# A Cooperative Negotiation Protocol for Physiological Model Combination

Nicola Gatti and Francesco Amigoni
Dipartimento di Elettronica e Informazione
Politecnico di Milano, Milano, Italy
{ngatti,amigoni}@elet.polimi.it

## Abstract

*The global model of a complex phenomenon can emerge from the cooperative negotiation of agents embedding local partial models of the phenomenon. We adopted this approach to model complex physiological phenomena, such as those related to the metabolism of glucose-insulin and to the determination of the heart rate (pacing). In this paper we formally describe and analyze the properties of a cooperative negotiation protocol we developed to allow the agents to produce a global coherent model of a physiological phenomenon. We concentrate on the study of the conditions under which an agreement is guaranteed to be reached. We also show details of an application concerning the pacing problem.*

## 1. Introduction

A system is considered complex when its behavior depends from the interaction of several heterogeneous subsystems [3, 13]. Complex systems are encountered in several disciplines, such as physics, chemistry, biology, psychology, economics, and political science. Physiological processes are a class of extremely complex systems to study and model [10]. This is mainly due to the fact that a physiological process usually emerges from the interaction of several elements belonging to an intricate network of relationships, where each element is involved in more processes [2].

In previous work [1], we developed a system, called *anthropic agency*, able to integrate a number of partial models of a physiological phenomenon in order to globally produce a comprehensive model of the phenomenon; in particular we addressed the glucose-insulin metabolism. Partial models are embedded in agents (called decisional agents) and the global model results from the interaction - a cooperative negotiation - of these agents. The negotiation process we implemented is mediated by an agent called *equalizer*. Negotiation offers a way to integrate partial models that goes beyond "putting pieces together": it can account for secondary inter-effects between the partial models and it can flexibly model physiological phenomena [4]. Among the advantages of the use of agents to represent partial models, the possibility to dynamically change their composition in a flexible and open way is particularly relevant.

The purpose of this paper is to formally describe and analyze the cooperative negotiation protocol used and experimentally validated in [1]. Although the protocol has demonstrated to work well in practice, we are interested in finding more formal justifications of its effectiveness. This formal study is especially needed because the global model emerging from the cooperative negotiation of the agents could be used to develop controllers of the modelled physiological phenomena. A very simple example of controller can be obtained by pushing a phenomenon toward the optimal state determined according to its global model. These regulating systems must be proved to satisfy some properties, the most important being the *stability* of the negotiation process to guarantee that an agreement is eventually reached and, as a consequence, that a smooth control action over the human body can be carried out. In this paper we formally define a cooperative negotiation paradigm and we present some results obtained from its formal analysis by providing constraints on the values of negotiation parameters that guarantee the stability. We note that, in addition to stability, other properties could be analyzed: for example, the convergence speed, the response time of the system, and the optimality of control. However, these aspects are outside the scope of this paper. In the following we will refer to the determination of the heart rate (pacing). Many models have been proposed for pacing, but each of them is effective in describing the normal sinus activity only under ideal and very restrictive conditions. Since the conditions are different for different models, the negotiation protocol presented in this paper allows the combination of the partial models in a global model. This is an application field different from that in [1] and shows the flexibility and the generality of the proposed cooperation negotiation paradigm, that can be applied to model different physiological phenomena. Briefly, the main original contributions of this paper are the defini-


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tion of a cooperative negotiation protocol to integrate partial models of physiological phenomena and the preliminary outline of a general methodology that can be employed to prove properties of negotiation protocols.

This paper is structured as follows. The next section will review the state of the art in cooperative negotiation, emphasizing its potential role in modelling complex physiological phenomena. Section 3 introduces our cooperative negotiation protocol and shows its effectiveness in the pacing application. Section 4 outlines a methodology that is applied to theoretically investigate some properties of the protocol. Finally, Section 5 concludes the paper.

## 2. Cooperative Negotiation

Cooperative negotiation is a particular kind of negotiation that takes advantages of the cooperative nature of the agents to maximize social utility [12]. For the purposes of this paper, cooperative negotiation is a way to perform (decentralized) distributed optimization for fully cooperative agents. In cooperative negotiations, each agent has a partial view of the problem and operates an optimization according to it, the results are put together via a negotiation trying to solve the conflicts risen by the partial views. The relevant benefits of cooperative negotiation for smoothing conflicts are the possibility to tackle dynamic environments and the reduction of the computation time [12]. Three dimensions can characterize a cooperative negotiation: the formal degree of the negotiation paradigm, the information about other agents, and the knowledge of the social utility.

Cooperative negotiation is currently adopted mainly in resource allocation field. For example, in [12] a cooperative negotiation approach for real-time control of cellular network coverage is proposed. Each base station is an agent that negotiates the antenna radiation pattern around traffic spot with the neighboring agents. The authors show that the local area real-time cooperative negotiation between base stations leads to a near-optimal coverage agreement in the whole cellular network. In [5] a cooperative negotiation approach is adopted to solve a distributed resource allocation problem in radar tracking of targets in an environment. Each radar platform has three distinct sensors and negotiates their orientation with the neighboring agents in order to better track a target. From the analysis of the cooperative negotiation paradigms for resource allocation presented in literature (see also [6, 9]), it emerges that they are usually non formal and that the agents have information about others agents; moreover, the negotiation can be with or without explicit social utility information. Although some analytical results have been derived for negotiations, they are usually referred to competitive negotiating agents.

Differently from what we found in literature, in our application we need a formal description of the cooperative negotiation paradigm in order to prove some properties (i.e., the stability); the agents have not information about other agents since (as stated above) we want a flexible architecture that allows for dynamic update of agents; and we do not know *a priori* the social utility, but we determine the social utility by finding the negotiation parameters that bring the agents to precisely mimic the behavior of the physiological phenomenon. This last issue is worth a more detailed discussion. In economic domains, *welfarism* is the principle according to which the relative attractiveness of social alternatives is determined by a total social ordering [17]. A social welfare functional assigns a social rank to the alternatives. In our application, the only available information is the set of local potential (utility) functions that evaluate the states from the local points of view of the agents. Hence, the possible global states of the multiagent system cannot be evaluated by any explicit social utility function nor ordered by any social welfare principle: the actual global state of the system is the result of the negotiation of the agents, as illustrated in the next section.

## 3. A Cooperative Negotiation Protocol for Physiological Modelling

The proposed negotiation protocol is inspired to [8]: we generalized the protocol presented there, intended for auction negotiations, to the case of cooperative negotiation.

### 3.1. Description of the Protocol

For the purposes of this paper, a negotiation involves $n$ contracting agents and a mediator, called equalizer. Each agent $i$ embeds a partial model referring to a space of parameters $A_i \in \mathbb{R}^{N_i}$ and its state is represented by a vector $\mathbf{p}_i \in A_i$. The spaces $A_i$ can overlap (i.e., they can have common dimensions) and the global parameter space $A$ of the system is $A = \bigcup_{i=1}^{n} A_i$. Each agent $i$ is associated with a potential function $\mathcal{U}_i(\cdot) : A_i \to [0, +\infty)$ that expresses the "distance" of the current state from the optimal curve of the model embedded by the agent. For example, Fig. 1 shows the potential function of an agent, called QT agent, that embeds the model [15] relating the heart rate (HR) and the length of the QT interval (a quantity inferred from ECG data). As another example, Fig. 2 shows the potential function of an agent, called RR agent, that embeds the model [16] relating HR and the respiration rate (RR), namely the number of respiration cycles per minute.

The two potential functions above have been calculated in the following way. We started from the models [15] and [16] and tuned their parameters to mimic the collected data relative to a particular patient (see Subsection 3.2). For instance, for patient 16265, we obtained:


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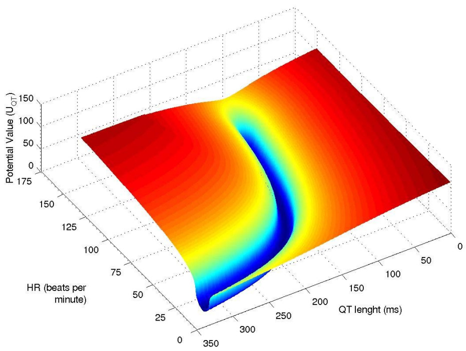


**Figure 1. Potential function $\mathcal{U}_{\mathbf{QT}}(\cdot)$**

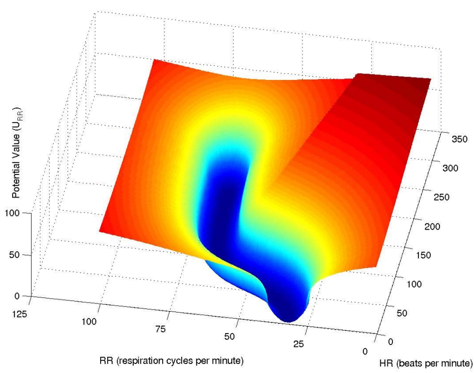


**Figure 2. Potential function $\mathcal{U}_{\mathbf{RR}}(\cdot)$**

$$\mathrm{HR} = -\frac{8.2}{\log\left(\dfrac{1930.4 - \mathrm{QT}}{1734.7}\right)} \qquad (3.1)$$

$$\mathrm{HR} = 26.98 \cdot \arctan\left(0.0995 \cdot \mathrm{RR} - 2.5947\right) + 95.02$$

These models describes two curves in the spaces $A_{QT}$, with dimensions HR and QT, and $A_{RR}$, with dimensions HR and RR, respectively. These curves represent the combination of values for which the heart has an optimal functioning for the patient 16265. Then, to every point $\mathbf{p} = \langle hr, qt \rangle$ of the space $A_{QT}$ we attributed a numerical value $\mathcal{U}_{\mathrm{QT}}(\mathbf{p})$:

$$\begin{cases} \dfrac{d(\mathbf{p})^2}{10} & d(\mathbf{p})^2 < 20 \\ 20 + 10 * \log\left(\left(\dfrac{d(\mathbf{p})^2}{10} - 20\right) + 1\right) & d(\mathbf{p})^2 > 20 \end{cases}$$

where $d(\mathbf{p})$ is the Euclidean distance of $\mathbf{p}$ from the curve (3.1). The same we did for the space $A_{RR}$ obtaining the potential function $\mathcal{U}_{\mathrm{RR}}(\cdot)$. The formulas above have been experimentally determined after several trials in order to define steep potential functions that assign high values to points that are far from the optimal curves. Indeed, the optimal curves are the only information we have on the physiological phenomenon: far from them, the heart functioning is not optimal; this aspect is captured in the above steep potential functions.

In addition, each agent $i$ is associated with a normalization parameter $\mathcal{N}_i \in [0, 1]$ that scales the potential function $\mathcal{U}_i(\cdot)$ in a global measure unit. This is needed because the ranges of the potential functions of different agents (for example, implemented by different designers) could be different.

We call $\mathbf{p}^t_{i\to e}$ the *offer* (or *proposal*) formulated by agent $i$ at time $t$ to the equalizer $e$. $\mathbf{p}^t_{i\to e}$ is a proposed state in the space $A_i$. We call $\mathbf{p}^t_{e\to i}$ the counter-offer formulated by the equalizer at time $t$ to the agent $i$. $\mathbf{p}^t_{e\to i}$ is the proposal of a new state in $A_i$ that is the expression of the negotiation step at time $t$. A *negotiation session* is a sequence of interleaved offers of the agents to equalizer and counter-offers of equalizer to the agents, starting at time $0$ and ending at time $\tau \in \mathbb{N}$. For example, the portion of a negotiation session regarding agent $i$ can be represented as follows:

$$\mathbf{p}^0_{i\to e} \succ \mathbf{p}^0_{e\to i} \succ \mathbf{p}^1_{i\to e} \succ \cdots \succ \mathbf{p}^\tau_{e\to i}$$

The anthropic agency of [1] models the glucose-insulin metabolism through a sequence of negotiation sessions. In the same way, the QT and RR agents described above model the heart rate of a patient through a sequence of negotiation sessions. Given the current state of the patient (the current values of QT and RR read by appropriate sensors), a negotiation session (with $\tau$ corresponding to $2s$) determines the heart rate that should exhibit the patient, the heart rate is compared with the actual heart rate of the patient, the new state of the patient is read, and a new negotiation session starts (see Fig. 3). A heart rate control system could be obtained by applying to the patient, using a pace-maker, the heart rate determined by the system. We now turn our attention to the description of a single negotiation session and, in particular, to the determination of the offers and counter-offers.

Given the vector $\mathbf{p} \in A$ representing the current state of the global phenomenon to be modelled (as read by sensors measuring physiological quantities), each agent $i$ formulates an initial offer as the result of a local optimal search performed with objective function $\mathcal{U}_i(\cdot)$ starting from the current state $\mathbf{p}_i$ of its internal model ($\mathbf{p}_i$ is the projection of $\mathbf{p}$ on $A_i$), for instance it could use a hill-climbing based technique:

$$\mathbf{p}^0_{i\to e} \leftarrow \mathcal{L}ocal\mathcal{S}earch\left(\mathbf{p}_i, \mathcal{U}_i(\cdot)\right)$$

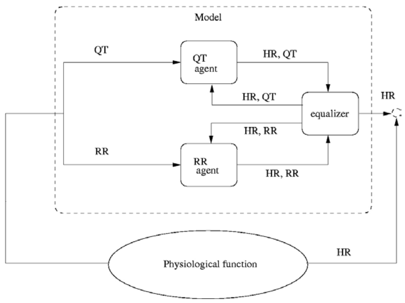


**Figure 3. The modelling system**

Usually the potential functions $\mathcal{U}_i(\cdot)$ constructed as described above do not have local minima (see, for example, Figs. 1 and 2): thus we can assume, without loss of generality, that hill-climbing based techniques efficiently find the global minimum of $\mathcal{U}_i(\cdot)$ (namely $\mathcal{U}_i(\mathbf{p}^0_{i\to e}) = 0$, for all $i$).

Each agent proposes its offer by sending the following triple to the equalizer: $\langle \mathbf{p}^t_{i\to e}, \mathcal{U}_i(\mathbf{p}^t_{i\to e}), \mathcal{N}_i \rangle$.

The equalizer receives the offers of all the agents at time $t$ and calculates their weighted average:

$$\mathbf{m}^t = \frac{\sum_{i=1}^n \mathbf{p}^t_{i\to e} \cdot \mathcal{N}_i \cdot [1 + \mathcal{U}_i(\mathbf{p}^t_{i\to e})]}{\sum_{i=1}^n \mathcal{N}_i \cdot [1 + \mathcal{U}_i(\mathbf{p}^t_{i\to e})]} \tag{3.2}$$

$\mathbf{m}^t$ is the *agreement* reached in the negotiation session at time $t$. Note that $\mathbf{m}^t$ is a point in $A$; in the above formula, the sum at the numerator is intended to sum only the corresponding elements of the vectors (for example, $\langle hr_1, qt\rangle + \langle hr_2, rr\rangle = \langle hr_1 + hr_2, qt, rr\rangle$). The equalizer communicates to the agents its counter-proposals $\mathbf{p}^t_{e\to i}$, that are the projections of $\mathbf{m}^t$ on the $A_i$, by sending to each one of them the value: $\langle \mathbf{p}^t_{e\to i}\rangle$.

After receiving the counter-proposal, every agent $i$ calculates its new offer to the equalizer $e$ as:

$$\mathbf{p}^{t+1}_{i\to e} = \mathbf{p}^t_{i\to e} + \|\mathbf{p}^t_{e\to i} - \mathbf{p}^t_{i\to e}\| (\alpha_i(\mathbf{p}_i)\mathbf{u}^{t+1}_i + \beta_i(\mathbf{p}_i)\mathbf{w}^{t+1}_i) \tag{3.3}$$

where $\alpha_i(\cdot)$ and $\beta_i(\cdot)$, called *negotiation parameters*, are two functions on $A_i$ to $\mathbb{R}$, $\|\ldots\|$ is the vector norm, and $\mathbf{u}^{t+1}_i$ and $\mathbf{w}^{t+1}_i$ are two versors defined below. (In our implementation of the negotiation protocol, the proposal $\mathbf{p}^{t+1}_{i\to e}$ is forced equal to $\mathbf{p}^t_{i\to e}$ when their difference is below a given threshold $\delta_i$.)

Considering the vector connecting $\mathbf{p}^t_{e\to i}$ to $\mathbf{p}^t_{i\to e}$ and normalizing it results in:

$$\mathbf{u}^{t+1}_i = \frac{\mathbf{p}^t_{e\to i} - \mathbf{p}^t_{i\to e}}{\|\mathbf{p}^t_{e\to i} - \mathbf{p}^t_{i\to e}\|} \tag{3.4}$$

This versor is headed toward the agreement (in $A_i$) at time $t$ (see Fig. 4). From (3.3), it can be seen that the next offer of the agent $i$ gets closer to the last agreement of the negotiation by a quantity $\alpha_i(\mathbf{p}_i)\|\mathbf{p}^t_{e\to i} - \mathbf{p}^t_{i\to e}\|$, proportional to the distance between the last offer of agent $i$ and the last agreement.

Considering the vector along the direction of the gradient of $\mathcal{U}_i(\cdot)$ in $\mathbf{p}^t_{i\to e}$ and normalizing it, we obtain the following versor:

$$\mathbf{v}^{t+1}_i = \frac{\nabla\mathcal{U}_i(\mathbf{p}^t_{i\to e})}{\|\nabla\mathcal{U}_i(\mathbf{p}^t_{i\to e})\|}$$

It points toward the direction of maximum increasing of $\mathcal{U}_i(\cdot)$ and every vector orthogonal to $\mathbf{v}^{t+1}_i$ is tangent to an equipotential curve (Fig. 4). We define the following versor that is along the direction orthogonal to the gradient direction and thus tangent to the level curves of the potential space (Fig. 4):

$$\mathbf{w}^{t+1}_i = \frac{\mathbf{u}^{t+1}_i - \left(\mathbf{v}^{t+1}_i \cdot \mathbf{u}^{t+1}_i\right)\mathbf{v}^{t+1}_i}{\|\mathbf{u}^{t+1}_i - \left(\mathbf{v}^{t+1}_i \cdot \mathbf{u}^{t+1}_i\right)\mathbf{v}^{t+1}_i\|}$$

From (3.3), it can be seen that the next offer of an agent tries to keep constant the potential value in the direction of the last agreement, namely it moves in the direction of $\mathbf{w}^{t+1}_i$ by a quantity $\beta_i(\mathbf{p}_i)\|\mathbf{p}^t_{e\to i} - \mathbf{p}^t_{i\to e}\|$. Among the infinite vectors orthogonal to the gradient direction, we choose $\mathbf{w}^{t+1}_i$ as the one the minimizes the angle with $\mathbf{u}^{t+1}_i$. This choice is justified by the fact that $\mathbf{w}^{t+1}_i$ has a component that is added to $\mathbf{u}^{t+1}_i$ and another component that is in the direction of the equipotential curve on which the last offer was.

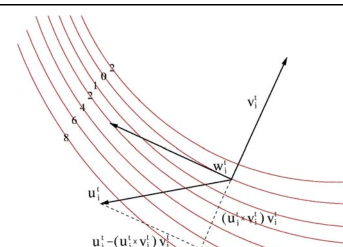

**Figure 4. The versors $\mathbf{u}^t_i$, $\mathbf{v}^t_i$, $\mathbf{w}^t_i$ in a bi-dimensional potential field**

To summarize, the next offer of an agent takes into account two components, the first one pushes toward the negotiation agreement, the second one pushes toward the equipotential curves. The first component contributes to accommodate the tendency of the society, while the second component contributes to keep the individual agent close to its optimal curve. The values $\alpha_i(\mathbf{p}_i)$ and $\beta_i(\mathbf{p}_i)$ determine the relative weights between these two components. For example, if $\alpha_i(\cdot)$ is small (namely, the weight of the component that pushes toward the negotiation agreement is small), the model embedded in agent $i$ has high confidence and its proposal moves only slightly from its optimal curve. We also note that the values of functions $\alpha_i(\cdot)$ and $\beta_i(\cdot)$ during a negotiation session depend only on $\mathbf{p}_i$, namely from the current state of the internal model of agent $i$, and thus are constant during the negotiation session. Hence, in the following, when it is clear that we refer to a single negotiation session, we will use $\alpha_i$ and $\beta_i$ instead of $\alpha_i(\mathbf{p}_i)$ and $\beta_i(\mathbf{p}_i)$, respectively.

### 3.2. Experimental Results

In order to validate the negotiation protocol described above in the pacing application, we compared the estimation of HR it produces with the estimations obtained with the single partial models and with the weighted average combination of the partial models. We used three 24-hours long ECGs, taken from [14], of patients 16265, 16420, and 17052, and we extracted from them the (24-hours long) temporal series of HR, QT, and RR, for each one of the three patients. We considered the average $E[e]$ and the standard deviation $var[e]$ of the error $e$, defined as the absolute value of the difference between the real value of HR (as read from the ECG) and the value of HR estimated by the system. We used the QT and the RR agents described above. Initially we calibrated (by estimating the parameters with a non-linear least-square technique) their models on the signals of every patient (see previous subsection).

When we use only the QT agent, we obtain $E[e] = 4.2 \pm 1.0$ (i.e., the average error $E[e]$ for the three patients is in the range $[3.2, 5.2]$) and $var[e] = 9.98 \pm 1.5$. When we use only the RR agent, we obtain $E[e] = 11.2 \pm 1.9$ and $var[e] = 47.9 \pm 3.0$.

In the case we use both the agents and their proposals are combined with a weighted average without negotiation, we have that (we report only the results corresponding to the weights, determined by a exhaustive search, that minimize $E[e]$): $E[e] = 3.6 \pm 0.3$ and $var[e] = 7.6 \pm 1.0$.

In the case we use both the agents and their proposals are combined with the negotiation protocol introduced in this paper (with $\mathcal{N}_1 = \mathcal{N}_2 = 1$), we have that (we report only the results corresponding to the negotiation parameters, determined by a exhaustive search, that minimize $E[e]$): $E[e] = 2.9 \pm 0.2$ and $var[e] = 6.5 \pm 0.3$. These results improve those obtained using weighted average combination of about 17% for $E[e]$ and 14% for $var[e]$. This means that negotiation provides a more accurate estimate (i.e., closer to the real value read from ECG) of the HR value.

Finally, we emphasize the role of $\beta_i$ during the negotiation: in Table 1 we report the results (for patient 16265) corresponding to the negotiation parameters that minimize $E[e]$ and, in Table 2, we report the results corresponding to the same parameters but with $\beta_1 = \beta_2 = 0$. It is clear that $\beta_1$ and $\beta_2$, introducing the contribution of $\mathbf{w}_i^{t+1}$, help in improving the reliability of the model.

| $\alpha_1$ | $\alpha_2$ | $\beta_1$ | $\beta_2$ | $E[err]$ | $var[err]$ |
|---|---|---|---|---|---|
| 0.010 | 0.152 | 0.010 | 0.240 | 2.918 | 6.794 |
| 0.010 | 0.150 | 0.010 | 0.248 | 2.918 | 6.808 |
| 0.010 | 0.150 | 0.010 | 0.250 | 2.918 | 6.814 |
| 0.010 | 0.154 | 0.010 | 0.244 | 2.918 | 6.816 |

**Table 1. An optimal case**

| $\alpha_1$ | $\alpha_2$ | $\beta_1$ | $\beta_2$ | $E[err]$ | $var[err]$ |
|---|---|---|---|---|---|
| 0.010 | 0.152 | 0.000 | 0.000 | 3.005 | 6.987 |
| 0.010 | 0.150 | 0.000 | 0.000 | 2.980 | 7.006 |
| 0.010 | 0.150 | 0.000 | 0.000 | 3.032 | 6.938 |
| 0.010 | 0.154 | 0.000 | 0.000 | 3.013 | 7.112 |

**Table 2. The same case with $\beta_1 = \beta_2 = 0$**

## 4. Formal Analysis of the Proposed Cooperative Negotiation Protocol

In this section, we first define the concept of stability of a negotiation session; then we show how the analysis of this stability can be reduced to the analysis of the asymptotic stability of a corresponding dynamical system; finally we exploit this approach to formally derive some results on negotiation stability.

### 4.1. Stability Property

The main property we are interested in theoretically investigating in this paper is the achieving of an agreement in a negotiation session. In particular we are interested in determining the constrains that guarantee the stability of the negotiation in order to facilitate the design of stable negotiations. To begin with, we formulate the above concepts more formally:

**Definition 4.1** *A succession of offers $\mathbf{p}_{i\to e}^t$ converges to $\bar{\mathbf{p}}_i \in A_i$ when $\exists \bar{t} > 0$ such that $\mathbf{p}_{i\to e}^t = \bar{\mathbf{p}}_i$ for $t > \bar{t}$.*

**Definition 4.2** *A succession of agreements $\mathbf{m}^t$ converges to $\bar{\mathbf{m}} \in A$ when $\exists \bar{t} > 0$ such that $\mathbf{m}^t = \bar{\mathbf{m}}$ for $t > \bar{t}$.*

**Definition 4.3** *A negotiation session is* stable *when the succession of agreements $\mathbf{m}^t$ converges to a $\bar{\mathbf{m}} \in A$.*

If a negotiation session is stable, then the agents involved are guaranteed to eventually reach an agreement if they are given enough time.

**Definition 4.4** *A negotiation session is* strongly stable *when the succession of offers* $\mathbf{p}^t_{i\to e}$ *converges to* $\bar{\mathbf{p}}_i \in A_i$*, for all agents* $i$.

It is easy to see that, if a negotiation session is strongly stable, then it is also (simply) stable.

## 4.2. Formulation of a Negotiation Session as a Dynamical System

To formally study the stability of a session of our cooperative negotiation protocol, it is convenient to cast the problem in the framework of dynamical systems [11]. Given (3.2), equation (3.3) can be written as:

$$\mathbf{p}^{t+1}_{i\to e} = \mathbf{p}^t_{i\to e} + \left(\alpha_i \mathbf{u}^{t+1}_i + \beta_i \mathbf{w}^{t+1}_i\right) \cdot \cdot \left\| \frac{\sum_{i=1}^n \mathbf{p}^t_{i\to e} \cdot \mathcal{N}_i \cdot [1 + \mathcal{U}_i(\mathbf{p}^t_{i\to e})]}{\sum_{i=1}^n \mathcal{N}_i \cdot [1 + \mathcal{U}_i(\mathbf{p}^t_{i\to e})]} - \mathbf{p}^t_{i\to e} \right\| \quad (4.1)$$

This can be seen as a dynamical system corresponding to the negotiation session, whose state at time $t$ is the collection of the $\mathbf{p}^t_{i\to e}$ (for all $i$). This dynamical system is asymptotically stable if and only if it exists a $\bar{t}$ such that $\mathbf{p}^{t+1}_{i\to e} = \mathbf{p}^t_{i\to e}$, for $t > \bar{t}$ and for all $i$. Hence, *the dynamical system above is asymptotically stable if and only if the corresponding negotiation session is strongly stable*, the equilibrium of the dynamical system being the collection of $\bar{\mathbf{p}}_i$ of Definition 4.4.

The dynamical system (4.1) can be studied in order to determine the conditions (on the negotiation parameters $\alpha_i$ and $\beta_i$) under which the system is asymptotically stable. We adopt an approach that considers geometrical properties of the proposals and of the agreements. Our approach is based on the Lyapunov criterion [11]: if we can find a "smooth" positive (equal to 0 in the equilibrium) monotonically decreasing function (called Lyapunov function) of the state of a dynamical system, then the system is asymptotically stable. To apply this criterion in our case, we need to determine a function that is positive and that decreases at each negotiation step.

## 4.3. Some Formal Results on Stability

In this section, we present some formal results on the stability of a negotiation session that can be obtained from the analysis of the corresponding dynamical system. To simplify the following discussion, we assume, without loss of generality, that all the $A_i$ have the same dimensions, namely $A_i = A$ for all $i$ and $\mathbf{p}^t_{e\to i} = \mathbf{m}^t$ for all $t$ and for all $i$.

**4.3.1. The Average Case.** Let us start from the case in which $\mathcal{N}_i = 1$ for all $i$ and $\mathcal{U}_i(\mathbf{p}^t_{i\to e}) = 0$ for all $t$ and $i$. In this case, $\mathbf{m}^t$ calculated according to (3.2) is the average of the proposals $\mathbf{p}^t_{i\to e}$:



$$\mathbf{m}^t = \frac{\sum_{i=1}^n \mathbf{p}^t_{i\to e}}{n}$$

Consider the following Lyapunov function:

$$\mathcal{L}(t) = \sum_{i=1}^n \|\mathbf{p}^t_{i\to e} - \mathbf{m}^t\|^2$$

It is a function of the state of the dynamical system corresponding to our negotiation protocol and it is trivially positive and equal to 0 in the equilibrium, namely when the proposals $\mathbf{p}^t_{i\to e} = \mathbf{m}^t$ (for all $i$). To demonstrate that it is also monotonically decreasing, we need to prove that $\mathcal{L}(t) > \mathcal{L}(t+1)$:

$$\sum_{i=1}^n \|\mathbf{p}^t_{i\to e} - \mathbf{m}^t\|^2 > \sum_{i=1}^n \|\mathbf{p}^{t+1}_{i\to e} - \mathbf{m}^{t+1}\|^2 \quad (4.2)$$

We prove it in two steps. Firstly, note that:

$$\sum_{i=1}^n \|\mathbf{p}^{t+1}_{i\to e} - \mathbf{m}^t\|^2 \geqslant \sum_{i=1}^n \|\mathbf{p}^{t+1}_{i\to e} - \mathbf{m}^{t+1}\|^2$$

since, in the space $A_i$, $\mathbf{m}^{t+1}$ is the average of the points $\mathbf{p}^{t+1}_{i\to e}$ and, by definition, the average is such that:

$$\mathbf{m}^{t+1} = \arg\min \sum_{i=1}^n \|\mathbf{p}^{t+1}_{i\to e} - \mathbf{m}^{t+1}\|^2$$

The second step consists in proving that:

$$\sum_{i=1}^n \|\mathbf{p}^t_{i\to e} - \mathbf{m}^t\|^2 > \sum_{i=1}^n \|\mathbf{p}^{t+1}_{i\to e} - \mathbf{m}^t\|^2 \quad (4.3)$$

A sufficient condition for the (4.3) to hold is that $\|\mathbf{p}^t_{i\to e} - \mathbf{m}^t\|^2 > \|\mathbf{p}^{t+1}_{i\to e} - \mathbf{m}^t\|^2$, for all $i$. To verify when this sufficient condition is satisfied, we write $\|\mathbf{p}^{t+1}_{i\to e} - \mathbf{m}^t\|$ as follows (according to (3.3) and (3.4)):

$$\|\mathbf{p}^t_{i\to e} + \alpha_i(\mathbf{m}^t - \mathbf{p}^t_{i\to e}) + \beta_i\|\mathbf{p}^t_{i\to e} - \mathbf{m}^t\|\mathbf{w}^{t+1}_i - \mathbf{m}^t\|$$

Hence, $\|\mathbf{p}^{t+1}_{i\to e} - \mathbf{m}^t\|^2$, considering that $\|\mathbf{w}^{t+1}_i\| = 1$ by definition of versor, is (after some math):

$$\|\mathbf{p}^t_{i\to e} - \mathbf{m}^t\|^2((1-\alpha_i)^2 + \beta_i^2 + + 2\beta_i(1-\alpha_i)\frac{(\mathbf{p}^t_{i\to e} - \mathbf{m}^t)\mathbf{w}^{t+1}_i}{\|\mathbf{p}^t_{i\to e} - \mathbf{m}^t\|})$$

Calling $\Delta_i = \|\mathbf{p}^t_{i\to e} - \mathbf{m}^t\|$ and $\Gamma_i = (1-\alpha_i)^2 + \beta_i^2 + 2\beta_i(1-\alpha_i)(\mathbf{p}^t_{i\to e} - \mathbf{m}^t)\mathbf{w}^{t+1}_i / \|\mathbf{p}^t_{i\to e} - \mathbf{m}^t\|$, inequality (4.3) can be rewritten as:

$$\sum_{i=1}^{N} \Delta_i^2 > \sum_{i=1}^{N} \Delta_i^2 \Gamma_i \qquad (4.4)$$

Since $\Delta_i \geq 0$ and $\Gamma_i \geq 0$ by definition, a sufficient condition to satisfy inequality (4.4) (and, consequently, (4.3)) is that $\Gamma_i < 1$ for all $i$. Namely:

$$(1-\alpha_i)^2 + \beta_i^2 + 2\beta_i \cdot \frac{\mathbf{m}^t - \mathbf{p}_{i\to e}^t}{\|\mathbf{p}_{i\to e}^t - \mathbf{m}^t\|} \cdot (\alpha_i - 1) \cdot \mathbf{w}_i^{t+1} < 1$$

Remembering the definition (3.4) of $\mathbf{u}_i^{t+1}$ and splitting the above inequality according to the sign of $\beta_i \cdot (\alpha_i - 1)$, we have:

$$\begin{aligned}
\beta_i \cdot (\alpha_i - 1) > 0 \Longrightarrow &\quad \mathbf{u}_i^{t+1} \cdot \mathbf{w}_i^{t+1} < \frac{2\alpha_i - \alpha_i^2 - \beta_i^2}{2\beta_i \cdot (\alpha_i - 1)} \\
\beta_i \cdot (\alpha_i - 1) < 0 \Longrightarrow &\quad \mathbf{u}_i^{t+1} \cdot \mathbf{w}_i^{t+1} > \frac{2\alpha_i - \alpha_i^2 - \beta_i^2}{2\beta_i \cdot (\alpha_i - 1)} \\
\beta_i = 0 \Longrightarrow &\quad 0 < \alpha_i < 2 \\
\alpha_i = 1 \Longrightarrow &\quad -1 < \beta_i < 1
\end{aligned}$$

These are the conditions on the negotiation parameters that guarantee that (4.3) (and (4.2)) holds and that the dynamical system is asymptotically stable.

Calling $\theta_i^{t+1}$ the angle between $\mathbf{u}_i^{t+1}$ and $\mathbf{w}_i^{t+1}$, we can write the two above inequalities as:

$$\begin{aligned}
\beta_i \cdot (\alpha_i - 1) > 0 \implies \cos(\theta_i^{t+1}) < \frac{2\alpha_i - \alpha_i^2 - \beta_i^2}{2\beta_i \cdot (\alpha_i - 1)} \\
\beta_i \cdot (\alpha_i - 1) < 0 \implies \cos(\theta_i^{t+1}) > \frac{2\alpha_i - \alpha_i^2 - \beta_i^2}{2\beta_i \cdot (\alpha_i - 1)}
\end{aligned}$$

Since $\cos(\cdot)$ has upper and lower limits, we have that $\cos(\theta_i^{t+1}) \leq 1$ and $\cos(\theta_i^{t+1}) \geq -1$ are inequalities always satisfied. Thus we can determine the ranges of $\alpha_i$ and $\beta_i$ in which inequality (4.2) is always satisfied as:

$$\begin{aligned}
\beta_i \cdot (\alpha_i - 1) > 0 \Longrightarrow &\qquad 1 < \frac{2\alpha_i - \alpha_i^2 - \beta_i^2}{2\beta_i \cdot (\alpha_i - 1)} \\
\beta_i \cdot (\alpha_i - 1) < 0 \Longrightarrow &\qquad -1 > \frac{2\alpha_i - \alpha_i^2 - \beta_i^2}{2\beta_i \cdot (\alpha_i - 1)}
\end{aligned}$$

And, with some trivial algebra, we obtain the conditions for which a negotiation session is strongly stable in the average case (for all $i$):

$$\begin{aligned}
&(0 < \beta_i < 1 \wedge \beta_i < \alpha_i < 2 - \beta_i) \vee \\
&\qquad \vee (-1 < \beta_i < 0 \wedge -\beta_i < \alpha_i < 2 + \beta_i) \qquad (4.5)
\end{aligned}$$

In a similar way, we determined also the conditions for which a negotiation session is not strongly stable. A graphical representation of these constraints for the average case is shown in Fig. 5.

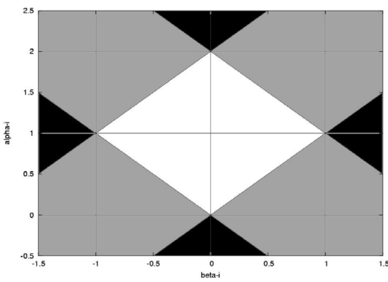


**Figure 5. Regions of space $(\alpha_i, \beta_i)$ where the negotiation is always strongly stable (white), the negotiation is not strongly stable (black), and nothing is said by our sufficient conditions (gray)**

**4.3.2. The Weighted Average Case.** Let us consider now the case in which $\mathcal{U}_i(\mathbf{p}_{i\to e}^t) = 0$ for all $t$ and $i$. In this case, $\mathbf{m}^t$ calculated according to (3.2) is the weighted average of the proposals $\mathbf{p}_{i\to e}^t$:

$$\mathbf{m}^t = \frac{\sum_{i=1}^n \mathbf{p}_{i\to e}^t \mathcal{N}_i}{\sum_i^n \mathcal{N}_i}$$

Note that the weights $\mathcal{N}_i$ are constant during the negotiation session.

Consider the following Lyapunov function:

$$\mathcal{L}(t) = \sum_{i=1}^{n} \|\mathbf{p}_{i\to e}^t - \mathbf{m}^t\|^2 \mathcal{N}_i$$

This function of the state of the dynamical system corresponding to our negotiation protocol is trivially positive and equal to 0 in the equilibrium. Following a reasoning similar to that of the previous case, it is possible to demonstrate that it is also monotonically decreasing when the conditions (4.5) hold. Hence, Fig. 5 graphically describes the strong stability results also for the weighted average case.

**4.3.3. Discussion.** Note that the above results are only *sufficient* conditions: a negotiation session can be (simply) stable although the offers of the agents do not converge but diverge at equal "speed" (see Fig. 6). The Lyapunov functions $\mathcal{L}$ defined above can be given an interesting geometrical interpretation. Consider the constellation of offers $\mathbf{p}_{i\to e}^t$ in the space $A$ and their convex hull $\mathcal{V}^t$ [7]. By definition, the weighted average $\mathbf{m}^t$ is a convex combination of the $\mathbf{p}_{i\to e}^t$ and thus, for a well-known property of convex hulls, it lies inside the volume $\mathcal{V}^t$. The monotonic decreasing of $\mathcal{L}$ can be interpreted as the reduction of this volume at each negotiation step, since the offers of the agents are the vertexes of the convex hull.

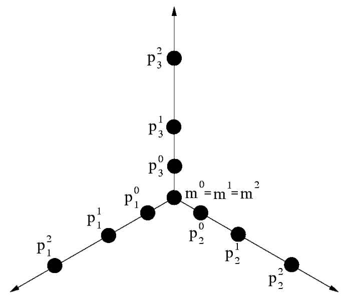


**Figure 6. A (simply) stable negotiation session**

## 5. Conclusions

In this paper we have presented a cooperative negotiation protocol for multiagent systems that model complex physiological phenomena. We have formally described and analyzed the negotiation protocol in order to determine the conditions under which the reaching of the agreement is guaranteed. We adopted a strategy that can be summarized as follows: $(a)$ describe the negotiation protocol as a dynamical system $D$ (Subsection 4.2), $(b)$ manage to reduce the strong stability of the negotiation to the asymptotic stability of $D$ (Subsection 4.2), $(c)$ find a suitable Lyapunov function that can be used to prove the stability of $D$ (Subsection 4.3). This strategy could be generalized to other negotiation protocols for proving their properties.

Future work will be devoted to define appropriate Lyapunov functions for studying the stability of our negotiation protocol in more general cases. Some very preliminary results show that a function similar to those used in this paper could work. Moreover, we aim at analyzing other properties of the negotiation protocol, such as the convergence speed, namely the time needed to reach an agreement in a stable negotiation session. Finally, we will apply the strategy outlined above to other negotiation protocols.

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