

## A ROBUST SIMULATION-BASED MULTICRITERIA OPTIMIZATION METHODOLOGY

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### ABSTRACT

This paper describes a methodology for solving Parameter Design (PD) problems in production and business systems of considerable complexity. The solution is aimed at determining optimum settings to system critical parameters so that each system response is at its optimum performance level with least amount of variability. When approaching such problem, analysts are often faced with four major challenges: representing the complex parameter design problem, utilizing an effective search method that is able to explore the problem's complex and large domain, making optimization decisions based on multiple and, often, conflicting objectives, and handling the stochastic variability of in system response as an integral part of the search method. To tackle such challenges, this paper proposes a solution methodology that integrates four state-of-the-art modules of proven methods: Simulation Modeling (SM), Genetic Algorithm (GA), Entropy Method (EM), and Robustness Module (RM).

### 1 INTRODUCTION

The focus of this paper is on achieving a high-level parameter design of production and business systems of considerable complexity. Such complexity often results in difficulties in representing as well as in seeking optimal performance for such systems. Further, the performance of the majority of production and business systems is often based on multiple and potentially conflicting objectives. This conflict makes the improvement in an objective only conceivable at the expense of degradation in one or more other objectives. The mentioned difficulties exist in a great number of applications in areas of Manufacturing, Business Operations, Product Development, and Enterprises Resource Planning (ERP).

To tackle such problem, simulation-based optimization methods, in general, apply a search algorithm on an optimization problem that is represented using a Discrete Event Simulation (DES) model. In this representation, the complex structure of the multi-criteria objective function and the problem constraints are evaluated by computer simula-

tion without the need to approximate a closed-form definition, if even possible, for the problem mathematical model.

Different approaches have been used in the literature to optimize or draw inferences from the output of a simulation model. Taguchi's Experimental Design using Orthogonal Arrays (OA) was applied to many simulation studies to seek a system-level parameter settings based on certain response signal that is evaluated through simulation runs (Madu and Madu, 1999). In addition to Taguchi's approach, some methods have utilized more efficient search engines from the field of Artificial Intelligence (AI). Examples of that include Genetic Algorithm (GA), Simulated Annealing (SA), Neural Networks (NNs), and Tabu Search (TS). A complete discussion of different simulation-based optimization methods is found in Carson and Maria (1997), Azadivar (1999), and Swisher et al. (2000).

Still, several challenges persisted as obstacles to current simulation-based optimization methods. Evans et al. (1991) Summarized those obstacles in three important respects:

1. The relationships between output variables and decision variables are not of a closed form.
2. The outputs may be random variables (probabilistic as opposed to deterministic nature).
3. The response surface may contain many local optima.

Furthermore, the challenges specified by Evans et al. (1991) are often dealt with in a Multi-Criteria Optimization (MCO) context. Thus, this paper proposes a simulation-based optimization methodology that is aimed at tackling these four challenges.

First, a Discrete Event Simulation (DES) model of the underlying system is utilized to capture the complex response of the relationships between output variables and design variables without the need for a closed form definition of these relationships. Second, Genetic Algorithm (GA) is utilized as a global randomized-search engine, which prevents the search from getting stuck at local optima of response surface. GA is considered by Goldberg

(1989) to be noted for *robustness* in searching complex spaces in a variety of application arenas.

Third, since simulation outputs are often random variables (stochastic as opposed to deterministic nature), incorporating robustness into the GA search is necessary to process the stochastic response and guide the search towards arriving at an optimal *robust* solution. Several methods were utilized to handle the stochastic variability of simulation modeling as an integral part of the search process. Examples of these methods include utilizing confidence intervals of performance metrics (Al-Aomar, 2000) and utilizing Taguchi's Loss Function (Sanchez, 2000) and Taguchi's Signal-to-Noise (S/N) ratio (Madu and Madu, 1999). Taguchi's robust design approach, which was initially originated in the field of quality engineering (Taguchi and Wu, 1980 and Taguchi, 1986), is based on combining the concepts of Signal-to-Noise (S/N) ratio and Quality Loss Function (QLF) with methods of Experimental Design using Orthogonal Arrays (OA). This approach was successfully applied to a wide range of product and process design applications (Ross, 1996). Hence, this paper proposes a Taguchi-based method, namely Signal-to-Noise (S/N) ratio, to handle the stochastic variability as an integral part of the GA search of the stochastic outputs of DES models.

Finally, to perform the GA search in a MCO context, an Entropy-based assessment of the decision-maker's Multi-Attribute Utility Function (MAUF) is utilized to establish an overarching criterion (utility) that guides the GA towards selecting the closest to optimal solution alternative based on multiple performance metrics. The Entropy module is developed based on a method taken from the information theory (Shannon, 1948). The method was used successfully to assess the set of weights associated with multiple performance metrics in several Multi-Criteria Decision-Making problems (Hwang and Yoon, 1981 and Zeleny, 1974).

Section 2 describes the structure of the Parameter Design (PD) problem. Section 3 discusses the four modules of the proposed solution methodology; GA search, Simulation Modeling (SM), Robustness Module (RM), and Entropy Method (EM). Finally, Section 4 concludes the paper, followed by a References list.

## 2 PARAMETER DESIGN PROBLEM

Structuring the Parameter Design (PD) problem is primarily based on forming two relationships; the first is between the system performance measures and design parameters and the second is between individual performance measures and the overall system performance/utility. The first relationship is expressed in a dynamic, stochastic, and nonlinear system response while the second one is expressed in terms of the decision-maker's utility function  $U$ . This structure is assumed to be amenable to parametric optimization and, therefore, formulated in terms of a set of system design parameters  $X$  and a set of system performance measures  $Y$ , as shown in Figure 1.

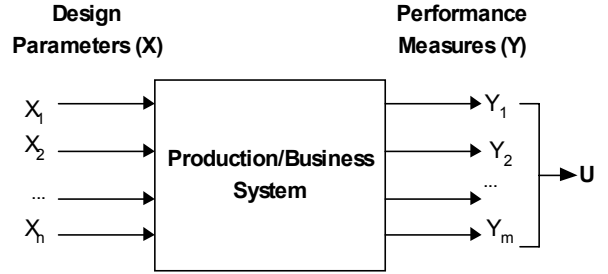


Figure 1: Parameter Design Structure

System design parameters  $X_1, X_2, \dots, X_n$  are those variables defined by designers as critical factors that impact the system performance and direct its actual behavior. Examples of such parameters in production systems include buffer sizes, number of production and maintenance resources, speeds and capacities of conveyance systems, and product-mix. The set of system performance metrics  $Y$ , on the other hand, represents the criteria on which improvement and efficiency of production systems are measured. Maximizing or minimizing levels of such metrics is, therefore, translated into design and improvement objectives. Examples of performance measures include profit, throughput, operating cost, lead-times, quality rates, and work-in-process (WIP) levels.

To represent real-world systems of considerable complexity, a DES model is often utilized to replace the mathematical approximation of the first relationship (between  $Y$  and  $X$ ). A decision-maker's utility function  $U$ , which consolidates multiple decision attributes into an overall score or value, is often used to map the objective function of multiple design objectives.  $U$  is often approximated in a linear or a nonlinear form. The linear form represents a weighted average of multiple criteria based on each criterion's relative importance weight whereas the nonlinear approximation is often presented in different mathematical shapes depending on the decision-maker's risk attitude.

The objective of the proposed simulation-based optimization is to arrive at the closest to optimal solution alternative (in terms of a set of system design parameters  $X_0$  and a set of system performance metrics  $Y_0$ ) at which the overall system utility score  $U$  is maximized.  $U$  is maximized by seeking maximum conceivable improvement to *all* design attributes simultaneously. Therefore, if  $U$  represents the decision-maker's utility function that consists of  $m$  decision (design) attributes ( $Y_1, Y_2, \dots, Y_m$ ), where each attribute is a function  $Y_i(X_j)$  of  $n$  design parameters ( $X_1, X_2, \dots, X_n$ ) and  $x_j$  is from the problem solution space  $S$ , then a general formulation of the system design problem can be defined as follows:

$$\begin{aligned} & \text{Max } \{U(Y_1, Y_2, \dots, Y_m)\} \\ & \text{s. t. } Y_i = f_i(X_1, X_2, \dots, X_n), 1 \leq i \leq m \\ & X_j \in S, 1 \leq j \leq n \end{aligned} \quad (1)$$

### 3 PROPOSED SOLUTION METHOD

This paper proposes a solution methodology to the defined parameter design problem that is based on an integration of four modules; Genetic Algorithm (GA), Simulation Modeling (SM), Robustness Module (RM), and Entropy Method (EM), as shown in Figure 2.

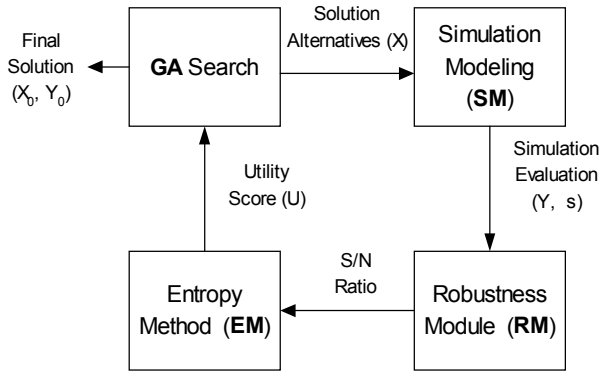


Figure 2: The Proposed Methodology

In the proposed method, GA is the core search engine that works on the parameter design problem and eventually converges to the optimal or near-optimal solution. Towards this end, the problem formulation shown in Formula 1 is coded into a solution space in the GA search process. GA standard operators and selection scheme are then applied on a population of feasible solution alternatives in each search iteration step until convergence.

For the GA to apply its selection scheme to different solution generations, three functions are needed; an evaluation of all generated solution alternatives based on multiple design attributes, a transformation of simulation evaluation of each attribute (in terms of response mean and variance) into a single fitness value, and an overall fitness score assignment to each solution alternative. The integration of GA with SM, RM, and EM modules provides the standard GA search with these three essential functions. In the SM module, a DES model whose built using the configuration, data, and logic of the underlying system is utilized by GA search to evaluate the set performance metrics ( $Y$ ) associated with each solution alternative ( $X$ ). The RM transforms simulation estimates of each performance measure (performance mean and variance) into a S/N ratio as a single fitness representation to each criterion. Finally, in the EM module, criteria' relative importance weights are assessed and a linear additive MAUF is used to assign each solution alternative an overall fitness score  $U$ .

Using each solution's overall utility score  $U$ , GA performs a MCDM in which solutions of high overall fitness scores ( $U$ ) are given higher chance for survival. Higher  $U$  values imply both higher performance and less encompassed variability. Using these scores, the preference made in the GA selection scheme will be based on a holistic con-

sideration of all performance metrics and their degree of robustness. The powerful GA search continues at each search iteration step until a certain termination condition, defined in the GA setup, is met. The final solution reached by GA is put in the form of a set  $X_0$  of optimal settings to system design parameters at which the best (high performance and less variability) set of system performance metrics  $Y_0$  is reached.

Based on the functionality shown in Figure 2, applying the proposed solution methodology to a Parameter Design problem requires the analyst to follow a systematic procedure of 5 steps, as summarized in Figure 3.

- Step 1: Structure the Parameter Design (PD) problem
- Step 2: Build, validate, and verify a system DES model.
- Step 3: Set up the parameters of the GA search
- Step 4: Structure the Robustness Module (RM) based on S/N ratio
- Step 5: Structure the MAUF based on the Entropy Method (EM)
- Step 6: Run GA-SM-RM-EM and obtain a robust optimal solution

Figure 3: A Proposed Methodology Procedure

#### 3.1 Genetic Algorithm (GA)

Genetic Algorithms (GAs) were developed by Holland in 1975. Holland (1992) considered them as a tool for searching out solutions to optimization problems of complex characteristics and large search spaces. GAs were essentially developed to emulate the "survival for fittest" principle introduced by Charles Darwin in his theory of evolution in 1830s. From this perspective and since optimization is analogous to fitness or the ability to survive real-world conditions, it made good sense to apply the GA approach for system improvement and optimization.

In GA optimization, the search for an optimal solution is achieved through the manipulation of randomly selected initial population size ( $N$ ) of string structures known as chromosomes. Each chromosome is a simple binary coding of one potential solution to the system design problem. Other non-binary encoding methods such as real-number encoding, integer or literal permutation, and general data structure encoding can also be used (Gen and Cheng, 2000). The fitness function in GA is often a single-objective and in a fixed closed-form. In the proposed method, however, the DES model represents the fitness objective through which multiple performance measures, associated with each solution alternative proposed by the GA search are evaluated in terms of mean and variance of multiple simulation replications. GA selection scheme, which works on single fitness values, is performed by combining mean and variance using S/N ratio in the Robustness Module. Finally, the Entropy method is used to combine multiple performance measures of each solution point into an overall utility score. GA selection scheme uses such score to make preference among potential solution candidates.

In its search for optima, GA utilizes a mix of exploration and exploitation strategies using *reproduction* (selection scheme) and *recombination* (*crossover* and *mutation* operators) mechanisms, respectively. Through the GA utility-based selection scheme, good candidates are reproduced and quality solutions are, therefore, exploited to next generations. According to Goldberg (1989), a good GA is one that is able to strike a balance between exploitation and exploration. This is achieved by balancing the GA parameter settings, which is a problem-specific effort, using a set-and-test approach at each GA application. A convergence test is conducted at the end of each search iteration step to check for termination condition. The test is based on reaching a defined maximum number of population generations or on converging to a certain rate of string bias, which measures the amount of similarity among solution chromosomes. A cycle of evaluation, reproduction, recombination, and convergence testing is repeated within each search iteration step until the termination condition is met. The GA search method is illustrated in Figure 4.

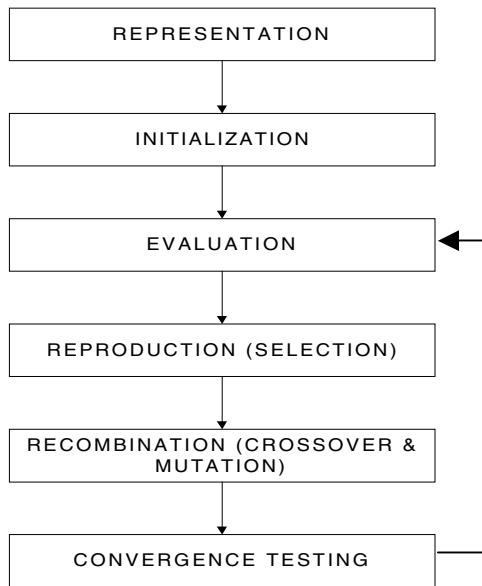


Figure 4: GA Search Operations

### 3.2 Simulation Modeling (SM)

By utilizing computer capabilities in logical programming, random generation, fast computations, and animation, Discrete Event Simulation (DES) modeling is capable of capturing the characteristics of the real-world process and estimating a system performance measures at different settings of its design parameters. To measure such performance, DES imitates the stochastic and complex operation of a real-world system as it evolves over time and seeks to describe and predict the system's actual behavior.

DES has undergone a tremendous development in the last decade. This development can be pictured through the

growing capabilities of simulation software tools and the application of simulation solutions to a variety of real-world problems. With the aid of DES, companies were able to design efficient production and business systems, validate and tradeoff proposed design solution alternatives, troubleshoot potential problems, and, consequently, cut cost, meet targets, and boost sales and profits. Examples of DES applications are found in Al-Aomar and Cook (1998), Law and Kelton (1991), and Pedgen et al. (1995).

The primary role of DES in the proposed methodology is to evaluate the performance measures of solution alternatives proposed by the GA search. Towards this end, system data, logic, and specifications are used in building the system simulation model. The simulation model is then utilized to estimate the set of system performance measures ( $Y$ ) at different settings of the system design parameters ( $X$ ). An abstraction of the DES process is illustrated in Figure 5.

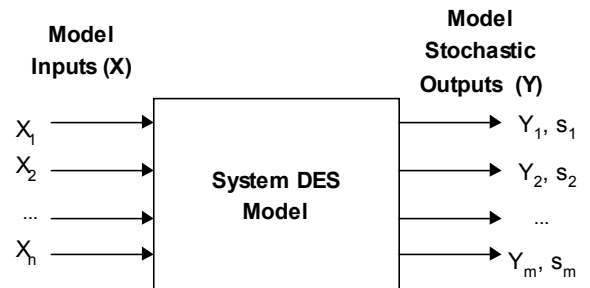


Figure 5: The DES Process

However, simulation outcomes, representing real-world systems, are often stochastic and leading inconsistent performance levels. We can classify the factors that contribute to variation in simulation outputs into controllable and uncontrollable (random/noise) factors. Model design factors such as buffer sizes, number of production resources, fixed cycle times, and speeds of conveyance systems are usually considered controllable factor. Different settings of system controllable factors may results in a different set of model outputs. Therefore, simulation-based optimization methods often aim at providing optimum settings to controllable factors so that model outcomes are at their best levels.

On the other hand, random factors whose individual values change by time based on system events and dynamics; are often presented in simulation models by a random generation from a certain probability distribution. Key distribution parameters are often estimated through statistical approximations by fitting a collected set of empirical data to one commonly used distribution. Because of variation in such factors, the behavior of DES models is, therefore, driven by the stochastic nature of real-world processes such as entities arrival process, in-system service process, and outputs departure process. Variability in those processes is caused by random factors such as arrival rate, service/processing rate, equipment mean-time-between failure

(MTBF) and mean-time-to-repair (MTTR), and percentages of scrap and rework. For example, the throughput of a production system that is subject to variability in material delivery, tools failure, and operators efficiency is represented by the DES model in a stochastic response. Based on the inherent fluctuations, the throughput could be a low value in one shift and then higher in another. Thus, simulation runs yield estimates of such performance measures in terms of means and variances to reflect such variability.

In addition to controllable and random factors, Sanchez (2000) suggested artificial factors to be a source of variation in simulation outputs. Artificial factors are those simulation-specific factors such as system initialization state, warm-up period, run length, termination condition, and random number streams. Changing the settings of such factors, from one simulation run to another, often results in changes in model outcomes. Hence, testing such factors in the model and providing proper settings to different model run controls to obtain a steady-state response is basically a prerequisite for applying simulation-based optimization methods.

### 3.3 Robustness Module (RM)

Incorporating Taguchi's robustness method as a module into the proposed simulation-based optimization method serves three goals. First, and most important, the stochastic variability of real-world simulation models is handled as an integral part of the GA search process. This is expected to lead GA towards arriving at an optimal solution design that is less sensitive to variations in model random and noise factors. Second, by using the response representation in terms of S/N ratio, GA selection scheme will be able to distinguish among solutions whose response results in overlapping confidence intervals. Finally, combining both response mean and variance into the single representation of S/N ratio provides the GA selection scheme with a single fitness value for each solution alternative, which permits the application of GA roulette wheel reproduction process.

Taguchi's Parameter Design (PD) approach aims at improving the uniformity of a product or a process without attempting to control or eliminate causes of variation. Products can be made robust to variation in materials, manufacturing process, and usage conditions. Manufacturing processes also can be made robust to variation in materials, environment, and machine parameters. Following a similar pattern, the proposed methodology aims at making high-level production/business processes, represented in DES models, robust to variation by setting system-level factors such as parts arrival rates, processing/service times, machine failures, and model-mixes to their optimum levels.

Taguchi classifies quality characteristics of product and processes into three types: Smaller-the-Better (SB), Nominal-is-Best (NB), and Larger-the-Better (LB). SB represents measurable quality characteristic with a target zero (the response is as small as possible) such as tool wear and process

downtime. NB represents measurable quality characteristic with a specific target value such as product dimensions and process yield. Finally, LB represents measurable quality characteristics with a target of infinity (the response is as large as possible) such as product life and process efficiency. The proposed methodology applies this classification to performance measures of simulated macro-level production systems (such as plants, assembly lines, and material handling systems) and business systems (such as banks, restaurants, logistics, and supply chains). In such context, a SB characteristic could be the lead-time in a production system or the operating cost in a business system. A NB characteristic could be the throughput of a production system or the Rate on Investment (ROI) of a business system. Finally, and a LB characteristic could be the efficiency of a production system or the profit of a business system.

#### 3.3.1 S/N Ratio in GA-SM

The original use of the term Signal-to-Noise ratio (S/N) was in the field of electronic communications to represent signals in terms of desired values and the noise around these values. The same analogy is used in Taguchi's Parameter Design (PD) since there is usually an undesirable aspect (noise) of any performance output combining the desired output level (signal) from the underlying system. Taguchi used the S/N ratio to measure the quality of system signals by consolidating the impact of the measured value of the signal and the noise around it. Thus, PD involves experimental design techniques using both Orthogonal Arrays (OA) and the S/N ratio. In these methods, factor levels corresponding to the highest S/N ratio would be selected to minimize variation and build robustness in the design of products and processes.

Taguchi (1986) defined S/N ratio, measured in decibels (db), as the reciprocal of the variance of the measurement error, so it is maximal for the combination of parameter levels that has the minimum error variance. In its simplest form, the S/N ratio is, therefore, the ratio of the mean (signal) to the standard deviation (the noise). Based on the type of system response, three standard forms of S/N ratio are often used in engineering applications: Smaller-the-Better (LB), Nominal-is-Best (NB), and Larger-the-Better (LB). Table 1 presents formulas for calculating the three standard forms of S/N ratio.

As seen in Table 1, S/N ratio formula offers a built-in tradeoff between the mean response (higher, lower, or nominal is best) and the variation in the response (least variation is always best). Therefore, for the SB characteristic, S/N objective is to reduce both the mean value and variation. For LB characteristic, on the other hand, the objective is to increase both the mean value and reduce variation. Finally, for the NB characteristic, the objective is to meet the target value and reduce variation. This is often approached in two ways; based on the variance only and

based on both mean and variance. The former aims at reducing excessive variability in response signal and the later aims at reducing variability in response and *then* bringing the mean response as close as possible to the target or nominal value. However, regardless of all situations, S/N ratio is always interpreted the same way: the larger the S/N ratio, the better.

Table 1: The Three Standard Forms of S/N Ratio

Performance Characteristic	S/N ratio formula (db)
Smaller-the-Better (SB)	$S/N = -10 \log \left[ \frac{1}{n} \sum_{i=1}^n y_i^2 \right]$
Nominal-is-Best (NB)	Mean and Variance $S/N = 10 \log \left( \frac{\bar{y}}{s} \right)^2$
	Variance Only $S/N = -10 \log (s^2)$
Larger-the-Better (LB)	$S/N = -10 \log \left[ \frac{1}{n} \sum_{i=1}^n \frac{1}{y_i^2} \right]$

### 3.4 MCDM with Entropy Method

Decision-making is an important part of any design process. The expertise of a decision-maker or a team of specialists is usually utilized to assess design alternatives, rank potential outcomes, and select best design strategies. Making decisions in the context of multiple and often competing objectives of a complex design problem is a typical challenge to decision-makers. Multi-Criteria Decision-Making (MCDM) requires determining the shape and structure of the decision-maker's Multi-Attribute utility Function (MAUF), with the absence of tradeoff information. Hence, the proposed methodology tackles these MCDM difficulties by forming a contextually adaptive linear additive MAUF and using this function as a multi-objective function in the GA selection scheme. Assessing the relative importance weights associated with individual attributes utilities is usually a key challenge when forming the linear additive MAUF. Therefore, entropy method is used to assess criteria' relative importance weights. At each iteration step, GA provides the EM module with a Decision Matrix (DM) that consists a population size of solution alternatives along with values of their performance metrics (valuated by SM module and and passed through the RM module). The EM module performs the entropy method on the DM to assess the set of attributes' relative importance weights  $W$  and form the MAUF. The MAUF is then used to transform performance multiple measures into a single overall utility

score  $U$ , on which GA selection scheme decide the survival chance of potential design solution alternatives.

#### 3.4.1 Forming the Utility Function

Steuer (1986) considers the utility function as the basis on which different settings (solution alternatives) to a MCO problem are judged. In this judgment, the greater the value of the utility score, the more preferred the solution alternative and its associated criterion vector. In the context of multiple objectives, a MAUF is often formed to judge solution alternatives. A tradeoff between these objectives is usually made to evaluate the utility value associated with any solution alternative. This tradeoff incorporates the contribution of each optimization objective into an overall system performance evaluation.

In the proposed methodology, a *linear additive* representation of the MAUF is used for assigning utility (GA fitness) scores to solution alternatives. A linear additive MAUF consists of two elements, individual utility values of decision attributes and relative importance weights associated with these attributes. The MAUF adds individual utility functions ( $U_1(y_1), U_2(y_2), \dots, U_m(y_m)$ ) for  $m$  different attributes ( $y_1, y_2, \dots, y_m$ ) to form an overarching system performance measure at any set of system design parameters  $X$ . Therefore, MAUF is simply a weighted average of these different utility functions. That is:

$$U(y_1, y_2, \dots, y_m) = w_1 U_1(y_1) + w_2 U_2(y_2) + \dots + w_m U_m(y_m) \\ = \sum_{i=1}^m w_i U_i(y_i) \quad (2)$$

Where  $w_1, w_2, \dots, w_m$  are the weights of relative importance assigned to attributes ( $i = 1, 2, \dots, m$ ) and  $\sum_{i=1}^m w_i = 1$  for convenience.

However, the following notes and assumptions are essential to address when forming the linear additive MAUF in the proposed methodology:

*First:* Although, a linear form of the MAUF is used in the proposed methodology as an overarching utility of all attributes, this linear form does not preclude attributes' individual utility functions ( $U_1(y_1), U_2(y_2), \dots, U_m(y_m)$ ) being nonlinear.

*Second:* For the MAUF to be additive, decision attributes should be mutually preferentially independent (Winston (1994)). That is, our preference to a certain attribute is not affected by the values of other attributes. Practically we often deal with interaction among decision attributes. Hence, Hwang and Yoon (1981) observed that when there are complementarities among the various decision attributes, the approach of weighted-sum in the linear additive MAUF might produce misleading results. To deal with this issue, the proposed methodology adopted a dynamic and contextually dependent MAUF to replace the traditionally used fixed form

of the MAUF. The structure of the MAUF is dynamically changed at each GA search iteration step by changing the values of attributes' weights. These weights are assessed using an entropy method at each search iteration step and under different decision contexts (as reflected by changes in the values of performance metrics obtained from simulation-evaluation). Therefore, The interaction among decision attributes is addressed by changing the values of criteria weights as one or more attribute value changes in the decision-matrix. This implies a practical consideration of dependencies among decision attributes while sustaining the linear additive form of the MAUF.

Finally, given that the linear additive MAUF may be useful for resolving tradeoffs, there exists the issue that, oftentimes, the units for expressing criteria or performance metrics will be different. These must be brought to a common aspiration orientation through normalization in order for solution alternatives to be comparable. Hence, determining utility scores starts by normalizing criteria values first and then deriving a common unit for all those criteria. This normalization aims at obtaining comparable scales in the evaluation of solution alternatives before applying the GA selection scheme.

### 3.4.2 Assessing Criteria' Weights

Forming the linear additive MAUF requires the assessment of the set of relative importance weights  $W$  associated with decision attributes. Typically, the method used for assessing criteria weights is primarily based on the nature of the problem and the available information. If the data of the Decision Matrix (DM), in terms of a set of solution alternatives and values of decision attributes associated with them, is unknown, approaches such as the Analytical Hierarchy process (AHP) (Saaty 1977) and Weighted Least Square Method (WLSM) (Chu et al. 1979) are usually adopted. In AHP, a pairwise comparison is established between pairs of decision attributes and solution alternatives using a preference scale and in WLSM a set of simultaneous linear algebraic equations are formed and solved using Saaty's matrix of pairwise comparison.

When the DM information is available, however, approaches such as Simple Multi-Attribute Rating Technique (SMART) (Edward 1986), LINEar programming techniques for Multidimensional Analysis of Preference (LINMAP) ((Srinivasan and Shocker 1973), and Entropy method (Nijkamp 1977 and Zeleny 1974) can be used to assess criteria' weights. In SMART, sometimes referred to as "swing weighting", the decision-maker ranks swings in the levels of decision attributes (Clemen 1996). An application of SMART in a Genetic Algorithm-Simulation Modeling (GASM) software tool was presented by Al-Aomar (2000). LINMAP proposes a Linear Programming (LP) model for estimation of the coordinates of an ideal point that denotes the decision-maker's most preferred stimulus and the weights

involved in the Euclidean distance measure in the multidimensional space.

The proposed methodology utilizes the EM for assessing criteria weights. The EM works on a Decision Matrix (DM). Solution alternatives of the DM are provided by GA search to the SM module for multiple performance evaluation. RM module assigns S/N ratios to the outcomes of SM module to provide values of their associate decision attributes. The availability of the DM information excludes the need for using AHP or WLSM. Further, the Entropy method does not require forming a LP model as the case in LINMAP and does not require information for ranking decision attributes or ranges of their values as the case in SMART. The method is, therefore, suitable for practical applications in parameter design problems, where little information is usually available to decisions-makers.

### 3.4.3 Entropy Method

Entropy is a subject that has played a central role in a number of areas such as Statistical Mechanics and Information Theory. The term Entropy is used in thermodynamics to describe a quantity accompanying a change from thermal to mechanical energy (Van Wylen and Sonntag, 1976). In information theory, entropy measures the *uncertainty* associated with random phenomena of the expected information content of a certain message (Shannon and Weaver, 1967). This uncertainty is represented by a discrete probability distribution  $p_j$ . The measure of uncertainty  $S$  in a probability distribution  $(p_1, p_2, \dots, p_n)$ , associated with  $n$  possible outcomes of a certain criterion, is given by Shannon (1948) as:

$$S(p_1, p_2, \dots, p_n) = -K \sum_{j=1}^n p_i \ln p_i \quad (3)$$

where  $K$  is a positive constant.

Since the term "entropy" and "uncertainty" are considered synonymous in information theory,  $S$  is called the entropy of the probability distribution  $p_i$ .  $S(p_1, p_2, \dots, p_n)$  takes its maximum value when the uncertainty in distribution outcomes is maximized, that is when all outcomes have the same probability  $p_i = 1/n$ . This establishes a useful rationale for utilizing the definition of entropy in criteria weights assessment. Hwang and Yoon (1981) mentioned that a criterion does not function much when all the alternatives have the similar outcomes for that criterion and if all values of a criterion are the same, we can eliminate the criterion. Therefore, the entropy idea, according to Hwang and Yoon (1981), is particularly useful to investigate contrasts between sets of data. These sets of data can be pictured as a set of solution alternatives in the Decision Matrix (DM) where each solution alternative is evaluated in terms of a set of outcomes of values of decision attributes.

The entries of the Decision Matrix (DM) with  $l$  solution alternatives and  $m$  decision attributes can be repre-

sented in a probability distribution  $p_{kj}$ , where ( $k = 1, 2, \dots, l$ ) and ( $j = 1, 2, \dots, m$ ). Each entry  $p_{kj}$  includes a certain information content, which can be measured by means of the entropy value. Therefore, if the DM of  $l$  solution alternatives and  $m$  decision attributes is:

$$DM = \begin{bmatrix} y_{11} & y_{12} & \dots & y_{1m} \\ y_{21} & y_{22} & \dots & y_{2m} \\ \dots & \dots & \dots & \dots \\ y_{l1} & y_{l2} & \dots & y_{lm} \end{bmatrix} \quad (4)$$

A probability value ( $p_{kj}$ ) for each entry in the DM can be simply determined by normalizing attribute values at each solution alternative. That is:

$$p_{kj} = y_{kj} / \sum_{k=1}^l y_k, \quad \forall k, j \quad (5)$$

Based on this, the  $p_{kj}$  matrix is formed as follows:

$$p_{ij} = \begin{bmatrix} p_{11} & p_{12} & \dots & p_{1m} \\ p_{21} & p_{22} & \dots & p_{2m} \\ \dots & \dots & \dots & \dots \\ p_{l1} & p_{l2} & \dots & p_{lm} \end{bmatrix} \quad (6)$$

The entropy  $S_j$  for a set of outcomes of a decision attribute  $j$  for  $l$  solution alternatives is determined as:

$$S_j = -K \sum_{k=1}^l p_{kj} \ln p_{kj}, \quad \forall j \quad (7)$$

where  $K$  represents a constant with a value of  $[1 / (\ln l)]$  at  $S_{max}$ , which guarantees that  $0 \leq S_j \leq 1$ .

Zeleny (1974) mentioned that the a *weight* assigned to an attribute is directly related to the average *intrinsic* information generated by a given set of alternatives at that attribute as well as to its subjective assessment. Based on this, the degree of diversification  $d_j$  of the information provided by an attribute  $j$  is defined as:

$$d_j = 1 - S_j, \quad \forall j \quad (8)$$

According to Hwang and Yoon (1981), if the decision-maker has no reason to prefer one attribute to another, the principle of insufficient reason (Starr and Greenwood 1977) suggests that each attribute should be equally preferred. Then the best weight set  $W$  associated with  $m$  decision at-

tributes the decision-maker can expect, instead of the equal weight, is as follows:

$$w_j = d_j / \sum_{j=1}^m d_j, \quad \forall j \quad (9)$$

If the decision-maker has a prior subjective weights ( $\lambda_1, \lambda_2, \dots, \lambda_m$ ), then subjective weights can be adapted using the set of calculated weights ( $w_1, w_2, \dots, w_m$ ) as follows:

$$w_j^0 = w_j \lambda_j / \sum_{j=1}^m w_j \lambda_j, \quad \forall j \quad (10)$$

#### 4 CONCLUSION

This paper has presented a robust simulation-based multi-criteria optimization methodology to solve Parametric Design (PD) problems in production and business systems of considerable complexity. The proposed methodology is based on an integration of four modules of proven methods: Genetic Algorithm (GA) search, Simulation Modeling (SM), Taguchi-based Robustness Module (RM), and Entropy-Method (EM). This synergic integration is aimed at tackling four common challenges in designing and optimizing complex, dynamic, and stochastic real-world production and business systems. This includes problem formulation/representation in terms of objective function and constraints (using SM), searching the often complex and large problem domain (using GA), handling the stochastic variability in system response (using RM), and performing optimization decisions based on multiple and often conflicting objectives in the absence of tradeoff information (using EM). GA search utilizes the SM to evaluate system multiple responses at different settings of design parameters. The RM module integrates robustness, Taguchi style, into the GA search engine by assigning a Signal-to-Noise (S/N) ratio to each SM simulation outcome. The EM module utilizes an Entropy method to assess the relative importance weights associated with the multiple decision attributes in a linear Multi-Attribute Utility Function (MAUF) and provide GA selection scheme with a unifying criterion to discriminate among feasible design alternatives. Until convergence, the form of the MAUF is updated at each GA search iteration step resulting in a dynamic and contextually dependent adaptation of the decision-maker's utility function.

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