The Effect of the Transmission Range on the Capacity of Ideal Ad Hoc Networks

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Abstract

We model stationary networks of randomly distributed devices that form an ad hoc network. Simulations are conducted in order to investigate the affects of the transmission range on the capacity of individual users. We load traffic into the system by calls, and measure a critical per node load that is needed to cause the blocking of a predefined fraction of calls. We derive an analytical framework on the estimation of the relationship of the capacity and the transmission range in order to find the global maximum of the critical per node load. Results in the literature show that for static networks the per node throughput decays as $O\left(\frac{1}{\sqrt{n}}\right)$ with n being the number of the nodes. In the present article we also investigate possibilities to eliminate this polynomial decrease, without restricting the devices to prefer short range calls. We found that the critical load stops decreasing with n in a non-trivial way if a number of special, long range nodes are introduced. The idea of an extended architecture is in no way new but the presented discussion bears interesting features.

Keywords

ad hoc networks, wireless networks

1. Introduction

An *ad hoc network* is a communication network of randomly distributed communication devices that lack centralized routing infrastructure. A number of recent studies are related to the investigation of throughput and traffic localization issues of ad hoc networks [1, 2, 3, 4]. One of the fundamental theoretical results is published by *Gupta* and *Kumar* in [5]. They have shown that for stationary ad hoc networks the per node throughput of the system decays as

$$O\left(\frac{1}{\sqrt{n}}\right)$$

with n being the number of nodes. Simply put, this is because the average path length of the calls grows as $O(\sqrt{n})$ and the total capacity achievable through a single transmission is proportional to n.

In accordance with the analysis quoted above, studies concerning ad hoc routing protocols agree that the bottleneck in capacity is the system's actual per node capacity. Every user entering the network adds more utilizable capacity to the grand total in the way that she can relay for others, but in peer-to-peer communication the major parameter affecting the throughput is still the node's own capacity. As a result, the actual per node capacity is one or two orders of magnitude smaller than the available grand total capacity. For example, *Broch* et al. found that using 2 Mbps interfaces only 50 kbps was actually usable [2]. These results suggest that large scale ad hoc networks have no or little use, if the traffic is not localized in some way. *Jinyang* et al. in [1] discuss the effects of local communication predominance based on IEEE 802.11 measurements.

The performance of the network is highly sensitive to the radio range used by the participating devices. If the radius is too low, then there might be too few different paths between two distant "parts" of the system and thus the nodes providing these few connections will reach their maximum usage fast and freeze . Increasing the radio range will lead to increase in overall throughput, as the load that used to drain those few will be distributed between more and more other nodes. However if the range is too high then every single transmission will be "heard" by too many nodes, which is clearly an exponential loss, as all recipients except the one intended will have to waste some of their utility due interference. A natural question is then that "Is there an optimum in capacity depending on the transmission range?"

In the present article we investigate the capacity of the network as the function of the common transmission range. We measure the capacity in the similar way as other papers do: the "bottleneck" capacity of individual nodes is taken, and we give an estimate on the per node capacity in general ideal ad hoc networks.

2. Model

We intend to find general features and limits of ad hoc networks and therefore a highly idealized model shall be used.

In our simulations the nodes are distributed randomly and independently over the unit square $[0;1]^2$. Direct transmission between two nodes is only possible if they reside within a fixed transmission range of each other. This way the network can be represented by a graph G in which the vertices and edges correspond to the nodes and the connections, respectively. In actual simulations we take the most populous connected component of G, in order to avoid calls where the destination nodes are isolated and thus unreachable.

Traffic is generated by calls that are initiated with equal probability between randomly chosen pairs of nodes. The calls are "put" into the system sequentially and thus no performance loss due to collision will be introduced. Every node is able to transmit and receive data in 100 units of capacity. The calls follow the shortest path between the source and destination occupying 1 unit of capacity of all nodes on the path. Interference is modeled as follows. If a node transmits then the node itself and all neighbors within the fixed radio distance use up 1 unit of their capacity. If a node has no free units left, when a new call hits it, then the call in question and all subsequent calls going through that particular node are blocked. We continue to generate calls until the fraction of blocked calls does not reach a predefined *blocking percentage*. The capacity of the network is defined by the *critical per node load* value: this is the number of calls a node needs to initiate in average to achieve the predefined blocking percentage.

The outcome of a single simulation run is sensitive to the actual realization of random parameters as in every numerical simulation. To the contrary of a number of "real world" traffic simulators in the present case a single run can yield almost no valuable information about the system investigated. Thus all derivations are made with the assumption that all variables are averaged over the ensemble, and likewise all simulations are repeated a sufficient number of times to make the variance of variables neglectable.

3. Analysis

In this Section we develop qualitative analysis of our model. The calculations used here are suitable for derivation of the transmission range dependence of the network capacity in ideal systems. For more realistic models the result should be modified according to the details of the actual implementation, however the main features, such as the existence of the maximum, still apply.

We carried out a number of simulations to provide data to control the findings. These data and comments are to be explained in Section 4.

3.1. Estimate of the Critical Per Node Load

To be able to generate and compare networks of similar connectivity properties but of different sizes we make all length parameters dimensionless. Length through the rest of the paper is measured in multiples of the *unit radius* r_0, which is defined by the size of the per node area in the whole system.

$$r_0 := \sqrt{\frac{A}{n\pi}} \tag{1}$$

where A is the area of the whole system ($\equiv 1$ in our case). The ratio of a given geometrical range r and the unit radius is called r's *normalized radius*:

$$r_N = \frac{r}{r_0} \tag{2}$$

Let $T(r)$ denote the number of nodes falling into a circle of radius r. The nodes are distributed uniformly randomly and thus the probability distribution of $T(r)$ will tend to the *Poisson*-distribution if $r \ll \sqrt{A}$, and using (2) we get:

$$\langle T(r) \rangle = \left(\frac{r}{r_0}\right)^2 = {r_N}^2 \tag{3}$$

(3) means that when a node transmits, then in average ${r_N}^2 - 1$ other nodes are receiving the signal and thus in average ${r_N}^2$ units of utility are used up in a single hop in total. If $\langle h \rangle$ is the expected length of a call in hops, then $\langle h \rangle \, {r_N}^2$ gives the utilization needed for a single call in average. We now can give an estimate on the total number of utilization units used up by the whole system when the blocking percentage is reached. If C is the total number of calls, then

$$\langle h \rangle \, {r_N}^2 \, C = n \, \langle u \rangle \tag{4}$$

where $\langle u \rangle$ is the average utilization of nodes.

$\langle h \rangle$ can be estimated by assuming that for sufficiently large n the path is a straight line between the two endpoints. In this case the geometrical length l_{call} of the path divided by the average distance between nodes that are in each others transmission range ($= r_1$) will estimate h. r_1 is the radius of a circle that has half the area of the circle with radius r_N, thus

$$r_1 = \frac{r_N}{\sqrt{2}} r_0 \tag{5}$$

$\langle l_{call} \rangle$ is the expectation value for the Euclidean distance of two randomly chosen points in $[0;1]^2$ and with elementary calculations $\langle l_{call} \rangle \simeq 0.52$. For the proof and validity check see Appendix A. Now for expectation values:

$$\langle h \rangle = \frac{\langle l_{call} \rangle}{r_1} \propto \frac{1}{r_N}. \tag{6}$$

Substituting (6) into (4) we have for the total number of calls initiated by the system:

$$\frac{C}{n} \propto \frac{\langle u \rangle}{r_N} \tag{7}$$

Furthermore the average free capacity of a node $100 - \langle u \rangle$ is clearly a polynomial function of r_N, as shown on Figure 2. Thus

$$100 - \langle u \rangle \propto \frac{1}{{r_N}^{\alpha}}$$

Parameter α is a function of the blocking percentage in the first order, and also of n in higher orders of approximation. We aim the further explanation of its exact value in a future work.

Now considering this dependence on r_N and (7) we get

$$\frac{C}{n} \propto \frac{{r_N}^{\alpha} - 1}{{r_N}^{\alpha+1}} \tag{8}$$

Figure 3 demonstrates that simulation can prove the derived relationship.

The function in (8) has a global maximum, so the main result of this Section is that if n and the blocking percentage are given, than the optimal radio transmission range can be calculated for a given network.

3.2. Power Control Added

Using (6) we are able to give a estimate concerning the effects of an ideal power control implementation on the system throughput. In this implementation the transmission ranges used by the nodes are adjusted individually. For every call and every node the radio range is chosen to be just enough to reach the next node on the path. As a result, the average transmission range will be reduced by $\frac{1}{\sqrt{2}}$ compared to the original radius without power control. This is implied by the fact that for a fixed node the normalized radius of the average distance of those other nodes within its transmission range is $r_{N,\mathrm{PC}} = \frac{r_N}{\sqrt{2}}$ and thus the right hand side of (6) will have an extra $\sqrt{2}$ multiplying factor:

$$\langle h_{\mathrm{PC}} \rangle \propto \frac{1}{r_{N,\mathrm{PC}}} = \frac{\sqrt{2}}{r_N} \tag{9}$$

This result implies that turning the power control on will yield a $\simeq 1.4$ increase in the throughput of the system, but otherwise the $\frac{1}{\sqrt{n}}$ fall will remain.

4. Numerical Results

In this Section we present the results of our numerical simulations. We have completed different tests for consistency checking the model and validating the analytical issues of Section 3.

4.1. Consistence

For our first test we aimed to obtain the capacity vs. node number relationship of our implementation. In this test we used n nodes and for every n a $10\times$ repeated average was taken. Blocking percentage was varied between 3-30%, but the actual value did not cause any difference in the n-dependence. The transmission range was set to $r_N = 3$, because at this value 99.9+% of all nodes will be in the connected component of the graph G with high probability B.

The calls per node throughput was measured in all cases. Figure 1 displays an example of our results, where the $1/\sqrt{n}$ fall is clearly detectable. It indicates, that our model confirms the result mentioned in the Introduction as expected: the capacity of an ad hoc networks – with users who “do not care” about the call distance – tends to zero, as the number of participants increases.

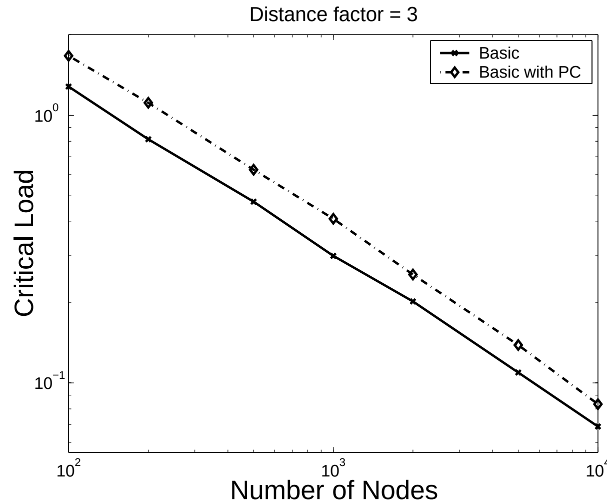

Figure 1: *Comparison plot for 10% blocking. The increase with power control is about $+35\%$.*

4.2. Power Control

We have repeated the same test of the previous Section, but the interference calculation was changed to the power controlled version (see Section 3.2). As expected by the predictions of (9), the outcome differs from the first case in a constant multiplier, but otherwise the $1/\sqrt{n}$ decrease remains the same. This supports our assumption, that power control does not affect the properties of the capacity in general. The result is summarized on Figure 1, where for the value of the constant we get $\simeq 1.35$.

4.3. Transmission Range Dependence

We now turn to the question of optimizing the radio range to be used to The actual number of nodes did not change the overall features of different simulation runs in the [500;50000] interval, so we decided to keep $n = 2000$. Blocking percentage is fixed to 10%.

In order to compare the results of Section 3 and especially to validate (8) we need to acquire first the r_N-

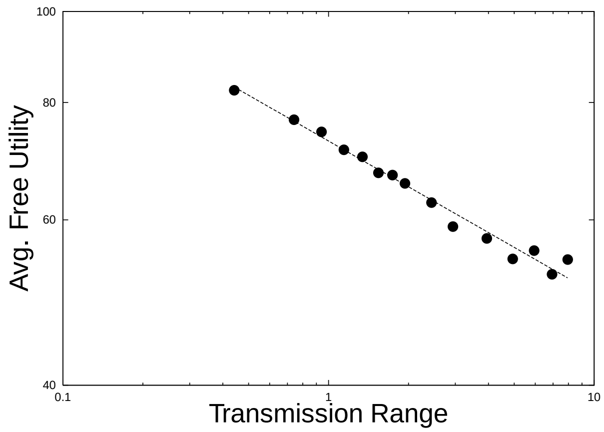

Figure 2:

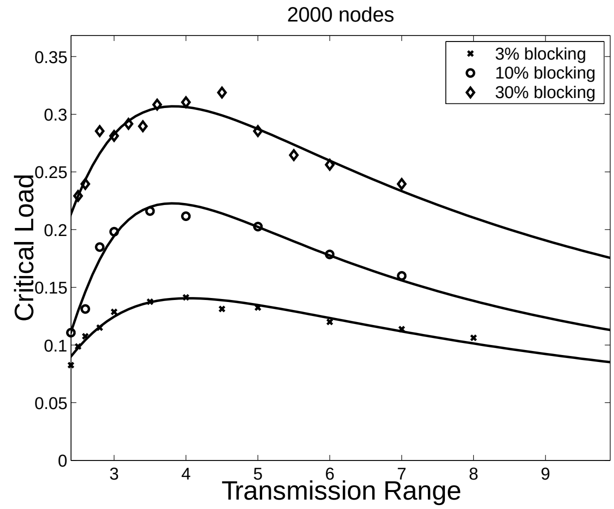

Figure 3: *Simulation data and fitted curves according* (8)

dependence of $\langle u \rangle$, the average number of utility units used by a single node. Fitting the data on Figure 2, we get for the average free utility

$$1 - \langle u \rangle \propto \frac{1}{(r_N - 2.03)^{0.16}}$$

Now we can us the obtained $\alpha = 0.16$ exponent to check (8)'s validity. We obtain the $\frac{C}{n}$ vs. r_N plot for the same parameters, as given above: $n = 2000$, blocking=10%, and we fit

$$a \frac{{r_N}^{0.16} - 1}{{r_N}^{1.16}} + b$$

to the acquired data. The data and fitted curves for this test and two other tests with different blocking values are shown on Figure 3.

The exponents α can bear information about the actual network parameter, however their detailed theory is under research and will be discussed in a future publication.

4.4. Meganodes and Capacity

In this Section we investigate the possibility of eliminating the $\frac{1}{\sqrt{n}}$ decrease in the capacity by stepping out of the model used in some way. The idea of an extended architecture is straightforward and in no way new but the present discussion yields novel points of view.

We modify the model described in Section 2 by introducing a second type of nodes. Type 1 nodes are the same as used in the basic model – they are equipped with a short range radio device (compared to the size of the whole network) and have 100 units of capacity. Type 2 nodes (*meganodes*) have two radio interfaces: a short range one for communicating with Type 1 nodes, and a longer range radio for reaching other meganodes. Furthermore the meganodes have more capacity available (we used $10^4 - 10^5$ units in our simulations), and they are much smaller in numbers (in the simulations presented in this paper two orders of magnitude in cardinality was used). This way new edges are inserted into the graph G of connections, but otherwise blocking percentage and calls per node are calculated the same way as for the basic case.

Now we implement our extended model to show the effect of the ad hoc "backbone". We again take n nodes but a small $x \ll 1$ part of them is chosen randomly to be Type 2 nodes with two interfaces, as described in Section 2. The later nodes have multiples of utility units of the Type 1 nodes. In this model we have two subsystems: the first one contains n Type 1 nodes, while the second has $x\,n$ Type 2s. We fix $r_N = 3$ for both Types, which yields that both subsystems are statistically almost connected for sufficiently large n-s (see Section B) independently of each other. In (1) for the first subsystem $n_{\text{Type1}} = n$ and for the second $n_{\text{Type2}} = x\,n$. Blocking percentage was again chosen to be 10%.

Now we obtain the throughput for n-s ranging from 10^2 to 10^4. Figure 4 displays typical results for different meganode radio capacities (10× and 200×) and x fractions. It can be seen that for $x = 0.02$ and $x = 0.05$ the systems behave the same in general. In contrast to the basic case a significant advance in calls per node is taking effect: above a threshold value of n the capacity decrease stops and $\frac{C}{n}$ freezes. Below the threshold the number of Type 2 nodes is not large enough to be statistically connected, which is supported by the result, that

$$1000 \simeq n_{\text{threshold}}(x = 0.05) > n_{\text{threshold}}(x = 0.02) \simeq 2000$$

The exact mechanism of the beneficial effect of the meganodes needs future investigation, but its evident, that they bestow a similar behavior on the network, as does the traffic localization.

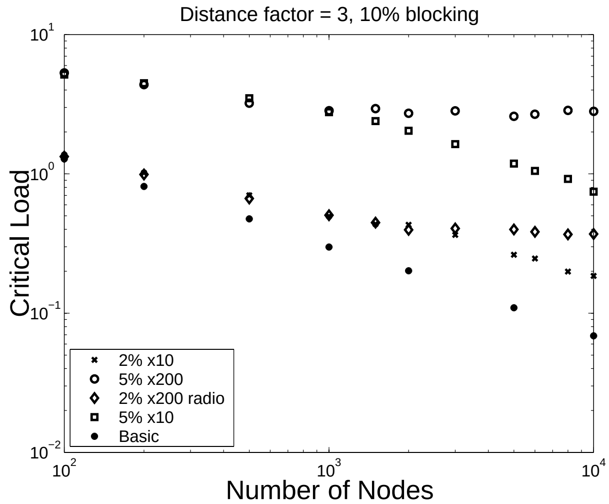

Figure 4: *Comparison for different meganode parameter sets. Note, that function values for the* $x = 0.05$ *case are multiplied by 3 for better visualization.*

5. Conclusion

In our paper we investigated wireless ad hoc networks from two aspects that are not accounted for in the related literature. The first one is the possibility of introducing an ad hoc backbone network based on special but still wireless nodes to co-exist and aid the base network. We have found that the $O(1/\sqrt{n})$ decrease in the throughput capacity of the system will cease in a non-trivial way at non-trivial parameter values.

The second attribute is the transmission range assignment. We found that it has an optimum value from the performance point of view, and we derived an estimate for it in general. Note that the exact value for the optimum can depend on the special features of the ad hoc model used.

6. References

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Appendix

A. Path Length Distribution

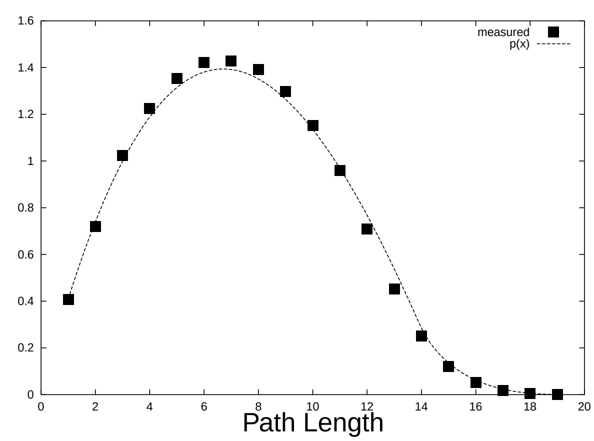

Figure 5: *Histogram for call length hop count distribution at* $n = 1000$ *and theoretical pdf from* (10).

Hereby we derive the calculation of the average hop count used during the construction of (6) and show a validity check for the equation.

Physical coordinates for the nodes in our model are generated by random variables of uniform distribution. Therefore the probability distribution of the distance of two nodes is the same, as the pdf of the d distance of two random nodes A and B in [0;1]^2. d is calculated by

$$d = \sqrt{(x_A - x_B)^2 + (y_A - y_B)^2},$$

where x and y denote the coordinates, which take values from the interval [0;1] with uniform distribution. Pdf of d can be obtained applying the theorems for convolution of random variables and pdf's of functions of random variables. Completing the somewhat rude but theoretically not much complicated calculations we get

$$p(d) = \begin{cases} 2d(d^2 - 4d + \pi), & d \le 1 \\ 2d(2\arcsin\dfrac{2-d^2}{d^2} + 4\sqrt{d^2-1} - d^2 - 2, & d > 1 \end{cases} \tag{10}$$

Now we are able to estimate the actual h hop counts for different node to node distances:

$$h = dr_1 \tag{11}$$

where r_1 is defined by (5).

Figure 5 is shown for checking the validity of (10). The Figure contains the plot of values of the analytical pdf $p(d)$ transformed by the inverse of (11), and simulation data. Simulation data was obtained as follow. For $n = 1000$ nodes and transmission range $r_N = 3$, the hop count of the shortest path for every pair of nodes was recorded. The procedure was repeated 100 times for averaging – this way a neglectable variance in intensities was achieved. As it is seen, the two plots are perfectly consistent, which proves that (6) is used rightfully.

B. Network Connectivity

In this section we present a method for the approximation of the transmission range needed for an almost fully connected network. By full connection here we mean that the undirected graph G representing the communication capabilities is connected.

For every realization of a random network in our model we can determine the S size of the largest connected component of G at a given transmission range r_N. The relationship $r_N(S)$ can be investigated by simulation means and it turns out that an analytical estimate can be derived using *random graphs* [9].

In [10] the authors present a method based on the generating function of the degree distribution of vertices of G. They derive a criterion for the formation of the giant cluster, which is a well measurable parameter in our case.

It is easily seen that the probability distribution of the number of nodes contained in any disc with radius $\mathrm{r_N}$ is a Poisson-distribution with expectation value $\mathrm{r_N}^2$. This means that

$$p_k = \frac{(\mathrm{r_N}^2)^k}{k!} e^{-\mathrm{r_N}^2} \tag{12}$$

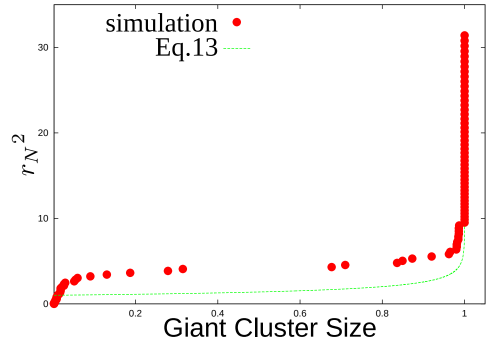

Figure 6: *The comparison of the clusterisation of our ad hoc graphs and the theory of random graphs.*

is the probability that a vertex will have $k-1$ neighbors (the -1 stands for the node itself).

Applying the result in [10] that the component sizes will diverge if

$$\sum_k k(k-2)p_k = 0$$

is true and substituting (12) here we can derive the relationship between the size of the giant component S and the transmission radius $\mathrm{r_N}$:

$$\mathrm{r_N}^2 = \frac{\log(1-S)}{-S}. \tag{13}$$

For $S = 0.999$ this relationship yields $\mathrm{r_N} \simeq 2.61$, which can be checked in a trivial way by simulation. Fig.6 shows that approaching the unity in S the random graph theory represented by (13) will yield a good approximation concerning the value of the transmission range to be used. According to these derivations a value

$$\mathrm{r_N} = 3 > 2.61$$

will generate a fully connected network with high probability.