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A PROCEDURE FOR HIGH REPRODUCIBLE MEASUREMENTS OF ADC SPECTRAL PARAMETERS

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Abstract

The evaluation of spectral parameters characterizing analog–to–digital converters (ADC) is addressed by applying a single or dual tone generator to the device input and by properly processing its output data stream. The coherent sampling condition, highly recommended by the IEEE standards 1057 and 1241 which list the most effective ADC testing procedures, is usually difficult to achieve, and sometimes even unfeasible. In fact, it requires a fine synchronization between the input and the sampling signals frequencies and it can not be achieved when spurious tones are present in the ADC output spectrum. Data windowing is usually employed to reduce the associated spectral leakage phenomenon. However, IEEE standards do not provide clear criteria for choosing the window to be used for testing a given b–bit converter. Therefore, a reduced measurement reproducibility can result. The European draft standard Dynad suggests the employment of one out of seven windows in accordance to the ADC resolution. However, each proposed sequence covers only a limited converter resolution range. In this paper an ADC testing procedure is described, suitable to yield highly repeatable and reproducible measurements also when non–coherent sampling applies. To this aim, the use of a class of windows is proposed, that uniquely applies to ADCs with arbitrarily high resolution. Finally, experimental results that validate its effectiveness are presented.

Keywords

Digitizer FFT test, FFT test procedure, DPSS windows

I. Introduction

Methodologies usually employed for the parameter estimation of analog–to–digital converters (ADCs) are based on the analysis of the ADC numerical output when a single or dual tone is used as input signal [1],

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[2]. The observed ADC output data can be often modeled as a set of single–tone components embedded in white zero–mean Gaussian noise. Therefore, the problem of evaluating the ADC frequency–domain performance reduces to the estimation of multiple sine parameters and of the broad–band noise–level.

Such an estimation problem, widely investigated and detailed in the scientific literature, can be addressed by employing both parametric or non–parametric procedures [3]–[6]. Although parametric procedures present an high frequency selectivity and statistical efficiency, the determination of the model order is often a difficult issue and iterative procedures must be used. Conversely, non–parametric testing procedures are characterized by low computational effort and robustness towards signal model inaccuracies. On the other hand, they present lower frequency selectivity and statistical efficiency [4], [5]. However, such performances reduction can be compensated for by increasing the observation interval length or the number of analyzed samples. Thus, non–parametric procedures can be widely applied for waveform digitizer testing.

Whatever the adopted testing methodology, the IEEE standards 1057 and 1241 [1], [2] recommend the use of coherent sampling in order to guarantee maximum estimation accuracy. However, such a condition can not be guaranteed with respect to spurious tones eventually present in the output spectrum. In such a situation, spectral granularity and leakage may affect the accuracy with which input signal parameters are estimated. Output–data windowing has been classically employed to reduce the effect of such phenomena [3]. Unfortunately, published results do not provide general criteria for the choice of the most appropriate window to be employed. Some suggestions are given in the draft standard Dynad [7], where it is recommended the selection of one out of well–known windows [3], in accordance to the resolution of the tested ADC. However, criteria to be followed for choosing the window that optimizes measurement accuracy are not explicitly given.

Previous works on this subject have shown that the employment of the zero–order Discrete Prolate

Spheroidal Sequences (DPSS) maximizes the measurement accuracy when non–coherent sampling applies [4], [9]. Windows belonging to this class of sequences are completely defined by their mainlobe width. Moreover, general criteria for selecting the window parameter that provides a given estimation accuracy for ADCs with any resolution have been given. This feature allows the optimization of the window selection process, thus improving testing automation. However, samples of the DPSS windows can be calculated only on the basis of numerical algorithms. As a consequence, different implementations applied to the same set of data can result in different estimates of ADC figures of merit, thus reducing measurement reproducibility.

In this paper, the use of a class of windows that can easily be calculated by using standard computational tools and that well approximates the DPSS sequences is proposed. In particular, such windowing sequences are employed in a DFT-based estimator of the main spectral figures of merit of a generic b–bit ADC. The algorithm is then detailed by itemizing steps to follow both when coherent or non–coherent sampling is used. Finally, the procedure is applied to a 16–bit acquisition board and the theoretical and experimental standard deviations are compared in order to validate both the proposed estimation process and the effectiveness of the employed window sequences.

II. DFT–BASED METHOD

Spectral figures of merit usually employed for characterizing ADCs in the frequency domain are the spurious–free dynamic range ($SFDR$), the signal–to–noise–and–distortion–ratio ($SINAD$), the signal–to–random–noise–ratio ($SRNR$) and the total harmonic distortion (THD). They require the power estimation of the wide–band noise σ_R^2 and of $H+S+1$ narrow–band components, $\sigma_{X_i}^2$, $i = 1, ..., H+S+1$, which are composed by the fundamental tone, H harmonics and S spurious.

When non–coherent sampling applies, the DFT–based method evaluates the power of the narrow– and wide–band components from the windowed ADC output spectrum obtained by using a Fast Fourier Transform (FFT) algorithm. To this aim, as described in detail in [9], [10] and [11], the output spectrum is divided in sets of frequency bins, each associated with one kind of spectral component, indicated with $\mathcal{B}_{X_i}$ for the narrow-band components and with $\mathcal{B}_{\mathcal{R}}$ for the wide-band noise.

Expressions for the power estimators of the wide–band noise, $\widehat{\sigma}^2_R$, and of the i–th narrow–band component, $\widehat{\sigma}^2_{X_i}$, are:

$$\widehat{\sigma}^2_R \triangleq \frac{1}{N_R N^2} \frac{1}{NNPG} \sum_{k \in \mathcal{B}_{\mathcal{R}}} |Y[k]|^2, \tag{1}$$

$$\widehat{\sigma}^2_{X_i} \triangleq \frac{2}{N^2} \frac{1}{NNPG} \sum_{k \in \mathcal{B}_{X_i}} |Y[k]|^2 - 2\frac{N_{X_i}}{N}\widehat{\sigma}^2_R. \tag{2}$$

In (1) and (2), N is the number of acquired samples, $Y[\cdot]$ is the DFT of the windowed acquired data samples, N_{X_i} is the number of samples in $\mathcal{B}_{X_i}$ and N_R is the number of samples in $\mathcal{B}_{\mathcal{R}}$. Moreover, $NNPG \triangleq \frac{1}{N}\sum_{n=0}^{N-1} w^2[n]$ is the window Normalized Noise Power Gain.

By defining $\widehat{\sigma}^2_{X_1}$, $\widehat{\sigma}^2_H \triangleq \sum_{i=2}^{H+1} \widehat{\sigma}^2_{X_i}$ and $\widehat{\sigma}^2_S \triangleq \sum_{i=H+2}^{S+1} \widehat{\sigma}^2_{X_i}$ as the estimators of the power of the fundamental, of the harmonics and of the spurious components, respectively, the estimators of the spectral figures of merit are [10]:

$$\widehat{SRNR} \triangleq \frac{N_R}{N_R + ENBW_0} \frac{\widehat{\sigma}^2_{X_1}}{\widehat{\sigma}^2_R}, \tag{3}$$

$$\widehat{SINAD} \triangleq \frac{\widehat{\sigma}^2_{X_1}}{\widehat{\sigma}^2_R + \widehat{\sigma}^2_H + \widehat{\sigma}^2_S}, \tag{4}$$

$$\widehat{SFDR} \triangleq \frac{\widehat{\sigma}^2_{X_1}}{\max_{i>1} \widehat{\sigma}^2_{X_i}}, \tag{5}$$

$$\widehat{THD} \triangleq \frac{\widehat{\sigma}^2_H}{\widehat{\sigma}^2_{X_1}}, \tag{6}$$

where $ENBW_0 \triangleq N\sum_{n=0}^{N-1} w^4[n] / (\sum_{n=0}^{N-1} w^2[n])^2$ represents the equivalent–noise bandwidth of the squared window, $w^2[\cdot]$.

The accuracy of each estimator can be separately optimized by suitably choosing the test parameters and, in particular, by carefully selecting the window. The zero–order DPSS sequences can be employed to obtain maximum estimation accuracy [10]. However, the procedure for calculating their samples requires the application of iterating numerical algorithms, thus potentially reducing measurement reproducibility.

The class of windows considered in this paper is defined in the frequency domain on the basis of the Dirichelet kernel as follows [12]:

$$W[k,\Lambda] \triangleq \frac{\sin\left[\frac{N}{2}\cos^{-1}\left(\gamma\cos\left(2\pi k\right)+\left(\gamma-1\right)\right)\right]}{\sin\left[\frac{1}{2}\cos^{-1}\left(\gamma\cos(2\pi k)+\left(\gamma-1\right)\right)\right]}, \qquad k=0,...,N-1 \tag{7}$$

where $W[\cdot,\Lambda]$ represents the DFT of $w[\cdot]$, $\gamma \triangleq \left(1+\cos\left(2\pi/N\right)\right)/\left(1+\cos(2\Lambda\pi/N)\right)$, and Λ is the window mainlobe width expressed in bins.

Such class of windows may easily be calculated from the knowledge of the record length N and of the needed mainlobe width Λ by using standard computational tools. Moreover, it has been demonstrated that they well approximates the zero–order DPSS sequences [12] with the same mainlobe width Λ. It follows that the window samples obtained by applying an inverse FFT (IFFT) algorithm to (7) assure optimal estimators accuracy, measurement reproducibility and testing automation.

In the next section, the related DFT–based testing procedure is step-by-step described. It allows the characterization of ADCs with any given resolution, by optimizing the available test–bench resources. In particular, indications for choosing the minimum number of samples which guarantees an upper bound, ε, on the relative type A uncertainty of the estimates of the spectral figures of merit, are explicitly given. Such bound has been derived by noticing that the normalized variance of the estimates of interest is equal to or lower than the normalized variance of (1), which is given by $\overline{\mathrm{var}}\{\widehat{\sigma}_R^2\} \simeq ENBW_0/N_R$ [9], [10]. As a consequence, the number of samples associated to the wide–band noise has to satisfy the condition:

$$N_R \geq \frac{ENBW_0}{\varepsilon^2}. \tag{8}$$

Moreover, indications for selecting the mainlobe window which guarantees optimum estimation accuracy is explicitly given.

III. The proposed procedure for ADC testing

Fig. 1 shows the steps to follow for estimating spectral parameters of a generic b-bit ADC with a given relative type A uncertainty ε. Grey boxes refer to calculations to be performed only when non–coherency applies. Whenever sampling is realized coherently, acquired data can be windowed by using a rectangular window and assuming $N_{X_i} = 1$.

The *algorithm input parameters* are related to both the characteristics of the measurement bench set–up and the device under test. The procedure requires knowledge of the maximum estimates accuracy, ε, the available memory–depth of the employed testing bench, N_{max}, the operating full scale range (FS) of the device under test, its sampling rate f_s and an a priori estimate of the expected $SRNR$ at the ADC output, γ_1. If γ_1 is not available, the $SRNR$ of an ideal b–bit quantizer can be employed.

The *data acquisition parameters* are then selected as follows: the number of acquired samples N_T, must satisfy the condition $N_T > \pi 2^b$ [1]. If $N_T > N_{max}$ results, the testing algorithm can be repeatly applied to R data records, each of length N, such that $RN \geq N_T$. Thus, the value of R can be calculated as $R = \lceil N_T/N \rceil$, where the operator $\lceil x \rceil$ rounds x to the closest upper integer. The parameters of interest can then be derived by averaging the R resulting estimates. It should be noticed that the employed number of samples is often set to a power of two in order to reduce the FFT computational time.

If non–coherent sampling applies, an optimum *window parameter*, Λ_{opt}, can then be calculated as [10]:

$$\Lambda_{opt} = 0.607 + 0.189 \log_{10} N_R + 0.378 \log_{10} \gamma_1, \tag{9}$$

where usually N_R can approximately be set equal to $N/3$. It is then possible to calculate the number of samples N_{X_i} associated to the estimate of each narrow–band component. The optimum value, obtained as a compromise between maximum estimator accuracy, maximum frequency selectivity and low computa-

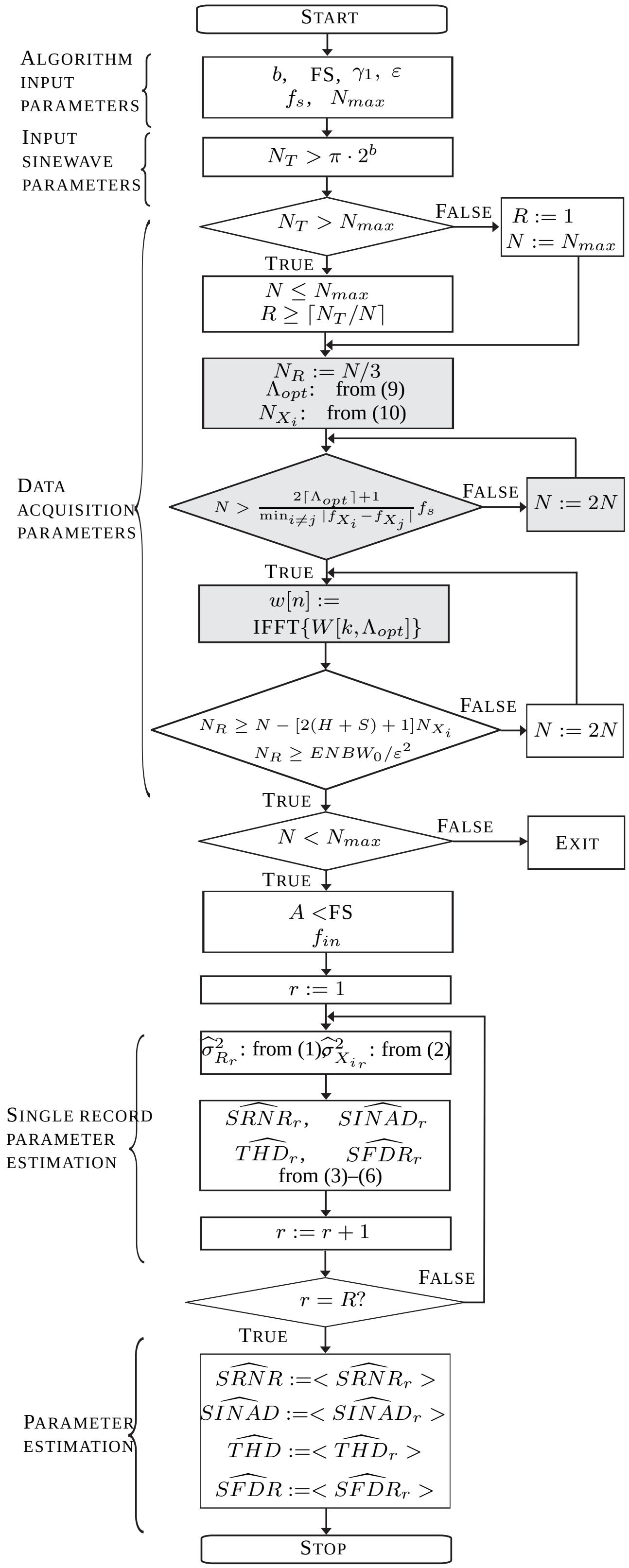

Figure 1. Flow chart of the DFT–based testing algorithm. Grey boxes indicate calculations to be performed only when non–coherent sampling applies. N is set to a power of two in order to apply classical FFT algorithms to the windowed data.

tional effort, is [4]:

$$N_{X_i} = 2[\Lambda_{opt}] + 1, \tag{10}$$

where the operator $[x]$ rounds x to the nearest integer.

In order to estimate accurately the narrow–band spectral components, it should be verified that the distance between the two closest components is greater than $2\lceil\Lambda_{opt}\rceil + 1$ bins. The condition to verify is then [10]:

$$N > \frac{2\lceil\Lambda_{opt}\rceil + 1}{\min_{i\neq j}|f_{X_i} - f_{X_j}|} f_s, \quad i, j = 1, ..., H + S + 1 \tag{11}$$

where f_{X_i} represents the frequency of the i–th narrow–band component expressed in Hz.

The window coefficients $w[\cdot]$ can then be calculated by substituting (9) in (7) and by applying the IFFT algorithm to the resulting expression.

Moreover, it should be verified that the total number of frequency bins associated to the $H + S + 1$ narrow–band components and to the wide–band noise do not exceed the number of available frequency bins, i.e. that

$$N_R \geq N - [2(H + S) + 1]N_{X_i} \tag{12}$$

It should be also verified if the required estimator accuracy is guaranteed, that is if (8) occurs.

If (8), (11) or (12) are not satisfied, N must be increased. Whenever the number of samples needed to perform the test is greater than the available memory–depth, i.e. $N > N_{max}$, the test can not be carried out because of limits on hardware resources.

The *input sinewave parameters* are then set. In particular, the amplitude A has to be chosen in order not to overload the ADC; moreover, a proper frequency f_{in} has to be selected. In particular, whenever coherency must be attained, $f_{in} = f_s L/N$, where L is the integer number of acquired sinewave cycles which has to be chosen prime with respect to N.

TABLE I

ALGORITHM PARAMETERS EMPLOYED FOR OBTAINING EXPERIMENTAL RESULTS OF FIG. 2

<table>
<tr><td rowspan="2">Algorithm input parameters</td><td>b</td><td>FS</td><td colspan="2">f_s</td><td>ε^2</td></tr>
<tr><td>16</td><td>10 V</td><td colspan="2">20 ksample/s</td><td>$2 \cdot 10^{-4}$</td></tr>
<tr><td rowspan="2">Input sinewave parameters</td><td>$\mathbf{A}$</td><td>$\mathbf{f_{in}}$</td><td colspan="3">γ_1 ideal</td></tr>
<tr><td>9.85 V</td><td>variable</td><td colspan="3">97.96 dB</td></tr>
<tr><td rowspan="2">Data acquisition parameters</td><td>N=N_{max}</td><td>N_T</td><td>N_R</td><td colspan="2">R</td></tr>
<tr><td>2^{14}</td><td>2^{18}</td><td>14448</td><td colspan="2">100</td></tr>
<tr><td rowspan="2">Window parameters</td><td>Λ_{opt}</td><td>$NNPG$</td><td>$ENBW_0$</td><td colspan="2">N_{X_i}</td></tr>
<tr><td>5.02</td><td>0.23</td><td>3.14</td><td colspan="2">11</td></tr>
</table>

Once the DFT–based algorithm parameters have been set, it is possible to proceed with the estimation of the desired ADC spectral parameters based on the r-th data record. In particular, the powers of the wide– and narrow–band components, $\widehat{\sigma}^2_{R_r}$ and $\widehat{\sigma}^2_{X_{i_r}}$, are evaluated by using (1) and (2), respectively. Finally, (3)–(6) can be applied to estimate the corresponding figures of merit. The overall ADC spectral performances can then be evaluated by taking the arithmetic average over R records, as indicated in Fig. 1 by the operator $< x_r > \stackrel{\triangle}{=} 1/R \sum_{r=1}^{R} x_r$.

IV. EXPERIMENTAL RESULTS

The algorithm described in section III has been applied to data acquired from the 16–bit data acquisition board AT-MIO-16XE-50, developed by National Instruments, for evaluating the $SRNR$ at various input frequency values. Such a board has a sampling rate of 20 ksample/s and a full-scale range equal to ± 10 V. The signal generator used as input stimulus is the Stanford Research System DS360 generator, which exhibits an $SFDR$ larger than 96 dBc. The amplitude of the input sinewave has been set equal to 9.85 V, the value at which the maximum $S\widehat{RN}R$ has been experimentally obtained by employing a 1 kHz input sinewave. The corresponding γ_1 value is 97.96 dB. The energy-based algorithm has been applied for input frequencies equal to 0.5, 1, 2, 3 and 4 kHz, and the $SRNR$ has been estimated.

The algorithm parameters employed for the specific case are reported in Tab. I and the various $SRNR$ estimates have been graphed in Fig. 2. In particular, the bolded line represents the average based on $R =$

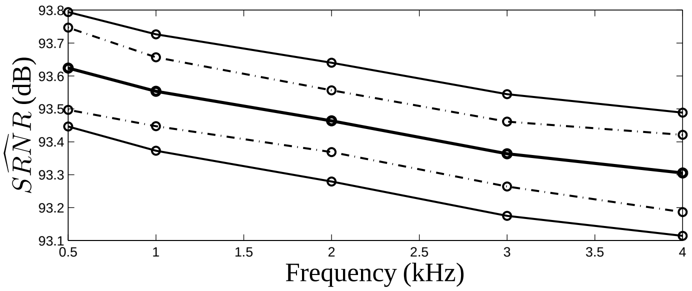

Figure 2. Bolded line represent the $\widehat{SRNR}$ of the AT–MIO–16–XE–50 data acquisition board estimated with the DFT–based algorithm by employing parameters shown in Tab. I and the proposed window (7). Solid and dash–dotted lines represent the average plus and minus the theoretical and experimental standard deviations, respectively.

100 data records, and dash-dotted lines represent the $SRNR$ plus and minus the experimental standard deviation.

In order to validate the proposed procedure, the value of the theoretical standard deviation of $\widehat{SRNR}$ have been calculated by means of [11]:

$$\mathrm{std}\left\{\widehat{SRNR}\right\} = \sqrt{\frac{ENBW_0}{N_R}}\gamma_1. \tag{13}$$

The experimental $SRNR$ plus and minus such a theoretical standard deviation has been also plotted in Fig. 2 with solid lines. The good agreement between the experimental and the theoretical lines confirms the effectiveness of the testing procedure and of the proposed class of windows.

It should be noticed that estimates accuracy achieved by using the class of windows proposed in this paper has the same order magnitude of that attainable by employing windows suggested by the classical scientific literature [3]. The advantage in using this new class of windows is that (7) can be employed for testing ADCs with any given resolution by setting only the Λ parameter.

V. Conclusions

The problem of optimizing the accuracy of the ADCs spectral figures of merit when non–coherent sampling applies, is not thoroughly investigated by the published scientific literature [1]–[3], [7]. In particular, the lack of a criterion for choosing the window which guarantees, for a given b–bit ADC, the maximum estimator accuracy, reduces measurements reproducibility.

In order to overcome this limitation, in this paper a DFT–based ADC testing algorithm has been detailed from a procedural point of view. Accordingly, it has been proposed the use of a class of windows that are completely characterized by setting only the mainlobe width value and that can be employed for testing any given resolution ADC. In order to improve measurement accuracy, a criterion for selecting the optimum window mainlobe width has been provided. Whenever coherent sampling can be attained, the proposed method can still be applied to un–windowed data.

The procedure has been used to measure the performance of a 16–bit data–acquisition board and experimental results, which validate both the testing method and the employed windows, have been presented.

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