Gravitation as an Inertial Process

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1. Introduction.

According to our common experience all our useful concepts of space and time are absolutely correlated with the existence and evolution of matter. This statement represents a qualitative definition of the strong interpretation of Mach's Principle (SIMP), and the primary purpose of the presented paper is to derive a formal quantitative expression of it. This purpose is achieved by a careful consideration of the 'inertia' concept, and the resulting formalism is a description of inertial processes given in terms of relations between our concepts of spacetime and our concepts of inertial matter. Thus, for example, the formalism suggests an understanding of the origin of 'inertial forces' which is directly analogous to our understanding of the electromagnetic force exerted on charged particles by electromagnetic fields.

The theory defines the metric properties of spacetime in terms of its inertial properties given as an 'inertial radiation field', and this latter field can be considered to represent a 'material vacuum', reminiscent of Dirac's 1951 proposal. A 'material vacuum' is a necessary component for any of the 'tired light' theories used to explain the cosmological redshifts. Such theories have been proposed, for example, by Zwicky (1929), Findlay-Freundlich (1953), Pecker, Roberts and Vigier (1972), and more recently, Pecker and Vigier (1986).

Of course, any description of the relation between concepts of spacetime and concepts of inertial matter is, in terms of the usual metaphor, a theory of gravitation; the presented theory represents the conventional gravitational processes as pure radiation processes in a thermodynamically evolving Universe. Thus, for example, a multipole analysis of a general spherically symmetric field can be readily performed, and shows that:

(1) The zero order term can be consistently interpreted as describing a 'heat death' Universe.

(2) The expansion up to the monopole term represents the 'weak field' case and leads to conformity with all the classical tests. Interestingly, the line element derived from these first two terms has exactly the same form as the Eddington form of the Schwarzschild line element of GR at $O(1/R^3)$;

(3) The dipole term is completely absent. This is crucial to the binary pulsar observations (Taylor *et al.*) because an interpretation of these in terms of a multipole analysis (Will, 1983) indicates the absence of dipole components to gravitational radiation.

2. General Considerations.

We are primarily concerned with obtaining a formal statement of SIMP, which states ... 'all our useful notions of space and time are absolutely correlated with the existence, and the evolution, of matter'.

This merely asserts the existence of an absolutely defined correlation between concepts of matter and concepts of space and time and, consequently, its formal expression will be a simple relational statement between the concepts. Naturally, a necessary prior condition for deriving such a formalism is that we have the appropriate space/time/matter concepts available; we can clarify the issues involved by reminding ourselves of the fundamental nature of an existing, very successful, description of matter/space/time relations - Newtonian mechanics.

The basic observation upon which Newtonian mechanics rests can be described as follows:- let two very smooth balls, b_1 and b_2 say, be in colinear collision on a very smooth flat surface; then, firstly, all changes appear to be colinear with the original motions and, secondly, if Δv_1 and Δv_2 are the changes in the respective velocities of b_1 and b_2, then $\Delta v_1 / \Delta v_2$ appears to be a constant, independent of the initial velocities of the two balls. This implies that if U_1 and U_2 are arbitrary colinear velocities of b_1 and b_2 respectively, then the quantity

$$[Q] = U_1 - \frac{\Delta v_1}{\Delta v_2} \cdot U_2 \tag{2.1}$$

is conserved through any collision involving b_1 and b_2. The Newtonian formalism is obtained by defining $\Delta v_1 / \Delta v_2$ to be the 'inertial mass' of b_2 relative to b_1, and 'linear momentum' as (inertial mass) × (velocity). The concept of 'inertial mass', which appears to be an intrinsic property of material, is interpreted as a model of material suitable for describing its behaviour in spacetime. With this formalism, the third law, and hence the whole of the Newtonian mechanics, follows directly from the conservation of $[Q]$ given at (2.1). This analysis exposes one particular point which allows us to focus clearly on the intrinsic nature of the formalized SIMP:-

> *Concepts of 'inertia' are fundamentally descriptions of certain particle properties expressed in terms of a* particular *model of spacetime. In the case of Newtonian mechanics, the 'inertia' concept is that of 'inertial mass', and the spacetime model is Galilean spacetime.*

Thus, in general, if we define the notation $S \equiv$ (spacetime model) and $I \equiv$ (inertial model), then $I \equiv I(S)$. Consequently, since a formal statement of SIMP represents a description of a given concept of spacetime expressed in terms of some concept of the inertial properties of material particles, then it must have the general form

$$f_1[S] \equiv f_2[I(S)], \tag{2.2}$$

where f_1 and f_2 represent the operations on S and I respectively which define the relation between S and I. So, in the final analysis, a formal statement of SIMP will be a description of spacetime given in terms of spacetime measurements—it is ultimately tautological! An uncritical reaction here is to suppose that the tautological nature of such a statement implies it can carry no information, and is therefore incapable of representing physical theory. However, this is not the case, for (2.2) carries information on two levels; firstly, the nature of (2.2) as a mathematical equivalence statement simply reflects the fact that it is a formal description of the fundamental equivalence between concepts of inertial spacetime and concepts of the inertial properties of material particles. The reality of this equivalence has been intuitively understood for at least two centuries, since Bishop Berkeley. Secondly, an explicit formulation of (2.2) will provide a definition of the relation between the concepts S and I; that is, the operators f_1 and f_2 will be defined. It is the knowledge of these operators which is added information, and which finally justifies designating the explicit statement of (2.2) as a non-trivial statement describing certain aspects of our experience.

3. Derivation of the Theory.

A necessary precondition for the explicit formulation of (2.2) is that we possess appropriate concepts of spacetime and inertial properties of matter. There are no reasons for supposing that the conventional Lorentzian concept of spacetime is not an adequate local model. Consequently, we make the assumption that $S \equiv$ Lorentzian spacetime and represent S by the corresponding metric tensor, Φ_{ij}, say. A consistent development of the theory requires that the corresponding concept of inertia must be formulated in terms of this spacetime model; thus, we must necessarily identify some inertial property of material particles which is relativistic with respect to Lorentzian spacetime.

An intuitive idea of the inertia concept is expressed by the statement 'inertia is resistance to change in motion', and this has the precise modern formulation that 'inertia is that property which, within an inertial frame, limits the possible states of motion for a particle to the interior of the locally defined light cone'. Using x_1, x_2, x_3 to denote spatial coordinates and x_4 to denote the temporal coordinate, then this latter statement can be formally expressed as 'inertia is that property which restricts the worldline of a particle in an inertial frame to be any trajectory satisfying

$$U(\underline{x}) \equiv \Phi_{ij} x^i x^j < 0, \tag{3.1}$$

where

$$\Phi_{11} = \Phi_{22} = \Phi_{33} = -\Phi_{44} = +1$$

and $\Phi_{ij} = 0$, otherwise, in the region of the coordinate origin'. That is, the essence of the particle's inertial nature is contained within the statement that the totality of all its possible trajectories is just contained within the surface (3.1). This is a statement of inertial properties which is invariant with respect to transformations in *S* and, consequently, suggests itself as a possible basis For the definition of inertial matter. As a first step, we define an 'inertial point particle' as the set of all the particle's future possibilities originating from the point at which it is defined; that is, we use (3.1) as the definition of an 'inertial point particle'. It is not, of course, a definition of 'inertial mass', since this latter concept is a description of relative inertial properties (*cf.* Newtonian inertial mass). It is, however, a description of the essential inertial properties of a single particle given in terms of the spacetime model, S, and so can be taken as a definition of the required inertial particle concept, I.

We are now in possession of a spacetime concept, S, and a concept of inertial particle, I, and must now consider the nature of the formal relation between them, so that an explicit statement of (2.2) can be given. If we assume, as an a priori truth, that any relationship between S and I must be covariant with respect to arbitrary coordinate transformations in S then we can easily see that

$$\Phi_{ab} = \left[\frac{d^2 U}{d x^a d x^b} \right] \tag{3.2}$$

where Φ_{ab} is the metric tensor of Lorentzian spacetime, U represents the inertial point particle, and where a factor ½ has been absorbed into U.

The formalism (3.2) is the required explicit statement of (2.2). In this form it represents a statement of the essential pointwise equivalence between the concepts 'inertial mass' and 'inertial reference frame', and provides total information on the nature of the relationship between our locally defined concepts of spacetime and inertia. A complete theory of inertial processes is obtained when we make the assumption that (3.2) is, in fact, the local expression of a *universal* equivalence between these two concepts. In this case, U must generalize from being a description of a 'point' inertial particle to being a description of an inertial particle field, and (3.2) must be replaced by its generally covariant form

$$g_{ab} = \frac{d^2U}{dx^a dx^b} - \Gamma^k_{ab}\frac{dU}{dx^k} \tag{3.3}$$

where

$$\Gamma^k_{ab} = -\frac{1}{2}g^{kr}\left[\frac{dg_{br}}{dx^a} + \frac{dg_{ra}}{dx^b} - \frac{dg_{qr}}{dx^r}\right]$$

as a description of the inertial processes in the Universe. If we now form the inner product $g_{ij}g_{ij}$,- from (3.3) we find

$$g^{ij} = \left[\frac{d^2U}{dx^i dx^j} - \Gamma^k_{ij}\frac{dU}{dx^k}\right] = 4 \tag{3.4}$$

which is the generally covariant wave equation for a scalar wave, U; that is, U is more properly described as an 'inertial radiation field'.

To summarize, the theory describes the geometric properties of spacetime purely in terms of an 'inertial radiation field' which is associated with the matter content of spacetime, and which is determined by the relativistic wave equation. We see that inertial matter is in the same relation to the inertial radiation field as charged matter is to the electromagnetic field. It follows that the 'inertial forces' experienced by accelerating inertial matter can be described in terms of the same metaphors used to explain electromagnetic forces on charged particles; that is, by the exchange of elementary particles mediating the interaction between inertial matter and the inertial field.

4. The Theory as a Gravitation Theory: General Discussion.

The theory has been derived as an explicit formal statement of the strong interpretation of Mach's Principle (SIMP) and, as a consequence, is properly designated as a theory of 'inertial processes' in the Universe. However, as a description of the relations between our concepts of spacetime and our concepts of matter it is, in terms of the usual metaphor, a theory of gravitation. So, it is necessary to consider to what extent the given theory conforms to the general requirements imposed on gravitation theories by modern perspectives.

The weakest of these requirements is that any viable theory must conform to the Einstein Equivalence Principle (EEP), and its metric must be coupled somehow to matter (weak Mach's Principle). The given theory uses a concept of 'inertial point particle', which is based on the idea that the totality of all possible particle trajectories is just bounded by the light cone. Of course, this can only be the case if, and only if, all possible individual trajectories are locally geodesic near the coordinate origin; that is, the concept of geodesic motion for inertial test particles is inherent to the given theory, and its conformity with the EEP follows automatically. The second requirement (weak Mach's Principle) is *a priori* satisfied. These weak properties of the theory are sufficient to ensure its consistency with the classical tests, as can be readily shown.

More stringent requirements arose in the late 1970's as a consequence of the binary pulsar observations (Taylor *et al.*); these can be expressed as the condition that any viable theory must conform to the Strong Equivalence Principle (SEP) defined, for example, by Will (1983). This principle is sufficient to classify virtually all past theories, with the singular exception of General Relativity (GR), as non-viable. The SEP can be qualitatively defined as the requirement that any viable theory of gravitation can only consist of a metric description of spacetime coupled directly to a description of its matter content, with no other field of any kind being involved. It is easy to see that the given theory does not violate this qualitative requirement, since it consists of a set of equations which relate the metric of spacetime directly to a particular representation of the matter content of spacetime (a description of its inertial properties), and that no other fields of any kind are involved. The requirement of consistency with the binary pulsar observations can also be interpreted as the requirement that for any given theory, approximated where possible as a radiation theory, the multipole expansion of its general spherically symmetric solution contains no dipole terms (Will). This property can be readily demonstrated. Thus it is unique, with GR, in conforming to the requirements of the SEP and, consequently, in being consistent with the binary pulsar observations.

To summarize, the given theory, describing 'inertial processes' and derived as a formal statement of SIMP, conforms to the stringent requirements imposed by modern perspectives on all viable theories of gravitation.

5. The Cosmic Background and Tired Light Theories

In reality, we have no conceptual difficulty in imagining an indefinitely (but not infinitely) extended flat spacetime. This suggests that we consider the existence of such a spacetime to represent some kind of limiting case for the Universe, and leads us naturally to ask what this implies about the nature of the corresponding U - the 'inertial radiation field'.

The assumption of globally flat spacetime means that (3.4) has the form of the flat spacetime wave equation (5.1)

$$\Box^2 U = 4 \,. \tag{5.1}$$

An arbitrary scalar field, *U*, satisfying this equation does not necessarily satisfy (3.2) since appropriate boundary conditions must be chosen. Assuming that homogeneous conditions at infinity are sufficient, then we find the solution

$$F(\underline{x}) = \int_{-\infty}^{+\infty} \frac{4d\left[(t-t_0) - R/c\right]}{R} dx_0^1 dx_0^2 dx_0^3 dt_0 \tag{5.2}$$

where

$$R^2 = \left(x^1 - x_0^1\right)^2 + \left(x^2 - x_0^2\right)^2 + \left(x^3 - x_0^3\right)^2$$

It is easily shown that this scalar field satisfies (3.2) so that the assumed boundary conditions are sufficient for the purpose. We now consider the general nature of this scalar field.

Since, according to (5.1), every point in spacetime can be considered as a source of equal strength then, with homogeneous conditions at infinity, U describes a perfectly homogeneous field which is in radiative equilibrium. If this field is quantized according to Bose statistics (which are appropriate to scalar fields), and if the usual laws of thermodynamics are assumed to be applicable, then the result will be an homogeneous scalar field with a perfect black-body spectrum. Thus, the given theory describes the properties of a globally defined flat inertial spacetime in terms of a globally defined scalar radiation field which is homogeneous, has black-body characteristics, and is interpreted as an 'inertial radiation field' describing the inertial properties of the associated matter distribution in spacetime. The character of this inertial field must, in some sense, reflect the character of the matter distribution with which it is associated, and the question arises 'what could this matter distribution be?'

As a means of arriving at a tentative answer to this latter question, consider our local experience; this relates that the locally flat spacetime of the free-fall observer is perfectly correlated with an electromagnetic field (the cosmic radiation background, CBR) which has an approximate black-body spectrum. If we tentatively assume that the CBR is an artifact of thermodynamic evolution in the universe (as it would be in any 'tired-light' theory), then, ultimately, this scalar matter field will evolve into a black-body state. In addition, it is now well known that, whilst in different inertial frames the CBR exhibits different Doppler shifts, there exists a frame of rest within which it appears isotropic. Correspondingly, any non-trivial scalar field derived from the CBR will also be isotropic in the frame. Moreover, the scalar nature of any such field ensures that, since it is isotropic in one inertial frame, it is isotropic in all inertial frames. Thus, according to our experience, the locally flat spacetime of the free-fall observer is perfectly correlated with a non-trivial scalar matter field which appears to be perfectly isotropic, and which has an approximate blackbody spectrum. If we tentatively assume that the CBR is an artifact of thermodynamic evolution in the Universe (as it would be in any 'tired light' theory) then, ultimately, this scalar matter field will evolve into a black-body state. Thus, given this latter assumption, the end-state CBR has properties which can be described by a scalar field which is perfectly isotropic and which has a perfect black-body spectrum. This scalar field has identical general properties to those possessed by the inertial radiation field of the limit case Universe, and therefore suggests itself as the matter field associated with this Universe; that is, the limit case Universe can be rationally associated with conditions in the 'heat death' Universe.

We have shown that assuming the CBR to be an artifact of thermodynamic evolution leads to the rational association of a 'heat death' Universe with a globally flat universe. If we accept this association to be unique, then it is possible to draw a closer connection between the given theory and the general class of ‘tired light’ theories: given this latter uniqueness, then a thermodynamically active Universe cannot be globally flat and, consequently, heat death and global flatness must be approached together. It follows directly that thermodynamic evolution must proceed through interaction with the inertial radiation field, and so this field plays the role of the ‘material vacuum’ required by the general class of ‘tired light’ theories. The CBR then has a direct interpretation as the product of a ‘tired light’ effect in which photons emitted from discrete sources interact with the inertial field; assuming the operation of the usual laws of thermodynamics this interaction will deplete photon energy, leading to a redshift effect, and give rise to a general redistribution of inertial and electromagnetic energy towards the black-body state. Theories of this nature, using hypothetical ‘material vacuums’, have most recently been suggested as the origin of the cosmic redshift by Pecker and Vigier (1986).

6. Closing remarks

A serious comment: for me, Newtonian mechanics represents the perfect example for the development of physical theory observations, concepts, formalism, theory. So, thank you Newton for inspiration, and thank you Pecker for encouragement.

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