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February 2004

Technical Report <u># DIT-04-042</u>

Effects of ADC Integral Non-Linearity on Digital Transmission

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Abstract: *This paper investigates the effects of Integral Non-Linearity (INL) on the performances of both A/D converters and Digital Communication Systems, which exploit Direct Digital Modulation. The performances of both PCM and Sigma-Delta converters affected by INL are considered and compared. Then, the effects of INL upon the BER performances of an OFDM system are evaluated and modeled. The accuracy of the theoretical model is discussed with respect to the ADC resolution and INL levels. It is shown that a multibit Sigma-Delta converter, operating at a low oversampling ratio, may outperform PCM converters.*

Keywords: *OFDM, Integral Non-Linearity, Sigma-Delta*

I. INTRODUCTION

Direct Digital Modulation (DDM) techniques, based upon A/D and D/A conversion of the modulated waveforms, are commonly used to implement modern Digital Communication Systems (DCS), achieving improved performances with respect to analog modulation schemes [1]. DDM also allows shifting signal-processing functions into the digital domain, thus obtaining more accurate and reproducible performances at a cost of more severe requirements for the involved A/D and D/A converters. Consequently, ADC and DAC unidealities may noticeably influence the overall system performance. It should be noticed that many DCSs, like Orthogonal Frequency Division Multiplexing (OFDM) systems or the downlink of Universal Mobile Telecommunication Systems (UMTS) [2], produce Gaussian distributed signals. In particular, OFDM is a multicarrier technique, adopted for several standards, like DVB-T [3], DAB [4], and ADSL [5], whose signals show a flat spectrum in the useful signal bandwidth [1]. Thus, characterizing the behavior of an A/D converter by means of Gaussian distributed testing signals may provide more useful results than the ones provided by a traditional sine wave test.

This paper analyzes the effects of Integral Non-Linearity (*INL*) upon the overall Bit Error Rate (*BER*) performance of an OFDM DCS. Both PCM and Sigma-Delta ($\Sigma\Delta$) converters are considered, and their performances are compared. Particular attention is given to multibit $\Sigma\Delta$s operating at a low oversampling ratio (*OSR*). In fact, in a wideband DCS, high *OSRs* may require an exceedingly high sampling rate. At first, the ADCs are considered as standalone components, and the effect of *INL* upon the output Signal to Noise and Distortion Ratio (*SINAD*) is analyzed. It is shown that a *b*-bit $\Sigma\Delta$ converter is more robust to *INL* than a *b*-bit PCM, while offering at the same time a better effective resolution. Then, the A/D converters are considered as a part of an OFDM receiver, and the influence of the ADC *INL* upon the OFDM *BER* performance is investigated. The performance requirements of A/D converters employed in OFDM systems have been evaluated in previous works [6]. Due to the high computational costs of low *BER* simulations, in this paper have been considered lower ADC resolutions. However, it has been verified that the presented results hold also for higher resolution ADCs. The *BER* analysis shows that DCSs robustness to ADC *INL* may depend not only on the ADC topology, but also on the DCS characteristics.

II. EFFECTS OF INTEGRAL NON-LINEARITY ON ADC PERFORMANCE

Fig. 1 shows the *SINAD* behavior of a PCM and two first order loop $\Sigma\Delta$ band-pass converters, fed with a white Gaussian distributed signal. The converters are assumed ideal, and *SINAD* is reported as a function of the input signal standard deviation σ_{IN} normalized to the ADC Full Scale *FS*. For $\Sigma\Delta$ converters, *FS* is related to the internal PCM. As the ADC stimulus is a wideband Gaussian noise, the *SINAD* cannot easily be evaluated by means of a Fourier analysis, usually performed when the input testing signal is a sine wave [7]. Thus, a time-domain approach has been adopted, that is based on the evaluation of the power of the quantizer error sequence. Each curve in Fig. 1 has a maximum, resulting from a tradeoff between granular noise, which

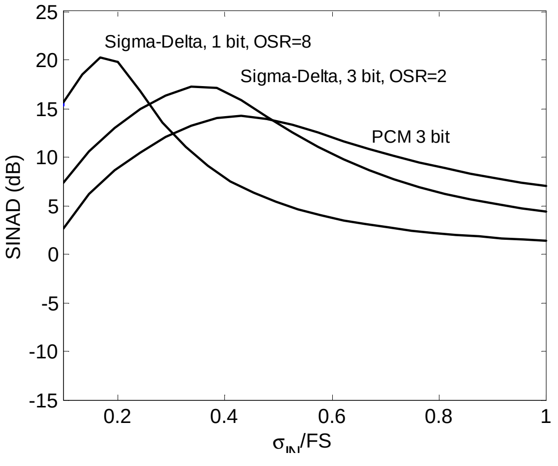

Fig. 1: *SINAD* performance of PCM and $\Sigma\Delta$ converters

grows with the ADC *FS*, and overload noise, which grows with the input signal dynamic range. Notice that the curve related to the PCM converter has been theoretically modeled [8]. It can be seen that, when optimal matching between input signal and ADC dynamic range is achieved, a 3-bit ΣΔ converter operating at *OSR*=2 provides a better *SINAD* than a 3-bit PCM. However, when overload is introduced, the *SINAD* of the 3-bit ΣΔ decreases faster than the 3-bit PCM one. Such a behavior is related to the feedback nature of ΣΔ converters, which may suffer overload even for amplitude limited input signals. The single-bit ΣΔ ADC operating at *OSR*=8 achieves a higher peak *SINAD*, but it shows an even higher sensitivity to overloading effects. Moreover, it can be seen that when the σ_{IN}/FS ratio deviates from the optimal value, the single-bit ΣΔ *SINAD* performance deteriorates faster than the 3-bit ΣΔ and PCM converters. As in a real transmission channel the ADC input signal dynamic range may vary quickly due to multipath and fading phenomena [9], it results that multibit ADCs are potentially a better solution for implementing a DDM based DCS receiver. In Fig. 2, the ratio between the *SINAD* of ideal ADCs and the *SINAD* of ADCs affected by *INL* is reported in dB as a function of σ_{IN}/FS, thus providing information on the performance reduction caused by *INL*. The quantizer has been modeled as a flash converter, and its resistors deviate from a nominal unit value by a Gaussian distributed offset, whose standard deviation σ_R equals 10% of the nominal resistance. Such a value, corresponding to large *INL* values, has been introduced to perform a worst-case analysis. It can be noticed how the 3-bit ΣΔ ADC shows a lesser performance degradation than the PCM converters. In fact, due to the ΣΔ feedback topology, small variations in the characteristics of the internal quantizer, which is located on the forward branch, do not have a great influence on the ADC performances. Moreover, due to the oversampling and noise shaping features, ΣΔ converters exhibit a greater accuracy with a lower quantizer resolution, that is, with less *INL* contributors.

III. EFFECTS OF ADC INTEGRAL NON-LINEARITY ON OFDM SYSTEM *BER*

In order to analyze the effects of ADC *INL* on the performances of a DCS, an OFDM system, similar to a DVB-T system operating in 2k-mode, has been considered [1],[3],[6]. Such a system uses 2048 QPSK modulated carriers, of which only 1705 are active [3]. Moreover, an Additive White Gaussian Noise (AWGN) transmission channel has been modeled.

Fig. 3 shows the system performance expressed in terms of *BER*, using both PCM and ΣΔ converters based on an ideal quantizer, as a function of Signal to Channel Noise Ratio (*SNR*). The results in Fig. 3 are obtained by optimally matching the ADC dynamic range to the standard deviation of its input signal, which is the sum of useful signal and channel noise, according to the results presented in Fig.1.

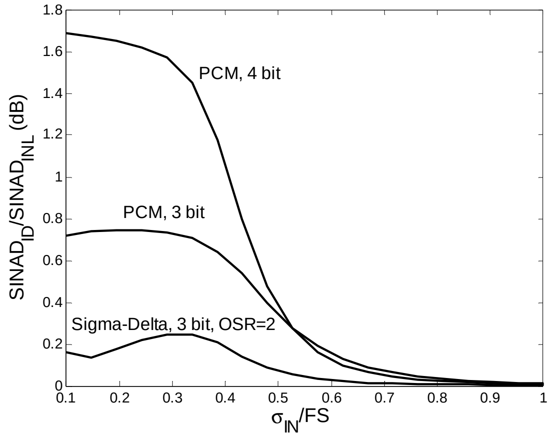

Fig. 2: *SINAD* worsening caused by ADC *INL* σ_{R}=0.1.

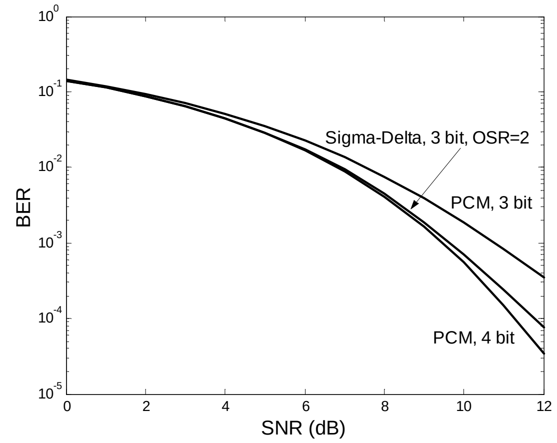

Fig. 3: *BER* vs. *SNR*, for ideal PCM and ΣΔ ADC.

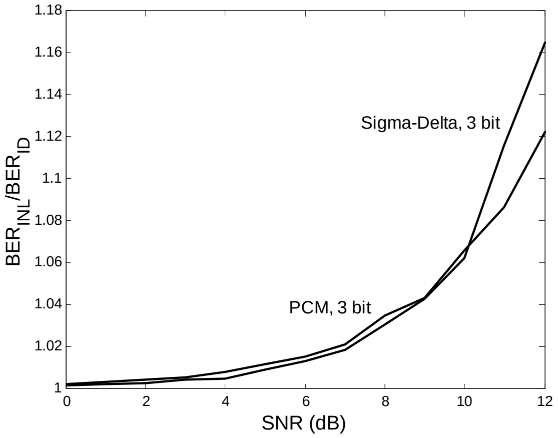

Fig. 4: *BER* worsening caused by ADC *INL*, for both PCM and ΣΔ ADC, σ_{R}=0.1.

Notice that, as signal and channel noise are uncorrelated, the ADC input power is the sum of the useful signal power and the AWGN power. In particular, Fig.3 shows that that the 3-bit ΣΔ ADC provides better performances than the 3-bit PCM one, and closely matches the performances of a 4-bit PCM ADC.

The loss of performance caused by quantizer *INL* is analyzed in Fig. 4 as a function of *SNR*. By comparing the *BER* variation of both PCM and ΣΔ ADCs, it can be observed that the 3-bit ΣΔ converter provides a slightly less robust performance to *INL* than the 3-bit PCM. Such a behavior may be explained with the interaction between the feedback topology of the ΣΔ converters and the non-linear features of the internal PCM, which introduce intermodulation noise in the useful signal bandwidth [1]. A similar phenomenon has been described in [1] and [9] in order to motivate the influence of ΣΔ ADC overload error on the *BER* performance of an OFDM receiver.

The effect of *INL* on the performance of the considered OFDM system has been modeled by assuming that the statistical properties of quantization noise do not significantly change when a moderate amount of *INL* is introduced. By generalizing the results reported in [1], under the hypothesis that quantization noise is white even in presence of ADC non-idealities, for a PCM conversion we obtain:

$$BER = \frac{1}{2} erfc\left(\left(\frac{N_A n_B}{N}\left[\frac{1}{SNR}+A\right]\right)^{-\frac{1}{2}}\right), \qquad (1)$$

$$A = \left(1+\frac{1}{SNR}\right)\frac{1}{SINAD(\sigma_{IN} / FS)},$$

where *erfc*() is the complementary error function, N is the number of OFDM carriers, N_A is the number of active carriers, and n_B is the number of bits transmitted by a single carrier in an OFDM symbol, which for QPSK modulations equals 2 [10]. The parameter $SINAD(\sigma_{IN}/FS)$ can be derived from Figs. 1 and 2 for a given value of σ_{IN}/FS. Equation (1) can be extended to ΣΔ converters, by keeping into account the noise-shaping feature. In fact, the overall *BER* may be obtained by averaging the *BER* of the OFDM carriers [1]. By assuming that the internal quantizer generates a white noise, the *BER* of the *i*-th carrier may be expressed by the following relationship:

$$BER_i = \frac{1}{2} erfc\left(\left(\frac{N_A n_B}{N}\left[\frac{1}{SNR}+\frac{|H_N(\omega_i)|^2}{\alpha}A\right]\right)^{-\frac{1}{2}}\right), \qquad (2)$$

where $H_N(\omega)$ is the ΣΔ noise transfer function, ω_i is the frequency of the *i-th* OFDM carrier, and α is the ratio between the in-band quantization noise power of the ΣΔ ADC and the quantization noise power of the ΣΔ internal quantizer. By defining α as:

$$\alpha = \frac{1}{2\pi}\int_{BW} |H_N(\omega)|^2 \, d\omega, \qquad (3)$$

where *BW* is the double sided signal bandwidth, the quantization noise contribution to *BER* is expressed as a function of the ΣΔ *SINAD*.

It should also be noticed that for an A/D converter operating in its granular region, *INL* effects on *SINAD* might be theoretically estimated. In fact, according to [7], the quantization noise power of a PCM converter affected by *INL* may be approximately expressed as

$$\sigma_q^2 = \sigma_{q0}^2 + \frac{1}{M}\sum_{k=1}^{M} inl_k^2, \qquad (4)$$

where σ_{q0}^2 is the quantization noise power of an ideal PCM converter, M is the number of quantizer thresholds and inl_k is the displacement of the *k-th* quantizer threshold due to *INL*. As (4) expresses the *INL* contribution to overall quantization noise power in a simple closed form, it is possible to estimate the *SINAD* variation induced by *INL*. It should be noticed that (4) has been derived in [7] under the assumption of uniformly distributed ADC input signal, and consequently it does not provide exact results when Gaussian distributed ADC stimuli are considered. In particular, it has been verified by means of meaningful simulations that such an approach introduces an error on the *SINAD* estimate which grows with the PCM resolution, exceeding 1 dB for a 6 bit flash PCM when σ_R=0.1. Fig. 5 reports the ratio between the *SINAD* obtained by applying Eq. (4) to the flash PCM described in section II and the *SINAD* obtained throughout simulations, expressed in dB as a function of σ_{IN}/FS. It is worth of notice that for an high σ_{IN}/FS, that is when deep overloading is introduced, *INL* does not affect anymore the ADC performances, and the reported curves show the same asymptotic behavior. It can be seen that, for the considered 3-bit and 4-bit flash PCM, the *SINAD* error introduced is negligible. Thus, by substituting the *SINAD* estimate in (1) and (2), it is possible to estimate the *INL* effects on the

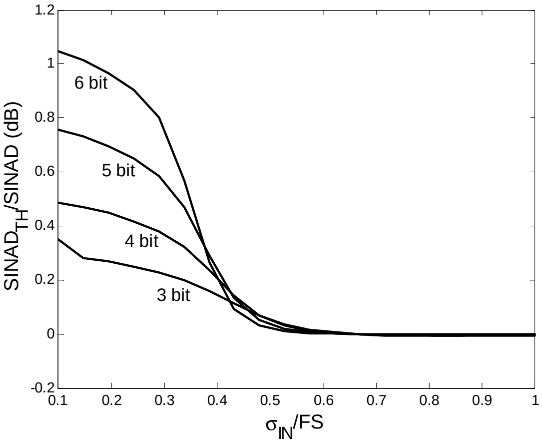

Fig. 5: *SINAD* estimation error introduced by applying Eq. (4) to a PCM affected by *INL* (σ_R=0.1), fed with a Gaussian distributed input signal.

system *BER*. Fig. 6 and 7, obtained for a 4 bit PCM and a 3 bit ΣΔ converter respectively, report the ratio between the *BER* estimate provided by (1)-(2) and the *BER* evaluated by means of simulations, as a function of the *SNR*. Various curves are reported, obtained for different levels of *INL*, that is for different values of σ_R. It can be noticed that the theoretical model overestimates the actual *BER* when large *INL* values are introduced. In fact, when *INL* is present, quantization error is no more a zero mean sequence. Consequently, a not negligible fraction of the overall quantization error power may be located on the DC component in the quantization error power spectrum. As the considered OFDM system performs bandpass A/D conversion, the DC component of quantization error is removed by the bandpass quantization noise filter. Thus, only a fraction of the quantization noise power introduced by *INL* actually affects the *BER* performances. This effect, as shown in Figs. 6-7, is more pronounced for high *SNR*, that is when quantization noise is dominant with respect to channel noise. It has also been verified that the accuracy of (1) and (2) is improved when higher ADC resolution are used, both for PCM and ΣΔ converters.

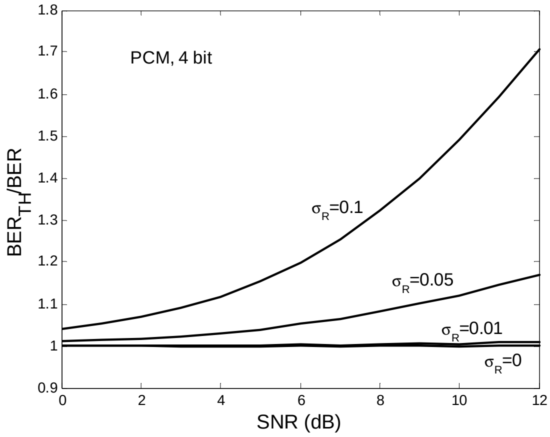

Fig. 6: Ratio between the predicted *BER* and the actual *BER*, evaluated for a 4 bit PCM ADC.

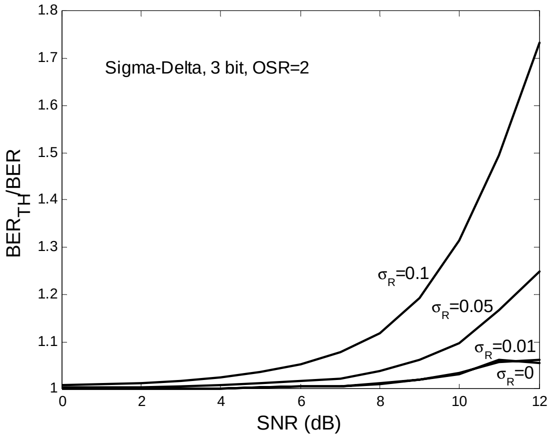

Fig. 7: Ratio between the predicted *BER* and the actual *BER*, evaluated for a 3 bit ΣΔ converter, *OSR=2.*

IV. CONCLUSIONS

The effects of *INL* upon the performance of ADCs and of an OFDM DCS exploiting DDM have been considered, showing that Sigma-Delta converters are more robust to *INL* than PCM ones. In particular, it is shown that a multibit Sigma-Delta ADC operating at a low *OSR* may outperform a PCM of the same resolution with respect to *SINAD* and *BER* performances. However, the robustness of the DCS to ADC *INL* may depend on both the ADC and DCS architectures. Consequently, a *SINAD* analysis alone may not conveniently describe the influence of A/D conversion on the performance of a DCS. An approximated theoretical model has been introduced, which conveniently describes the effects of *INL* upon both *SINAD* and *BER* performances, and its accuracy has been evaluated. Future developments are a more accurate modeling of the effects of *INL* and the extension of the analysis to other DCSs.

REFERENCES

[1] A. Moschitta, D. Petri, "Analysis of Bandpass Sigma-Delta conversion in OFDM Systems," proc. of the 8th IEEE Int. Conf. on Electronics, Circuits and Systems, Malta, 2-5 September 2001, pp 1387-1390.

[2] H. Holma and A.Toskala, *WCDMA for UMTS*, John Wiley & Sons, 2001.

[3] ETS 300 744, "Digital Video Broadcasting (DVB); Framing structure, channel coding and modulation for digital Terrestrial television (DVB-T)," available on the Internet at www.etsi.org.

[4] ETS 300 401, "Radio Broadcasting systems; Digital audio Broadcasting (DAB) to mobile, portable and fixed receivers," available on the Internet at www.etsi.org.

[5] ANSI T1.413, "Asymmetric Digital Subscriber Line (ADSL) Metallic Interface," 1995.

[6] A. Moschitta, D. Petri, "Performance Requirements of Bandpass Sigma-Delta Converters in OFDM Systems," proc. of 6th Euro Workshop on ADC Modelling and Testing, Lisbon, Portugal, 13-14 September 2001, pp. 125-129.

[7] P. Carbone, D. Petri, "Noise Sensitivity of the ADC Histogram Test," *IEEE Trans. Instrumentation and Measurements*, vol. 47, no. 4, August 1998.

[8] F.H. Irons, K.J.Riley, D. M. Hummels, G. A. Friel, "The Noise Power Ratio," *IEEE Trans. Instrumentation and Measurement*, vol. 49, no. 3, June 1999.

[9] J. G. Proakis, *Digital Communications*, Mc. Graw Hill, 1983.

[10] A. Moschitta, D. Petri, "Wideband Communication System Sensitivity to Quantization Noise," accepted for presentation at the IMTC 2002 conference, to be held in Anchorage, AK, USA, 21-23 May 2002.