Rule Based Fuzzy Cognitive Maps: Fuzzy Causal Relations

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Abstract: Rule Based Fuzzy Cognitive Maps (RBFCM) are proposed as an evolution of Fuzzy Causal Maps (FCM) that allow a more complete representation of cognition, since relations other than monotonic causality are made possible. Their structure is based on traditional fuzzy systems with feedback. The main problem to solve while trying to implement a RBFCM is the causal relation itself, since traditional fuzzy operations can not implement causality as it is usually defined in causal maps.

This paper introduces Rule Based Fuzzy Cognitive Maps and presents a method to implement Fuzzy Causal Relations. This method allows a great flexibility in the addition and removal of concepts and links among concepts, and introduces a new Fuzzy Operation that simulates the "accumulative" property of causal relations – the Fuzzy Carry Accumulation (FCA).

Keywords: Rule Based Fuzzy Cognitive Maps (RBFCM); Causality; Fuzzy Causal Relations (FCR); Fuzzy Carry Accumulation (FCA); Fuzzy sets.

1. Introduction

Decision makers usually face serious difficulties when approaching significant, real-world dynamic systems. Such systems are composed of a number of dynamic concepts or actors which are interrelated in complex ways, usually including feedback links which propagate influences in complicated chains. Axelrod work on Cognitive Maps (CMs)[1] introduced a way to represent these systems, and several methods and tools exist to analyse the structure of causal maps like [2] [3]. However, complete, efficient and practical mechanisms to analyse and predict the evolution of data in CMs are necessary [4] but not yet available for several reasons. System Dynamics tools like [5] could be a solution, but since numerical data may be uncertain or hard to come by, and the formulation of a mathematical model may be difficult, costly or even impossible, then efforts to introduce knowledge on these systems should rely on natural language arguments in the absence of formal models. Fuzzy Cognitive Maps (FCM), as introduced by Kosko [6], are a qualitative alternative approach to dynamic systems. However, a FCM is indeed a man-trained Neural Network (Multilayer perceptron) which is not Fuzzy in a traditional sense, and doesn't explore usual Fuzzy capabilities. FCM don't share the properties of other fuzzy systems and can not be mixed with traditional fuzzy rules and operations. Although this is not a problem by itself since FCM perform rather well when trying to represent the evolution of management, organisational or socio-economic problems, FCM are limited to the representation of simple monotonic causal relations between concepts. FCM are indeed Fuzzy Causal Maps.

Causal Maps advantages [7] reside essentially in the fact that causal associations are the major way in which understanding about the world is organised. However, in the area of social sciences and/or psychology we can find that Cognitive Maps should use other kind of relations between concepts (like the Cognitive Base Schemes defined in [8] and [9]) in order to allow a better representation of real world systems that involve cognition.

FCM also have other weaknesses. One of them, uncertainty, can be partially solved by a proper use of FCM [10], but others, like contradictory forces, the simple monotonic relationships or the absence of time delays [7] are out of FCM league.

The use of Fuzzy sets, logic and inference in its traditional rule based form, as introduced by Zadeh [11] and developed throughout these 30 years is particularly more adequate to represent qualitative knowledge involved in cognitive maps due to its linguistic nature [12]. So, it seems a straightforward solution to try to implement Rule Based Fuzzy Cognitive Maps starting from a traditional fuzzy architecture with feedback in order to overcome FCM weaknesses. However, since traditional fuzzy operations can not emulate the effects of causality as it is usually defined in causal maps, there is one important problem to solve while trying to create a RBFCM: the implementation of fuzzy mechanisms that allow causal relations.

2. Rule Based Fuzzy Cognitive Maps (RBFCM)

A RBFCM is essentially a standard rule based fuzzy system where we add feedback and mechanisms to deal with causal relations. It consists of nodes (representing concepts like Inflation or Police_Vigilance), and fuzzy rule bases (which relate and link concepts). Each concept contains several membership functions. The membership functions represent either the concept's possible values or the possible values of its change

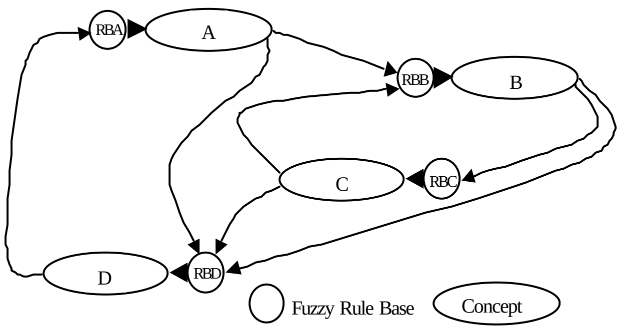

Figure 1 – RBFCM structure

Any kind of relation that can be represented by fuzzy rules is allowed: opposition, similarity, implication, traditional fuzzy reasoning, etc. With the introduction of Fuzzy Causal Relations, RBFCMs can also represent causality. Inputs can be combined using most fuzzy operations (and, or, etc.).

A RBFCM allows the answer to *what-if* questions in cognitive maps. What happens to a system if some of its concepts change, or if new concepts are introduced or removed. The evolution of a RBFCM is iterative. The current value of each concept is computed with its inputs previous values. The concept can be represented either by a crisp or fuzzy value. Eventually, given certain conditions [13] [14], a RBFCM might reach equilibrium, converging to a certain state or entering a cycle of states. With the introduction of fuzzy time delays, RBFCM can become a tool to introduce uncertainty to System Dynamics.

3. Fuzzy Causal Relations (FCR)

3.1 Causal Relations in Cognitive Maps and Fuzzy Logic

There is a causal relation between two given concepts whenever a change in one of those concepts affects the other one. For example, there is a causal relation between Police vigilance and Robbery: a major increase in Police Vigilance will probably cause a decrease in Robbery.

Causal relations in causal maps always involve change: the result of a causal effect is always a variation in one or more concepts. Therefore, causal maps show the variation of a concept's value, not the concept absolute value.

Another property of causal relations is that, by nature, causality is "accumulative". For example, using "natural language" we can say that if concept A and concept B each cause concept C to increase "little", then C will increase "more than a little". If two concepts A and B have the exact opposite effect on C, then C will not change. If A affects "little" and B affects "much", then C will increase "more than much". The effect when both decrease is similar.

In contrast, a traditional fuzzy relation tends to reinforce itself. If A and B cause C to increase "little" with a belief of 0.3 and 0.6, then concept C will increase "little" with a stronger belief (0.9). If A causes C to increase "little" and B causes C to increase "much", then C would increase somewhere between "little" and "much". Opposed effects tend to nullify each other (as in a causal relation).

This essential intrinsic difference causes a total incompatibility in the use of classical fuzzy to represent causal relations. Therefore, in order to introduce causal relations in fuzzy systems, it is necessary to find new ways to make both worlds compatible.

It is important to note that several fuzzy additive systems exist (like Kosko's SAMs [6] for instance), which are not accumulative in the above sense. Those systems add the beliefs of the variables (Y-Axis), not the values in their universe of discourse (UoD – X-Axis), which means that they can not be used to emulate the intended causal behaviour.

To represent causality in FCM, Kosko [6] used a different approach that is not compatible with classic fuzzy systems: To obtain the value of a concept, the value of each of its inputs (concepts) [-1..1] is multiplied by a weight [-1..1]; then the results are added and passed by a non-linearity, just like a common neuron in a Neural Network.

3.2 Causal Rules and Membership Functions

Since in a causal map all concepts represent change, in a RBFCM all concepts can be defined by the same set of membership functions (mbf), with each mbf representing a certain amount of change ranging for example from *Decrease_Very_Much* (*DVM*) to *Increase_Very_Much* (IVM). Figure 2 shows one possible set of mbf for a given concept. In order to guarantee causality, mbf in fuzzy causal systems must abide by certain conditions which are presented in [14].

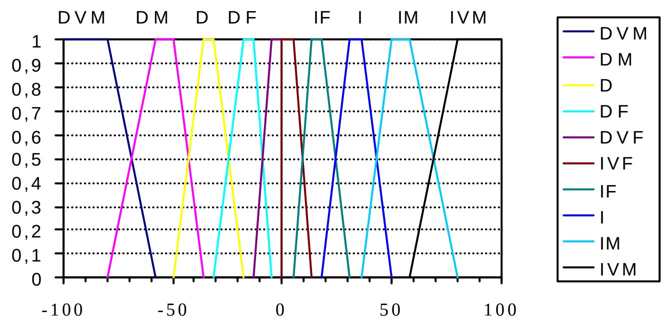

Figure 2 Membership functions

The causal effect of each concept on another one is represented by fuzzy **If…then** rules like:

*“**If** Police_Vigilance Increases **then** Robbery Decreases_Few”*

*“**If** Police_Vigilance Increases_Much **then** Robbery Decreases”*

In FCM we have two kinds of causality: positive, and negative. If an increase in concept A causes an increase in concept B then causality is positive. If an increase in concept A causes concept B to decrease, then causality is negative. The amount of change is defined by a weight between concepts. In RBFCM we also have positive and negative causality, but the amount and type of change is defined by the rules relating the involved concepts. For instance, we can implement rules to define monotonic causal relations (A has a large effect on B, A has a small negative effect on B, …), to define exponential causal relations or even asymmetric causal relations. The set of rules obviously depend on the amount of membership functions and intended meaning.

Although FCM can be more precise than RBFCM (weight 4.5 causes more change than weight 4.4), they are not flexible since even with the use of different non-linear functions in the map, each concept can only be associated with one non-linear function that affects all its inputs. This limits FCM to the representation of simple monotonic causality.

3.3 Fuzzy Carry Accumulation (FCA)

Whenever more than one concept affects another one, then the result of the effects of each concept must be accumulated in order to produce the intended causal effect. The Fuzzy Carry Accumulation (FCA) is a fuzzy operation that provides this effect. It is commutative and associative [14] [15] allowing the introduction and removal of rules and concepts in existing systems (which is essential when we want to analyse the effect of the introduction of new variables in existing systems).

The FCA is based on the principle of the overflow of the belief. This principle can be easily explained using singletons. Let us imagine that the fuzzy set *Increase* of concept *Robbery* is a singleton at *Robbery* = 0.6 (Figure 3) and we have the following rules:

- **If** Police_Vigilance *Decrease* **Then** Robbery *Increase*;
- **If** Wealth_of_Residents *Increase_Much* **Then** Robbery *Increase*

If the consequent of the application of the two rules is *Increase* (μ=0.7) and *Increase* (μ=0.5), how should these consequents be combined in order to produce the accumulative effect (as we saw above, the result should represent a variation larger than Increase)?

In a traditional fuzzy system, the result would always be (after defuzification) *Robbery* = 0.6, even when the sum of the beliefs is greater than 1. In a Fuzzy Causal System, we have the following effect:

- If the sum of the beliefs is lower or equal to 1, then we have a standard fuzzy operation - If we have *Increase* (μ=0.3) and *Increase* (μ=0.5), then the result is *Increase* with μ=0.8, since "I do not believe completely" that the result should be more than *Increase*.

However,

- If the sum of the beliefs is greater than 1, then there is an overflow of the reminder (just like a carry in a sum operation) towards a value representing a larger variation ("I believe it should be larger than Increase").

Figure 3 represents this operation. With the overflow of the excess, the increase of *Robbery* will be larger than the initial value of 0.6.

If the rules involved represent a decrease, then the carry is performed in the opposite direction.

If there are several rules that cause large reminders, whenever the reminder exceeds 1 there is another overflow towards a larger variation.

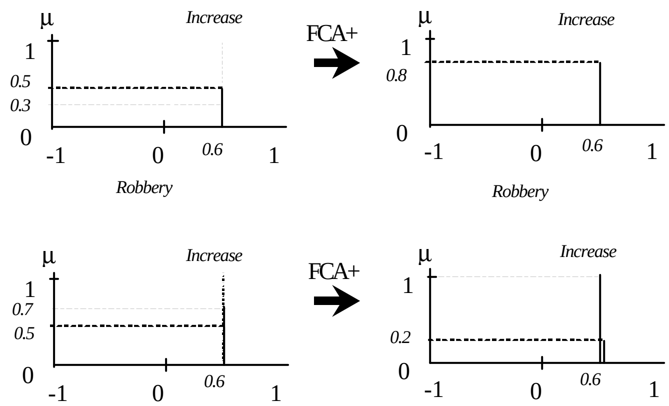

Figure 3 Fuzzy Carry Accumulation with singletons

When the consequents involve different fuzzy sets – for example *Increase* and *Increase_Much* – then since we know that the result must be larger than the largest consequent, the solution is to shift the smaller consequent towards the former. This shift operation arises several problems and constraints that will affect the implementation of fuzzy causality, since there must be a way of retaining and distinguish the original value of the shifted set. Due to this constraints, fuzzy causal operations can not be implemented with singletons. Singletons were used as a simpler way to show the FCA principles.

When we use typical fuzzy sets, the FCA operation is applied at every point of the UoD. The overflow of the sum at each point is added to the sum of the μ of both sets at the next point of the UoD. While overflow exists, it is carried over to the next point, provoking an increase of the value of the resulting set centroid.

In [14] we present details, characteristics and constraints of the Fuzzy Carry Accumulation, and conditions for its implementation and application.

4. Applications, results and considerations

RBFCM were developed because we felt the need for a tool that would allow us to analyse and predict the evolution of negotiation processes' cognitive maps. Other existing tools did not provide mechanisms to implement relations other than monotonic causal relations, or even mechanisms to model human behaviour or abstract concepts in such systems.

RBFCM are still evolving but FCR tests already show the intended behaviour: different rule bases allow flexible causal relations; there seems to be no restrictions to the number of concepts affecting another concept; introduction and removal of inputs is possible [14] [15].

Stability of RBFCM using FCR is highly dependent on the system, not on the FCR itself. The use of different mbf has an high impact on system stability, but mbf that follow the constraints presented in [14] (like those depicted in Figure 2) usually lead most systems to converge to a finite cycle of states [13] (even in systems with a relatively large number of concepts and loops).

In [13] we compare FCM (using a study by Mohr [16]) and RBFCM (a paper with this subject will be submitted for publication soon). The results show that systems which are apparently unstable when allowing continuous values for each concept (like the Crime and Punishment FCM [16]), become stable (converge to a finite cycle of states) when using RBFCM. Since the use of FCM has been most of the times restricted to systems with bivalent (increase/decrease) or trivalent (increase/maintains/decrease) concepts (probably due to its inherent instability with continuous concepts), we can say that RBFCM can improve the efficiency of FCM in the analysis of most systems.

Finally, one of the greatest improvements of RBFCM is the ability to deal with non monotonic and/or asymmetric causal relations. In many real world systems, causal relations can not be described by a single number or sentence. For instance let us see the following example: A City council is biased against polluting industries. If the industrial pollution increases in a given period, then the City council will attempt to increase preventing measures against industrial pollution. If however pollution decreases, the measures will not diminish by the same amount (or may even not diminish at all). FCM can not model this situations, but a simple set of fuzzy rules can easily do it:

If IndPollution	Then PrevMeasures		If IndPollution	Then PrevMeasures
Decrease_Much	Decrease_Few		Increase_Few	Increase_Much
Decrease	Maintain		Increase	Increase_Much
Decrease_Few	Increase_Few		Increase_Much	Increase_Much
Maintain	Increase			

Although RBFCM might not have the simplicity, elegance and ease of use of FCM, their potential is by far superior, and given enough development they might become a valid and useful tool to model and predict the evolution of real world systems which are difficult or impossible to model using a mathematical approach.

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