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# MODELING A CONTROLLED CONVEYOR NETWORK WITH MERGING CONFIGURATION

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## ABSTRACT

Simulation with Arena is used to analyze a controlled *conveyor network with merging configuration* (CNMC). We use simulation to realize the logic in a *queueing-theoretic model* (QTM), and to analyze the behavior of CNMCs under various conditions. We also examine the performance of QTM while keeping or violating the QTM assumptions and constraints. Simulation experiments are designed for the special features of CNMC operations. Various situations are investigated to identify the behavior of CNMCs as well as the robustness of QTM. A case study is reported where mainline and induction-line speeds change proportionally.

## 1 INTRODUCTION

Conveyor systems, an essential component of material-handling systems, are widely used in transportation and manufacturing, such as mail hubs, airports, distribution centers, cargo carriers, warehouses, and other sortation or delivery facilities. In many of these systems, the first and most popular situation to handle is a merging operation. After merging, cargo will be transported to downstream operations, such as sorting, splitting, or more merging. The system, or portion of a system, that exclusively handles merging operations is called a *conveyor network with merging configuration* (CNMC).

### 1.1 Conveyor Networks with Merging Configuration

CNMCs play a key role in the performance of conveyor systems, since cargo conveyed on induction lines may be delayed due to contention for space when inducted into the main line. In some systems, such as distribution centers, warehouses, and airports where throughput is the primary concern, space contention on the mainline is intense, which decreases the operational efficiency of the whole system. In this case, the performance of CNMCs is critical to the performance of the whole system. Thus, it is important to improve the performance of CNMCs.

The CNMC discussed in this paper is in Fig. 1. In such a system, several induction conveyor lines connect into the main conveyor line at consecutive places. Cargo is loaded at the up ends of induction lines, transported into the mainline, and then downstream. There is an operator assigned to each induction line. Each operator attempts to load at a given rate. The operator could be a person, a machine, or an upstream conveyor. If enough space is available on the induction conveyor, the operator places a parcel on the conveyor and then begins to unload the next package. Each parcel is random in size and requires a different amount of space in the induction conveyor. Since the output of this CNMC could be the input to an induction line of another CNMC, several CNMCs can form a complicated network.

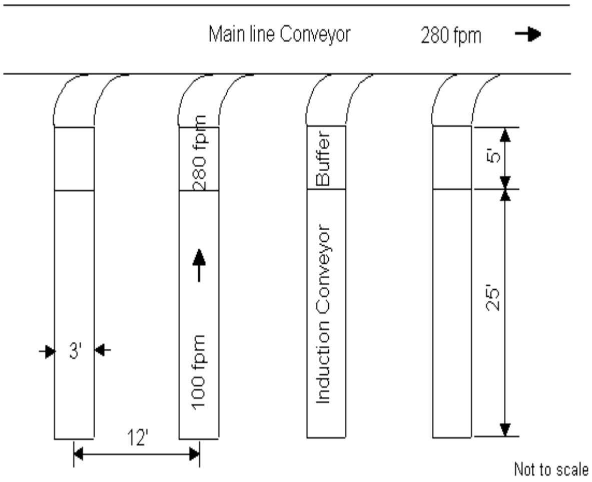


Figure 1: A Conveyor Network with Merging Configuration

Performance of a CNMC is primarily measured by its main-line throughput and utilization. High throughput and high utilization are desired. But high utilization increases contention for space, causing imbalance of throughput among induction lines and decreasing main-line throughput. Parameters need to be carefully chosen for a CNMC to reach high performance.

### 1.2 Controlled CNMCs

A major problem in CNMCs is imbalance of throughput among induction lines. Since lines upstream have the advantage in seizing space, they are more likely to reach higher throughput than are those downstream, provided they have the same arrival rate. Balanced throughput rates among induction lines are desirable for even distribution of workload among the induction conveyors and corresponding staff, or for balanced downstream demand, or other reasons. For example, there might be several flights checking in at the same time, and baggage is checked in from different induction lines but transferred through the same main conveyor line. None of the flights or check-in stations should be blocked; the best way to do this is via a similar throughput rate.

The imbalance problem is caused by natural contention under no control. In advanced systems, the merging operations of induction lines are under control. The philosophy of control is illustrated in Fig. 2. There is a section, called a *buffer* (queue 1, 2, 3, 4, …), at the end of each induction line that connects to the main line. A detection system, which detects the size of parcels, is located at the entrance to each buffer. Based on the detection, a control system allocates appropriate spaces (*windows*) on the main line for parcels at a location in front of merging points called the *window assignment station* (WAS). A parcel is held in the buffer until the window assigned for it arrives at its merging point, when it is released for merging. When the merging operation occurs, the parcel enters the main line and takes the space reserved for it.

Technically, the buffer sections operate at higher speeds than the induction section to pull gaps between the parcels. One-inch gaps are necessary for detection systems to work. Due to limited capacity, a buffer might be full, in which case it blocks (stops) the induction conveyor and reduces the throughput of the induction line. The blocked induction line will be resumed once a merging event occurs that allows the next parcel to enter the buffer. By controlling parameters such as the buffer sizes, one can influence the blocking rate of each induction line, so all lines can reach a balanced throughput.

Arantes and Deng (1996) devised an algorithm (called *QTM*) based on queueing theory and this control philosophy to design the system so that different induction lines can reach a balance while maintaining high throughput. The QTM can identify proper buffer sizes based on the number of induction lines, arrival rates, conveyors' speeds, parcel size, and the distance between buffers and the WAS. Since there are approximations and restrictive assumptions in QTM, how this algorithm works under various situations remains questionable. This problem motivated us to use simulation to analyze the behavior of controlled CNMCs.

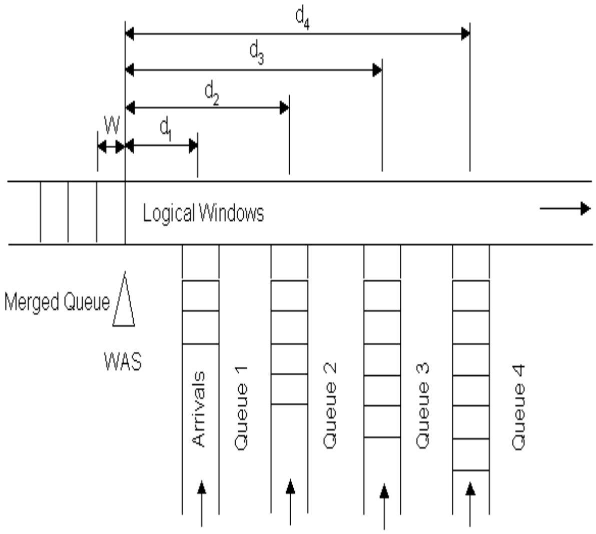


Figure 2: A Queueing-Theoretic Representation of CNMCs

## 2 A SIMULATION MODEL FOR CNMCS BASED ON QUEUING THEORY

Simulation has been increasingly applied to conveyor-system analysis with the rapid improvement of simulation software. Yannopoulos, Jenness and Hawaleshka (1991) used the simple animated simulator PCmodel to simulate an automated paint-line conveyor system; Bartlett and Harvey (1995) used SIMAN to simulate a CIM cell in which two conveyors were considered; Gunal and Williams. (1996) modeled chain conveyors in Automod. So far, an application that focuses on a controlled conveyor network with merging configuration described here has not been observed.

One special feature of controlled CNMC application is that nominated windows, which correspond to specific parcels, need to be generated on the main line. This is handled in our model by duplicating a dummy entity for the merged queue (WAS) once an entity enters a buffer, then disposes it after merging.

## 2.1 Types of CNMCs

There are two different types of control systems for window assignment. One is to assign windows from a fixed window assignment station (WAS), called *fixed WAS*. An alternative is to allocate the closest available window in front of the merging point for a parcel. This is called *moving WAS*.

There are also different kinds of control logic to assign windows. The logic could be first in first out (FIFO), longest queue first (LQF), highest priority first, random, natural (no control), or cyclic (round robin).

There are two sizing styles for assigning windows, fixed length or variant length. For the fixed-length style, all windows have the same length. The length of the windows should be large enough to carry the longest package.

The capacity of buffers could be measured in one of two ways: number of parcels or length of occupied space. The induction conveyor could also be one of two styles: accumulating and non-accumulating. There is no restriction on the distributions of arriving parcels.

## 2.2 Simulation Elements

A CNMC can be broken into four kinds of basic parts: induction line, buffer, WAS, and main line. The simulation model can be integrated by four kinds of submodels: induction line, WAS queue, merging point, and exit. Duplicating induction-line and merging-point submodels can generate CNMCs with an unlimited number of induction lines, while duplicating WAS submodels can generate CNMCs with multiple main lines. Some auxiliary elements are also needed for specifying the simulation experiment, the output statistics, and the animation.

### 2.2.1 Conveyor

A conveyor is the basic element in a CNMC. In Arena, conveyors are aggregated by multiple conveyor units. A conveyor unit is indivisible; thus, no matter how small the unit, the conveyor must be an integer multiple of its unit in length. Also, the position on the conveyor must be counted discretely. This feature affects the simulation results. Too small a unit decreases simulation efficiency, while too large a unit decreases precision. Different conveyor unit lengths have been tested for our model. Experimental results have indicated that good precision and efficiency are attainable if the conveyor unit length is about 1/10 of its longest parcel size.

### 2.2.2 Animation

For the same reason as for conveyors, animation requires that entities be presented in a discrete manner. Parcel sizes, which fall into a continuous interval, have to be clustered into picture sets in which the length is discrete. Based on our experiments, picture sets with six sizes and different colors for different lines are used.

### 2.2.3 Criteria for Evaluation

Criteria used to evaluate the performance of CNMCs are as follows:

- *Throughput* of the main line or induction lines: 1/(average time between outputs).
- *Main-line utilization*: (occupied space)/(available space) on the main line at any moment.
- *Utilization of induction line*: 1 – (blocking probability).
- *Balk rate* of induction lines: (nominal arriving number – real entering number) / (nominal arriving number).
- *Time in system*: average residence time in the system.
- *Time in buffer*: average waiting time in the merging queue.
- *Time between output*: average time between consecutive outgoing entities.

## 2.3 Submodels

Figures 3 through 6 show the logic controlling the induction-line submodel, the WAS submodel, the merging-point submodel, and the exit submodel. This logic governed the construction of our Arena model.

## 2.4 The QTM Based Simulation Model for CNMCs

A general simulation model for CNMCs (Fig. 7) has been built and tested in Arena by using the above submodels. Arena (Kelton, Sadowski and Sadowski, 1998) has been chosen as our primary simulation tool since its lower-level modeling features give us the necessary flexibility.

The particular configuration used in this example comes from a real problem, which includes four non-accumulating induction lines merging at 5, 17, 29, and 41 feet from the WAS. The lengths of the induction lines are 25 feet. The main line runs at 280 feet per minute (ft/m) while induction lines run at 100 ft/m. Packages arrive at the rate of 16 packages per minute. The system may have a fixed window size, such as a tilt conveyor system, or a variant window size. In both cases, a fixed gap must exist between consecutive packages; we used 12 inches. Hence, the window length, in the fixed-window-size case, was 60 inches, which is determined by the length of the longest package size plus the fixed gap length. The window length in the variant-window-size case is the package's length plus 12 inches.

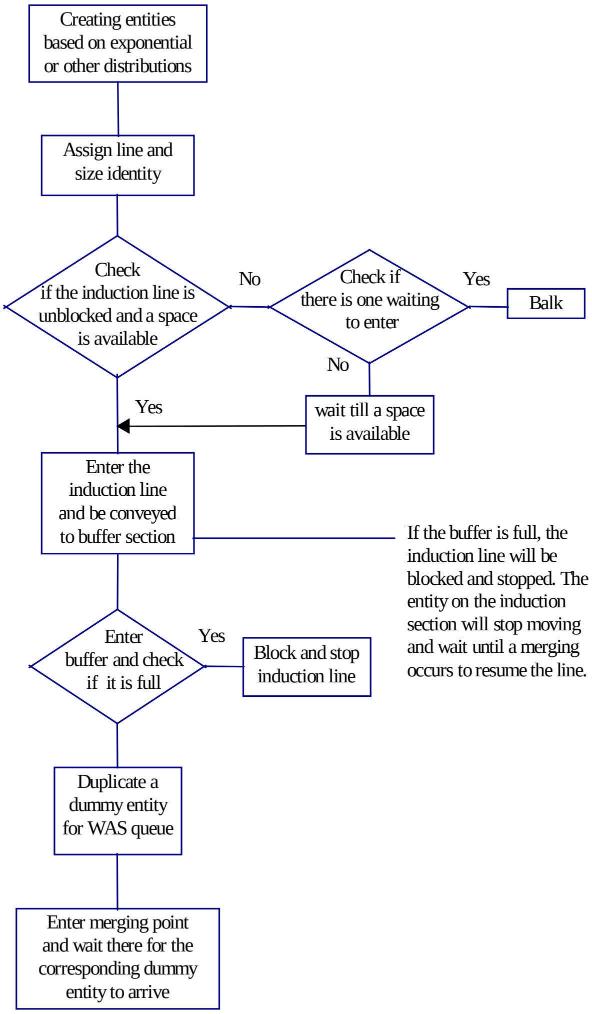


Figure 3: Induction Line Submodel

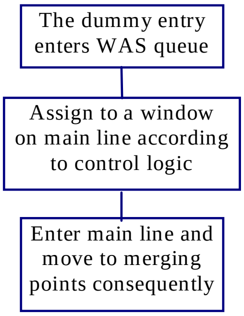


Figure 4: WAS Submodel

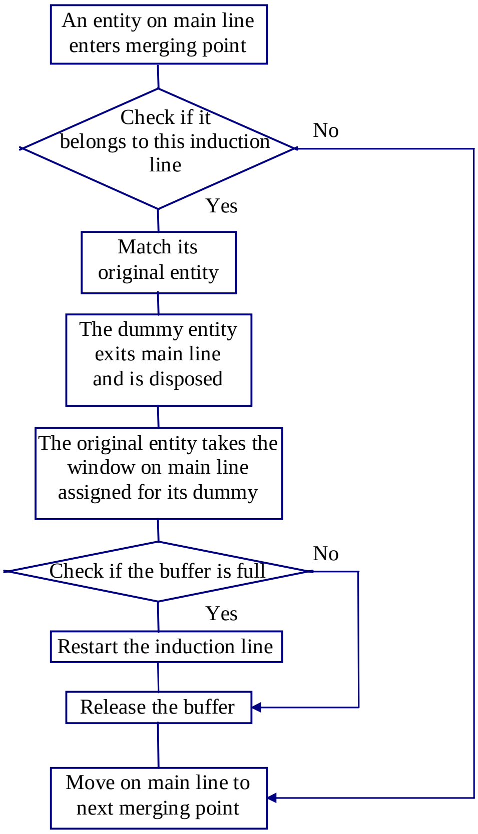


Figure 5: Merging Point Submodel

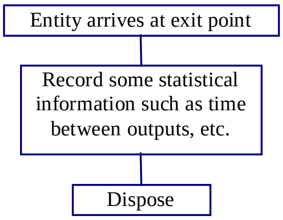


Figure 6: Exit Submodel

There is no restriction on the distributions of inter-arrival times; by default, a stationary Poisson process is used. There is no restriction on the distribution of parcel size; by default, an empirical distribution collected from the real world is used in our analysis. Induction conveyors can be either accumulative or non-accumulative. By changing distributions, other arrival or size patterns could be investigated. The capacity of buffers is by default measured in the number of parcels. By slightly modifying the induction-line submodel, the buffer could be measured in length. By modifying the WAS submodel, other control logic could be used. The model cannot handle a moving WAS situation. In summary, this model is a fairly general one for CNMCs.

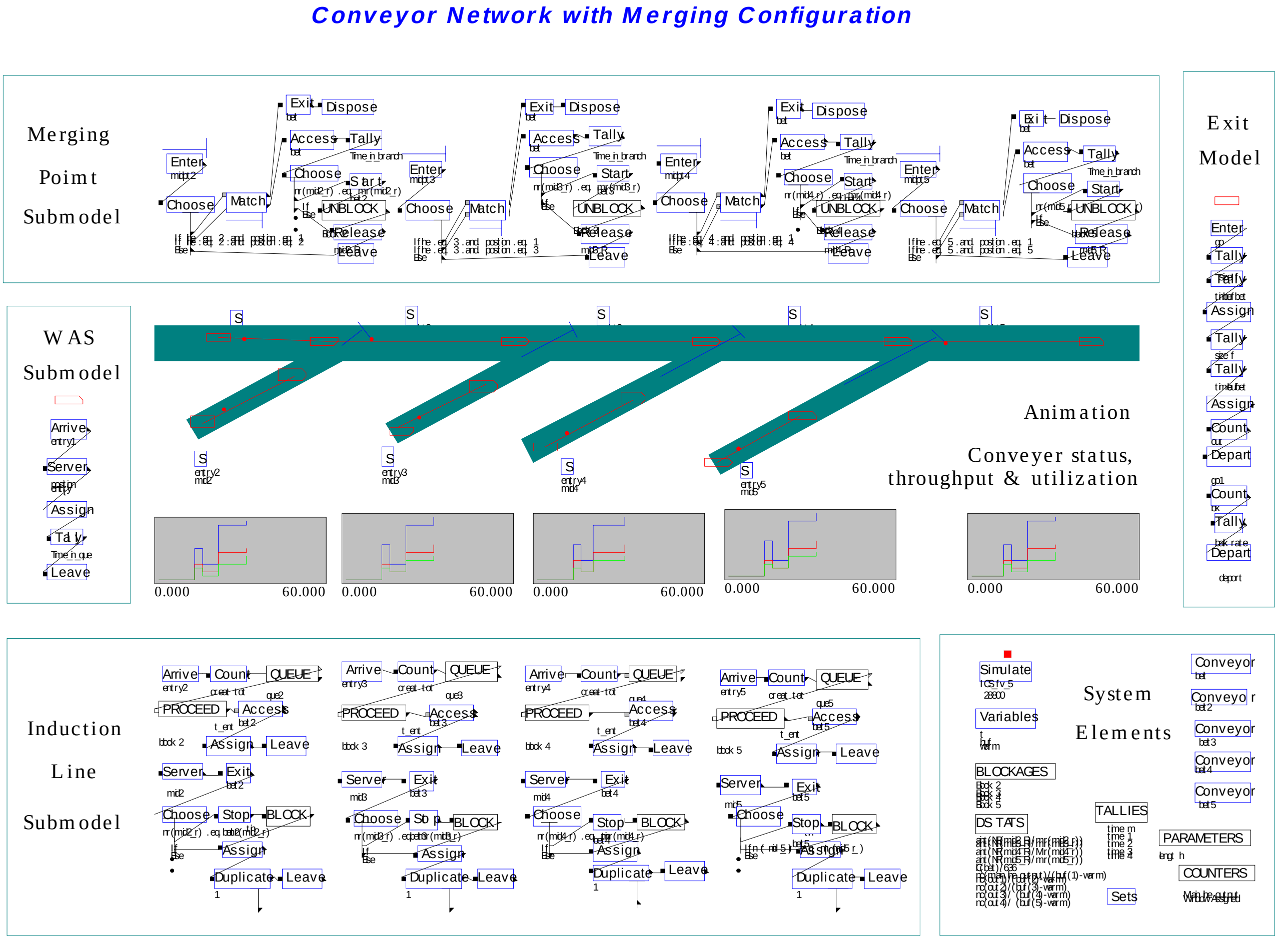


Figure 7: A General Simulation Model for CNMCs

## 3 DESIGN OF EXPERIMENTS AND OUTPUT ANALYSIS

We are interested in the steady-state behavior of a CNMC with a specified buffer size. We want to compute a point estimate and confidence interval for the mean of the criteria mentioned earlier. We chose batch means for confidence-interval formation (Law and Kelton, 1991).

### 3.1 Batching Experimental Design

The run length covers at least ten batches, while each batch covers at least ten significant correlation lags (Pegden, Shannon and Sadowski, 1995) as shown in Fig. 8.

The length of a non-terminating simulation run in our case is selected to be 8 hours, corresponding to a work shift. The simulation run is split into 24 batches of 1200 seconds each. The first 600 seconds corresponds to the warm-up period and is therefore excluded from data collection; the first batch is also excluded from statistical analysis. The design of the simulation experiments is the same for all the cases referring to non-terminating systems. This facilitates analysis while providing the necessary precision. A case study shows that this design produces simulation results with adequate precision. The CNMC is studied as a terminating system later for the investigation of non-stationary arrival processes. The thinning method (Law and Kelton, 1991) is used to generate this non-stationary Poisson arrival process.

### 3.2 Investigating CNMC Behavior and QTM Performance Under Various Situations

With the simulation model and the experimental design, system performance under various conditions is investigated. The configuration defined by QTM is investigated first, and then some QTM assumptions and constraints are violated to assess the robustness of QTM.

The situation that obeys the QTM assumptions corresponds to the default set-up of our simulation model. This situation is for fixed FIFO WAS and non- accumulative

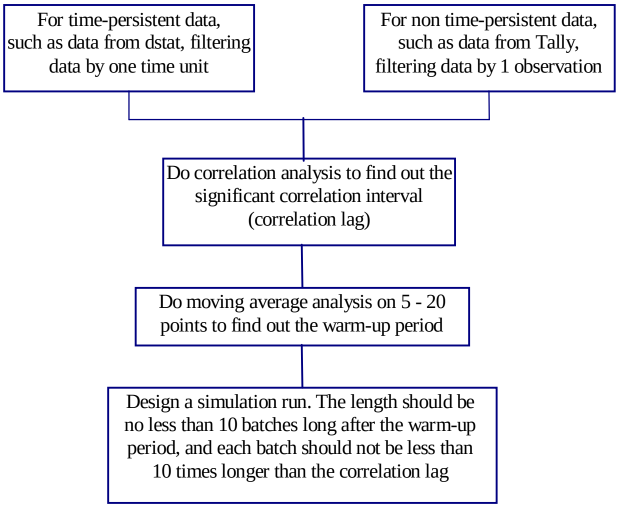


Figure 8: Batching Experimental Design and Output Analysis

conveyors with buffers measured by the number of parcels. For each of the induction lines, the distribution of interarrival times between consecutive parcels is assumed to be exponential.

Other situations investigated include non-stationary arrival rates, different arrival distributions, buffers measured in length, different parcel-size distributions, changing mainline and induction-line speeds, and changing the number of branches. The buffer sizes identified by QTM are used unless other requirements are specified. These investigations, as well as the detailed numerical comparison between QTM and simulation results, are presented elsewhere (Jing, Arantes and Kelton, 1998; Arantes, Jing and Houshmand, 1998).

## 4 CASE STUDY: MAINLINE AND INDUCTION-LINE SPEEDS CHANGE PROPORTIONALLY

As an example, we explore a situation where mainline and induction-line speeds change proportionally. In practice, the speed of the mainline and that of the induction lines are more likely to be adjusted synchronously. We want to find out how the buffer sizes, identified by QTM for the primary speed setting, work in this situation. The effect of induction-line speed is ignored in QTM but can be considered in simulation.

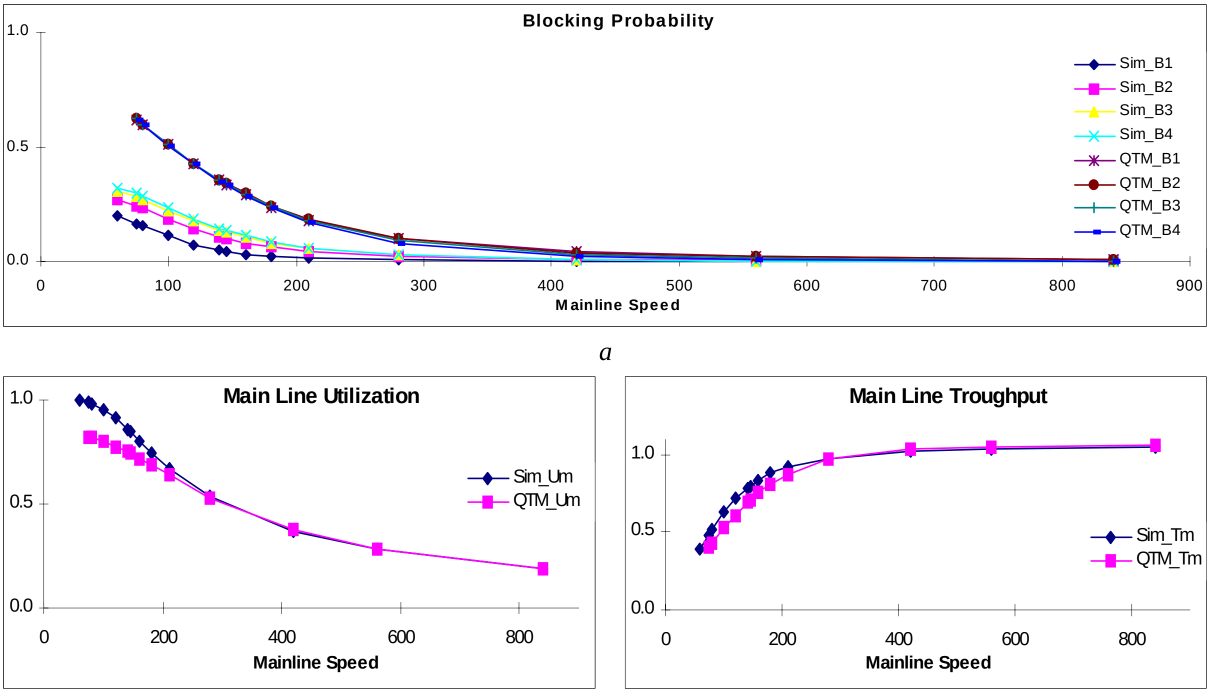
The results for some parameters are in Fig. 9, where (*a*), (*b*) and (*c*) represent induction-line blocking probabilities ($B_i$, $i$=1,…,4), mainline utilization $U_m$, and throughput $T_m$, respectively. In Fig. 9, simulation results are denoted by "*Sim_*" while QTM results are denoted "*QTM_*". Compared with those of mainline-speed-change-only, the simulation results are closer to those of QTM. The two most significant differences are:

- $T_m$ for simulation is more consistent with that of QTM. The values of $T_m$ from the two methods fit very well at high speed, starting from the primary speed where $U_m \cong 0.5$
- Blocking probabilities for simulation drop at low speeds, forming a steadier trend for an upstream line to have a lower blocking probability. The explanation for the blocking probabilities' dropping is that the low induction-line speed retards buffer filling, which QTM cannot detect. The agreement of the values for $T_m$ is due to the fact that the arrival rate increases while the induction-line and mainline speeds increase.

A break point exists around $U_m$ = 0.8, below which the results for the two methods are significantly different. The lower the speeds the more significant the difference. Above this point, $U_m$ and $T_m$ are almost the same. The difference is expected and is because the QTM approximation deteriorates at high mainline utilization.

There is also a break point with respect to mainline speed, above which the mainline throughput is bounded by the arrival rate, and below which the mainline throughput is bounded by its capacity. The turning-point speed can be obtained analytically by imposing $N\lambda=\mu_0$, where $N$ is the number of induction lines, $\lambda$ is the nominal arrival rate, and $\mu_0$ is the service rate at the merged queue for the WAS. The turning point is unimportant here because the arrival rate and throughput capacity change with the speed changes.

The inconsistency between the blocking probabilities and the ($U_m$, $T_m$) values from simulation is bigger at lower speeds. The same thing happens in QTM. However, the overall performance ($U_m$, $T_m$) from simulation and from QTM gets closer. The biggest relative difference still occurs around the break point. The consistency at high speed but inconsistency at low speed reveals that the approximation error in QTM is small at high speeds (low $U_m$), but is large at low speeds (high $U_m$). Thus, the QTM design (buffer sizes) is less sensitive (compared to mainline-speed-change-only) to the decrease, and is not sensitive to the increase when the two speeds increase simultaneously.



*a*

*b* *c*

Figure 9: Mainline and Induction Line Speeds Change Proportionally

## 5 CONCLUSIONS

With our simulation model, the performance of CNMCs under various situations has been investigated, and thus the robustness of QTM has been tested. Overall, our findings are:

- CNMCs have short warm-up periods and reach steady state quickly, so a terminating system can be treated as non-terminating with reasonable precision.
- QTM is quick and conservative in finding a reasonably good initial design for CNMCs to reach a balanced throughput. With reasonable relative precision on simulation-generated confidence intervals, QTM generally works well. Otherwise, the results may need to be improved by other means such as simulation for high precision.
- QTM is sensitive to neither the assumption of a stationary arrival process nor to the interarrival-time distribution nor to the way in which buffer lengths are measured. Further simulations showed that QTM deteriorates as the main-line utilization is high. As a result, under the same conditions, QTM fits VWS better than FWS since VWS has lower main-line utilization.
- QTM is more stable against changes of branches than changes in speeds. The QTM results and simulation results agree very well under a variable number of branches. There is a turning point with mainline speed beyond which the mainline throughput will be bounded by the arrival rate, and below which the mainline throughput will be bounded by its capacity. The turning-point speed can be obtained analytically.
- There is a threshold for induction-line speed above which we confirmed the conclusion from the analytical results, that the induction speed does not have a significant impact on the performance of the conveyor network studied. We conclude that the value of the mainline speed at the turning point is actually the optimal mainline speed. The induction-line speed at the threshold is also optimal since we want to reach reasonably high throughput with reasonably low speeds.

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