

Traffic Grooming, Routing, and Wavelength Assignment in Optical WDM Mesh Networks

J.Q. Hu
Boston University
15 St. Mary's Street
Brookline, MA 02446
Email: hqiang@bu.edu

Brett Leida
Sycamore Networks
220 Mill Road
Chelmsford, MA 01824
Email: brett.leida@sycamorenet.com

Abstract—In this paper, we consider the traffic grooming, routing, and wavelength assignment (GRWA) problem for optical mesh networks. In most previous studies on optical mesh networks, traffic demands are usually assumed to be wavelength demands, in which case no traffic grooming is needed. In practice, optical networks are typically required to carry a large number of lower rate (sub-wavelength) traffic demands. Hence, the issue of traffic grooming becomes very important since it can significantly impact the overall network cost. In our study, we consider traffic grooming in combination with traffic routing and wavelength assignment. Our objective is to minimize the total number of transponders required in the network. We first formulate the GRWA problem as an integer linear programming (ILP) problem. Unfortunately, for large networks it is computationally infeasible to solve the ILP problem. Therefore, we propose a decomposition method that divides the GRWA problem into two smaller problems: the traffic grooming and routing problem and the wavelength assignment problem, which can then be solved much more efficiently. In general, the decomposition method only produces an approximate solution for the GRWA problem. However, we also provide some sufficient condition under which the decomposition method gives an optimal solution. Finally, some numerical results are provided to demonstrate the efficiency of our method.

Keywords: System design, Mathematical programming/optimization.

I. INTRODUCTION

Wavelength division multiplexing (WDM) is now being widely used for expanding capacity in optical networks. In a WDM network, each fiber link can carry high-rate traffic at many different wavelengths, thus multiple channels can be created within a single fiber. There are two basic architectures used in WDM networks: ring and mesh. The majority of optical networks in operation today have been built based on the ring architecture. However, carriers have increasingly considered the mesh architecture as an alternative for building their next generation networks. Various studies have shown that mesh networks have a compelling cost advantage over ring networks. Mesh networks are more resilient to various network failures and also more flexible in accommodating changes in traffic demands (e.g., see [7], [12], [25] and references therein). In order to capitalize on these advantages, effective design methodologies are required.

In the design of an optical mesh network, traffic grooming, routing, and wavelength assignment are some of the most

important issues that need to be considered. The problem of traffic grooming and routing for mesh networks is to determine how to efficiently route traffic demands and at the same time to combine lower-rate (sub-wavelength) traffic demands onto a single wavelength. On the other hand, the problem of wavelength assignment is to determine how to assign specific wavelengths to lightpaths, usually under the wavelength continuity constraint. In previous studies on the routing and wavelength assignment (RWA) problem (e.g., see [17, Chapter 8] and references therein), the issue of grooming has largely been ignored, i.e., it has been assumed that each traffic demand takes up an entire wavelength. In practice, this is hardly the case, and networks are typically required to carry a large number of lower rate (sub-wavelength) traffic demands.

The traffic grooming problem has been considered by several researchers for ring networks (e.g. see, [4], [6], [8], [9], [10], [13], [14], [18], [19], [20], [22], [24]), and is only considered recently in [23] for mesh networks. The objective considered in [23] is either to maximize the network throughput or to minimize the connection-blocking probability, which are operational network-design problems. Alternatively, a strategic network-design problem is to minimize the total network cost.

Typically, the cost of a nation-wide optical network is dominated by optical transponders and optical amplifiers. If one assumes that the fiber routes are fixed, then the amplifier cost is constant, in which case one should concentrate on minimizing the number of transponders in the network. Grooming costs should also be considered. However, under realistic assumptions of either a low-cost interconnect between grooming equipment and transport equipment, or integrated (long-reach) transponders on the grooming equipment, then the relative cost of the grooming switch fabric is negligible, and minimizing the number of transponders is still the correct objective. In addition, the advent of Ultra Long-Haul transmission permits optical pass-through at junction nodes, hence, requiring transponders only at the end of the lightpaths.

Though the number of transponders has been used as an objective function in many studies on ring networks, it has not been considered at all for mesh networks. The objective functions that have been considered for mesh networks so far include: the blocking probability, the total number of

wavelengths required, and the total route distance.

In this paper, we consider the problem of traffic grooming, routing, and wavelength assignment (GRWA) with the objective of minimizing the number of transponders in the network. We first formulate the GRWA problem as an integer linear programming (ILP) problem. Unfortunately, the resulting ILP problem is usually very hard to solve computationally, in particular for large networks. To overcome this difficulty, we then propose a decomposition method that divides the GRWA problem into two smaller problems: the traffic grooming and routing (GR) problem and the wavelength assignment (WA) problem. In the GR problem, we only consider how to groom and route traffic demands onto lightpaths (with the same objective of minimizing the number of transponders) and ignore the issue of how to assign specific wavelengths to lightpaths. Similar to the GRWA problem, we can formulate the GR problem as an ILP problem. The size of the GR ILP problem is much smaller than its corresponding GRWA ILP problem. Furthermore, we can significantly improve the computational efficiency for the GR ILP problem by relaxing some of its integer constraints, which usually leads to quite good approximate solutions for the GR problem. Once we solve the GR problem, we can then consider the WA problem, in which our goal is to derive a feasible wavelength assignment solution.

We note that the WA problem has been studied by several researchers before (e.g., see [5], [1], [15], [16], [17], [21], [2] and references therein). However, the objective in all these studies has been to minimize the number of wavelengths required in a network, in some cases by using wavelength converters. In general, the use of additional wavelengths in a network only marginally increases the overall network cost as long as the total number of wavelengths used in the network does not exceed a given threshold (the wavelength capacity of a WDM system). This is mainly because the amplification cost is independent of the number of wavelengths. In recent years, the wavelength capacity for optical networks has increased dramatically. For example, with most advanced techniques, a single WDM system on a pair of fibers can carry up to 160 10G-wavelengths or 80 40G-wavelengths. Of course, once the wavelength capacity is exceeded, then a second parallel system (with another set of optical amplifiers) needs to be built, which would then substantially increase the network cost. Therefore, assuming a single WDM system on all fiber routes fixes the amplifier cost, then one should focus on minimizing the number of transponders in the network, which is already taken into consideration in the GR problem. In this setting, the objective in our WA problem is to find a feasible wavelength assignment solution under the wavelength capacity constraint.

It is clear that in general the decomposition method would not yield the optimal solution for the GRWA problem. However, we will provide a sufficient condition under which we show that the decomposition method does produce an optimal solution for the GRWA problem. This is achieved by developing a simple algorithm that, under this sufficient condition, finds an optimal wavelength assignment.

The rest of this paper is organized as follows. In Section 2, we present the GRWA problem and demonstrate how it can be formulated as an ILP problem. In section 3, we first present our decomposition method. We then provide an ILP formulation for the GR problem and develop an algorithm for solving the WA problem. We also discuss under what condition the decomposition method produces an optimal solution for the GRWA problem. Some numerical results are provided in Section 4. Finally, a conclusion is given in Section 5.

II. THE GRWA PROBLEM

An optical network architecturally has two layers: a physical layer and an optical layer. The physical layer consists of fiber spans and nodes and the optical layer consists of lightpaths (optical links) and a subset of nodes contained in the physical layer. A lightpath in the optical layer is a path connecting a pair of nodes via a set of fiber spans in the physical layer. Throughout this paper, we assume that lightpaths and their routes in the physical layer are given. In practice, the selection of lightpaths is another important design issue that needs to be addressed, which is beyond the scope of this paper.

We use graph $G_f = (V_f, E)$ to represent the physical layer, where E is the set of edges representing fiber spans and V_f is the set of nodes representing locations which are connected via fiber spans. We use graph $G_o = (V_o, L)$ to represent the optical layer, where L is the set of edges representing lightpaths and $V_o \subset V_f$ is a subset of locations that are connected via lightpaths. Each edge in L corresponds to a path in G_f . In this paper, we treat each lightpath as a logical connection between a pair of nodes (not just a single wavelength), therefore, one lightpath can contain multiple wavelengths. For ease of exposition, we first assume that G_o is a directed graph (i.e., the lightpaths are unidirectional). The extension to the undirected graph case is quite straightforward and will be discussed later in this section (basically, we can simply replace every undirected edge with two directed edges).

The GRWA problem concerned in our study can be described as follows. Assuming that a set of traffic demands are given (some of them are of low rate, i.e., sub-wavelength), our goal is to find an optimal way to route and groom these demands in the optical layer, G_o , and also to assign a set of specific wavelengths to each lightpath so that the total number of transponders required is minimized. There are two key constraints we need to take into consideration in this problem: 1) the wavelength capacity constraint for each fiber span, and 2) the wavelength continuity constraint for every lightpath, i.e., the same wavelength(s) needs to be assigned to a lightpath over the fiber spans it traverses. In this problem setting, the number of transponders required for each lightpath is equal to twice the number of wavelengths assigned to it (one transponder for each end of each wavelength on a lightpath). Therefore, by grooming several low rate demands onto a single wavelength, we can potentially reduce the total number of wavelengths required by the lightpaths, thus the number of transponders.

The GRWA problem can be formulated as an integer linear programming (ILP) problem. First, we need to introduce some

necessary notation:

W : the set of wavelengths available at each fiber;

D : the set of traffic demands;

s_d : the size of demand $d \in D$;

g : the capacity of a single wavelength;

A : = $[a_{v,l}]_{|V_o| \times |L|}$, the node-edge incidence matrix of graph G_o , where $a_{v,l} = 1$ if lightpath l originates from node v , -1 if lightpath l terminates at node v , and 0 otherwise;

B : = $[b_{e,l}]_{|E| \times |L|}$, the fiber-lightpath incidence matrix, where $b_{e,l} = 1$ if fiber span e is on lightpath l , and 0 otherwise;

u_d : = $[u_{v,d}]_{v \in V_o}$, the source-destination column vector for $d \in D$, where $u_{v,d} = 1$ if v is the starting node of d , -1 if v is the end node of d , and 0 otherwise;

x_d : = $[x_{l,d}]_{l \in L}$, the column vector containing lightpath routing variables for $d \in D$, where $x_{l,d} = 1$ if demand d traverses lightpath l , and 0 otherwise;

y_w : = $[y_{l,w}]_{l \in L}$, the column vector containing wavelength assignment variables for $w \in W$, where $y_{l,w} = 1$ if wavelength w is assigned to lightpath l , and 0 otherwise (note that in our setting each lightpath l is treated as a logical connection between a pair of nodes, hence it can be assigned with multiple wavelengths, i.e., it is possible that $\sum_{w \in W} y_{l,w} \geq 1$);

$\mathbf{1}$: = $[1, 1, \dots, 1]$, the unit column vector.

Then the GRWA problem can be formulated as the following ILP problem (which we shall refer to as the GRWA ILP problem):

$$\begin{aligned} \min \quad & \sum_{w \in W, l \in L} y_{l,w} \\ \text{s.t.} \quad & Ax_d = u_d \quad d \in D \quad (1) \\ & By_w \leq \mathbf{1} \quad w \in W \quad (2) \\ & \sum_{d \in D} s_d x_{l,d} \leq g \sum_{w \in W} y_{l,w} \quad l \in L \quad (3) \\ & x \text{ and } y \text{ are binary variables.} \end{aligned}$$

where the objective function $\sum_{w \in W, l \in L} y_{l,w}$ is the total number of wavelengths assigned to all lightpaths, which is equivalent to minimizing the total number of transponders needed. The three constraints are

- (1) is the flow balance equation, which guarantees that the lightpaths selected based on x_d constitute a path from the starting node of d to the end node of d .
- (2) implies a single wavelength along each fiber span can be assigned to no more than one lightpath.
- (3) is the capacity constraint for lightpath l , since $\sum_{d \in D} s_d x_{l,d}$ is the total amount of demands carried by lightpath l , and $g \sum_{w \in W} y_{l,w}$ is the total capacity of lightpath l .

We refer the type of the network considered above as the basic model. There are several variations of the basic model, which include

- 1) Networks with both protected and unprotected demands;
- 2) Networks in which lightpaths are undirected;
- 3) Networks with non-homogeneous fibers where different types of fiber may have different wavelength capacities;

- 4) Networks in which demand exceeds a single WDM system per fiber pair.

III. A DECOMPOSITION METHOD

In the previous section, we formulated the GRWA problem as an ILP problem, however, it may not be computationally feasible to solve the ILP problem, particularly for large networks (e.g., see numerical results in Section 4). Therefore, it is necessary to find more efficient ways to solve the GRWA problem. In this section, we propose a decomposition method that divides the GRWA problem into two smaller problems: the traffic grooming and routing (GR) problem and the wavelength assignment (WA) problem. In the GR problem, we only consider how to groom and route demands over lightpaths and ignore the issue of how to assign specific wavelengths to lightpaths. Based on the grooming and routing, we can then derive wavelength capacity requirements for all lightpaths. Similar to the GRWA problem, we formulate the GR problem as an ILP problem. The size of the GR ILP problem is much smaller than its corresponding GRWA ILP problem. Furthermore, we can significantly improve the computational efficiency for the GR ILP problem by relaxing some of its integer constraints, which usually leads to approximate solutions for the GR problem. Once we solve the GR problem, we can then consider the WA problem, in which our goal is to derive a feasible wavelength assignment solution that assigns specific wavelengths to lightpaths based on their capacity requirements derived in the GR problem.

It is obvious that in general the decomposition method would not yield the optimal solution for the GRWA problem. However, we will provide a sufficient condition under which we show that the decomposition method does produce an optimal solution for the GRWA problem. We also develop a simple algorithm that finds a wavelength assignment solution under this sufficient condition.

A. The GR Problem

Let $t = [t_l]_{l \in L}$, a column vector containing lightpath capacity decision variables, where $t_l = \sum_{w \in W} y_{l,w}$ is the number of wavelengths needed for lightpath $l \in L$. Then, the GR problem can be formulated as:

$$\begin{aligned} \min \quad & \sum_{l \in L} t_l \\ \text{s.t.} \quad & Ax_d = u_d \quad d \in D \quad (4) \\ & Bt \leq |W|\mathbf{1} \quad (5) \\ & \sum_{d \in D} s_d x_{l,d} \leq g t_l \quad l \in L \quad (6) \\ & x \text{ binary variable and } t \text{ integer variable.} \end{aligned}$$

We refer the above ILP problem as the GR ILP problem. We now present the following result:

Proposition 1: If x and y are feasible solutions for the GRWA ILP problem, then x and t are feasible solutions for the GR ILP problem, where $t = \sum_{w \in W} y_w$.

Proof: We first note that by summing over $w \in W$ in (2) it leads to (5). Secondly, (3) is the same as (6). Hence, the result follows. ■

Based on Proposition 1, we have

Proposition 2: If x^* and t^* are the optimal solutions of the GR ILP problem, and there exists a binary y^* such that $\sum_{w \in W} y_w^* = t$ and $By_w^* \leq \mathbf{1}$ for $w \in W$, then x^* and y^* are the optimal solutions of the GRWA ILP problem.

Proof: Suppose x and y are feasible solutions for the GRWA ILP problem, then based on Proposition 1, x and $t = \sum_{w \in W} y_w$ are feasible solutions for the GR ILP problem. Since x^* and t^* are the optimal solutions of the GR ILP problem, we have $\sum_{w \in W, l \in L} y_{l,w}^* = \sum_{l \in L} t_l^* \leq \sum_{l \in L} t_l = \sum_{w \in W, l \in L} y_{l,w}$. Therefore, the conclusion follows. ■

Obviously, the GR ILP problem is much easier to solve than the GRWA ILP problem since it has fewer integer variables and fewer constraints (e.g., see numerical examples in Section 4). More importantly, we can now relax the integer constraint on t in the GR ILP problem and solve a relaxed mixed ILP problem and then round up the values of t to obtain a solution for the GR problem. This would dramatically improve the computational efficiency. On the other hand, the relaxation approach is much less effective for the GRWA ILP problem since all its decision variables are binary. In general, if most lightpaths have relatively high wavelength counts (i.e., the values of their corresponding components in t are large), then the relaxed GR ILP problem often produces very good solutions for the GR problem, as illustrated by our numerical examples in Section 4. This is simply because that if the optimal value of t_l is large, then the error of rounding up is relatively small.

B. The WA Problem

The WA problem of our interest is to find a binary solution y such that

$$\sum_{w \in W} y_w = t \quad \text{and} \quad By_w \leq \mathbf{1} \quad \text{for } w \in W,$$

where t is a feasible (or optimal) solution of the GR problem. This problem can be viewed as an ILP problem (without an objective function), which is much easier to solve than the GRWA ILP and the (relaxed) GR ILP problems. For example, it can be solved for networks with a few hundred nodes and lightpaths in seconds or minutes by using commercially available LP software, e.g., CPLEX. Based on Proposition 2, we know that if x and t are optimal solutions of the GR problem and the WA problem has a feasible solution y , then x and y are optimal solutions of the GRWA problem. In case when we cannot find a feasible solution for the WA problem, we can either increase the number of wavelengths in W in the WA problem (note that we can always find a feasible solution for the WA problem if W has enough wavelengths), or we can use $W^* \subset W$ in the GR problem (specifically, replace $|W|$ with $|W^*|$ in (5)) but still use W in the WA problem. Obviously, the latter approach is preferred in which case the decomposition method provides a feasible solution

for the GRWA problem. An alternative approach is to use wavelength conversion via lightpath regeneration, which is equivalent to modifying L by breaking some lightpaths into two or more lightpaths. In addition, there are other possible remedies available to alleviate the infeasibility of the WA problem.

Though the WA problem can be solved as an ILP problem, it is also possible to solve it directly based on some heuristic algorithms (e.g., see [5]). In what follows, we consider a special type of the GRWA problem, in which the lightpaths satisfy a certain condition. Under such a condition, we show that a feasible solution for the corresponding WA problem can always be found, and we also develop an algorithm for finding a feasible solution. Without loss of generality, we assume that the capacity of every lightpath is one wavelength (i.e., $t_l = 1$ for every $l \in L$). For a lightpath whose capacity is more than one wavelength, we can treat it as several identical parallel lightpaths, each of which has capacity of one wavelength. Let p_e ($e \in E$) be the number of lightpaths that traverse fiber span e , and $p = \max_{e \in E} p_e$, which is the minimum number of wavelengths required for the network.

Define:

$$E_l = \{e \in E \mid e \text{ is on lightpath } l\}, \quad l \in L;$$

$$L_e = \{l \in L \mid l \text{ traverses fiber span } e\}, \quad e \in E;$$

We now present the following algorithm for the WA problem.

Algorithm 1 (for the WA problem)

- 1) Select an initial lightpath $l_0 \in L$ (arbitrarily), and assign a wavelength to l_0 .
- 2) Suppose $E_{l_0} = \{e_1, e_2, \dots, e_k\}$. Set $L_0 = \{l_0\}$. For $i = 1$ to k , do
 - a) Assign a wavelength to every lightpath $l \in L_{e_i} \setminus \cup_{0 \leq j < i} L_j$ such that no two lightpaths in L_{e_i} share the same wavelength (note that $L_{e_i} \setminus \cup_{0 \leq j < i} L_j$ is a subset of lightpaths in L_{e_i} to which wavelengths have not been assigned yet).
 - b) Let

$$L_i = L_{e_i} \setminus \cup_{0 \leq j < i} L_j,$$

$$E_i = \cup_{l \in L_i} E_l \setminus \{e_i\}.$$

We note that L_i is the set of lightpaths to which wavelengths are assigned in Step 2(a) and E_i is the set of fiber spans that are on at least one lightpath in L_i (excluding fiber span e_i).

- 3) For $i = 1, 2, \dots, k$, apply the procedure in Step 2 to E_i (with E_{l_0} being replaced with E_i), and continue until all the lightpaths in L are assigned (note that since all the fiber spans in E_{l_0} have been considered already in Step 2, we can simply replace E_i by $E_i \setminus E_{l_0}$).

To study some useful properties associated with Algorithm 1, we first introduce the following terminologies:

Definition 1) We say a lightpath l and a fiber span e are connected (via fiber spans $\{e_1, \dots, e_m\}$ and lightpaths $\{l_1, \dots, l_m\}$) if there exist a set of fiber spans

- $\{e_1, \dots, e_m\}$ and a set of lightpaths $\{l_1, \dots, l_m\}$ such that $e_i \in E_{l_{i-1}}$ for $i = 1, \dots, m+1$ (where $l_0 \equiv l$ and $e_{m+1} \equiv e$) and $l_i \in L_{e_i}$ for $i = 1, \dots, m$.
- 2) We say two lightpaths l_0 and l_m are connected (via fiber spans $\{e_1, \dots, e_m\}$ and lightpaths $\{l_1, \dots, l_{m-1}\}$) if there exist a set of fiber spans $\{e_1, \dots, e_m\}$ and a sequence of lightpaths $\{l_1, \dots, l_{m-1}\}$ such that $e_i \in E_{l_{i-1}}$ and $l_i \in L_{e_i}$ for $i = 1, \dots, m$.
 - 3) We say a set of lightpaths $\{l_1, \dots, l_m\}$ is a lightpath cycle if $E_{l_i} \cap E_{l_{i+1}} \neq \emptyset$ (i.e., lightpaths l_i and l_{i+1} share at least one common fiber span) for $i = 1, \dots, m$ ($l_{m+1} \equiv l_1$).
 - 4) We say a lightpath cycle $\{l_1, \dots, l_m\}$ is a complete lightpath cycle if $E_{l_1} = \dots = E_{l_m}$, otherwise it is a non-complete lightpath cycle.

To help understand what is a lightpath cycle, consider the network depicted in Figure 1. The network has four nodes (A, B, C, D), three fiber spans (A–B, B–C, C–D), and three lightpaths (A–B–C, C–B–D, D–B–A). It is clear that the three lightpaths (A–B–C, C–B–D, D–B–A) constitute a lightpath cycle, however it is a non-complete cycle.

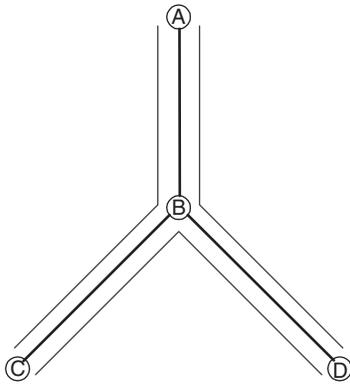


Fig. 1. A 4-Node Network

We now present the following properties associated with Algorithm 1.

Proposition 3:

- 1) Every fiber span in E_i is on at least one lightpath in L_i .
- 2) For $1 \leq j \leq i$, $e_j \notin E_i$;
- 3) $L_i \cap L_j = \emptyset$ ($i \neq j$);
- 4) If $l \in L_i$, then it does not traverse fiber spans $\{e_1, \dots, e_{i-1}\}$;
- 5) If $E_i \cap E_j \neq \emptyset$ ($i \neq j$), then there exists a lightpath cycle with one lightpath in L_i and one lightpath in L_j ;
- 6) If a lightpath in L_i is connected to another lightpath in L_j in two different ways via lightpaths in $L \setminus \cup_{0 \leq h \leq k} L_h$ and fiber spans in $E \setminus E_{l_0}$, then there exists a lightpath cycle $\{l_1, \dots, l_m\}$ such that $E_i \cap (E_{l_{i_1}} \cap E_{l_{i_1+1}}) \neq \emptyset$ and $E_j \cap (E_{l_{i_2}} \cap E_{l_{i_2+1}}) \neq \emptyset$, where $1 \leq i_1 < i_2 \leq m$.

Proof: We want to iterate the fact that assign wavelengths are assigned to lightpaths in $L_i \subset L_{e_i}$ in Step 2(a).

- 1) By definition.

- 2) By definition, $e_i \notin E_i$. For $1 \leq j < i$ and $e \in E_i$, it is clear we have assigned wavelengths to all the lightpaths in L_{e_j} before Step 2(a) while at least one lightpath in L_e has not been assigned by a wavelength before Step 2(a). Hence, $e_j \notin E_i$.
- 3) All the lightpaths in L_j are assigned by wavelengths at the end of Step 2(a) and they will not be considered again in later iterations.
- 4) By the same argument as in (2).
- 5) Suppose $e \in E_i \cap E_j$. Based on (1), e is on one lightpath in L_i , say l_i , and on another lightpath in L_j , say l_j . Furthermore, l_i and l_j traverse e_i and e_j , respectively, which are both on lightpath l_0 . Therefore, we have a lightpath cycle $\{l_i, l_0, l_j\}$.
- 6) The same argument used in (5) can be applied here as well.

In general, one needs to be careful about what wavelengths to use in Step 2(a) of Algorithm 1, otherwise it is possible that it may not produce a feasible solution for the WA problem. For example, consider the following example in which $E_{l_0} = \{e_1, e_2\}$, $L_1 = \{l_1\}$, $L_2 = \{l_2\}$, and $E_1 = E_2 = \{e\}$. If we assign the same wavelength to l_1 and l_2 , then we end up with assigning one wavelength to l_1 and l_2 on fiber span e , which is not permissible. Therefore, we have to assign u_1 and u_2 with different wavelengths.

It is clear that the number of different wavelengths needed in the WA problem is at least p . In what follows, we provide a sufficient condition under which p different wavelengths are enough to solve the WA problem.

Theorem 1: If a network does not contain any non-complete lightpath cycle, then Algorithm 1 can produce a feasible solution for the WA problem which only needs p wavelengths.

Proof: Since the network does not contain any non-complete lightpath cycle, based on (4), (5), and (6) in Proposition 3 we have (i) $E_i \cap E_j = \emptyset$, and (ii) no lightpath in L_i is connected to lightpath in L_j ($i \neq j$). Hence, when doing wavelength assignment for lightpaths in L_i in Step 2(a) we can use arbitrary wavelengths, and it guarantees that it is permissible (i.e., no two lightpaths that traverse the same fiber span would be assigned by the same wavelength). By repeating this argument, we can show that in Algorithm 1 we can use arbitrary wavelengths in Step 2(a) and obtain a feasible solution for the WA problem. Since wavelengths used in Step 2(a) can be arbitrary, the maximum number of different wavelengths needed throughout Algorithm 1 should be no more than p . This completes our proof. ■

Theorem 1 implies that if a network does not contain any non-complete lightpath cycle, we can find a solution for the WA problem which only needs p wavelengths. In [5], the problem of whether the WA problem can be solved with p wavelengths was also studied. However, we believe that the result there (Theorem 2 in [5]) is incorrect, which states that if a network is acyclic then its WA problem can be solved with p wavelengths. The network in Figure 1 is a counterexample to this result. It is a tree (hence acyclic). Clearly we have $p = 2$,

but need three wavelengths for its WA problem.

Since t in the WA problem is a feasible solution for the GR problem, i.e., $Bt \leq |W|\mathbf{1}$, we have $p \leq |W|$. This, together with Theorem 1, leads to the following result:

Theorem 2: If a network does not contain any non-complete lightpath cycle, Algorithm 1 produces a feasible solution for the WA problem, and the decomposition method gives an optimal solution for the GRWA problem.

To test whether a network contains any non-complete lightpath cycle, one can obviously use the exhaustive search method: finding all lightpath cycles and then test if any of them is non-complete. Clearly the complexity of this exhaustive search method grows exponentially. Currently, we do not have an efficient method to verify if a network contains any non-complete lightpath cycle. In fact, this problem itself could be NP-complete, just like the wavelength assignment problem.

In case that a network contains non-complete lightpath cycles, let c^* be the minimum number of lightpaths that need to be removed from the network so that the remaining portion of the network does not contain any non-complete lightpath cycles. Then we have

Theorem 3: There exists a feasible solution for the WA problem which requires at most $c^* + p$ wavelengths. Therefore, if $c^* + p \leq |W|$, then we can find a feasible solution for the WA problem and the decomposition method still gives an optimal solution for the GRWA problem.

Before closing this section, we should point out that if the result in Theorem 3 can be further refined, then it can lead to better upper bounds on the number of wavelengths required for the WA problem.

IV. NUMERICAL RESULTS

In this section, we present four sets of numerical examples. All ILPs and mixed ILPs were solved by using CPLEX 7.0 on a Dell Precision 420 PC with two 1GHz processors. We compare the numerical results obtained based on the three methods proposed in the previous two sections: the GRWA ILP formulation, the decomposition method combined with the GR ILP formulation, and the decomposition method combined with the relaxed GR ILP formulation. The run time for the decomposition method includes the run times for both the (relaxed) GR ILP problem and the WA problem. The run time for the WA problem in all four examples is very fast (it is less than a second in the first three cases and less than 3 seconds in the last case). Our numerical results clearly indicate that the decomposition method combined with the relaxed GR ILP formulation produces quite good results with reasonably small run times.

Example 1. This is relatively small network with 12 nodes, 17 fiber spans, 24 lightpaths, and 104 traffic demands (with different sizes). For this example, we were able to obtain the optimal solution based on the GRWA ILP formulation. The results are presented in Table 1.

	Run Time	Solution
GRWA ILP	400 seconds	128
GR ILP	80 seconds	128
Relaxed GR ILP	2 seconds	136

Table 1: Numerical results for Example 1.

Example 2. The network we consider in this example has 30 nodes, 38 fiber spans, 47 lightpaths, and 242 demands (with different sizes). The results are presented in Table 2. For the GRWA ILP problem, we stopped the CPLEX program after 75 hours and obtained a feasible solution with objective value 249.

	Run Time	Solution
GRWA ILP	>75 hours	249
GR ILP	37 hours	189
Relaxed GR ILP	12 seconds	202

Table 2: Numerical results for Example 2.

Example 3. The network in this example has 49 nodes, 75 fiber spans, 155 lightpaths, and 238 demands (with different sizes). It is a medium size network. For this example, the decomposition method based on the relaxed GR ILP problem produced a solution with value 345 in about 13 minutes, while the CPLEX program did not even return a feasible solution for the GRWA ILP and GR ILP problems after 40 hours (at which point we stopped the program). From the CPLEX program, we were also able to obtain a lower bound (based on the GR ILP problem) 328 for the objective function. Hence, the solution provided by the relaxed GR ILP based decomposition method is within 5% of the lower bound. We note that the WA problem was solved in 0.37 seconds for this example.

	Run Time	Solution
GRWA ILP	>40 hours	No Solution
GR ILP	>40 hours	No Solution
Relaxed GR ILP	13 minutes	345

Table 3: Numerical results for Example 3.

Example 4. The network in this example has 144 nodes, 162 fiber spans, 299 lightpaths, and 600 demands (with different sizes). It is a relatively large network (a typical size for a nation-wide network). For this example, the method based on the relaxed GR ILP problem produced a solution in about 38 minutes, and the CPLEX program did not even return a feasible solution for the GRWA ILP and GR ILP problems after 100 hours (at which point we stopped the program). The WA problem in this case was solved in 2.67 seconds for this example.

	Run Time	Solution
GRWA ILP	>100 hours	No Solution
GR ILP	>100 hours	No Solution
Relaxed GR ILP	38 minutes	431

Table 4: Numerical results for Example 4.

V. CONCLUSION

We studied the GRWA problem for optical mesh networks and proposed a decomposition method based on both ILP formulation and its relaxed version. In the decomposition method, we divided the GRWA problem into two smaller problems: the GR problem and the WA problem, both of which are much easier to solve compared to the original GRWA problem. We also provided a sufficient condition under which we proved that the decomposition method in fact produces an optimal solution for the GRWA problem. In general, our numerical results showed that the decomposition method produces quite good approximate solutions with relatively short run times and it can be used to solve the GRWA problem for large optical mesh networks (with a few hundred nodes and fiber spans).

ACKNOWLEDGMENT

J.Q. Hu is supported in part by the National Science Foundation under grant EEC-0088073.

REFERENCES

- [1] A. Aggarwal, A. Bar-Noy, D. Coppersmith, R. Ramaswami, B. Schieber, and M. Sudan, "Efficient routing and scheduling algorithms for optical networks," In *Proceedings of the 5th ACM-SIAM Symposium on Discrete Algorithms*, pp. 412-423, Jan. 1994.
- [2] V. Auletta, I. Caragiannis, C. Kaklamanis, and P. Persiano, "Random path coloring on binary trees," In *Proceedings of the 3rd International Workshop on Approximation Algorithms for Combinatorial Optimization Problems (APPROX'00)*, LNCS 1913, Springer, pp. 60-71, 2000.
- [3] D. Banerjee and B. Mukherjee, "A practical approach for routing and wavelength assignment in large wavelength-routed optical networks," *IEEE Journal on Selected Areas in Communications*, Vol. 14, No. 5, pp. 903-908, 1996.
- [4] R. Berry and E. Modiano, "Reducing electronic multiplexing costs in SONET/WDM rings with dynamic changing traffic," *IEEE Journal on Selected Areas in Communications*, Vol. 18, No. 10, pp. 1961-1971, 2000.
- [5] I. Chlamtac, A. Ganz, and G. Karmi, "Lightpath communications: an approach to high bandwidth optical WAN's," *IEEE Transactions on Communications*, Vol. 40, No. 7, pp. 1171-1182, 1992.
- [6] A. Chiu and E. Modiano, "Traffic grooming algorithms for reducing electronic multiplexing costs in WDM ring networks," *Journal of Lightwave Technology*, Vol. 18, No. 1, pp. 2-12, 2000.
- [7] L. A. Cox, J. Sanchez, and L. Lu, "Cost savings from optimized packing and grooming of optical circuits: mesh versus ring comparisons," *Optical Networks Magazines*, pp. 72-90, May/June, 2001.
- [8] R. Dutta and G. Rouskas, "On Optimal Traffic Grooming in WDM Rings," *Proc. of SIGMETRICS/PERFORMANCE 2001*, June 16-20, 2001, Cambridge, MA.
- [9] O. Gerstel, P. Lin, and G. Sasaki, "Wavelength assignment in a WDM ring to minimize the cost of embedded SONET rings," *Proc. Infocom*, San Francisco, CA, April 1998.
- [10] O. Gerstel, P. Lin, and G. Sasaki, "Combined WDM and SONET network design," *Proc. Infocom*, New York, NY, April 1999.
- [11] O. Gerstel, R. Ramaswami, and G. Sasaki, "Cost effective traffic grooming in WDM rings," *Proc. Infocom*, San Francisco, CA, April 1998.
- [12] W. Grover, J. Doucette, M. Clouqueur, D. Leung, "New options and insights for survivable transport networks," *The Communications Magazine*, Vol. 40, No. 1, pp. 34-41, January 2002.
- [13] J. Q. Hu, "Optimal traffic grooming for WDM rings with all-to-all uniform traffic," *Journal of Optical Networks*, Vol. 1, 32-42, 2002. (<http://www.osa-jon.org/abstract.cfm?URI=JON-1-1-32>.)
- [14] J. Q. Hu, "Traffic Grooming in WDM Ring Networks: A Linear Programming Solution," White Paper, Sycamore Networks, Inc., April 2001.
- [15] P. Raghavan and E. Upfal, "Efficient routing in all-optical networks," In *Proceedings of the 26th ACM Symposium on Theory of Computing*, pp. 134-143, May 1994.
- [16] M. Mihail, C. Kaklamanis, and S. Rao, "Efficient access to optical bandwidth-wavelength routing on directed fiber trees, rings, and trees of rings," In *Proceedings of the 36th Annual Symposium on Foundations of Computer Science*, pp. 548-557, Milwaukee, Wisconsin, 23-25 October, 1995.
- [17] R. Ramaswami and K. Sivarajan, *Optical Networks: A Practical Perspective*, Morgan Kaufmann Publishers, 1998.
- [18] G. Sasaki, O. Gerstel, and R. Ramaswami, "A WDM ring network for incremental traffic," *Proc. Thirty-Sixth Annual Allerton Conference on Communication, Control, and Computing*, Monticello, IL, September 1998.
- [19] J. Simmons, E. Goldstein, and A. Saleh, "On the value of wavelength add/drop in WDM rings with uniform traffic," *Proc. OFC*, San Jose, CA, February 1998.
- [20] J. Simmons, E. Goldstein, and A. Saleh, "Quantifying the benefits of wavelength add-drop WDM rings with distance independent and dependent traffic," *Journal of Lightwave Technology*, Vol. 17, No. 1, pp. 48-57, 1999.
- [21] S. Subramaniam, M. Azizoglu, and A. Somani, "On optimal converter placement in wavelength-routed networks," *IEEE/ACM Transactions on Networking*, Vol. 7, No. 5, pp. 754-766, 1999.
- [22] J. Wang, W. Cho, V. Vemuri, and B. Mukherjee, "Improved approaches for cost-effective traffic grooming in WDM ring networks: ILP formulations and single-hop and multihop connections," *Journal of Lightwave Technology*, Vol. 19, No. 11, pp. 1645-1653, 2001.
- [23] K. Zhu and B. Mukherjee, "Traffic grooming in an optical WDM mesh network," *Proc. of IEEE International Conference on Communications*, pp. 721-725, Helsinki, Finland, June, 2001.
- [24] X. Zhang and C. Qiao, "An effective and comprehensive solution to traffic grooming and wavelength assignment in WDM rings," *SPIE Proc. of Conference on All-Optical Networking*, Vol. 3531, pp. 221-232, Boston, MA, November 1998.
- [25] "The intelligent optical core: improving bandwidth efficiency," Sycamore Networks, white paper, 2001.