Scalable Coordinated Uplink Processing in Cloud Radio Access Networks

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***Abstract*—Featured by centralized processing and cloud based infrastructure, Cloud Radio Access Network (C-RAN) is a promising solution to achieve an unprecedented system capacity in future wireless cellular networks. The huge capacity gain mainly comes from the centralized and coordinated signal processing at the cloud server. However, full-scale coordination in a large-scale C-RAN requires the processing of very large channel matrices, leading to high computational complexity and channel estimation overhead. To resolve this challenge, we show in this paper that the channel matrices can be greatly sparsified without substantially compromising the system capacity. Through rigorous analysis, we derive a simple threshold-based channel matrix sparsification approach. Based on this approach, for reasonably large networks, the non-zero entries in the channel matrix can be reduced to a very low percentage (say $0.13\% \sim 2\%$) by compromising only 5% of SINR. This means each RRH only needs to obtain the CSI of a small number of closest users, resulting in a significant reduction in the channel estimation overhead. On the other hand, the high sparsity of the channel matrix allows us to design detection algorithms that are scalable in the sense that the average computational complexity per user does not grow with the network size.**

I. Introduction

The explosive growth in mobile data traffic threatens to outpace the infrastructure it relies on. To sustain the mobile data explosion with low bit-cost and high spectrum/energy efficiency, a revolutionary wireless cellular architecture, Cloud Radio Access Network (C-RAN) emerges as a promising solution [1]. C-RAN separates remote radio heads (RRHs) from baseband processing and migrates the latter to a centralized data center by using a high-bandwidth, low-latency optical transport network. This keeps RRHs light-weight, thereby allowing them to be deployed in a large number of small cells with low costs. Meanwhile, high-bandwidth, low-latency transport links and centralized processing allow RRHs to seamlessly cooperate with each other for flexible interference management, coordinated signal processing, and *etc.*. In this way, C-RAN opens up possibilities for significant system-capacity enhancement and cost reduction.

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The exciting opportunities come hand in hand with new technical challenges. Existing studies on multiuser MIMO systems suggest that the highest system capacity is achieved when all RRHs form a large-scale virtual antenna array that jointly detects the users' signals. The full-scale cooperation, however, requires the processing of a very large channel matrix consisting of channel coefficients from all mobile users to all RRHs. The computational complexity typically grows polynomially with the matrix size. This implies that the average computational complexity per mobile user grows with the network size, which fundamentally limits the scale of RRH cooperation. In addition, the full-scale RRH joint signal processing requires to estimate the large-scale channel matrix, causing significant channel estimation overhead. In [2], it is shown that the benefit of cooperation is fundamentally limited by the overhead of pilot-assisted channel estimation. As such, it is critical to find scalable signal processing algorithms, where the computational complexity and channel estimation overhead do not explode with the size of the network.

A. Contributions

The above-mentioned challenges lie in the estimation and processing of a very large channel matrix. In C-RAN, only a small fraction of the entries in the channel matrix have reasonably large amplitudes, because a user is only close to a small number of RRHs in its neighborhood, and vice versa. Thus, ignoring the small entries in the channel matrix would significantly sparsify the matrix, which can potentially lead to significant reduction in the computational complexity and channel estimation overhead. The question is to what extent can the channel matrix be sparsified without substantially compromising the system performance. In this paper, we attempt to address this question. In particular, we propose a threshold-based channel matrix sparsification method, where the matrix entries are ignored according to the distance between the users and RRHs. Through rigorous analysis, we derive a closed-form expression describing the relationship between the threshold and the SINR loss due to channel sparsification. The result shows that a vast majority of the channel coefficients can be ignored with a very small percentage of SINR loss. Our analysis serves as a convenient guideline to set the threshold subject to a tolerable SINR loss.

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B. Related Work

Existing solutions to reduce channel estimation overhead and computational complexity in distributed antenna or C-RAN systems include antenna selection [3]–[5] and clustering algorithms [6]–[8]. By controlling the number of serving antennas or the cluster size, these two methods can limit the estimation overhead and complexity to a low level. However, such approaches inevitably reduce the system capacity. This is because by limiting the scale of cooperation to a small antenna cluster, the centralized processing power of C-RAN is not fully exploited. A recent work by Shi *et al.* [9] proposed a framework consisting of a compressive CSI acquisition and stochastically coordinated downlink beamforming. Similar to our work here, the channel matrix estimated in [9] is sparsified. However, [9] does not make use of the sparsity of channel matrix to reduce the complexity of beamforming algorithm. Moreover, no theoretical analysis is provided to quantify the performance loss due to channel sparsification. According to their simulations, to guarantee a low performance loss, a very high percentage of CSI was required, say 60%, which means that the computational complexity and channel estimation overhead can still be very high when the network size is large. In contrast, in our work, the amount of CSI needed per user is constant, implying a scalable system where the complexity and overhead per user does not increase with the network size.

The rest of paper is organized as follows: in Section II, we describe the system model and propose the channel matrix sparsification method. In Section III, we derive a closed-form expression of the sparsification threshold, and analyse the effect of system parameters, such as user density and RRH density, on the selection of a threshold. We also discuss how significantly the sparsification method can reduce the estimation overhead and computational complexity in Section III. Numerical results and conclusions are shown in Section IV and Section V respectively.

II. System Model

A. System Setup

We consider the uplink transmission of a C-RAN with N single-antenna RRHs, and K single-antenna mobile users uniformaly located over the entire coverage area. The received signal vector $\mathbf{y} \in \mathbb{C}^{N\times 1}$ at the RRHs is

$$\mathbf{y} = \mathbf{H}\mathbf{P}^{\frac{1}{2}}\mathbf{x} + \mathbf{n}, \tag{1}$$

where $\mathbf{H} \in \mathbb{C}^{N\times K}$ denotes the channel matrix, with the (n,k)th entry $H_{n,k}$, being the channel coefficient between the kth user and the nth RRH. $\mathbf{P} \in \mathbb{R}^{N\times N}$ is a diagonal matrix with the kth diagonal entry P_k being the transmitting power allocated to user k. $\mathbf{x} \in \mathbb{C}^{K\times 1}$ is the vector of the transmitted signal from the K users and $\mathbf{n} \sim \mathcal{CN}(\mathbf{0}, N_0\mathbf{I})$ is the vector of noise received by RRHs. The transmit signals are assumed to follow an independent complex Gaussian distribution with unit variance, i.e. $E[\mathbf{x}\mathbf{x}^H] = \mathbf{I}$. Specifically, $H_{n,k} = \gamma_{n,k} d_{n,k}^{-\frac{\alpha}{2}}$, where $\gamma_{n,k}$ is the i.i.d Rayleigh fading coefficient with zero mean and variance 1, $d_{n,k}$ is the distance between the nth RRH and the kth user, and α is the path loss exponent. Then, $d_{n,k}^{-\alpha}$ is the path loss from the kth user to the nth RRH.

Without loss of generality, let us consider user k. The receive beamforming vector is

$$\mathbf{v}_k = P_k^{\frac{1}{2}}(\mathbf{H}\mathbf{P}\mathbf{H}^H + N_0\mathbf{I})^{-1}\mathbf{h}_k, \tag{2}$$

and the decision statistics of x_k is

$$\widehat{x}_k = \mathbf{v}_k^H \mathbf{h}_k P_k^{\frac{1}{2}} x_k + \mathbf{v}_k^H \sum_{j\neq k} \mathbf{h}_j P_j^{\frac{1}{2}} x_j + \mathbf{v}_k^H \mathbf{n}, \tag{3}$$

where $\mathbf{h}_k \in \mathbb{C}^{N\times 1}$ is the kth column of the channel matrix $\mathbf{H}$. The SINR for user k is

$$\mathrm{SINR}_k = \frac{P_k|\mathbf{v}_k^H\mathbf{h}_k|^2}{\sum_{j\neq k} P_j|\mathbf{v}_k^H\mathbf{h}_j|^2 + N_0\mathbf{v}_k^H\mathbf{v}_k}. \tag{4}$$

Notice that to calculate the detection vector $\mathbf{v}_k$, the full channel matrix $\mathbf{H}$ needs to be acquired and processed. In particular, the complexity of calculating inverse of $\mathbf{H}\mathbf{P}\mathbf{H}^H + N_0\mathbf{I}$ is as high as $O(N^3)$ by using Gaussian elimination. The polynomial increase in computational complexity causes a serious scalability problem. That is, the average computational complexity per user increases quadratically with the network size, if K scales in the same order as N, rendering the full-scale RRH cooperation very costly in large-scale C-RANs. To address the issue, we will argue in the next subsection that most entries in $\mathbf{H}$ are insignificant, and thus can be ignored. The effect of sparsifying matrix $\mathbf{H}$ by ignoring insignificant entries will be quantified analytically in Section III.

B. Channel Sparsification

Since the RRHs and users are distributed over a large area, an RRH can only receive reasonably strong signals from a small number of nearby users, and vice versa. Thus, ignoring the small entries in $\mathbf{H}$ would significantly sparsify the matrix, hopefully with a negligible loss in system performance. In this paper, we propose to ignore the entries of $\mathbf{H}$ based on the distance of links. In other words, the entry $H_{n,k}$ is set to 0 when the link length $d_{n,k}$ is larger than a threshold d_0. The resulting sparsified channel matrix, denoted by $\widehat{\mathbf{H}}$, is given by

$$\widehat{H}_{n,k} = \begin{cases} H_{n,k}, & d_{n,k} < d_0 \\ 0, & \text{otherwise.} \end{cases} \tag{5}$$

Note that we propose to sparsify the channel matrix based on the link distance instead of the actual values of entries, which are affected by both the link distance and fast channel fading. In practice, link distances vary much more slowly than fast channel fading. The distance-threshold-based approach leads to a relatively stable structure of $\widehat{\mathbf{H}}$. The received signal $\mathbf{y}$ can then be represented as

$$\mathbf{y} = \widehat{\mathbf{H}}\mathbf{P}^{\frac{1}{2}}\mathbf{x} + \widetilde{\mathbf{H}}\mathbf{P}^{\frac{1}{2}}\mathbf{x} + \mathbf{n}, \tag{6}$$

where $\widetilde{\mathbf{H}} = \mathbf{H} - \widehat{\mathbf{H}}$. Treating the interference signal $\widetilde{\mathbf{H}}\mathbf{P}^{\frac{1}{2}}\mathbf{x}$ as noise, the detection vector becomes

$$\widehat{\mathbf{v}}_k = P_k^{\frac{1}{2}}\left(\widehat{\mathbf{H}}\mathbf{P}\widehat{\mathbf{H}}^H + \mathbf{\Gamma} + N_0\mathbf{I}\right)^{-1}\widehat{\mathbf{h}}_k, \tag{7}$$

where $\widehat{\mathbf{h}}_k$ is the kth column of $\widehat{\mathbf{H}}$, and

$$\mathbf{\Gamma} = \mathrm{E}\left[\sum_{j\neq k} P_j \left(\widetilde{\mathbf{h}}_j \widehat{\mathbf{h}}_j^H + \widehat{\mathbf{h}}_j \widetilde{\mathbf{h}}_j^H + \widetilde{\mathbf{h}}_j \widetilde{\mathbf{h}}_j^H\right)\right], \quad (8)$$

is the variance of the $\widetilde{\mathbf{H}}\mathbf{P}^{\frac{1}{2}}\mathbf{x}$.

With this, the SINR becomes

$$\widehat{\mathrm{SINR}}_k(d_0) = \frac{P_k|\widehat{\mathbf{v}}_k^H \mathbf{h}_k|^2}{\sum_{j\neq k} P_j |\widehat{\mathbf{v}}_k^H \mathbf{h}_j|^2 + N_0 \widehat{\mathbf{v}}_k^H \widehat{\mathbf{v}}_k}. \quad (9)$$

Notice that when the distance threshold d_0 is small, the matrix $\widehat{\mathbf{H}}$ can be very sparse, leading to a significant reduction in channel estimation overhead and processing complexity. The key question is: how small d_0 can be without significantly affecting the system performance. This quesiton will be answered in the next section.

III. Distance Threshold Analysis

In this section, we show by rigorous analysis how to set the distance threshold d_0 if a high percentage of full SINR is to be achieved. Specifically, we wish to set d_0, such that the SINR ratio, defined as

$$\rho(d_0) = \frac{\mathrm{E}[\widehat{\mathrm{SINR}}_k(d_0)]}{\mathrm{E}[\mathrm{SINR}_k]} \quad (10)$$

is larger than a prescribed ρ^*, where the two expectations are taken over entries in $\mathbf{H}$, which are affected by path loss and Rayleigh fading.

In the following, we first derive a lower bound of $\rho(d_0)$. We then derive a closed-form expression of d_0 as a function of the target SINR ratio ρ^*. Finally, we discuss the possibility of reducing the estimation overhead and processing complexity based on the sparsified matrix.

A. Lower Bound of $\rho(d_0)$

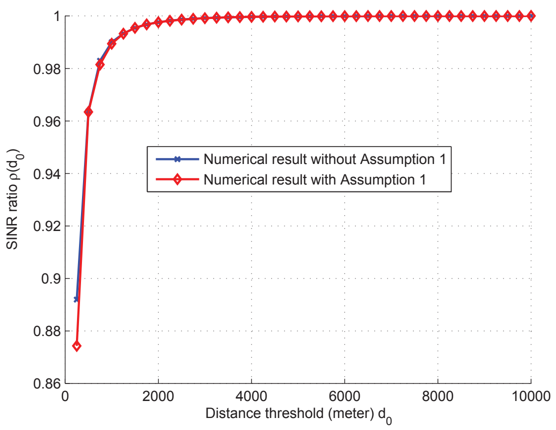

Fig. 1. Average SINR ratio vs distance threshold when $N = 1000, K = 600$.

In this subsection, we derive a lower bound of the SINR performance ratio, $\rho(d_0)$. First of all, we make two approximations to make the analysis tractable:

Assumption 1: The distances $d_{n,k}$, for all n, k are mutually independent.

As shown in Fig. 1, we plot the SINR ratio for systems with and without Assumption 1. The system area is assumed to be a circle with radius 2.5 km. The figure shows that the gap between the SINR ratio is very small, which validates the independence assumption.

Assumption 2: Conditioning on the distance threshold d_0, the matrices $\widehat{\mathbf{H}}$ and $\widetilde{\mathbf{H}}$ are mutually independent.

Note that $\mathrm{E}[\widehat{\mathbf{H}}\widetilde{\mathbf{H}}^H] = \mathrm{E}[\widetilde{\mathbf{H}}\widehat{\mathbf{H}}^H] = \mathbf{0}$, which means that $\widehat{\mathbf{H}}$ and $\widetilde{\mathbf{H}}$ are uncorrelated. With the independence assumption, the equality $\mathrm{E}_{\mathbf{H}}[\widehat{\mathrm{SINR}}_k(d_0)] = \mathrm{E}_{\widehat{\mathbf{H}}}\left[\mathrm{E}_{\widetilde{\mathbf{H}}}[\widehat{\mathrm{SINR}}_k(d_0)]\right]$ holds. This assumption will be verified in our numerical results in Fig. 2, which shows that the gap between the simulated SINR ratio and the lower bound of $\rho(d_0)$ derived based on this assumption is small.

Based on these two approximations, we see that $\mathbf{\Gamma} = N_1\mathbf{I}$, where $N_1 = \mathrm{E}[\sum_{j\neq k} P_j|\widetilde{h}_{n,j}|^2]$ for arbitrary RRH n.

The following lemma gives a lower bound on the expectation of $\widehat{\mathrm{SINR}}_k(d_0)$.

Lemma 1:

$$\mathrm{E}\left[\widehat{\mathrm{SINR}}_k(d_0)\right] \geq P_k\widehat{\mu}\mathrm{E}\left[\mathrm{tr}\left(\sum_{j\neq k} P_j \widehat{\mathbf{h}}_j \widehat{\mathbf{h}}_j^H + N_1\mathbf{I} + N_0\mathbf{I}\right)^{-1}\right], \quad (11)$$

where $\widehat{\mu} = \mathrm{E}[\widehat{H}_{i,j}\widehat{H}^*_{i,j}], \forall i, j$

Proof: See Appendix. ■

Likewise, we obtain

$$\mathrm{E}\left[\mathrm{SINR}_k\right] = P_k\mu\mathrm{E}\left[\mathrm{tr}\left(\sum_{j\neq k} P_j \mathbf{h}_j \mathbf{h}_j^H + N_0\mathbf{I}\right)^{-1}\right], \quad (12)$$

with $\mu = \mathrm{E}[H_{n,k}H^*_{n,k}], \forall n$.

Then a lower bound on $\rho(d_0)$, denoted by $\underline{\rho(d_0)}$, is given in Theorem 1.

Theorem 1:

$$\rho(d_0) \geq \underline{\rho(d_0)} \triangleq \frac{\widehat{\mu}N_0}{\mu\left((\mu - \widehat{\mu})\sum_{j\neq k} P_k + N_0\right)}. \quad (13)$$

When a same amount of power P is allocated to each user, the lower bound becomes

$$\underline{\rho(d_0)} = \frac{\widehat{\mu}N_0}{\mu\left((\mu - \widehat{\mu})(K-1)P + N_0\right)}. \quad (14)$$

Proof:

$$\rho(d_0) \tag{15a}$$
$$=\frac{\mathrm{E}[\widehat{\mathrm{SINR}}_k(d_0)]}{\mathrm{E}[\mathrm{SINR}_k]} \tag{15b}$$
$$\geq\frac{P_k\widehat{\mu}\mathrm{E}_{\widehat{\mathbf{h}}_j,\forall j\neq k}\left[\mathrm{tr}\left(\sum_{j\neq k}P_j\widehat{\mathbf{h}}_j\widehat{\mathbf{h}}_j^H+N_1\mathbf{I}+N_0\mathbf{I}\right)^{-1}\right]}{P_k\mu\mathrm{E}_{\mathbf{h}_j,\forall j\neq k}\left[\mathrm{tr}\left(\sum_{j\neq k}P_j\mathbf{h}_j\mathbf{h}_j^H+N_0\mathbf{I}\right)^{-1}\right]} \tag{15c}$$
$$\geq\frac{P_k\widehat{\mu}\mathrm{E}_{\mathbf{h}_j,\forall j\neq k}\left[\mathrm{tr}(\sum_{j\neq k}P_j\mathbf{h}_j\mathbf{h}_j^H+N_1\mathbf{I}+N_0\mathbf{I})^{-1}\right]}{P_k\mu\mathrm{E}_{\mathbf{h}_j,\forall j\neq k}\left[\mathrm{tr}\left(\sum_{j\neq k}P_j\mathbf{h}_j\mathbf{h}_j^H+N_0\mathbf{I}\right)^{-1}\right]} \tag{15d}$$
$$=\frac{\hat{\mu}}{\mu}\mathrm{E}\left[\frac{\sum_{i=1}^{N}\frac{1}{\lambda_i+N_1+N_0}}{\sum_{i=1}^{N}\frac{1}{\lambda_i+N_0}}\right] \tag{15e}$$
$$\geq\frac{\hat{\mu}N_0}{\mu(N_1+N_0)}, \tag{15f}$$

where $\lambda_1,\lambda_2,\cdots,\lambda_N$ in (15e) are the eigenvalues of the positive semidefinite matrix $\sum_{j\neq k}P_j\mathbf{h}_j\mathbf{h}_j^H$. (15f) holds since $N_1\geq 0$, $N_0\geq 0$ and $\lambda_i\geq 0,\forall i$.

Substituting $N_1=(\mu-\widehat{\mu})\sum_{j\neq k}P_k$ in (15f), we have

$$\rho(d_0)\geq\frac{\widehat{\mu}N_0}{\mu\left((\mu-\widehat{\mu})\sum_{j\neq k}P_k+N_0\right)}. \tag{16}$$

When the users transmit equal power, i.e., $P_1=P_2=\cdots=P_K=P$, we have

$$\rho(d_0)\geq\frac{\widehat{\mu}N_0}{\mu\left((\mu-\widehat{\mu})(K-1)P+N_0\right)}. \tag{17}$$

■

B. Analysis of Distance Threshold

To express $\underline{\rho(d_0)}$ as an explicit function of d_0, we need to calculate $\hat{\mu}=\mathrm{E}[\widehat{H}_{n,k}\widehat{H}_{n,k}^*]$ and $\mu=\mathrm{E}[H_{n,k}H_{n,k}^*]$, as detailed in this subsection.

The expectations in $\widehat{\mu}$ and μ are taken over both path loss and Rayleigh fading. Especially, the expectation over the path loss coefficients depends on the distribution of distance between mobile users and RRHs. In [10], distance distributions are derived for different network area shapes, such as circle, square and rectangle. Take, for example, a circular network area with radius r. In this case, the distance distribution between two random points is [10]

$$f(d,r)=\begin{cases}\int_0^{r_0}\frac{2d}{r^2}\left(\frac{2}{\pi}\arccos\left(\frac{x}{2r}\right)-\frac{x}{\pi r}\sqrt{1-\frac{x^2}{4r^2}}\right)\mathrm{d}x, & d=r_0,\\ \frac{2d}{r^2}\left(\frac{2}{\pi}\arccos\left(\frac{d}{2r}\right)-\frac{d}{\pi r}\sqrt{1-\frac{d^2}{4r^2}}\right), & 0<d<2r,\end{cases} \tag{18}$$

where r_0 is the minimum distance between RRHs and users.

Then, $\hat{\mu}$ and μ can be calculated as

$$\widehat{\mu}=\int_{d=r_0}^{d_0}d^{-\alpha}f(d,r)\mathrm{d}d, \tag{19}$$

$$\mu=\int_{d=r_0}^{2r}d^{-\alpha}f(d,r)\mathrm{d}d. \tag{20}$$

When the network radius r becomes very large, (18) can be approximated as

$$f(d,r)=\begin{cases}\frac{r_0^2}{r^2}, & d=r_0,\\ \frac{2}{r^2}d, & r_0<d<r.\end{cases} \tag{21}$$

Substituting (21) into (19) and (20), we have $\widehat{\mu}$ and μ as follows

$$\widehat{\mu}=\frac{\alpha r_0^{2-\alpha}-2d_0^{2-\alpha}}{(\alpha-2)r^2}, \tag{22}$$

$$\mu=\frac{\alpha r_0^{2-\alpha}-2r^{2-\alpha}}{(\alpha-2)r^2}. \tag{23}$$

Then, we obtain

$$\rho(d_0)\geq\underline{\rho(d_0)}=\frac{\alpha r_0^{2-\alpha}-2d_0^{2-\alpha}}{\left(\alpha r_0^{2-\alpha}-2r^{2-\alpha}\right)\left(\frac{2(d_0^{2-\alpha}-r^{2-\alpha})(K-1)P}{(\alpha-2)r^2N_0}+1\right)}. \tag{24}$$

From (24), the distance threshold to acheive an SINR ratio ρ^* can be derived as

$$d_0(\rho^*)\leq\left(r^{2-\alpha}+\frac{(\alpha r_0^{2-\alpha}-2r^{2-\alpha})(1-\rho^*)N_0}{2N_0+\frac{2\rho^*(\alpha r_0^{2-\alpha}-2r^{2-\alpha})(K-1)P}{(\alpha-2)r^2}}\right)^{-\frac{1}{\alpha-2}}. \tag{25}$$

Notice that the above result can be easily extended to the case with other distance distributions. Even though, the number of RRHs has effect on the actual SINR ratio, surprisingly, (25) shows that the number of RRHs has no effect either on the lower bound of $\rho(d_0)$ or on the upper bound of $d_0(\rho^*)$. On the other hand, increasing the number of users or the transmit power leads to an increase of the upper bound of $d_0(\rho^*)$ since the interference caused by neglected entries in the channel matrix becomes larger, and more RRHs should be included to reduce the interference. To analyse the effect of the network size, we denote the RRH density and user density as $\beta_N=\frac{N}{\pi r^2}$ and $\beta_K=\frac{K}{\pi r^2}$ respectively. When the network size goes to infinity, the lower bound of the SINR ratio becomes

$$\rho(d_0)\geq\underline{\rho(d_0)}=\frac{(\alpha-2)(\alpha r_0^{2-\alpha}-2d_0^{2-\alpha})N_0}{\alpha r_0^{2-\alpha}\left(2P\pi\beta_K d_0^{2-\alpha}+(\alpha-2)N_0\right)}. \tag{26}$$

Consequently,

$$d_0(\rho^*)\leq\left(\frac{2N_0(\alpha-2)+2\alpha r_0^{\alpha-2}\rho^*\pi\beta_K P}{\alpha r_0^{2-\alpha}N_0(1-\rho^*)(\alpha-2)}\right)^{\frac{1}{\alpha-2}}. \tag{27}$$

TABLE I
PERCENTAGE OF NON-ZERO ENTRIES IN THE CHANNEL MATRIX WITH $\beta_K=10/\mathrm{KM}^2$, $\frac{P}{N_0}=80$DB AND $\rho^*=0.95$

r (km)	5	10	15	20
d_0 (meter)	694	705	707	708
Percentage of non-zero entries (%)	1.93	0.50	0.20	0.13

As we can see from (27), the distance threshold converges to a constant when the network radius goes to infinity. This means that the number of non-zero entries per row (i.e., corresponding to each RRH) or per column (i.e., corresponding

TABLE II
PERCENTAGE OF NON-ZERO ENTRIES IN THE CHANNEL MATRIX WITH $\beta_K = 10/\text{KM}^2$, $\frac{P}{N_0} = 80\text{DB}$ AND $r = 10\text{KM}$

ρ^*	0.90	0.93	0.96	0.99
d_0 (meter)	456	572	807	1812
Percentage of non-zero entries (%)	0.21	0.33	0.65	3.28

to each mobile user) in $\widehat{\mathbf{H}}$ does not scale with the network radius r in a large C-RAN. In Table I, both the distance threshold d_0 and the percentages of non-zero entries in matrix $\widehat{\mathbf{H}}$ are listed for different network sizes, with $\beta_K = 10/\text{km}^2$, $\frac{P}{N_0} = 80\text{dB}$ and $\rho^* = 0.95$. It can be seen that, when r is large, d_0 does not change much with the network radius r. Moreover, as shown in Table I, only a very low percentage entries (say $2\% \sim 0.13\%$) in $\widehat{\mathbf{H}}$ are non-zero. That is, each RRH only needs to obtain CSI of a small number of closest users and the channel estimation overhead can be significantly reduced. If a larger SINR loss can be tolerated, the amount of CSI needed can be further reduced as shown in Table II, which lists the percentages of non-zero entries in $\widehat{\mathbf{H}}$ for different ρ^*, with $\beta_K = 10/\text{km}^2$, $\frac{P}{N_0} = 80\text{dB}$ and $r = 10\text{km}$. We can see that the percentage of non-zero entries can be reduced from 3.28% to 0.21% by decreasing the SINR performance from 99% to 90%, which means the sparsity of $\widehat{\mathbf{H}}$ can be increased a lot and the estimation overhead can be reduced.

C. Discussion

As shown in preceding subsection, we can calculate the multiuser detection vector $\hat{\mathbf{v}}$ based on a very sparse matrix $\widehat{\mathbf{H}}$, and yet achieve a high percentage of the original full SINR. The high sparsity of $\widehat{\mathbf{H}}$ allows us to design scalable detection algorithms, where the average computational complexity per user does not grow with the network size. Due to the page limit, the design of a scalable detection algorithm will be reported in a long version of this paper. Instead, we briefly discuss the intuition of the algorithm in the following.

By ignoring the channel coefficients of long-distance links, a user is effectively served by the neighboring RRHs only, and thus the detection complexity should be limited to its local area instead of the whole network. Indeed, we have found that, based on the localization property of the system, the sparsified channel matrix $\widehat{\mathbf{H}}$ can be permutated to some special matrices, such as a bordered block diagonal matrix or a multiple-band matrix. Making use of the special structures of the matrix, we can reduce the overall computational complexity of MMSE detection from $O(N^3)$ to $O(N)$, or equivalently from $O(K^3)$ to $O(K)$ if K scales at the same rate as N. This is a very encouraging result, as it implies that the joint multiuser detection in C-RAN can be made scalable through channel sparsification, in the sense that both the computational complexity and channel estimation overhead per user do not grow with the network size.

IV. NUMERICAL RESULTS

In this section , we first verify our analysis through numerical simulations. We then illustrate the effect of SINR ratio requirement on the choice of the distance threshold. Unless stated otherwise, we assume that the minimum distance between RRHs and users is 1 meter, the path loss exponent is 3.7, and the average transmit SNR at the user side equals to 80dB. That is $\frac{P}{N_0} = 80\text{dB}$.

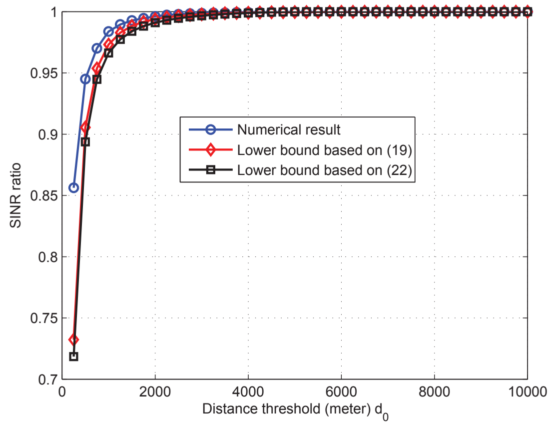

Fig. 2. Average SINR ratio vs distance threshold when $N = 1200, K = 1000, r = 5\text{km}$.

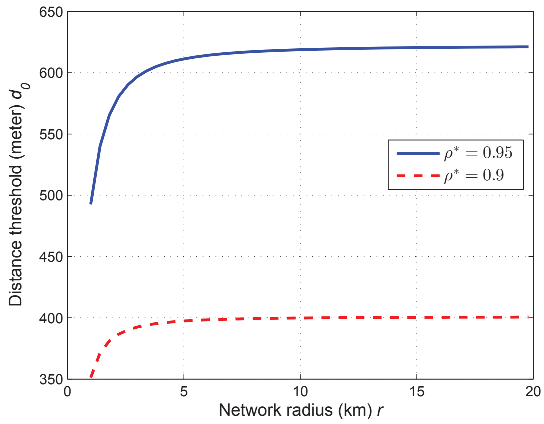

Fig. 3. Distance threshold d_0 vs area radius r when the user density $\beta_K = 8/\text{km}^2$.

A. Verification

We first verify Theorem 1 in Fig. 2, where $N = 1200$, $K = 1000$ and $r = 5$ km. Fig. 2 plots the average SINR performance ratio against the distance threshold. The simulated SINR ratio is plotted as the blue curve and the lower bound $\underline{\rho(d_0)}$ derived based on the distributions in (18) and (21) are plotted as the red curve and black curve, respectively. It can be seen that the ratio calculated based on (18) is a lower bound of the numerical result. Moreover, we can see that the gap between the lower bound based on (18) and that based on (21) is negligible, which means our distribution approximation is accurate.

We then verify in Fig. 3 our claims in (27). That is, the distance threshold converges to a constant when the network radius r becomes large. Here, the user density is $\beta_K = 8/\text{km}^2$, and the SINR ratio requirement is set to $\rho^* = 0.95$ and $\rho^* = 0.9$, respectively. As expected, the distance threshold converges quickly to a constant when the network radius increases. Indeed, the convergence is observed even when the network radius is as small as 5 km for both $\rho^* = 0.9$ and $\rho^* = 0.95$.

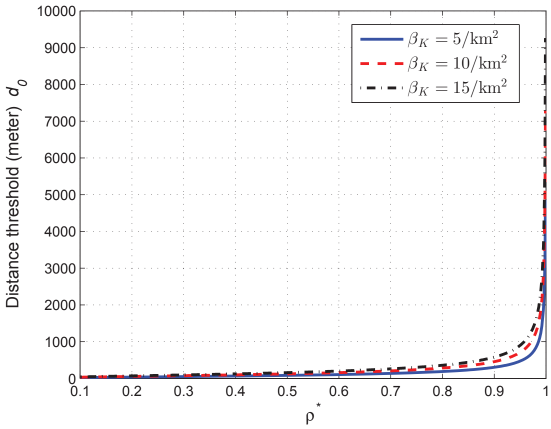

Fig. 4. Distance threshold vs SINR ratio.

B. Discussion on the Distance Threshold

In Fig. 4, we plot the distance thresholds against the SINR ratio requirements with user density $\beta_K = 5, 10$ and $15/\text{km}^2$, respectively. The network radius is assumed to be very large. We can see that the distance threshold is very small for a wide range of ρ^*, i.e. when ρ^* is smaller than 0.95. There is a sharp increase in d_0 when ρ^* approaches to 1. This implies an interesting tradeoff: if the full SINR is to be obtained, we do need to process the full channel matrix $\mathbf{H}$ at the cost of extremely high complexity when the network size is large. On the other hand, if a small percentage of SINR degradation can be tolerated, the channel matrix can be significantly sparsified, leading to low-complexity and scalable detection algorithms. We would like to emphasize that the SINR degradation may not imply a loss in the system capacity. This is because the overhead of estimating the full channel matrix can easily outweight the SINR gain. A small compromise in SINR (say reduce from 100% to 95%) may lead to a higher system capacity eventually.

V. Conclusion and Future Work

In this paper, we proposed a threshold-based channel matrix sparsification method, and derived a closed-form expression describing the relationship between the threshold and the SINR loss due to channel sparsification. It was shown that a vast majority of the channel coefficients can be ignored with a small percentage of SINR loss. According to our simulations, by compromising only 5% SINR loss, the CSI acquisition of each RRH can be reduced from all users to a small number of closest users. Thus, the estimation overhead is significantly reduced. Based on the high sparsity of channel matrix, scalable detection algorithms are briefly discussed in this paper. The detailed design of the algorithm will be reported in a long version of this paper.

Appendix

Proof of Lemma 1:

$$\mathrm{E}\left[\widehat{\mathrm{SINR}}_k\left(d_0\right)\right] \tag{28a}$$

$$=\mathrm{E}\left[\frac{P_k\left(|\widehat{\mathbf{v}}_k^H\widehat{\mathbf{h}}_k|^2+\widehat{\mathbf{v}}_k^H\left(\widehat{\mathbf{h}}_k\widetilde{\mathbf{h}}_k^H+\widetilde{\mathbf{h}}_k\widehat{\mathbf{h}}_k^H+\widetilde{\mathbf{h}}_k\widetilde{\mathbf{h}}_k^H\right)\widehat{\mathbf{v}}_k\right)}{\widehat{\mathbf{v}}_k^H\left(\sum_{j\neq k}P_j\mathbf{h}_j\mathbf{h}_j^H+N_0\mathbf{I}\right)\widehat{\mathbf{v}}_k}\right] \tag{28b}$$

$$\geq\mathrm{E}_{\widehat{\mathbf{H}}}\mathrm{E}_{\widetilde{\mathbf{h}}_j,\forall j\neq k}\left[\frac{P_k|\widehat{\mathbf{v}}_k^H\widehat{\mathbf{h}}_k|^2}{\widehat{\mathbf{v}}_k^H\left(\sum_{j\neq k}P_j\mathbf{h}_j\mathbf{h}_j^H+N_0\mathbf{I}\right)\widehat{\mathbf{v}}_k}\right] \tag{28c}$$

$$=\mathrm{E}_{\widehat{\mathbf{H}}}\left[\frac{P_k|\widehat{\mathbf{v}}_k^H\widehat{\mathbf{h}}_k|^2}{\widehat{\mathbf{v}}_k^H\left(\sum_{j\neq k}P_j\widehat{\mathbf{h}}_j\widehat{\mathbf{h}}_j^H+N_1\mathbf{I}+N_0\mathbf{I}\right)\widehat{\mathbf{v}}_k}\right] \tag{28d}$$

$$=\mathrm{E}_{\mathbf{H}}\left[\frac{1}{1-\widehat{\mathbf{v}}_k^H\widehat{\mathbf{h}}_k}-1\right] \tag{28e}$$

$$=\mathrm{E}_{\widehat{\mathbf{H}}}\left[P_k\mathrm{tr}\left(\widehat{\mathbf{h}}_k\widehat{\mathbf{h}}_k^H\left(\sum_{j\neq k}P_j\widehat{\mathbf{h}}_j\widehat{\mathbf{h}}_j^H+N_1\mathbf{I}+N_0\mathbf{I}\right)^{-1}\right)\right] \tag{28f}$$

$$=\mathrm{E}_{\widehat{\mathbf{h}}_j,\forall j\neq k}\left[P_k\mathrm{tr}\left(\widehat{\mu}\mathbf{I}\left(\sum_{j\neq k}P_j\widehat{\mathbf{h}}_j\widehat{\mathbf{h}}_j^H+N_1\mathbf{I}+N_0\mathbf{I}\right)^{-1}\right)\right] \tag{28g}$$

$$=P_k\widehat{\mu}\mathrm{E}\left[\mathrm{tr}\left(\sum_{j\neq k}P_j\widehat{\mathbf{h}}_j\widehat{\mathbf{h}}_j^H+N_1\mathbf{I}+N_0\mathbf{I}\right)^{-1}\right], \tag{28h}$$

where $\widehat{\mu} = \mathrm{E}\left[\widehat{H}_{i,j}\widehat{H}_{i,j}^*\right], \forall i, j$, and (28d) follows the fact that $\widehat{\mathbf{v}}_k^H\left(\sum_{j\neq k}P_j\left(\widetilde{\mathbf{h}}_j\widehat{\mathbf{h}}_j^H+\widehat{\mathbf{h}}_j\widetilde{\mathbf{h}}_j^H+\widetilde{\mathbf{h}}_j\widetilde{\mathbf{h}}_j^H\right)\right)\widehat{\mathbf{v}}_k$ converges to $N_1\widehat{\mathbf{v}}_k^H\widehat{\mathbf{v}}_k$ as N goes to infinity based on the law of large numbers.

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