Remarks on Photon-Hadron Interactions

E. Comay

School of Physics and Astronomy, Raymond and Beverly Sackler Faculty, of Exact Sciences, Tel Aviv University, Tel Aviv 69978, Israel

Theoretical aspects of VMD and related approaches to real photon-hadron interaction are discussed. The work relies on special relativity, properties of linearly polarized photons, angular momentum conservation and relevant experiments. It is explained why VMD and similar approaches should not be regarded as part of a theory but, at most, as phenomenological models. A further experiment pertaining to this issue is suggested.

Keywords: Photon, Hadron, VMD

Introduction

The discovery of the photon in the early years of the previous century has identified it as a pure electromagnetic object. Many years later it has been observed that hard photons interact with hadrons in a manner which is akin to strong interactions and appears to be independent of the electric charges of the hadronic

target. The following experimental results can be used as an illustration of this conclusion.

The cross section of hard photons scattered from a proton target is practically the same as that from a neutron target (see [1], pp. 292-293). Another kind of data is the ratio between the numbers of hadrons and leptons emitted from a photon-proton interaction region. Here the number of hadrons is greater by four orders of magnitude with respect to the leptonic number. (This point is easily inferred from the data discussed in [2], pp. 1567-1568 and from the table on p. 323 of [1].) Thus, it is concluded that "there is ample evidence which shows that the photon's hadronic structure plays a significant role in its interactions" (see the abstract of [1]).

An approach attempting to explain experimental results of photon-hadron interactions claims that a physical photon is composed of a pure electromagnetic component *and* a hadronic component. According to this claim, the wave function of a physical photon takes the form

$$\mid \gamma > = c_0 \mid \gamma_0 > + c_h \mid h > \tag{1}$$

where $\mid \gamma >$ denotes the wave function of a physical photon, $\mid \gamma_0 >$ denotes the pure electromagnetic component of a physical photon and $\mid h >$ denotes its hadronic component. c_0 and c_h are appropriate numerical coefficients. Relation (1) means that a real photon fluctuates between a pure electromagnetic state and a hadronic state. Moreover, this fluctuation is an *inherent property of the photon and is independent of its distance from the hadronic target*. The relative time allotted to each state is proportional to the absolute value of the square of the corresponding coefficient of (1). This approach takes several ramifi-

cations, many of which are known as Vector Meson Dominance (VMD) or Vector Dominance Models (VDM). These approaches assume that the hadronic part of (1) is a neutral vector meson, which has the same spin, parity and charge conjugation quantum numbers as the photon (see eg. eq (2.1) on p. 271 of [1] and eq. (10.104) on p. 298 of [3]). A related claim states that $| h >$, the hadronic part of (1), may belong to a larger set of hadronic states[4]. All these approaches are called below Photon's Hadronic Structure Approaches (PHSA). The present work examines critically the theoretical meaning of the common idea of PHSA, which is manifested in (1).

A brief discussion of common properties and of differences between the notions of a theory and a model is helpful for a clarification of the main point of this work. The distinction presented below between these notions should be regarded as a suggestion which is useful for the case discussed here. Obviously, other definitions may be used, if they appear helpful in other circumstances.

The following properties are common to a theory and to a model.

A. Both provide a scheme leading to mathematical formulas which describe experimental data. The scheme should be mathematically self-consistent.

B. Both are acceptable within an appropriate domain of validity. (See [5] for a discussion of the notion of a validity domain of a theory.)

C. Both require a knowledge of certain constants which are determined experimentally.

On the other hand, the following properties distinguish between a theory and a model.

D. Within the corresponding validity domain, predictions of a theory should be very precise whereas a model is acceptable even if it yields just reasonably approximate predictions.

E. The physical constants used in a theory can be determined by means of any set of experiments, provided they are carried out within the theory's validity domain. Another aspect of this point is that in the case of a theory, after fixing the required constants, one can apply *extrapolation*, into far regions, provided they are included in the theory's validity domain. (Thus, for example, after measuring the mass of a macroscopic body, one may use Newtonian mechanics for all velocities which are much smaller than the speed of light.) Contrary to this, a model is generally useful only within a small domain where its constants have been determined. In other words, a model is useful in cases where *interpolation* is applied and deteriorates as it is extrapolated into far regions.

F. A model is tested by its practical benefit. If problems arise, a model may be improved by an addition of certain corrections. (Thus, for example, the nuclear liquid drop model is improved by an addition of nuclear shell model terms, which account for nuclear magic numbers.) By contrast, a theory is tested by its *correctness*. In other words, a model is regarded as useful or not very useful for certain applications whereas a physical theory can be *refuted* if it

does not fit either experimental data or well established
theories which have been confirmed by many experiments.

It is explained in the rest of this paper why PHSA formulas
belong to models and do not constitute a part of a theory. This
is probably the common belief of the physical community as
seen from the term VDM (Vector Dominance Models) and from
its inclusion in the phenomenological sections of PACS and of
hep-ph@arXiv.org.

The present work discusses only the theoretical side of PHSA.
On the other hand, the problem of its usefulness as a model is
beyond the scope of the paper. In the second section it is shown
that PHSA is inconsistent with some well established theoreti-
cal results. Experiments relevant to this matter are discussed in
the third section. Concluding remarks are the contents of the
last section. Expressions are written in units where $\hbar = c = 1$.
Energy-momentum units are MeV and $f^{-1} \simeq 197 MeV$. The
cross section unit is $mb = 0.1\, f^2$.

Theoretical Problems of the Photon's Hadronic Structure Approach

Several theoretical difficulties of PHSA are pointed out here.

Let us examine the implications of Lorentz transformations
on the coefficients c_0 and c_h of (1). For this end, consider
Wigner's analysis of the Poincare group[6,7]. The analysis shows
that a massive particle can be regarded as an irreducible repre-
sentation of this group, characterized by its self mass and spin.
Massless particles, like the photon, belong to a special case where
spin is replaced by helicity.

This analysis is used in quantum field theory. It proves that

photons and hadrons are distinct objects. Since a quantum mechanical state of a particle is characterized by the eigenvalues of the self mass and the spin (or helicity), one concludes that *every term* of its wave function should have the same eigenvalues of these operators. It follows that (1) *cannot* represent a quantum mechanical state of a particle because the first term on the right hand side is massless and the second term is massive. This conclusion proves that PHSA is inconsistent with relativistic quantum field theory.

Another relativistic point is the behavior of the coefficients c_0 and c_h of (1) under Lorentz transformations. It appears that in PHSA, it is assumed that Lorentz transformations do alter these quantities, because the hadronic part of soft photons is assumed to be negligible (see [3], p. 298). Thus, following this assumption, one does not expect that optical photons (or the blackbody radiation photons, etc.) interact strongly with hadrons. The assumption that the relative size of the coefficients c_0 and c_h of (1) depends on the photon's energy is denoted below as the energy dependence assumption.

It is not clear how the energy dependence assumption is embedded in a relativistic theory. Indeed, assume that one measures energetic photons and finds that for 10% of the time they interact like hadrons and for 90% of the time they interact like pure electromagnetic objects. Moreover, as claimed by PHSA, this ratio is an inherent property of the photon and is independent of its proximity to an hadronic target. Hence, relativity tells us that this ratio must be conserved for Lorentz transformations in general and for a Lorentz transformation into a frame where the photon's energy is small, in particular. This matter can be restated as follows. By their definitions, the coefficients

c_0 and c_h of (1) are the transition probabilities from a state of a physical photon to that of a pure electromagnetic photon and that of a hadron, respectively. Now, "the transition probability has an invariant physical sense" (see [6], top of p. 150). This outcome is inconsistent with the energy dependence assumption. Experimental aspects of this point are discussed in the next section.

Another issue is related to transverse properties of photons. Thus, let us take a linearly polarized photon moving parallel to the z-axis, while its electric field is parallel to the x-axis. Experiments measuring the interactions of such a photon with unpolarized target of protons are discussed below. Properties of linearly polarized photons clearly do not satisfy cylindrical symmetry around the z-axis, because a rotation around this axis alters the direction of its electric and magnetic fields. For photons of this kind, the vector potential **A** is parallel to the electric field. Hence, since the interaction term of the electromagnetic Lagrangian density is [8,9]

$$L_{int} = -j^\mu A_\mu, \tag{2}$$

one finds that a linearly polarized photon interacts with matter in a manner which breaks cylindrical symmetry around the z-axis.

Let us turn to the interaction of the assumed hadronic part of this photon. Angular momentum conservation is utilized and calculations carried out below show that, *under this restriction*, the assumed hadronic part of a photon interacts with an unpolarized target in a manner which conserves cylindrical symmetry. Special emphasis is put on the M values of the angular momenta, namely on their projection on the z-axis.

Due to angular momentum conservation, the angular momentum part of the photon's hadronic state should be the same as that of the helicity of the (ordinary) electromagnetic state of the photon. Thus, since we have a linearly polarized photon, its spin part has an equal amount of positive and negative helicity (see [8], pp. 114-116; [9] pp. 273-275 and [10]) and is written as a sum of two terms

$$| SM >= (| 11 > + | 1 - 1 >)/\sqrt{2}. \qquad (3)$$

Here S denotes spin and M denotes its projection on the z-axis. Due to angular momentum conservation, (3) describes also the spin state of the assumed hadronic part of the photon. Let us examine this state under a rotation by $\pi/2$ around the z-axis. (This rotation exchanges the directions of the undulating electric and magnetic fields of the linearly polarized photon.) Under this rotation, each term of the wave function is multiplied by $e^{-im\phi}$[11]. Thus, in the present case the corresponding factor is $e^{\mp\pi/2} = \mp i$ and we have in the rotated frame

$$| SM >_{rot}= -i(| 11 > - | 1 - 1 >)/\sqrt{2}. \qquad (4)$$

The scattering process is determined by the evaluation of the Hamiltonian. Let $p_{\pm 1/2}$ denotes the wave function of the proton at the target whose spin m-value is $\pm 1/2$, respectively. Thus, the full wave function associated with (3) is

$$(| p_{\pm 1/2}11 > + | p_{\pm 1/2}1 - 1 >)/\sqrt{2}. \qquad (5)$$

Using the Hamiltonian operator and (5), one finds the fol-

lowing four terms

$$\frac{1}{2}(< 11p_{\pm 1/2} \mid H \mid p_{\pm 1/2}11 > +$$
$$< 11p_{\pm 1/2} \mid H \mid p_{\pm 1/2}1 - 1 > +$$
$$< 1 - 1p_{\pm 1/2} \mid H \mid p_{\pm 1/2}11 > +$$
$$< 1 - 1p_{\pm 1/2} \mid H \mid p_{\pm 1/2}1 - 1 >). \qquad (6)$$

The corresponding expression pertaining to (4) is

$$\frac{1}{2}(< 11p_{\pm 1/2} \mid H \mid p_{\pm 1/2}11 > -$$
$$< 11p_{\pm 1/2} \mid H \mid p_{\pm 1/2}1 - 1 > -$$
$$< 1 - 1p_{\pm 1/2} \mid H \mid p_{\pm 1/2}11 > +$$
$$< 1 - 1p_{\pm 1/2} \mid H \mid p_{\pm 1/2}1 - 1 >). \qquad (7)$$

Observing (6) and (7), one realizes that they differ by the sign of the second and the third terms. The following argument proves that each of these terms vanishes. Therefore, (6) and (7) are the same and if the PHSA assumption holds then the scattering is expected to be invariant under a rotation around the z-axis.

Three kinds of angular momenta are involved in the process: that of the assumed hadronic part of the photon, (3), that of a proton at the target (having $s = 1/2$ and $m_s = \pm 1/2$) and the spatial angular momentum between the incoming vector meson and the proton participating in the interaction. Since the linear momentum of the photon and of its assumed associated vector meson is parallel to the z-axis, the projection of the spatial angular momentum on this axis vanishes $(\mathbf{r} \times \mathbf{p}) \cdot \mathbf{p} = 0$.

Using this property of the spatial angular momentum, one finds that the overall M-value of the *bra* of the second term of (6) is

$$M(< 11p_{\pm 1/2} |) = 1 \pm 1/2, \tag{8}$$

whereas the corresponding value of the *ket* is

$$M(| p_{\pm 1/2}1 - 1 >) = -1 \pm 1/2. \tag{9}$$

Now, since the Hamiltonian H is a scalar in the 3-dimensional space, one finds that the M-values do not match. Hence, the second term of (6) and of (7) vanishes. By the same token, this result holds also for the third term of these expressions. This conclusion completes the required proof.

This discussion shows that a pure electromagnetic linearly polarized photon interacts with matter in a manner which breaks cylindrical symmetry, as seen in the interaction part of the Lagrangian (2) and remembering the x-direction of the vector potential **A**. On the other hand, the assumed hadronic part of such a photon conserves this symmetry. This result means that a transverse information of the photon, namely - its linear polarization, *disappears* as the physical photon is assumed to fluctuate into a hadronic state. This property clearly reduces the theoretical appeal of the VMD hypothesis. As shown in the next section, it can also be used in an experimental test of its validity.

Experimental Considerations

Let us turn to experimental aspects of the topics discussed in the previous section. First, the behavior of the coefficients c_0 and c_h of (1) under Lorentz transformations is examined. Two

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alternatives are discussed where the energy dependence assumption holds or fails, respectively.

Assume that the energy dependence assumption holds. This assumption is probably made in order to settle problems of expected soft photon interactions with hadrons, which otherwise emerge from the VMD assumption. However, it leads to problems as one takes soft photons and examines them in another inertial frame where these photons are very energetic.

Photon-photon interaction is probably most suitable for this purpose, because, unlike massive targets whose rest frame may appear preferential, photons have no rest frame. Consider an inertial frame Σ and two sources of soft photon rays (see fig. 1)
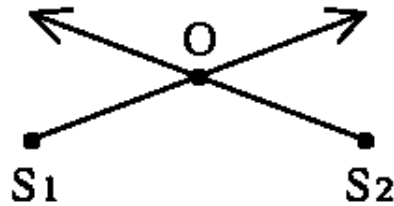
Figure 1. Two rays of light are emitted from sources S_1 and S_2 which are located at $x = \pm 1$, respectively. The rays intersect at point O which is embedded in the (x, y) plane.

Here the photons interact electromagnetically and, as far as the linearity of electrodynamics and Maxwell equations hold, the photon-photon interaction vanishes.

Now, let us examine the process in another frame Σ'. In Σ, Σ' is seen moving parallel to the negative direction of the y-axis and its velocity is not much smaller then the speed of light.

Hence, in Σ', the photons emitted from S_1 and S_2 are very energetic. Now, if VMD and (1) hold then *these photons should have a hadronic part*. Thus, in Σ', the photon-photon interaction is expected to consist of two kinds of dynamical processes. The first one is the pure electromagnetic process which is obtained from a Lorentz transformation of what is found in Σ and yields a null quantity. The second process is the hadron-hadron interaction which should take place under the assumption examined here. This is a contradiction because the percentage of events where photons interact and exchange energy-momentum should be the same in all inertial frames. This result proves that the assumption stating that the coefficient c_h of (1) tends to zero if one examines the photon in a frame where it is very soft leads to a contradiction. Indeed, here one may take the opposite course and look at a collision of very soft photons in another frame where both are very energetic.

The second case is the ordinary quantum mechanical approach where c_0 and c_h of (1) conserve their absolute value under a Lorentz transformation (see [6], top of p. 150). For examining this issue, let us take, for example, the Compton scattering of 1 MeV photon colliding with an electron of a hydrogen atom. In this example, calculations refer to the backwards direction $\theta = \pi$. The Compton process is well known[12]. The angular dependence and the energy of the emitted photon are obtained from the Compton relation

$$k_{out} = \frac{k_{in}}{1 + (2k_{in}/m)sin^2(\theta/2)}. \qquad (10)$$

Putting $k_{in} = 1$ MeV, $m = 0.511$ MeV and $\theta = \pi$, one finds for

the scattered photon

$$k_{out} \simeq 0.2 MeV. \tag{11}$$

The Compton unpolarized cross section is[12]

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{2m^2} \left(\frac{k_{out}}{k_{in}} \right)^2 \left(\frac{k_{out}}{k_{in}} + \frac{k_{in}}{k_{out}} - sin^2(\theta/2) \right), \tag{12}$$

where $\alpha \equiv e^2 \simeq 1/137$. In the present experiment, one finds

$$\frac{d\sigma}{d\Omega}(\theta = \pi) \simeq 6.5 mb. \tag{13}$$

Let us turn to the photon-proton interaction. Here, the Compton process (12) can be ignored because the proton/electron mass ratio is about 2000 and the cross-section is smaller by a factor of 1/4000000. On the other hand, if VMD-PHSA holds and the photon has a hadronic component, then one expects another process which is a meson-proton scattering. Since, in this case, the proton's mass is 938 times that of the photon's energy, one should have here a scattering process where the photon's energy is (nearly) conserved.

The effective radius of a meson-proton interaction region is less than $10f$ and the photon's momentum is $1 MeV \simeq 1/197 f^{-1}$. Hence, one finds that the spatial angular momentum practically vanishes and, in a partial wave analysis, only the S-wave contributes to the process.

A crude estimate of the vector meson-proton cross section can be obtained for the case discussed here from the data on π-proton cross section[13]. Here one finds that in the low energy

limit

$$\sigma \simeq 7mb. \tag{14}$$

Relying on a quark count, one concludes that a vector meson-proton cross section is of the same order of magnitude as that of the π-proton (14). Hence, since we have here an S-wave, the expected differential cross section is obtained from a division of (14) by 4π

$$\frac{d\sigma}{d\Omega} \simeq 0.6mb. \tag{15}$$

Due to the assumption discussed here, where c_h of (1) is not negligible, one compares (15) with the backwards Compton scattering differential cross section (13). Thus, it is found that if this version of VMD-PHSA takes place, then a certain percentage of the photons scattered backwards in an actual Compton experiment should conserve the energy of the incoming photon and violate the Compton relation (10) which yields (11). In other words, for $\theta = \pi$, the Compton scattering yields outgoing photons whose energy is $0.2\ MeV$, whereas the assumed VMD-PHSA effect should yield $1\ MeV$ ones.

To the best of the author's knowledge, this effect has never been reported. Evidently, due to their energy difference, a distinction between these kinds of scattered photons can be easily made. It is interesting to carry out such a test of VMD-PHSA in an experiment dedicated to this problem.

Another issue is the test of cylindrical symmetry in a scattering process of linearly polarized photons on protons. As shown in the previous section, electrodynamics breaks this symmetry

whereas the assumed vector meson is expected to interact with protons in a manner which conserves it. Related experiments have been carried out a long time ago[14-16]. These experiments use linearly polarized photons and measure outgoing pions in γp and γn collisions. The results prove that cylindrical symmetry is not conserved, contrary to what is expected from VMD.

Concluding Remarks

This work examines theoretical aspects of VMD and related approaches. It is shown above that VMD can be no more than a phenomenological model. Evidently, if its merits are extended and it is regarded as a part of a theory then it should stand refutation tests. As a matter of fact, results of theoretical and experimental tests show that VMD is inconsistent with some well established theories and with experiments. A further experiment dedicated to this issue can be carried out as discussed in the third section.

The conclusions of this work are in accordance with earlier criticism of VMD. Thus, in his Nobel prize lecture, J. I. Friedman shows an example of inconsistency of VMD with experimental data (see pp. 617-618 of [17]). The reader is also referred to the rather exceptional humoristic-sarcastic poster published at the bottom of page 267 of [1]. This poster reflects theoretical difficulties as well as disbelief in VMD.

Another result of this work is that the hadronic features of real photon-hadron interaction await theoretical interpretation.

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