = 0$ and $\gamma = \infty$, we have $S_x(f, 0) = 2\mathrm{rect}(2f)$ and $S_x(f, \infty) = 8\mathrm{rect}(8(f - 0.3125))$, respectively; these corresponds (assuming $f_s = 1$) to symmetric PSDs having large and moderate spectral widths, respectively. For intermediate values of γ, $S_x(f, \gamma)$ exhibits non-null skewness, whose maximum occurs in the neighborhood of $\gamma = 6$.

The star and diamond symbols, in Fig. 3, are sample bias estimates based on 20 independent runs (sample size is $N = 64$). Despite the low number of independent runs, it is clear that both PP estimates are highly biased compared with the BL ones, which seem not to be biased (as far as it can be concluded from such a small number of independent runs).

V. Conclusions

A new nonparametric technique for spectral moments estimation was proposed. By assuming that the power spectral density function is banlimited, the exact maximum likelihood estimator of the spectral moments was derived. This estimator is a simple function of the maximum likelihood covariance estimate sequence. Replacing this sequence by the sample covariance, a suboptimal spectral moment estimator, suitable to weather radar applications, was also obtained. It exhibits

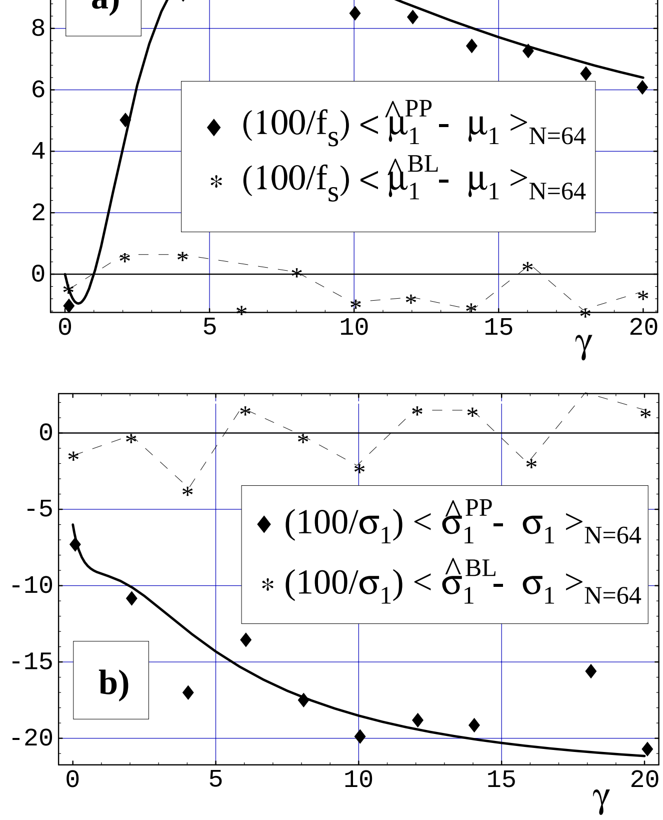

Figure 3: PP and BL bias comparison.

the following features: (1) unbiased for symmetric spectra; (2) asymptotically unbiased for general shaped spectra; (3) consistent; (4) asymptotically efficient. In the case of Gaussian shaped spectrum and a sample size as low as 32, the mean velocity estimate has minimum variance for the typical signal to noise ratios and spectral widths used in weather radar. The spectral width also reaches minimum variance for low signal to noise ratios.

Compared with the pulse pair and the periodogram based estimates, the proposed approach exhibits a negligible bias and for generic shaped spectra the total error (bias plus standard deviation) is consistently smaller.

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