Adaptive Coded Modulation for Fading Channels

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***Abstract*— We apply coset codes to adaptive modulation in fading channels. Adaptive modulation is a powerful technique to improve the energy efficiency and increase the data rate over a fading channel. Coset codes are a natural choice to use with adaptive modulation since the channel coding and modulation designs are separable. Therefore, trellis and lattice codes designed for additive white Gaussian noise (AWGN) channels can be superimposed on adaptive modulation for fading channels, with the same approximate coding gains. We first describe the methodology for combining coset codes with a general class of adaptive modulation techniques. We then apply this methodology to a spectrally efficient adaptive M-ary quadrature amplitude modulation (MQAM) to obtain trellis-coded adaptive MQAM. We present analytical and simulation results for this design which show an effective coding gain of 3 dB relative to uncoded adaptive MQAM for a simple four-state trellis code, and an effective 3.6-dB coding gain for an eight-state trellis code. More complex trellis codes are shown to achieve higher gains. We also compare the performance of trellis-coded adaptive MQAM to that of coded modulation with built-in time diversity and fixed-rate modulation. The adaptive method exhibits a power savings of up to 20 dB.**

***Index Terms*—Adaptive coded modulation, coset codes, fading channels, spectral efficiency.**

I. Introduction

HIGH-SPEED wireless data transmission requires robust and spectrally efficient communication techniques for fading channels. Transmission techniques which do not adapt to the fading channel require a fixed link margin or coding to maintain acceptable performance in deep fades. Thus, these systems are effectively designed for the worst-case channel conditions, resulting in insufficient utilization of the full channel capacity. If the channel fade level is known at the transmitter, then Shannon capacity is achieved by adapting the transmit power, data rate, and coding scheme relative to this fade level [1]. Based on these results, an adaptive variable-rate variable-power transmission scheme using uncoded M-ary quadrature amplitude modulation (MQAM) was proposed in [2]. This adaptive technique is 17 dB more power efficient than nonadaptive modulation in fading. Adaptive modulation has also been studied in [3]–[6] and the references therein. These adaptive schemes all use uncoded modulation, with the exception of [3], which uses a form of the adaptive coded modulation we describe below.

In this paper we show that coded modulation techniques designed for the additive white Gaussian noise (AWGN) channel can be superimposed on a very general class of adaptive modulation schemes, with the same approximate coding gain. Trellis and lattice codes, which are special cases of coset codes, are particularly well suited for adaptive coded modulation since the code design and modulation design are separable [7], [8]. Thus, we can vary the size, power, and symbol time of the transmitted signal constellation to maximize the average data rate without affecting the bit-error rate (BER) or the coding gain. We can also take advantage of the extensive work in trellis and lattice code designs for AWGN channels to obtain good codes.

The end-to-end system for adaptive coded modulation is shown in Fig. 1. We assume a flat-fading channel with AWGN $n(t)$ and a stationary and ergodic channel gain $\sqrt{g(t)}$. Let $\bar{S}$ denote the average transmit signal power, $N_0/2$ denote the noise density of $n(t)$, B denote the received signal bandwidth, and $\bar{g}$ denote the average channel gain. With appropriate scaling of $\bar{S}$ we can assume that $\bar{g} = 1$. For a constant transmit power $\bar{S}$, the instantaneous received signal-to-noise ratio (SNR) is $\gamma(t) = \bar{S}g(t)/(N_0B)$ and the average received SNR is $\bar{\gamma} = \bar{S}/(N_0B)$. We denote the fading distribution of γ by $p(\gamma)$. If the transmit power $S(t)$ is adapted relative to $g(t)$ or, equivalently, to $\gamma(t)$, then the SNR at time t is given by

$$\mathrm{SNR}(t) = \frac{\gamma(t)S(\gamma(t))}{\bar{S}} = \frac{g(t)S(g(t))}{N_0B}. \quad (1)$$

When the context is clear, we will omit the time reference t relative to γ and $S(\gamma)$.

Adaptive coded modulation does not require interleaving since error bursts are eliminated by adjusting the power, size, and duration of the transmitted signal constellation relative to the channel fading. However, adaptive modulation does require accurate channel estimates at the receiver which are fed back to the transmitter with minimal latency. In this paper we assume perfect channel estimates ($\hat{\gamma}(t) = \gamma(t)$) at the receiver with zero delay ($\tau_f = 0$) in the feedback path. The effects of estimation error and feedback path delay on adaptive modulation were analyzed in [2], where it was found that an estimation error less than 1 dB and a feedback path delay less than $0.001/f_D$ results in minimal performance degradation, for $f_D = v/\lambda$ the Doppler frequency of the fading channel. The effect of estimation error and feedback path delay for

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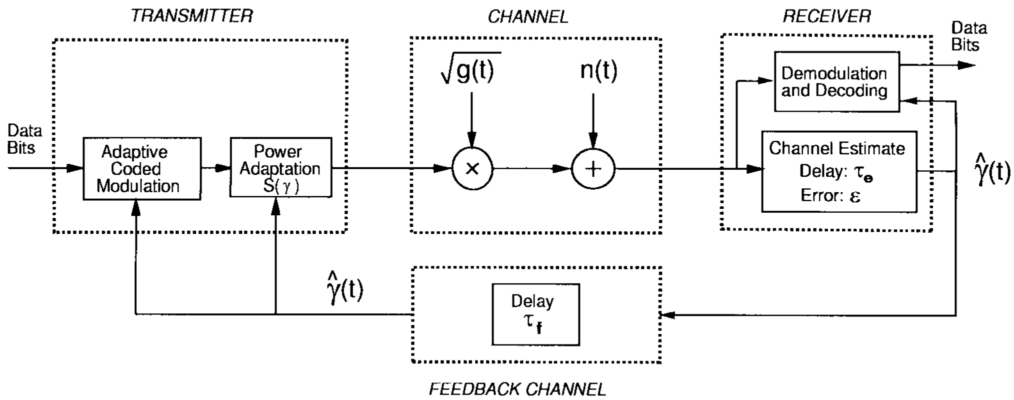

Fig. 1. System model.

adaptive coded modulation is similar, yielding the same set of requirements for minimal performance degradation. These requirements are easily met on slowly varying channels.

Another practical consideration in our adaptive coded modulation scheme is how quickly the transmitter must change its constellation size. Since the constellation size is adapted to an estimate of the channel fade level, several symbol times may be required to obtain a good estimate. In addition, hardware and pulse shaping considerations generally dictate that the constellation size must remain constant over tens to hundreds of symbols. We showed in [2] that this requirement translates mathematically to the requirement that $\overline{\tau_j} >> T\,\forall j$, where T is the symbol time and $\overline{\tau_j}$ is the average time that our adaptive modulation scheme continuously uses the constellation M_j. Since each constellation M_j is associated with a range of fading values called the fading region $\mathcal{R}_j$, $\overline{\tau_j}$ is the average time that the fading stays within the region $\mathcal{R}_j$. The value of $\overline{\tau_j}$ is inversely proportional to the channel Doppler and also depends on the number and characteristics of the different fade regions. It was shown in [2] that in Rayleigh fading with an average SNR of 20 dB and a channel Doppler of 100 Hz, $\overline{\tau_j}$ ranges between 0.7–3.9 ms, and thus for a symbol rate of 100 ksymbols/s, the signal constellation remains constant over tens to hundreds of symbols. Similar results hold at other SNR values.

The flat-fading assumption in our model implies that the signal bandwidth B is much less than the channel coherence bandwidth $B_c = 1/T_M$, where T_M is the root-mean-square (rms) delay spread of the channel. For Nyquist pulses $B = 1/T$, so flat fading occurs when $T >> T_M$. Combining $T >> T_M$ and $\overline{\tau_j} >> T$, we see that

$$\overline{\tau_j} >> T >> T_M \tag{2}$$

must be satisfied to have both flat fading and the signal constellation constant over a large number of symbols. In general, wireless channels have rms delay spreads less than 30 μs in outdoor urban areas and less than around 1 μs in indoor environments [11]. Taking the minimum $\overline{\tau_j} = .7$ ms we see that based on (2), rates on the order of tens of ksymbols/s in outdoor channels and hundreds of ksymbols/s in indoor channels are practical for our adaptive scheme.

The remainder of this paper is organized as follows. The method for combining a coset code with a general class of adaptive modulation techniques is outlined in Section II. Section III describes the superposition of trellis codes onto the adaptive modulation proposed in [2] and obtains the corresponding spectral efficiency. Analytical and simulation results for the BER and spectral efficiency of this trellis-coded adaptive MQAM are presented in Section IV. We compare the performance of trellis-coded adaptive MQAM with that of fade-resistant nonadaptive coded modulation in Section V.

II. Coset Codes with Adaptive Modulation

In this section we show how the separability of code and modulation design inherent to coset codes can be used to combine coset codes with adaptive modulation. We restrict our attention to cubic signal constellations, so the coset codes have zero shape gain.

The general structure of the adaptive coded modulation is shown in Fig. 2. Comparing this figure to the standard block diagram for nonadaptive coded modulation [7, Fig. 10] we see that the *channel coding* segments in each case are identical. Specifically, in both cases a binary encoder E operates on k uncoded data bits to produce $k + r$ coded bits, and then the coset (subset) selector uses these coded bits to choose one of the 2^{k+r} cosets from a partition of the signal constellation. For the nonadaptive modulation of [7], the *modulation* segment then uses $n - k$ additional uncoded bits to choose one of the 2^{n-k} signal points in the selected coset, which is then transmitted via the modulator. These steps essentially decouple the channel coding from the modulation. Specifically, the fundamental coding gain is a function of the minimum squared distance between signal point sequences, which is determined by the encoder (E) properties and the subset partitioning, independent of the modulation. This minimum distance is given by $d_{\min} = \min\{d_s, d_c\}$, where d_s is the minimum distance between coset sequences and d_c is the minimum distance between coset points. For square MQAM signal constellations, both d_s and d_c are proportional to d_0, the minimum distance between constellation points before partitioning, as shown in Fig. 3. The number of nearest neighbor codewords also impacts the effective coding gain, as we discuss in more detail below.

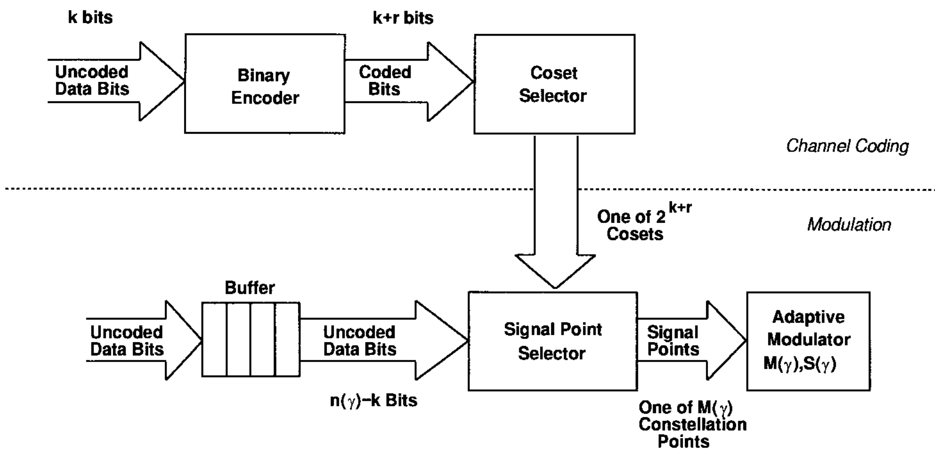

Fig. 2. General structure for adaptive coded modulation.

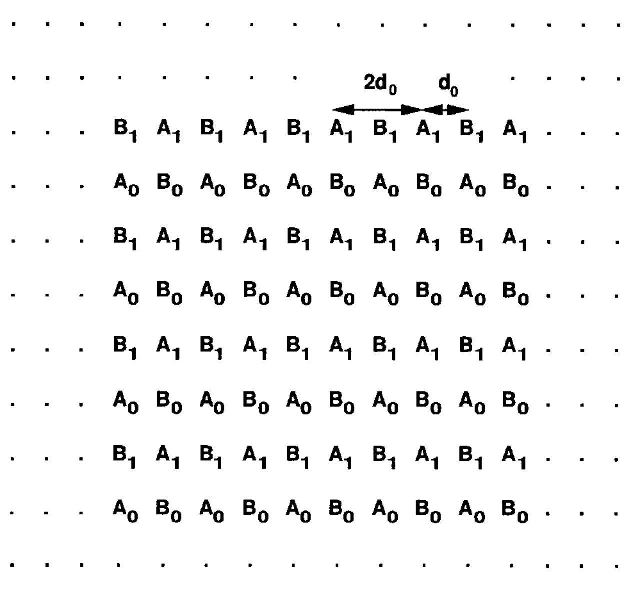

Fig. 3. Two-level set partitioning.

In a fading channel the instantaneous SNR varies with time, which will cause the distance $d_0(t)$ in the received signal constellation, and, therefore, the corresponding distances $d_c(t)$ and $d_s(t)$, to vary. The basic premise for using adaptive modulation with coset codes is to keep these distances constant by varying the size $M(\gamma)$, transmit power $S(\gamma)$, and/or symbol time $T(\gamma)$ of the transmitted signal constellation relative to γ, subject to an average transmit power constraint $\bar{S}$ on $S(\gamma)$. By maintaining $d_{\min}(t) = \min\{d_c(t), d_s(t)\} = d_{\min}$ constant, the adaptive coded modulation exhibits the same coding gain as coded modulation designed for an AWGN channel with minimum codeword distance $d_{\min}$.

The *modulation* segment of Fig. 2 works as follows. The channel is assumed to be slowly fading so that $\gamma(t)$ is relatively constant over many symbol periods. During a given symbol period $T(\gamma)$ the size of each coset is limited to $2^{n(\gamma)-k}$, where $n(\gamma)$ and $T(\gamma)$ are functions of the channel SNR γ. A signal point in the selected coset is chosen using $n(\gamma) - k$ uncoded data bits. The selected point in the selected coset is one of $M(\gamma) = 2^{n(\gamma)+r}$ points in the transmit signal constellation [e.g., MQAM, M-ary phase-shift keying (MPSK)]. By using appropriate functions for $M(\gamma)$, $S(\gamma)$, and $T(\gamma)$, we can maintain a fixed distance between points in the received signal constellation $M(\gamma)$ corresponding to the desired minimum distance $d_{\min}$. The variation of $M(\gamma)$ relative to γ causes the information rate to vary, so the uncoded bits used for signal point selection must be buffered until needed. Since r redundant bits are used for the channel coding, $\log_2 M(\gamma) - r$ bits are sent over the symbol period $T(\gamma)$ for a received SNR of γ. The average rate of the adaptive scheme is thus given by

$$R = \int_{\gamma_0}^{\infty} \frac{1}{T(\gamma)} (\log_2 M(\gamma) - r) p(\gamma)\, d\gamma \tag{3}$$

where $\gamma_0 \geq 0$ is a cutoff fade depth below which transmission is suspended $(M(\gamma) = 0)$. This cutoff value is a parameter of the adaptive modulation scheme. Since γ is known to both the transmitter and receiver, the modulation, encoding, and decoding processes are suspended while $\gamma < \gamma_0$.

At the receiver the adaptive modulation is first demodulated, which yields a sequence of received constellation points. Then the points within each coset that are closest to these received points are determined. From these points the maximum-likelihood coset sequence is calculated and the uncoded bits from the *channel coding* segment are determined from this sequence in the same manner as for nonadaptive coded modulation in AWGN. The uncoded bits from the *modulation* segment are then determined by finding the points in the maximum-likelihood coset sequence which are closest to the received constellation points and applying standard demodulation to these points.

The adaptive modulation described above consists of any mapping from γ to a constellation size $M(\gamma)$, power $S(\gamma)$, and symbol time $T(\gamma)$ for which $d_{\min}(t)$ remains constant. Proposed techniques for adaptive modulation maintain this constant distance through adaptive variation of the transmitted power level [9], symbol time [10], constellation size [4], [5], or any combination of these parameters [2], [3], [6]. The *modulation* segment of Fig. 2 can use any of these adaptive modulation methods. In the next section we use the adaptive modulation proposed in [2] to obtain trellis-coded adaptive MQAM. This modulation yields two advantages over other adaptive modulation techniques. First, the modulation maintains a constant symbol period $T(\gamma) = T$, since varying the symbol period is difficult to implement in practice. In addition, this modulation maximizes the spectral efficiency of the fading channel, so it achieves higher average data rates than any other uncoded adaptive modulation method.

III. Trellis-Coded Adaptive MQAM

We now apply the general method of adaptive coded modulation described in Section II to the specific modulation proposed in [2]. We use trellis codes of varying complexity for the *channel coding*. The trellis structure is designed by specifying the constellation subsets corresponding to each state transition, and we use Ungerboeck's heuristics [12] to obtain the trellis structure and signal point assignment for simple four-state and eight-state codes. These designs result in coding gains of 3 and 3.6 dB, respectively. Higher gains are obtained using more complex codes, where the trellis structure and signal point assignment are designed using more sophisticated techniques [8].

The four-state code is designed as follows. We first partition the MQAM constellation twice to obtain four subsets A_0, A_1, B_0, B_1 with twice the minimum distance between points as in the original constellation. This partition is illustrated in Fig. 3. We see from this figure that if either the transmit power or the channel gain increases, then we can increase the number of points in each of the four subsets A_0, A_1, B_0, B_1 without changing the distance d_0 between subset points in the received signal constellation. Therefore, an increase in transmit power and/or channel gain allows us to increase the size of each subset or, equivalently, to transmit a larger signal constellation, while maintaining a constant d_0. Thus, we transmit more uncoded bits without affecting the minimum distance of the code or its corresponding coding gain.[1] Since the adaptive MQAM of [2] is designed based on this premise, we can superimpose the trellis code design onto this modulation in a straightforward manner.

We assume that the modulation uses ideal Nyquist data pulses with a fixed symbol period $T = 1/B$. We also restrict $M(\gamma)$ to square MQAM constellations of size $M_0 = 0$ and $M_j = 2^{2(j-1)}, j = 2, \cdots, J$. Thus, at each symbol time a constellation from the set $\{M_j : j = 0, 2, \cdots, J\}$ is used—the choice of constellation depends on the fade level γ over that symbol time. Choosing the M_0 constellation corresponds to no data transmission. Since the constellation set is finite, there will be a range of γ values over which a particular constellation M_j is used. Within that range the power must also be adapted to maintain the desired d_0. Thus, for each constellation M_j the power adaptation $S_j(\gamma)$ associated with that constellation is a continuous function of γ.

The mapping for $M(\gamma)$ which maximizes spectral efficiency while maintaining the desired d_0 for the constellation restriction $\{M_j\}$ is derived in [2]. This mapping is given as follows.

[1] Changing the constellation size may impact the number of nearest neighbor constellation points, which in turn can affect the number of nearest neighbor codewords or, equivalently, the code's error coefficient [8].

Define

$$M(\gamma) = \frac{\gamma}{\gamma_K^*} \tag{4}$$

where $\gamma_K^* \geq 0$ is a parameter which is optimized relative to the fade distribution to maximize spectral efficiency, as shown in (6) and (7) below. For $\gamma < \gamma_K^* M_2$ the channel is not used. The constellation size M_j used for a given $\gamma \geq \gamma_K^* M_2$ is the largest M_j for which $M_j \leq M(\gamma)$. The range of γ values for which $M(\gamma) = M_j$ is thus $M_j \leq \gamma/\gamma_K^* < M_{j+1}$, with $M_{J+1} \triangleq \infty$. We call this range of fading values the fading region $\mathcal{R}_j$ associated with constellation M_j.

Once the regions and associated constellations are fixed, we must find the power adaptation policy $S_j(\gamma)$ which maintains the desired d_0 for the constellation M_j as γ varies from $\gamma_K^* M_j$ to $\gamma_K^* M_{j+1}$. Let $P_j(d_0)$ denote the required transmit power to achieve a minimum distance d_0 between the constellation points of M_j in the absence of fading $(g(t) = 1)$. Since in fading the transmitted signal is attenuated by $\sqrt{g(t)} = \sqrt{\gamma(t)/\bar{\gamma}}$, the power adaptation policy which maintains the desired d_0 inverts this attenuation: $S_j(\gamma) = P_j(d_0)\bar{\gamma}/\gamma$.

In general the desired value of d_0 is determined from the desired BER of the system. Therefore, instead of finding the power adaptation policy which maintains a given d_0, it is more direct to find the policy which achieves the desired $\mathrm{BER} = \mathrm{BER}_0$. This policy was derived in [2] based on a tight approximation for the BER of square MQAM constellations. If we use this policy with the superimposed trellis code, then, since the coding introduces an effective power gain of G_e, we can reduce the transmit power $S(\gamma)$ by this gain and still maintain the target BER_0. Thus for trellis-coded adaptive MQAM we use the power adaptation policy [2, eq. (29)] with power reduced by G_e

$$\frac{S_j(\gamma)}{\bar{S}} = \begin{cases} (M_j - 1)\frac{1}{\gamma K}, & M_j \leq \frac{\gamma}{\gamma_K} \leq M_{j+1} \\ 0, & M_j = 0 \end{cases} \tag{5}$$

where $K = -1.5G_e/\ln(5\mathrm{BER}_0)$. Note that G_e equals the effective coding gain of the trellis code, which is based on the code's fundamental coding gain G_f and its error coefficient n_0 [8].

The average spectral efficiency of our adaptive policy equals R/B, the average information rate (R b/s) normalized by the signal bandwidth B. For each γ, one redundant bit per symbol is used for the channel coding, so the number of information bits per symbol is $\log_2 M(\gamma) - 1$. Thus the information rate for a single γ is $R_\gamma = [\log_2 M(\gamma) - 1]/T$ b/s, and the corresponding spectral efficiency is $R_\gamma/B = \log_2 M(\gamma) - 1$, since we use Nyquist pulses ($B = 1/T$). We obtain the spectral efficiency of our adaptive policy by averaging the spectral efficiency for each γ weighted by its probability

$$\frac{R}{B} = \sum_{j=2}^{J} (\log_2 M_j - 1) p(M_j \leq \gamma/\gamma_K < M_{j+1}) \tag{6}$$

where γ_K is picked to maximize (6), subject to the average power constraint

$$\sum_{j=2}^{J} \int_{\gamma_K M_j}^{\gamma_K M_{j+1}} S_j(\gamma) p(\gamma)\, d\gamma = \bar{S}. \tag{7}$$

In general the optimal γ_K^*, which depends on the fading distribution $p(\gamma)$, cannot be found in closed form and must be determined using numerical search techniques. Analytical and simulation results for (6) as a function of $\bar{\gamma}$ will be given in the next section.

More complex trellis codes are superimposed onto the adaptive modulation of [2] in a similar way. Specifically, we first find a trellis code designed for an AWGN channel with an effective coding gain G_e; many code designs and their corresponding coding gains can be found in [8]. The *channel coding* segment of Fig. 2 then uses same trellis structure and signal point mapping as the AWGN code. The adaptive modulation uses the mapping $M(\gamma)$ described above where the constellation set of $M(\gamma)$ is restricted to $\{M_j : j = 0, j_{\min}, \cdots, J\}$, with $M_0 = 0$ (no transmission) and $M_{j_{\min}}$ equal to the smallest signal constellation which contains at least one point from each coset. For example, in an eight-state code we require a minimum square constellation size of $M_j = 16$ to represent all eight cosets. The power adaptation policy $S_j(\gamma)$ is given by (5) and is parameterized by the effective coding gain G_e. The spectral efficiency for these more complex codes is given by (6) maximized subject to (7), with the summations in both of these equations starting at $j_{\min}$.

The BER of uncoded MQAM in AWGN with square signal constellations and Gray encoding is well approximated at large SNR's by [18, eq. (4.2.144)].

$$\mathrm{BER} = \alpha_M \mathrm{erfc}\left(\sqrt{\beta_M \gamma}\right) \tag{8}$$

where γ is the SNR (E_s/N_0)

$$\alpha_M = \frac{2}{\log_2 M}\left(1 - \frac{1}{\sqrt{M}}\right) \tag{9}$$

$$\beta_M = \frac{3}{2(M-1)}. \tag{10}$$

Based on this formula the BER of uncoded adaptive MQAM is derived in [2, eqs. (39)–(41)]. For trellis-coded adaptive MQAM the BER depends on all possible error events and their associated probability. This computation is prohibitively complex, so we consider only the error probability resulting from nearest neighbor codewords, which is an accurate approximation at large SNR's. The average BER is then just the weighted average of BER's associated with each signal constellation. If we assume that the minimum distance of the code is equal to the minimum distance between parallel transitions, then this average is given by

$$\mathrm{BER} = \sum_{j=j_{\min}}^{J} n_0(M_j)\mathrm{erfc}\left(\sqrt{\beta_{M_j} G_f \frac{(M_j - 1)}{K}}\right) \cdot \int_{\gamma_K^* M_j}^{\gamma_K^* M_{j+1}} p(\gamma)\, d\gamma \tag{11}$$

where we use the instantaneous SNR (1) in (8) and $n_0(M_j)$ equals the number of nearest neighbor codewords for the trellis-coded M_j-QAM code design. If the BER is dominated by parallel transitions, then $n_0(M_j)$ can be replaced by α_{M_j} in (11). In this case we see after some substitution that (11) is the same as the BER without coding [2, eq. (41)], except for an extra term G_f/G_e multiplying β_{M_j}. Since the fundamental coding gain G_f is approximately equal to the effective coding gain G_e when parallel transitions dominate the error probability, we expect the BER of uncoded and coded adaptive MQAM to be approximately equal in this case.

IV. Performance Results

In this section we obtain numerical results for BER and spectral efficiency of the four-state trellis-coded adaptive MQAM described in the previous section. These results are obtained using both the formulas derived in Section III and by simulation. The simulations were developed using COSSAP under the assumption of perfect channel side information (CSI) of the channel amplitude and phase at the transmitter and receiver. Rayleigh fading and lognormal shadowing were simulated using the modules in the COSSAP library [19], with velocity entered as a parameter. The velocity was chosen so that, over a symbol period T, $\gamma(t)$ is approximately constant. The MQAM signal constellations were restricted to $M_j \in \{0, 4, 16, 64\}$ in Rayleigh fading and to $M_j \in \{0, 4, 16, 64, 256\}$ in lognormal shadowing.

In the four-state code the BER is dominated by parallel transitions. Therefore, we compute analytical results for BER using (11) with $n_0(M_j) = \alpha_{M_j}$. These analytical results match closely with the simulation results and, as expected, these results are similar to those in the uncoded case, shown in [2, Figs. 6 and 7]. Specifically, the simulated BER is slightly below the analytical BER of (11), and both BER values are below the target BER_0 on which the power adaptation (5) is based.

In Fig. 4 we show the simulation results for the spectral efficiency of the four-state trellis-coded adaptive MQAM at a target BER of 10^{-3} in Rayleigh fading. For comparison, the analytical results for spectral efficiency with and without coding, given by (6) and [2, eq. (31)], respectively, are also shown. We see that the simulation results do not exhibit the 3-dB gain predicted by (6)—we obtain an average coding gain of just 1.5–2 dB relative to the uncoded scheme. This results from the fact that the 3-dB gain of a four-state trellis code is achieved asymptotically at large SNR's, where the probability of error is dominated by the probability of mistaking a given codeword for one of its nearest neighbors. The SNR which achieves a BER of 10^{-3} is moderate, so the effect of codewords other than the nearest neighbor codewords decreases the effective coding gain.

This effect is illustrated more clearly in Fig. 5, where we show the coding gain of a four-state trellis code in AWGN for each of our signal constellations. The coding gain for each signal constellation at a BER of 10^{-3} lies between 1–3 dB, with the smallest gain exhibited by the largest signal constellation, where the total number of codewords contributing to the error probability is largest. From this figure we see that the asymptotic 3-dB coding gain for all of the constellations is achieved at a lower BER than 10^{-3}, and

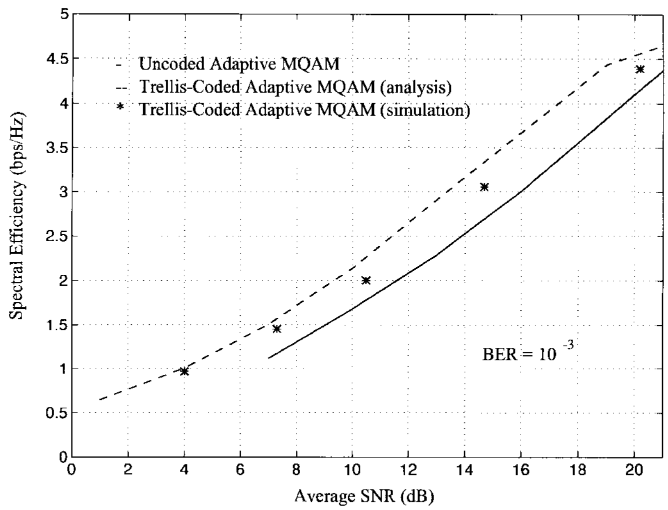

Fig. 4. Coding gain of four-state code ($\mathrm{BER} = 10^{-3}$).

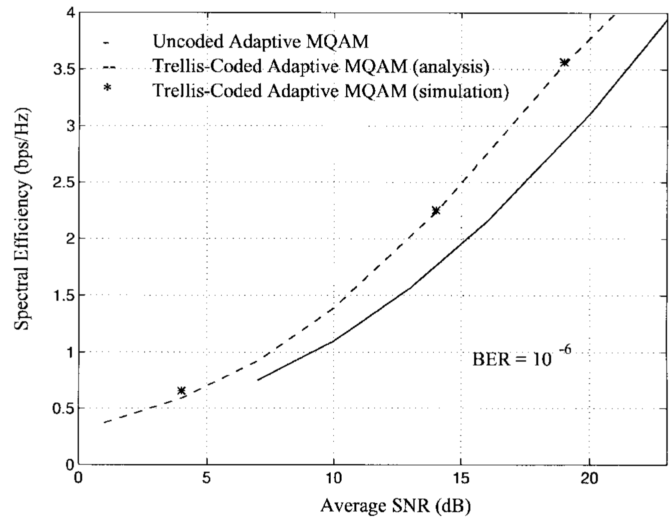

Fig. 6. Coding gain of four-state code ($\mathrm{BER} = 10^{-6}$).

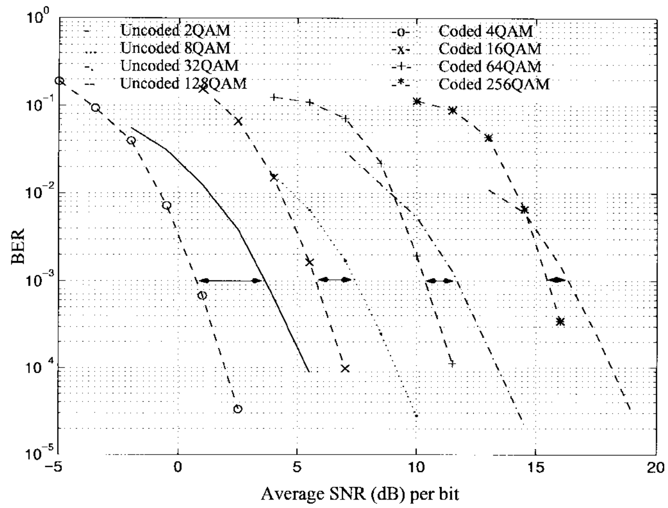

Fig. 5. Coding gain of four-state code in AWGN.

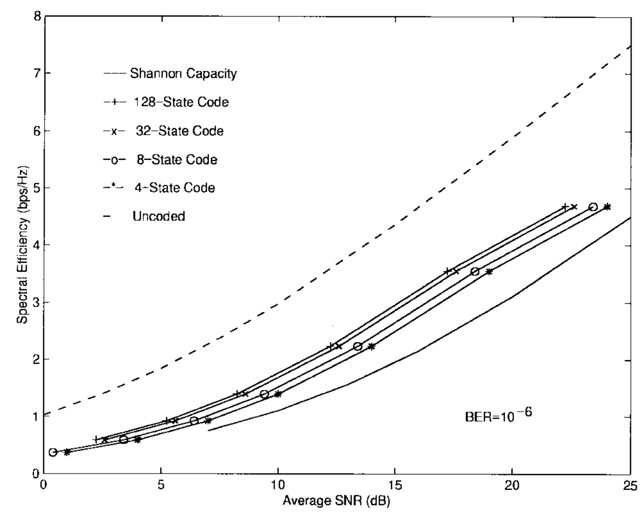

Fig. 7. Coding gain in Rayleigh fading.

therefore we expect that by sufficiently lowering the BER target of our adaptive coded modulation we should achieve the asymptotic coding gain. This expectation is confirmed in Fig. 6, where we achieve the asymptotic coding gain of 3 dB for a 10^{-6} BER in Rayleigh fading. Of course, the spectral efficiency at this lower BER is smaller than in Fig. 4.

For any target BER, higher coding gains can be achieved by increasing the number of trellis states. The effective coding gain G_e affects the power adaptation (5) which in turn impacts spectral efficiency (6). Fig. 7 shows the spectral efficiency (6) and corresponding coding gain for trellis-coded adaptive MQAM with higher complexity codes in Rayleigh fading. The Shannon capacity of the fading channel with transmitter adaptation [1, eq. (7)] and the spectral efficiency of uncoded adaptive MQAM [2, eq. (31)] are also shown for reference. We see that a 128-state code with $G_e = 4.74$ dB comes within approximately 6 dB of the Shannon limit at low SNR's. At high SNR's the restriction to square constellations of size 64 or less causes some additional loss relative to Shannon capacity. Although increasing the number of trellis states above 128 will yield even higher coding gains, particularly if the number of constellations is increased as well, the decoding complexity of the code will increase exponentially with the number of trellis states. In fact, for most applications the complexity constraints on current hardware designs typically limit trellis codes to eight or fewer states.

Spectral efficiency for trellis-coded adaptive MQAM in lognormal shadowing was also simulated and compared with the uncoded adaptive modulation results in [2]. At low BER's both simulation and analysis yield a coding gain of 3 dB for the four-state code and a 3.6 dB gain for the eight-state code. We plot the spectral efficiency for lognormal shadowing in Fig. 8. From this figure we see that, as in the Rayleigh case, not all of the coding gain is realized at high BER's due to the restriction on constellation size.

V. Comparison with Fade-Resistant Trellis Code Designs

In this section we compare the spectral efficiency of trellis-coded adaptive MQAM with that of fixed-rate trellis codes

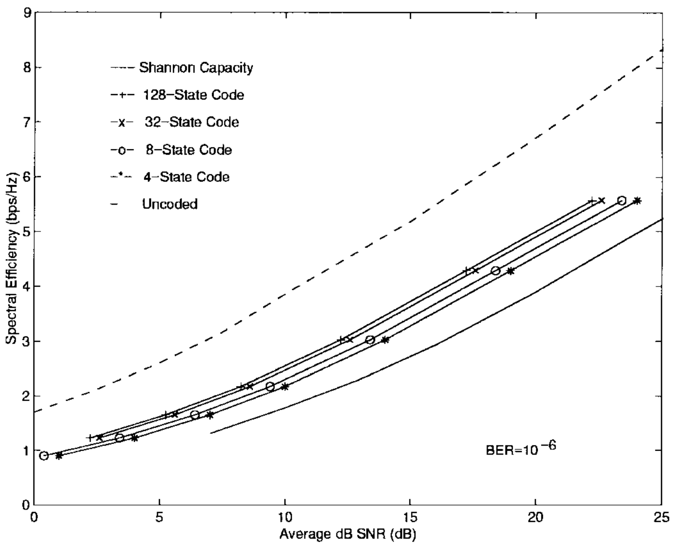

Fig. 8. Coding gain in lognormal shadowing.

TABLE I
COMPARISON OF ADAPTIVE AND NONADAPTIVE TECHNIQUES

Spectral Efficiency (bps/Hz)	BER	Trellis States	Average SNR (dB)	
			Adaptive	Nonadaptive
2	10^{-3}	4	10.5	18.5
		128	9.0	13.7
	10^{-6}	4	13	36.0
		128	11.3	21.0
3	10^{-3}	8	14.2	20.8
		128	13.1	16.8
	10^{-6}	8	16.4	36.5
		128	15.3	24.8

designed for Rayleigh-fading channels. The fixed-rate codes we use for our comparison are the 8 phase-shift keying (8PSK) and 16 quadrature amplitude modulation (16QAM) Du–Vucetic codes with optimized effective code length and product distance [16], [17]. These codes have a higher efficiency than the original Ungerboeck codes [12], as well as the Wilson–Leung codes [14] and the Schlegel–Costello codes [15].

Table I shows the average SNR (E_s/N_0) needed for the given spectral efficiency and BER, for both the adaptive and nonadaptive techniques. The required SNR for the nonadaptive techniques is obtained from the simulation results for 8PSK (2 b/s/Hz) and 16QAM (3 b/s/Hz) reported in [16] and [17], using the approximation $\mathrm{SNR} = \rho E_b/N_0$, where ρ is the number of information bits per signal. These simulations assume perfect channel phase and amplitude information (CSI) at the receiver, as well as infinite interleaving. The SNR values for the nonadaptive techniques at a 10^{-6} BER are obtained by extrapolating the simulation results in [16] and [17]. We see that the adaptive modulation exhibits a minimum 5 dB of power savings, with more than 20 dB of power savings for low-complexity codes (four to eight states) with low required BER's (10^{-6}).

VI. CONCLUSION

Coset codes are a natural choice to use with adaptive modulation since the code design and modulation design are separable. We present the general principles of combining coset codes with a general class of adaptive modulation techniques designed for flat-fading channels. We then apply these techniques to a spectrally efficient adaptive MQAM technique to obtain trellis-coded adaptive MQAM. At low BER's this adaptive coded modulation exhibits a 3-dB gain using a four-state code, a 3.6-dB gain using an eight-state code, and comes within 6 dB of the Shannon capacity limit of the fading channel with adaptive transmission using a 128-state code. These coding gains, which are obtained both in Rayleigh fading and in lognormal shadowing, are in addition to the gain of adaptive MQAM relative to nonadaptive modulation. We also compare the performance of the trellis-coded adaptive MQAM with that of nonadaptive trellis codes designed for Rayleigh-fading channels and show that for moderate complexity codes at low BER's, the adaptive technique has up to 20 dB of power savings.

The adaptive coded modulation we describe has many potential enhancements. Constellation shaping may yield 1 dB or more of shape gain. In addition to providing shape gain, cross constellations and other constellation shapes may be more robust to channel estimation errors. Rotationally invariant code designs would eliminate the need for a coherent phase reference [20]. However, the use of a time-varying constellation will make constellation shaping and rotational invariance more difficult to achieve than in the case of fixed constellations. Finally, coded modulation using turbo codes has achieved rates very close to the Shannon capacity limit of an AWGN channel [21]. Using the multilevel turbo code construction proposed in [22], we can design adaptive coded modulation with turbo codes using the same techniques we have described for trellis codes. We believe that this combination will come quite close to the Shannon capacity limit of fading channels with transmitter adaptation.

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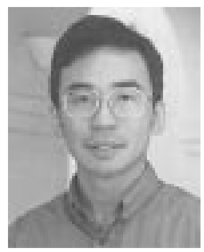

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